

# High Integrity Carrier Phase Navigation for Future LAAS Using Multiple Civilian GPS Signals

Jaewoo Jung  
*Stanford University*

## BIOGRAPHY

Jaewoo Jung is a Ph.D. candidate in the Department of Aeronautics and Astronautics Engineering at Stanford University. His research is focused on local-area differential GPS system design, analysis and application. Mr. Jung received his BS in Aerospace Engineering in 1992 from Boston University and MS in Aeronautics and Astronautics in 1994 from Stanford University.

## ABSTRACT

In near future, there will be two more civilian GPS signals, at 1227.60 MHz and 1176.45 MHz, in addition to the present one at 1575.42 MHz. Of many possible benefits from the three civilian signals, this paper focuses on how to achieve high integrity and continuity local area differential navigation by using multiple combinations of the carrier wave frequencies of the soon available signals. It also investigates the advantages and disadvantages of the current location of the third civilian frequency.

The carrier frequencies of the three civil signals can be used to form measurements with different wavelengths, both longer and shorter than their own, depending on the specific combination used. By using these combinations in cascade manner, first finding the integer of the signal with the longest wavelength then moving on to a shorter one, the L1 integer can be resolved without using redundant measurements. Since integrity and continuity of differential carrier phase navigation depends on the validity of the integer solution, an integer rounding criteria based on wavelength and measurement noise is developed to verify the solution.

Analysis of the integer rounding criteria shows that multiple levels of integrity and continuity for differential carrier phase navigation are possible, depending on the measurement noise level. Sensitivity analysis is carried out to

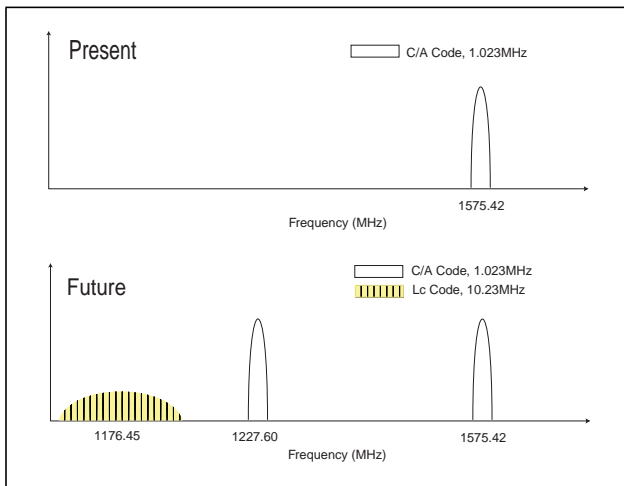
study the effect of receiver noise, multipath delay, ionosphere delay and change in distance between an user and a reference receiver. The analysis is done over a region of L band since location of the third frequency, although now decided, also affects the performance to some extent. An optimal location for the third frequency, with a constraint that it will be located in Aeronautical Radio Navigation Services (ARNS) band and with assumption that the second frequency will be located at the L2 frequency, is found using this analysis.

## INTRODUCTION

### Present GPS Signal Configuration

In the present configuration, GPS satellite constellation broadcasts three signals. One with the Coarse/Acquisition code (C/A) modulation, and the other two with the Precision code (P) modulation. The P code, with chipping rate of 10.23 MHz, modulates its carrier at 1575.42 MHz (L1) and at 1227.60 MHz (L2). They are encrypted, and are intended for military usage. Civilians are limited with using the C/A code and its carrier frequency for their applications. The C/A code, with chipping rate of 1.023 MHz, modulates carrier at the L1 frequency with 90 degree phase shift from the P code.

In the present configuration, the accuracy of GPS position estimates can range from tens of meters to centimeters depending upon the augmentation. With code measurements, real-time position estimation with accuracy in order of tens of meters can be obtained from the un-augmented Standard Positioning Service (SPS). To obtain better performance, augmentations to the GPS signal are required. With about 30 reference stations distributed over U.S., GPS/WAAS would provide meter-level accuracy with sufficient integrity, continuity and availability for Category I aircraft landing for the entire U.S. airspace. With a reference station at an airport, GPS/LAAS would



**Figure 1: Civil GPS Signals, The Present and Future**

provide sub-meter accuracy with Category III capability within about 20nm of the airport. Both GPS/WAAS and GPS/LAAS utilize differential code measurement in their augmentation system. Using carrier phase measurements, the position of a user relative to a local reference station may be determined with centimeter-level accuracy in real time. However, a robust system offering such accuracy with high level of integrity and continuity is yet to be demonstrated.

### Benefits of Multiple Civil GPS Signals

On January 25, 1999, Vice president Gore announced a new GPS modernization initiative. It will provide the second civil signal at the L2 frequency along with the current military signal, and the third civil signal at 1176.45 MHz (Lc). Figure 1 shows the details of the signals. This frequency diversity will alleviate concerns over accidental interference, and would constitute the key step toward achieving robustness of service. The civil signal at the L1 frequency will remain unchanged, thereby ensuring that all of the fielded GPS receivers will continue to operate. Two-frequency signaling would allow civil users to estimate the ionospheric propagation effect and, in the post-SA era, would offer positioning accuracy of better than 10 meters without any augmentation.

While two frequencies would improve accuracy for the civil users significantly, the third frequency would bring about an improvement of an order of magnitude. With three frequencies, the user equipment can multiply and filter the individual signals to create three beat frequency signals. The L1 and L2 measurements can be processed to create a beat frequency signal with a wavelength of approximately 86 centimeters. Since this wavelength is significantly greater than the wavelength for either L1 or L2, the corresponding measurement is commonly known as a widelane observable. Two more observables can be

generated by multiplying the third signal with the L1, and the L2 carrier frequency. The combination of the L1 and the Lc carrier frequency yields a signal with approximately 75 centimeters in wavelength, and the L2 and the Lc carrier frequency yields one with approximately 5.9 meters. Table 1 shows the details of these combinations.

**Table 1: Civil GPS Signal Components, Near Future**

Component	Frequency (MHz)	Wave Length (m)
L1 Carrier	1575.42	.190
L2 Carrier	1227.60	.244
Lc Carrier	1176.45	.255
L1-L2 Carrier, Widelane (WL)	347.82	.862
L1-Lc Carrier, Mediumlane (ML)	348.97	.751
L2-Lc Carrier, Extra Widelane (EWL)	51.15	5.86
L1 Code	1.023	293
L2 Code	1.023	293
Lc Code	10.23	29.3

### Paper Organization

One of the major difficulty with the present carrier phase based positioning is that although it can yield highly accurate, centimeter level results, the number of waves that lie between the user and satellites must be known in order for it to work. Currently, there are many methods which address this difficulty. One of the method utilizes the aforementioned L1-L2 widelane. Due to its longer wavelength, the effort for finding the right number of waves, or the 'integers', reduces greatly for the widelane measurement than for the L1 or the L2 carrier phase measurement. However, finding the right integer does still require some work, such as lengthy time-averaging of code measurement with the carrier measurement, or waiting for the satellite geometry to change to use redundant satellite measurements. In either technique, one requires to truncate, or round, the real number of widelane waves to become an integer. This is a crucial step since any error in widelane integer will induce pseudorange error in order of 86 centimeters. It is almost impossible to get the right L1 or L2 integers if the widelane integers are not correct.

This paper investigates the possibilities of finding the correct carrier phase integers with high integrity and continuity, without using redundant measurement. The idea is

simple; since there is more than one widelane available with three civil GPS frequencies, one uses the similar widelane technique with the multiple beat frequency observables. Assuming the integers of the Lc code, with the wavelength of 29.3 meters, are known, one can estimate the integers of the measurement with the next longest wavelength, the L2-Lc combination ( $\sim 5.9$  m, Extra Widelane), by using the Lc code measurement. The same procedure can be used for finding the L1-L2 ( $\sim 86$  cm, Widelane) integers, the L1-Lc ( $\sim 75$  cm, Mediumlane) integers, and finally the L1 integers. The integrity of this cascade integer finding method is verified by applying an integer rounding criteria in each refining step. This criteria, in turn, is based on the wavelength of the observable and noise of the measurement that is used to estimate the integer. The key to do this cascade integer finding with high integrity and continuity (possibly high enough for GPS/LAAS use) is low measurement error. Two separate sensitivity analysis are carried out to study the effect of baseline independent measurement error, such as receiver noise and multipath delay, and baseline dependent error such as ionosphere delay. Since the location of the third civil frequency, although now fixed, decides the available beat frequencies, it also affects the performance of the cascade integer finding method. The sensitivity analyses are thus carried out over a region of L band which contains ARNS frequency bands

## CARRIER PHASE DIFFERENTIAL POSITIONING WITH WIDELANE OBSERVABLE

### Widelane Combinations

The widelane signal makes it easier to resolve the integer ambiguities due to its increased wavelength over the L1 and the L2 carrier phase measurement. Consider the following double difference L1 carrier phase measurement equation.

$$\phi_{ABL_1}^{ij} = \frac{1}{\lambda_{L_1}} \rho_{AB}^{ij} + N_{ABL_1}^{ij} - I_{ABL_1}^{ij} + \gamma_{ABL_1}^{ij} \quad (1)$$

Where A and B indicates the reference and the user receiver, i the pivot satellite used in double difference, and j the satellites. Wavelength of the L1 carrier phase measurement is indicated by  $\lambda_{L_1}$ , the pseudorange by  $\rho$ , and integer by  $N$ . Term  $I_{L_1}$  is for the Ionosphere delay, which is a function of the square of the carrier frequency. The Troposphere delay, which is independent of the carrier frequency, is included explicitly in the pseudorange term. Finally,  $v_{L_1}$  includes multipath delay and receiver noise.

A simplified version of Equation 1, where double difference indexes such as A, B, i and j are removed, can be used to formulate the following measurement equations. Note that P represents code measurements.

$$P_{L_1} = \rho + I_{L_1} + \mu_{L_1} \quad (2)$$

$$P_{L_2} = \rho + \alpha I_{L_1} + \mu_{L_2} \quad (3)$$

$$P_{L_c} = \rho + \beta I_{L_1} + \mu_{L_c} \quad (4)$$

$$\lambda_{L_1} \phi_{L_1} = \rho + \lambda_{L_1} N_{L_1} - I_{L_1} + v_{L_1} \quad (5)$$

$$\lambda_{L_2} \phi_{L_2} = \rho + \lambda_{L_2} N_{L_2} - \alpha I_{L_1} + v_{L_2} \quad (6)$$

$$\lambda_{L_3} \phi_{L_c} = \rho + \lambda_{L_c} N_{L_c} - \beta I_{L_1} + v_{L_c} \quad (7)$$

$$\lambda_{EWL} \phi_{EWL} = \rho + \lambda_{EWL} N_{EWL} - \Gamma I_{L_1} + v_{EWL} \quad (8)$$

$$\lambda_{WL} \phi_{WL} = \rho + \lambda_{WL} N_{WL} - \tau I_{L_1} + v_{WL} \quad (9)$$

$$\lambda_{ML} \phi_{ML} = \rho + \lambda_{ML} N_{ML} - \Upsilon I_{L_1} + v_{ML} \quad (10)$$

$$\alpha = \frac{f_{L_1}^2}{f_{L_2}^2} = 1.65 \quad \beta = \frac{f_{L_1}^2}{f_{L_c}^2} = 1.79$$

$$\Gamma = \lambda_{EWL} \left( \frac{\alpha}{\lambda_{L_2}} - \frac{\beta}{\lambda_{L_c}} \right) = -1.72$$

$$\tau = \lambda_{WL} \left( \frac{1}{\lambda_{L_1}} - \frac{\alpha}{\lambda_{L_2}} \right) = -1.28$$

$$\Upsilon = \lambda_{ML} \left( \frac{1}{\lambda_{L_1}} - \frac{\beta}{\lambda_{L_c}} \right) = -1.34$$

These measurement equations yields eight unknowns,  $\rho$ ,  $I_{L_1}$ , and the integers for each carrier phase and beat frequency measurements, and nine equations. Although estimation of each unknowns is possible by using these set of measurements, this paper focuses on estimating and rounding only the integers, thus including Ionosphere delay as a source of measurement error. Since the differential Ionosphere delay grows over distance, its effect is investigated in the later section.

### Cascade Integer Finding

Using any of the available code measurements, the integer for the extra widelane,  $N_{EWL}$ , can be found by following manner.

$$N_{EWL} = \phi_{EWL} - \frac{P_{L_c}}{\lambda_{EWL}} \quad (11)$$

Equation 11 yields a floating solution of  $N_{EWL}$ , which contains the measurement error from both  $\phi_{EWL}$  and  $P_{L_c}$ . By rounding the float solution to a correct integer, the extra widelane measurement now can be used to find the widelane integer,  $N_{WL}$ , without the added measurement error from the code measurement, which was used to get the floating solution of  $N_{EWL}$ .

$$N_{WL} = \phi_{WL} - \frac{\lambda_{EWL}}{\lambda_{WL}} (\phi_{EWL} - N_{EWL}) \quad (12)$$

Using the similar manner, one can eventually find the integer for the L1 carrier phase measurement.

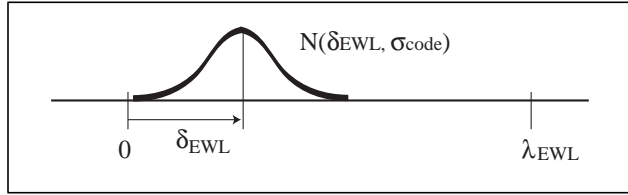
$$N_{ML} = \phi_{ML} - \frac{\lambda_{WL}}{\lambda_{ML}}(\phi_{WL} - N_{WL}) \quad (13)$$

$$N_{L1} = \phi_{L1} - \frac{\lambda_{ML}}{\lambda_{L1}}(\phi_{ML} - N_{ML}) \quad (14)$$

Equations 11-14 mathematically expresses the cascade integer finding method, where one starts from the code measurement, then successively solve for an integer of the measurement with the shorter wavelength than the measurement being used. Assuming the third civil frequency will carry a code with higher modulation than the C/A code, it should be used to begin the method since it should have lower measurement noise than the C/A code measurement. For the cascade integer finding method to be successful, each of the refining step must resolve the integer ambiguity correctly before each of the carrier phase measurement is used for the next step. If not, using this method to get the correct L1 integer will not be possible.

## ROUNDING CRITERIA

### Criteria Development



**Figure 2: Rounding a Float Integer Solution**

As described in the earlier section, getting a correct integer in each step, Code to EWL, EWL to WL, and WL to ML are all very important. Each of the integers is used to calculate a float integer solution for the next carrier phase measurement with smaller wavelength. Therefore, the integrity of integer solution of each step needs to be verified. To do so, an integer rounding criteria based on the measurement error and the wavelength is developed.

Assume that a floating solution is  $\delta$  away from the true integer, which is indicated at 0 in Figure 2, and  $\delta$  is positive. Given the distribution of  $\delta$  with the standard deviation of  $\sigma$ , the conditional probability of rounding to the correct integer is then the following.

$$P(0|\delta, \sigma) \cong \frac{\exp\left(\frac{-1}{2\sigma^2}\delta^2\right)}{\exp\left(\frac{-1}{2\sigma^2}\delta^2\right) + \exp\left(\frac{-1}{2\sigma^2}(\lambda - \delta)^2\right)} \quad (15)$$

$$P_{err} = 1 - P(0|\delta, \sigma) \cong \frac{\exp\left(\frac{-1}{2\sigma^2}(\lambda - \delta)^2\right)}{\exp\left(\frac{-1}{2\sigma^2}\delta^2\right)}$$

$$P_{err} = \exp\left(\frac{-1}{2\sigma^2}(\lambda^2 - 2\lambda\delta)\right) \quad (16)$$

The probability of rounding to an incorrect integer,  $P_{err}$ , is shown in Equation 16. Note that Equation 16 only holds when  $P_{err} \ll 1$ . It is re-arranged as below to find requirements for a desired level of integrity.

$$\exp\left(\frac{-1}{2\sigma^2}(\lambda^2 - 2\lambda\delta)\right) < (P_{err})_{required}$$

$$\frac{\delta}{\sigma} < \frac{1}{2}\left(\frac{\lambda}{\sigma} + \frac{2\log(P_{err})_{required}}{\lambda/\sigma}\right) \quad (17)$$

Given a desired integrity,  $(P_{err})_{required}$ , a suitable level of continuity can be assigned by setting the  $\delta/\sigma$  ratio, then the required  $\lambda/\sigma$  ratio can be calculated. Table 2 shows a few sample calculations of parameters for the rounding criteria for different levels of integrity and continuity requirements.

**Table 2: Examples of Integrity and Continuity Parameters**

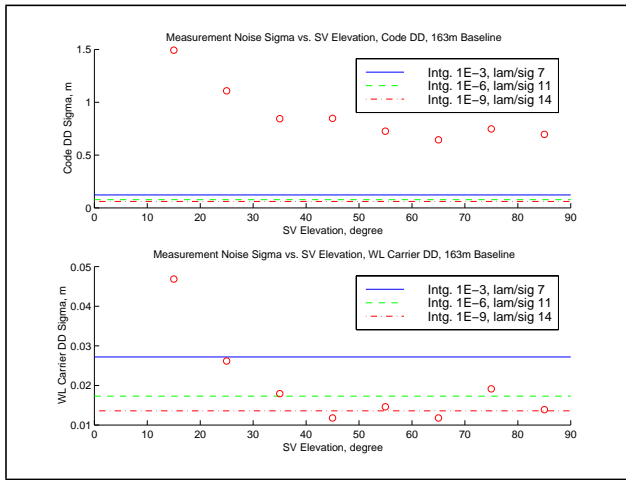
Integrity	Continuity	$\delta/\sigma$	$\lambda/\sigma$
$10^{-9}$	$10^{-8}$	5.5	14
$10^{-6}$	$10^{-5}$	4.2	11
$10^{-3}$	$10^{-2}$	2.5	7

### Cascade Integer Finding with Rounding Criteria: The Present Signal Structure

**Table 3: The  $\lambda/\sigma$  ratio of the Present GPS Signal (all units in m)**

	WL $\lambda .86$	L1 $\lambda .19$
Code $\sigma$ 1.1	0.78	.17
WL $\sigma$ 0.026		7.3

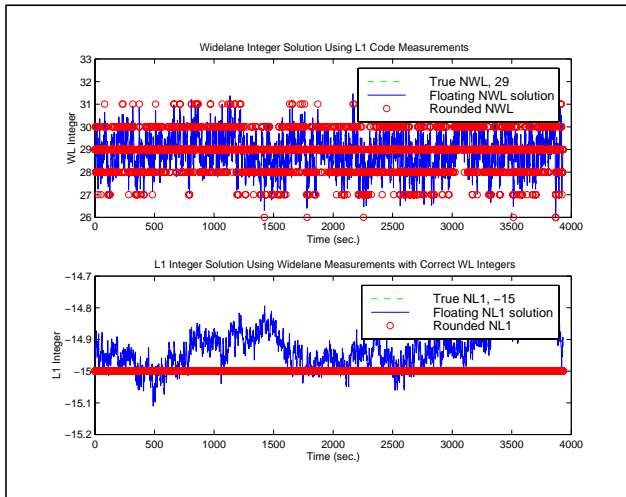
The cascade integer finding method can be used with the current civil GPS signal if one is equipped with dual frequency receivers which can generate the L1-L2 widelane. The method would be solving for the WL integer using the code measurement then the L1 integer using the WL measurement. Table 3 shows the ratio between available wavelength and measurement noise standard deviation,  $\lambda/\sigma$ , of the present civil GPS signal structure. The double difference  $\sigma$  used in this calculation is based on an hour of data collected at 1Hz interval using dual frequency receivers. Small amount of carrier aided averaging is done, 2 sec-



**Figure 3: Double Difference Measurement Noise  $\sigma$  vs. Satellite Elevation**

onds to the L1 and 20 seconds to the L2 code measurement. Ten degrees of the satellite elevation mask angle is used in both the reference and the user receiver. The distance between the two antennas is 163m. To select a reasonable double difference  $\sigma$  for the code and widelane measurement, relation between the satellite elevation, which affects signal to noise ratio and multipath delay, and  $\sigma$  is studied, as seen in Figure 3. The figure also shows the required  $\sigma$  for different levels of integrity and continuity for each of the wavelength.

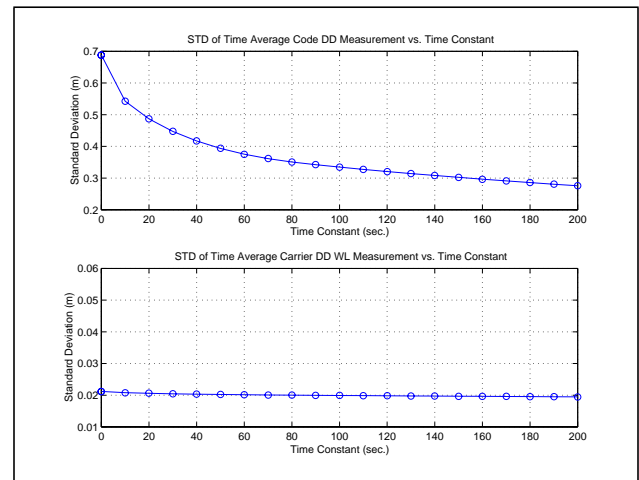
Table 3 suggests that getting the widelane integer from the



**Figure 4: Integer Solutions Using Cascade Integer Finding, With Present GPS Signal**

code measurement with reasonable integrity is not possible, since the  $\lambda/\sigma$  ratio between the widelane wavelength and the code measurement noise standard deviation is far less than 7, which is the threshold for  $10^{-3}$  integrity. Fig-

ure 3 also suggests the same. Across the entire elevation, the widelane  $\sigma$  is much too high for a reasonable integrity and continuity requirements. However, both Table 3 and Figure 3 also suggest that getting the L1 integer from the widelane measurement can be done with certain degrees of integrity and continuity once the correct widelane integers are known. Figure 4 shows the result of the cascade integer finding using the present civil signal. As expected, the widelane floating solution is rounded into incorrect integers quite often, due to low  $\lambda/\sigma$  ratio. However, it also shows that L1 floating solution is rounded into the correct integer, in all 3887 cases, due to relatively high  $\lambda/\sigma$  ratio, which is also an expected result. The correct widelane integer, which is calculated separately, are used to get the floating L1 solution.

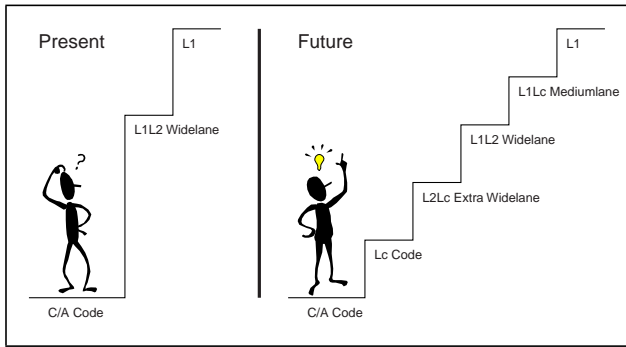


**Figure 5: Effect of Time Average**

The  $\lambda/\sigma$  ratio of the present GPS signal can be improved if time averaging of the measurement is used. Figure 5 shows change in the standard deviation of the double difference code and carrier phase widelane measurements when different time constants are used for the averaging. For an example, if one uses 200 seconds to average the measurements before applying it to the cascade integer finding method, the  $\lambda/\sigma$  ratio of the code to widelane integer step will show about factor of two (0.78 to 1.56) improvement. However, it is still very low, compared with 7 for  $10^{-3}$  integrity requirement. The  $\lambda/\sigma$  ratio of the widelane to L1 integer step shows very small improvement since the time averaging is ineffective in reducing the  $\sigma$  of carrier phase measurement, where the majority of noise, such as multipath delay, is biased.

### Cascade Integer Finding with Rounding Criteria: Multiple Civil Signal Structure

With three available beat frequencies, the extra widelane,



**Figure 6: Cascade Integer Finding, the Present and Future**

the widelane and the mediumlane, and with assumption that the third civil frequency will carry a code with higher chipping rate at 10.23MHz, than the present C/A code at 1.023MHz, the future GPS signal structure shows much promise in successful cascade integer finding with the higher integrity than which the present signal structure could offer. Figure 6 conceptually illustrates the difference between the application of the cascade integer finding method on the present and future civil signal structure. With the present signal, it is possible to solve for the L1 integer using the widelane measurement, but getting the correct widelane integer using the code measurement is not possible using the method (i.e. the person can climb up the top stair but not the bottom one). With the multiple signals, the same difficulty is divided among three more steps, which should allow the cascade integer finding method with high integrity to work.

Table 4 shows the result when the rounding criteria is applied to the multiple signals structure. The measurement

**Table 4:  $\lambda/\sigma$ , Multiple Civil Signals,  $\sigma$  is 3% of measurement wavelength (all units in m)**

	EWL $\lambda$ 5.9	WL $\lambda$ .86	ML $\lambda$ .75	L1.19
LcCode $\sigma$ 0.90	6.6	0.96	0.83	0.21
EWL $\sigma$ 0.18		4.9	4.2	1.1
WL $\sigma$ 0.026			29	7.4
ML $\sigma$ 0.023				8.4

**Table 5:  $\lambda/\sigma$ , Multiple Civil Signals,  $\sigma$  is 1% of measurement wavelength (all units in m)**

	EWL $\lambda$ 5.9	WL $\lambda$ .86	ML $\lambda$ .75	L1.19
LcCode $\sigma$ 0.30	20	2.9	2.5	0.63
EWL $\sigma$ 0.059		15	13	3.2
WL $\sigma$ 0.0086			87	22

noise of each double difference measurement is set at 3% of their wavelength, which is derived from the  $\sigma$  versus satellite elevation angle study in the earlier section. Note that cascade integer finding is possible, but with very low integrity (much less than  $10^{-3}$ ) due to the  $\lambda/\sigma$  ratio of 4.9 at the step where the widelane integer is calculated using the extra widelane measurement. However, the  $\sigma$  value of 3% of wavelength is derived from the experimental data where no significant effort was made to reduce the multipath delay, which is quite possible to reduce to a lower level. For an example, according to Minimum Aviation System Performance Standards (MASPS) for the Local Area Augmentation System (LAAS), it is possible to reduce the pseudorange error six times less in lower satellite elevation angle and two and a half times less in higher elevation when multipath limiting antenna in conjunction with narrow correlator technology is applied. In addition, time average can be used to reduce some portion of the measurement noise, as seen in earlier. The effect of multipath limiting technique and time averaging on the performance of the cascade integer finding method will be investigated in-depth in future.

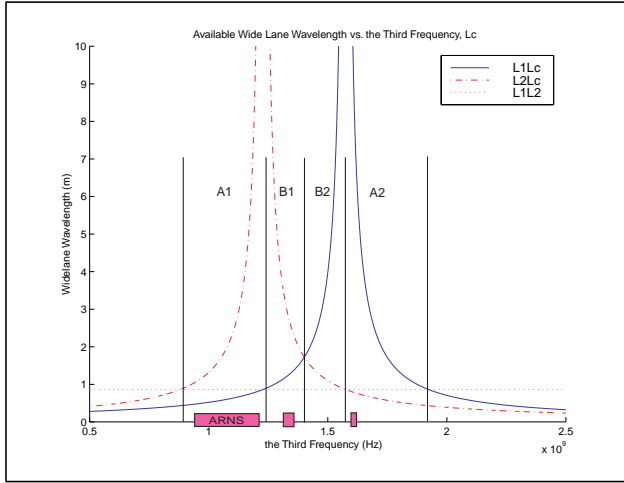
Table 5 shows the  $\lambda/\sigma$  ratio of the multiple signals when the measurement noise  $\sigma$  is set at 1% of their wavelength. The table suggests that getting the L1 integer by the cascade integer finding method with high integrity is possible since all of the integer rounding steps have a higher than 14  $\lambda/\sigma$  ratio. Since the level of integrity of the rounding criteria depends on both the wavelength of measurement and the standard deviation of its noise, analysis on both of them are carried out in the next section.

**Table 5:  $\lambda/\sigma$ , Multiple Civil Signals,  $\sigma$  is 1% of measurement wavelength (all units in m)**

ML $\sigma$ 0.0075				25
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## ROUNDING CRITERIA PARAMETER ANALYSIS

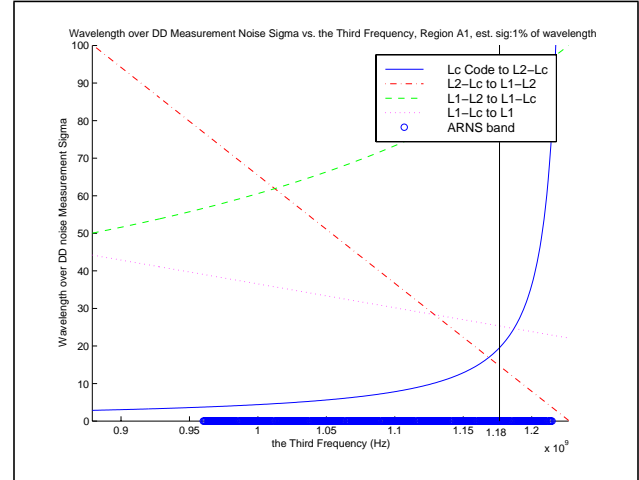
### Location of the Third Frequency



**Figure 7: Available Beat Frequency vs. the Third Frequency**

Although the third civil frequency is now set at 1176.45 MHz, the performance of rounding criteria will be affected if a different frequency were selected. This is true because both of the parameters in the  $\lambda/\sigma$  ratio will change since a different third frequency means change in the wavelength of the available beat frequencies. Since the measurement noise level is related with the wavelength, the longer wavelength will result in a higher level of  $\sigma$ , and the shorter one a lower level. Figure 7 shows the available beat frequency over a region of L band with assumption that the second civil frequency will be located at the L2. It is divided into four regions, A1, A2, B2 and B2. In the A region, wavelength of the L2-Lc beat frequency is larger than the L1-L2 (.86m), and wavelength of the L1-Lc is smaller than the L1-L2. In the B region, both the L2-Lc and the L1-Lc wavelengths are larger than the L1-L2. The region A2 and B2 are the same as A1 and B1, except the relative magnitude between the L2-Lc and the L1-Lc wavelength is reversed.

Figure 8 shows the details of the region A1 where the  $\lambda/\sigma$  ratio of the each integer rounding step is plotted over the third frequency. The solid line represents the level of integrity, or the  $\lambda/\sigma$  ratio of rounding a float solution of the L2-Lc extra widelane integer by using the Lc code measurement. As the third frequency increases toward the L2 frequency, the wavelength of the extra widelane increases much faster than the increase in  $\sigma$ , which results



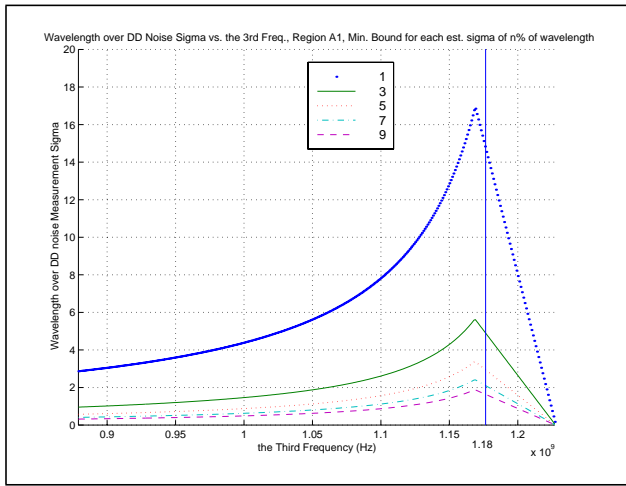
**Figure 8: The  $\lambda/\sigma$  ratio of the Region A1**

the higher  $\lambda/\sigma$  ratio. Using the same analogy, rounding the floating solution of L1-L2 integer by using the EWL measurement, which is plotted in dash-dot, is shown to become less reliable as the third frequency increases toward the L2 frequency. The  $\lambda/\sigma$  ratio of this step decreases since wavelength of the L1-L2 widelane is fixed at 0.86 meters while  $\sigma$  of the EWL measurement increases as the third frequency becomes higher. Similar relationships exist in the rest of plot. A minimum boundary for the  $\lambda/\sigma$  ratio over frequency can be formed using these lines, which can be used to locate an optimal place for the third frequency.

According to Figure 8, the optimal place for the third frequency would be at 1170.0 MHz where the local maximum of the  $\lambda/\sigma$  ratio is located, at the peak of the triangle that is bounded by the code to the EWL (solid line) and the EWL to the WL (dash dot line) step. The peak  $\lambda/\sigma$  ratio, 16.8, is 11% higher than the ratio at the selected third frequency, 15.1.

### Multipath and Receiver Noise

For a given wavelength, it is crucial to have low noise measurement in order to round the float integer solution with high integrity. As described in Equation 1, double difference measurement error mainly comes from multipath delay, receiver noise and Ionosphere delay. These error sources can be roughly divided into two groups. One group includes Ionosphere delay, which decorrelates as the distance between the reference and the user receiver increases. The other group includes multipath delay and



**Figure 9: Minimum  $\lambda/\sigma$  over Frequency, Effect of Multipath and Receiver Noise**

receiver noise, which depends on the type of receiver and the location of antenna, but are independent of the distance between the reference and the user location. Because these error groups are independent, the following equation can be used to model the double difference measurement noise,  $\sigma$ .

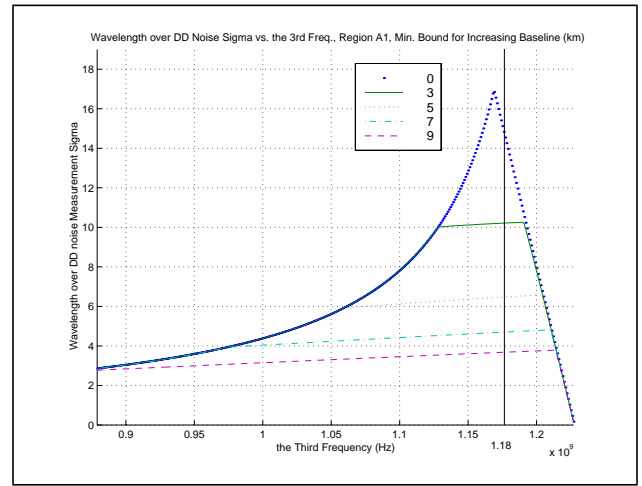
$$\sigma^2 = \sigma_{mp}^2 + \sigma_I^2 \quad (18)$$

To simplify the analysis, the effect of different levels of multipath delay and receiver noise,  $\sigma_{mp}$ , on the rounding criteria is investigated first. Figure 9 shows how the  $\lambda/\sigma$  ratio of the four steps, Lc Code to EWL, EWL to WL, WL to ML and ML to L1, changes as the  $\sigma_{mp}$  increases from 1% to 9% of each of the wavelength. Only the minimum boundary of the  $\lambda/\sigma$  ratio is shown in the plot. As expected, the  $\lambda/\sigma$  ratio, or the available level of integrity, decreases as the  $\sigma_{mp}$  increases. Since the second and third civil signals are not available yet, it is hard to verify which, if any, minimum boundary in the plot represents the true value. Site dependency of multipath delay also increases the difficulty of choosing the correct values. However, with advance in multipath limiting technology, both in hardware and software, it should be possible to decrease the  $\sigma_{mp}$  to a few percent of the measurement wavelength.

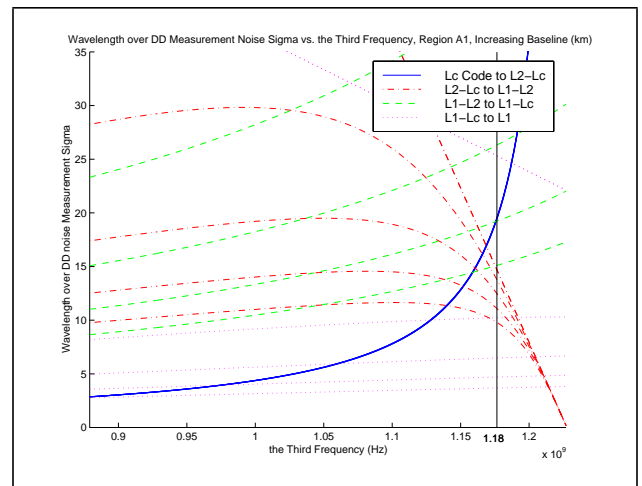
### Ionosphere Delay and Change in Baseline

Assuming that value of the  $\sigma_{mp}$  is fixed at 1% of each of the measurement wavelength over different distances, effect of the differential Ionosphere delay,  $\sigma_I$ , on the rounding criteria over different distances between the reference and the user receiver can be investigated using the following equation,

$$\sigma_I(f) = k(f)\nabla\sigma_{IL1}S \quad (19)$$



**Figure 10: Minimum  $\lambda/\sigma$  Ratio over Frequency, Effect of Ionosphere Delay**



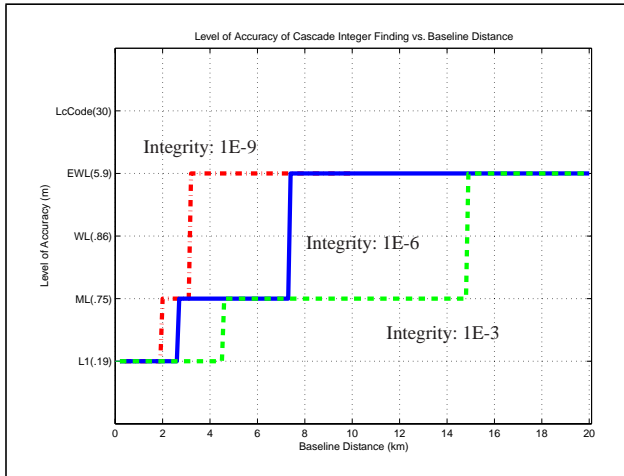
**Figure 11: The  $\lambda/\sigma$  Ratio over Frequency, Effect of Ionosphere Delay**

Equation 19 shows how the standard deviation of differential Ionosphere delay for each of the measurement is calculated. The spatial gradient of Ionosphere at L1,  $\nabla\sigma_{IL1}$ , is multiplied by the distance between two receivers,  $S$ , then scaled by a frequency factor  $k(f)$  for the measurements with a different frequency than the L1 frequency. Since the cascade integer finding method uses differential measurements, it is important to know how much Ionosphere delay decorrelates over a distance. Unfortunately there is not a fixed number that can be used for  $\nabla\sigma_{IL1}$ . It varies from as small as 2mm per km [Parkinson], to as large as 55mm per km in extreme cases, such as in Antarctica [Goad].

Figure 10 shows the effect of Ionosphere delay as the distance between the reference and the user receiver increases from 0 to 9 km, with the  $\nabla\sigma_{IL1}$  of 3mm per km. The result



shown in the figure suggests that at a distance larger than 3 km, the cascade integer finding method will produce very low integrity integers. Figure 11 shows the details of changes in the  $\lambda/\sigma$  ratio of the each rounding step. In the figure, the dotted line reduces the minimum  $\lambda/\sigma$  ratio boundary the most as the distance increases. This line represents the step which rounds the L1 integer from a float solution obtained by using the medium lane measurement. The relatively large growth in the differential Ionosphere delay over distance, compared with its wavelength (.75m), causes a significant drop in the  $\lambda/\sigma$  ratio, making it the performance limiting step. Note that getting the widelane



**Figure 12: Level of Accuracy and Integrity of the Cascade Integer Finding method over Distance**

integer using the extra widelane measurement (dash dot line) is the next performance limiting step. Getting the mediumlane integer using the widelane measurement (dash dash line) is the next.

The threshold distance for each of the integer rounding steps for a given integrity requirement is shown in Figure 12. For an example, with  $10^{-9}$  integrity and  $10^{-8}$  continuity requirements, an user can expect the L1 carrier phase measurement level of accuracy up to 2 km away from the reference receiver, the mediumlane level up to 3.2 km away, and the extra widelane level of accuracy beyond that. With  $10^{-3}$  integrity and  $10^{-2}$  continuity requirements, an user can expect the L1 level of accuracy up to 4.6 km, the ML level up to 15 km, and the EWL level beyond.

## CONCLUSION

In this paper, possibility of high integrity and continuity carrier phase navigation using the multiple civil GPS signals is shown. By using the multiple beat frequency observables, such as the extra widelane, the widelane and

the mediumlane, one can quickly resolve the L1 integer ambiguity by applying the cascade integer finding method. The integrity and continuity of this method is verified by the integer rounding criteria, which is based on the wavelength and the measurement noise standard deviation. The performance of the cascade integer finding method is investigated over different available beat frequencies and measurement noise  $\sigma$  levels, including changes in the receiver noise, multipath delay and differential Ionosphere delay.

The benefits to the users from the multiple civil GPS signals in the form of improved accuracy and robustness of service would come slowly over the next 10-20 years as new satellites with expanded capabilities are produced and launched to replace the current satellites. Once the multiple civil GPS signal structure is in place, an user can utilize the cascade integer finding method, first with relatively low integrity and accuracy when located far from the reference receiver, then with the higher integrity and accuracy, eventually up to  $10^{-9}$  integrity with the L1 carrier phase level accuracy, as the user approaches near the reference receiver.

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