Loran Availability and Continuity Analysis for Required Navigation Performance 0.3

Sherman Lo, Department of Aeronautics and Astronautics, Stanford University,
496 Lomita Mall, Durand 250, Stanford, CA USA
daedalus@stanford.edu

Co Authors: Ben Peterson, Peterson Integrated Geopositioning, USA
Per Enge, Department of Aeronautics and Astronautics, Stanford University, USA
Robert Wenzel, Booz Allen Hamilton, USA
G. T. Gunther, Booz Allen Hamilton, USA
Peter Morris, Northrop Grumman Information Technologies
Lt. Kevin Carroll, US Coast Guard Loran Support Unit, USA
Lt. David Fowler, US Coast Guard Navigation Center, USA

1 ABSTRACT

The Loran Integrity Performance Panel (LORIPP) is to complete its 18-month study of whether an enhanced Loran system, which includes modernized equipment, new receiving equipment, and updated operating procedures, could meet aviation requirements for Required Navigation Performance 0.3 (RNP 0.3) in March 2004. There are four primary areas that must be examined in order to demonstrate RNP 0.3 – integrity, availability, continuity and accuracy. This paper discusses the availability and continuity analysis.

A determination of availability and continuity must start with an examination of the system hazards. It is important to understand these hazards since their effects manifest themselves in many ways. A coverage area tool was developed to incorporate the effects of each primary Loran hazard. They affect the availability of a solution by reducing the confidence level on cycle resolution or increasing the bound on the protection level. The coverage tool models these effects by incorporating the algorithms and integrity equation that a Loran receiver will need to employ. A description of the coverage tool and its design will be given in the paper.

The analysis of transmitter availability and continuity is one of the fundamental values used in determining the overall availability and continuity. The paper will discuss how these values are derived for the modernized Loran system upgraded by the current Loran Recapitalization Program and proposed changes to both the system specification and operating procedures. These values are incorporated into the coverage tool to determine overall availability and continuity.

The coverage tool was design to be flexible especially since the bounds on errors due to various hazards are being refined. Furthermore, it design allows it to be used for analyzing maritime harbor entrance and approach (HEA) needs. The coverage tool thus is the system integration tool for analyzing availability and continuity for Loran across multiple applications. The paper will show the results of using this tool to

analyze the expected availability and continuity of Loran for both RNP and HEA throughout the conterminous US (CONUS).

2 INTRODUCTION

Robustness and redundancy for position, navigation and timing (PNT) has always been valued. The events of recent years have only amplified this desire, particularly for safety critical or economically valuable operations. As the Global Positioning System (GPS) has become integral to the critical infrastructure in many safety and economic activities, it has become increasingly necessary to determine an efficient means of retaining most of the safety or economic benefits derived from GPS the event that GPS is unavailable. These thoughts are echoed in the Volpe National Transportation Safety Center (VNTSC) Report on GPS Vulnerability [1]. Its recommendations include examining various alternatives to providing redundancy to GPS, particularly in safety critical applications. Redundancy provides benefits by providing the ability to maintain operations with little or no degradation. Furthermore, it acts as a deterrent against malicious interference. Thus, the various agencies with in the United States Department of Transportation (USDOT) have engaged in evaluating the need for redundancy and how to meet the need.

The Long Range Navigation system or Loran is one of the systems being considered for providing redundancy. It is one of the few systems being evaluated that is capable of meeting this need across many modes of application. A study on the ability of Loran-C, conducted under the direction of the United States Federal Aviation Administration (FAA) has just been completed. It evaluated the ability of an enhanced Loran (*eLoran*) system to mitigate the impact of a GPS outage on GPS position, navigation, and time applications [2]. In terms of position and navigation, the goal was to determine whether *eLoran* could provide the capacity and efficiency to continue commercial operations. The paramount FAA Loran issue is whether it can support non-precision approach (NPA). The preferred NPA is Required Navigation Performance 0.3 (RNP 0.3). For the United States Coast Guard (USCG), the application of concern is Harbor Entrance and Approach (HEA). For timing and frequency, it is Stratum 1 frequency and highly synchronized (20 nanosec) time to UTC.

The Loran study evaluation team, composed of members from government (FAA and USCG), academia, and industry members, was assembled to conduct an evaluation of the current and potential capabilities of Loran system. From this membership, the Loran Integrity Performance Panel (LORIPP) was formed with its primary purposes is to investigate ability of Loran to meet RNP 0.3 using the underlying structure of the current Loran system along with planned upgrades and reasonable modifications. Another group, the Loran Accuracy Performance Panel (LORAPP) was charged with the same task for HEA. The groups had common membership and analysis from one application often applied to the other. However, there are distinct differences as highlighted by the name of the groups – integrity was seen as the primary challenge to RNP while accuracy was seen as the primary obstacle to HEA. This paper presents details of the availability and continuity analysis conducted by members of the report evaluation team. While it focuses on RNP, HEA will be discussed as well.

3 LORAN BACKGROUND

Loran is a low frequency (LF), terrestrial, pulsed, hyperbolic, horizontal navigation system operating between 90-110 kHz. Due to the nature of LF signals and the power of Loran transmissions, the signals have a long range. Users at distances of 800 km or more can receive these signals which makes then useful for a long-range navigation system. There are 24 (27) stations in the US (North American) Loran chain which provide coverage to CONUS and a significant portion of Alaska. More details on Loran can be found in numerous papers and books [3].

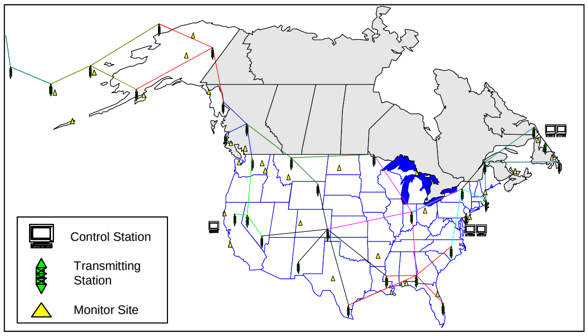

Figure 1. North American Loran-C System

3.1 Loran Navigation

A traditional method of navigating with Loran is to use the time difference of arrival (TDOA) for signals from different stations. This generally requires that the measured stations are transmitting at the same rate; i.e. they are in the same chain. However, since all stations do not transmit at the same rate, this method limits the number of stations that can be used. Time of arrival (TOA) measurements allows for using all (signals) in view (AIV). Using TOA is akin to using GPS pseudoranges.

Regardless of whether TDOA or TOA is used, either measurement requires that the Loran pulse and timing can be tracked. A nominal pulse is shown in Figure 2. A specific tracking point is selected with the point being a balance between signal power and mitigating skywave interference[1]. Typically the tracking point is set around 30 microseconds (or the sixth zero crossing) after the start of the pulse. Noise makes it difficult to identify the start of the pulse (and hence the 30 μsec point) and a two-step process is used to accomplish the tracking. First, the envelope of the signal is used for a "coarse" identification. The goal is identify the correct cycle of the Loran signal to track. Cycle identification, usually using envelope slope, is generally not difficult provided that the signal is not too corrupted by noise. After the desired cycle has been

---

[1] Skywave interference comes from ionospheric reflections of the Loran signal. It is akin to GPS multipath.

identified, the location of the desired tracking point is determined by examining the zero crossing. This then yields the timing or phase estimate used to determine range.

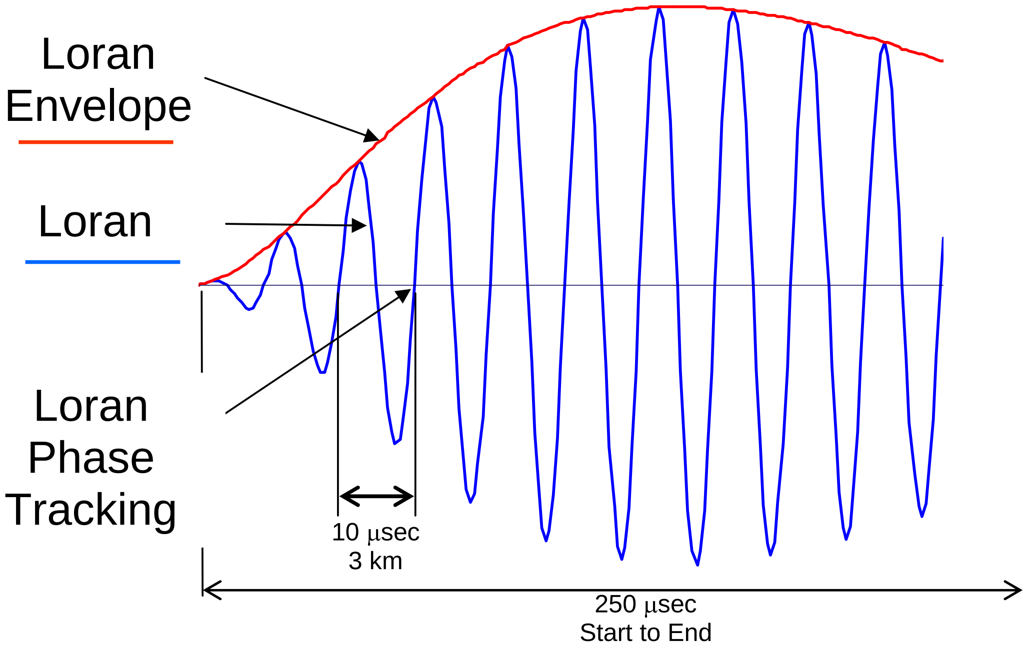

Figure 2. Normal Loran Pulse

3.2 Hazards

There are numerous hazards that can affect the precision of the ranging measurement as well as the overall availability of the signal. Some hazards affect the ability to measure the envelope, resulting in incorrect cycle determination (with an accompanying large range error). For example, the envelope may shift relative to the underlying carrier (and hence the tracking point). This shift is termed the envelope to cycle difference (ECD). Some hazards, such as noise, affect the phase measurement by obscuring the location of the zero crossing or delaying the overall signal. Other hazards, such as transmitter outages, precipitation static or early skywave, make the signal unavailable for use.

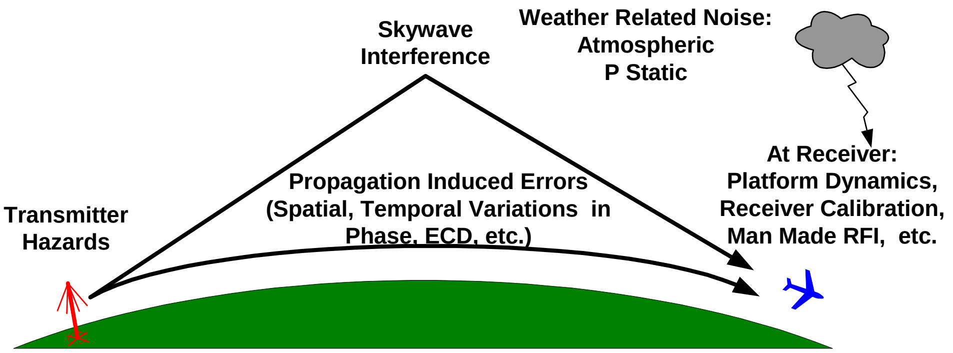

Figure 3. Loran Hazards

Category	Hazard
Transmitter	Timing and Frequency Equipment Transmitter and Antenna Coupler Transmitter Equipment Monitoring
Propagation	Spatial variation of phase along approach path Temporal variation of phase Spatial variation of ECD along approach path Temporal variation of ECD Temporal variation of SNR
Receiver	Platform dynamics Atmospheric Noise Precipitation Static Skywaves Cross-Rate Interference Man-made RFI Structures Receiver Calibration

Table 1. Hazards and What They Affect

The evaluation team had to identify and assess each of these hazards to determine their effect on the system. The hazards are listed in Table 1 and pictorially depicted in Figure 3. This paper will not detail the hazards which are described in other papers [4].

3.3 Requirements & Modernized Loran

An assessment of whether the system can meet the requirements for HEA and RNP 0.3 can be made once the hazards that can affect the signal are identified. These requirements are quantified in Table 2[2]. Integrity is the fidelity of the system. It is the ability of the system to alert a user when a signal or a solution should not be used. This must occur within the time to alert (TTA). Hence, for the solution to be available, it must have already met integrity requirements. Continuity is the probability that the system remains available throughout the operation presuming that availability initially exists.

---

[2] Availability and continuity are expressed in a range of values from minimum to maximum. The "target" requirements listed in the table are derived from the U.S. standard for GPS that the Loran program is trying to achieve. The "minimum" requirements represent the ICAO standards that must be met.

Performance Requirement	RNP Value	HEA Value
Accuracy (target)	307 meters	20 m, 2 drms
Monitor/Alert Limit (target)	556 meters	50 m, 2 drms
Integrity	10^{-7}/hour	3 x 10^{-5}
Time-to-alert	10 seconds	10 seconds
Availability (minimum)	99.9%	99.7%
Availability (target)	99.99%	
Continuity (minimum)	99.9% over 150 seconds	99.85% over 3 hours
Continuity (target)	99.99%	

Table 2. RNP 0.3 and HEA Requirements

Since the current Loran system does not meet the requirements shown in Table 2, the evaluation team had to define and describe the capability of a modernized Loran system that will meet RNP, HEA and timing and frequency needs. Sometimes termed enhanced Loran (eLoran), the group defined:

The modernized Loran system continues to be a low-frequency, terrestrial navigation system operating in the 90- to 110-kHz frequency band and synchronized to coordinated universal time. However, this modernized Loran system has a recapitalized infrastructure and a new communication modulation method that enables operations that satisfy the accuracy, availability, integrity, and continuity performance requirements for non-precision approaches and harbor entrance and approaches, as well as the requirements of non-navigation time and frequency applications. Required changes to the current system include modern solid-state transmitters, a new time and frequency equipment suite, modified monitor and control equipment, and revised operational procedures that new receiver technology can exploit [2].

It is this system that the availability and continuity analysis described below applies to.

4 COVERAGE MODEL

The performance of Loran can vary greatly within the coverage area since the signals available, their strengths, noise level and other factors vary spatially. The coverage model represents the high level system integration of the results of the assessment. The basic calculation is shown in Figure 4. First, signal to noise ratio (SNR) is calculated. Fundamental noise and signal propagation models are used to determine baseline SNRs. Next, cycle selection and HPL are calculated. The calculation incorporates the algorithms that will be used to provide the required level of integrity. This means utilizing the results of the investigation of each hazard such as integrity bounds, availability reduction, and other effects that a hazard may cause. Finally, the calculation is repeated for various noise levels and station availabilities and the results are compiled. The following sections will discuss each of these three steps.

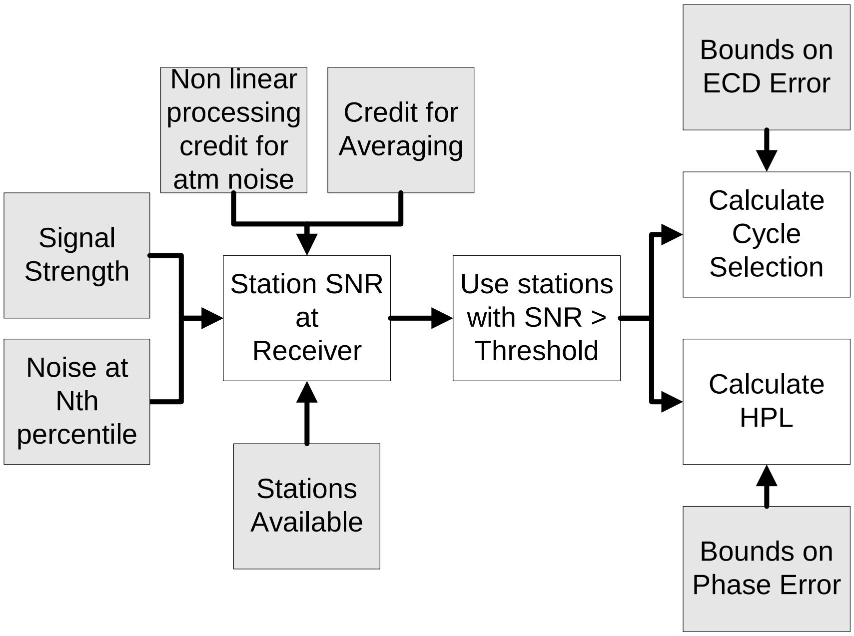

Figure 4. Calculation Process of Basic Coverage Analysis

4.1 Signal Strength and Noise

Signal strength (SS) for each station is calculated using Millington's method for smooth inhomogeneous earth [5]. The results for each station are stored in an array.

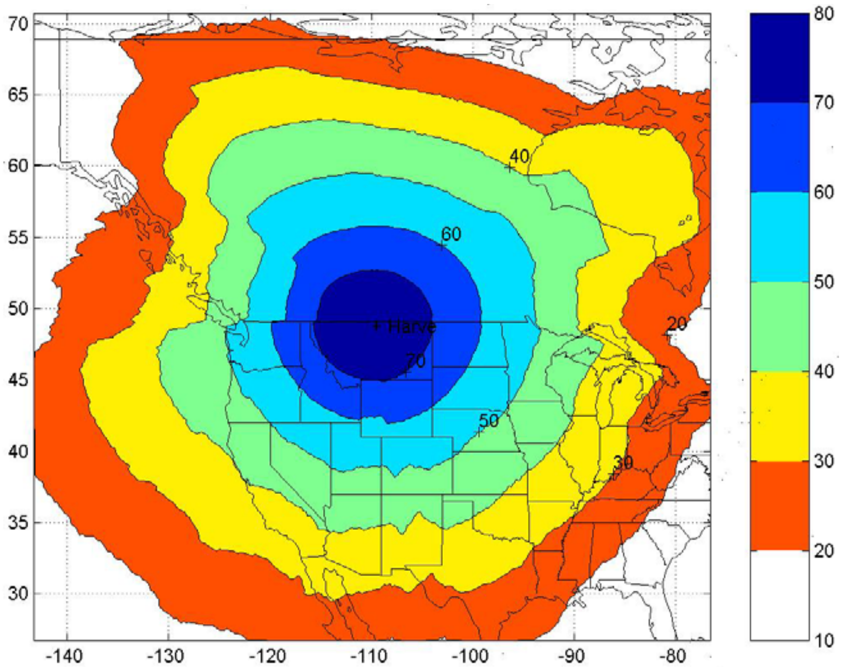

Figure 5. Sample Signal Strength from Havre in dB re 1uv/m for 400 kW

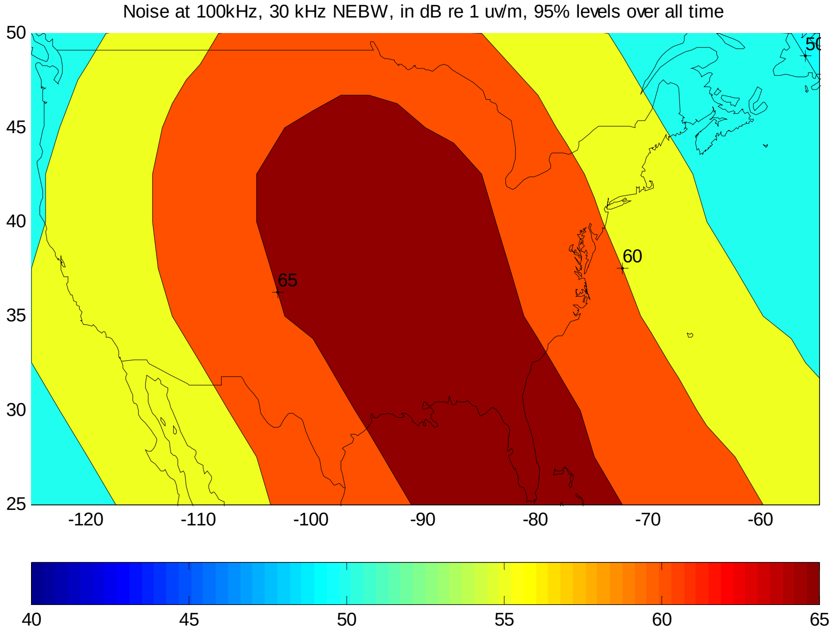

Figure 6. 95% CONUS Noise Map (Annual Average)

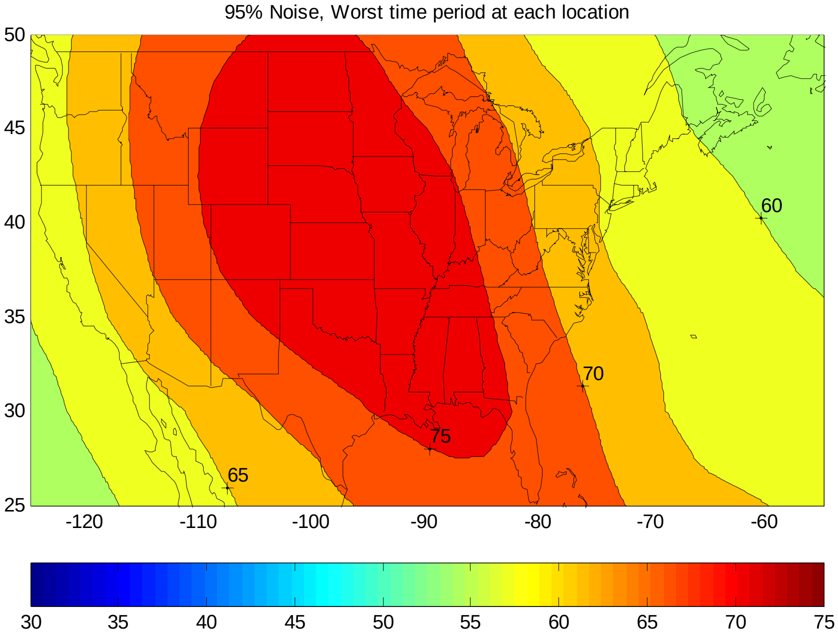

Figure 7. 95% CONUS Noise Map During Worst Period at Each Location

Noise levels are based on the widely accepted model from International Telecommunication Union (ITU), formerly the International Radio Consultative Committee (CCIR) [6]. The noise levels vary as a function of location, time of day and season. The noise at various levels from 50th to 99.9th percentile are calculated and stored. Both the annual average and worst case values are stored for a given location. Figure 6 and Figure 7 show the annual average and worst case 95th percentile noise diagrams for conterminous United States (CONUS), respectively. There is a significant difference (10 dB) between these two values at the 95% level and this difference increases as the noise percentile increases.

The omnipresent noise measured by ITU and shown above is atmospheric noise – noise due to lightning discharge. At higher levels, noise displays more and more impulsive strikes. The impulsiveness is characterized by a quantity termed V_d which is the ratio between the root mean square noise and the average noise in dB. Non linear processing such as clipping and blanking can greatly mitigate the effect of highly impulsive noise and hence credit can be taken for such processing. Some details of this work are given in [7]. Figure 8 shows the processing credit as a function of V_d based on data collected. Since the median V_d for high noise is roughly 12, a conservative credit of 12 dB is taken. This value is consistent with prior research [8].

The SNR in the receiver calculated is then a function of the signal strength from Millington, ITU noise figures and processing credit. A threshold of -12 dB is set as the lowest usable receiver SNR and only signals above this level are used for a position solution.

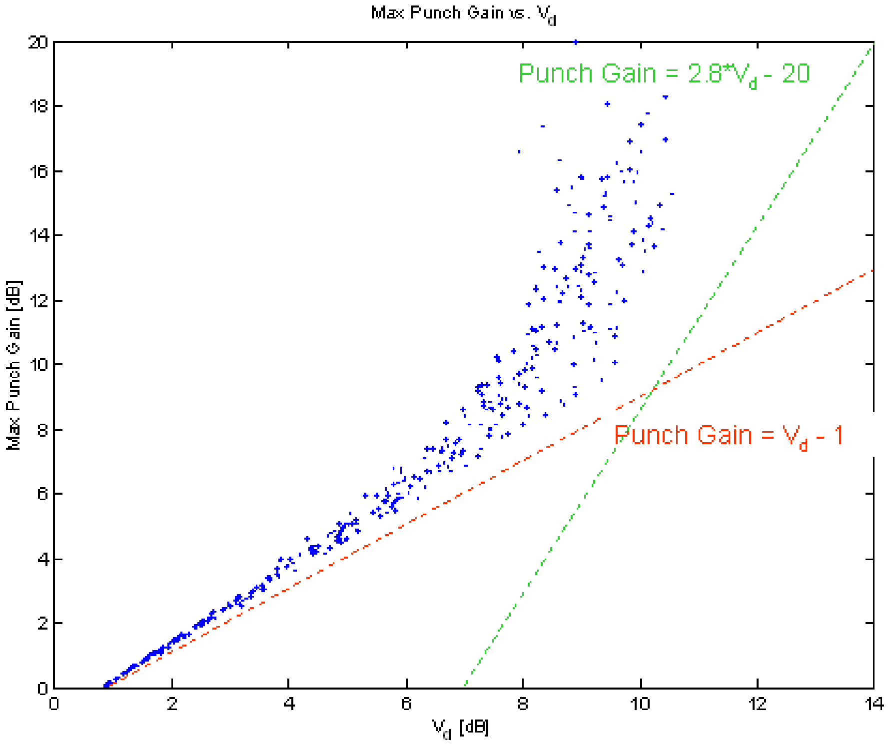

Figure 8. Processing Gain vs. V_d (dB)

4.2 Cycle and HPL

Once the stations available have been identified, these stations are then used to generate a Loran navigation solution. The coverage tool determines the solution and whether it meets the requirements for availability. For the navigation solution to be available, the horizontal protection level (HPL), a high fidelity/integrity/confidence bound on horizontal position error (HPE), must be below the requirement horizontal alert limit (HAL). The HAL is the maximum allowable level for the HPL for a given application. As the position solution depends on a range measurement, providing integrity associated with that position requires two steps. First, there must be adequate confidence on the selected cycle. Second, there must be adequate confidence on the bound on phase.

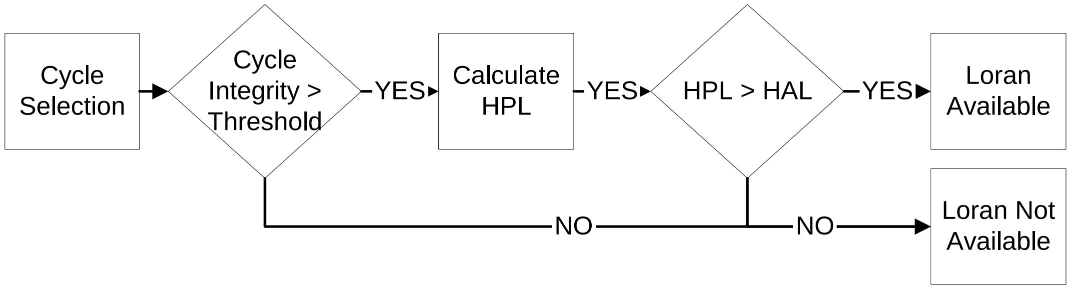

Figure 9. Determining System Availability

Cycle selection is the receiver's confidence in tracking the signal. Factors that affect cycle selection include ECD bias and noise in the envelope. ECD bias can be due to the transmitter, the receiver, or temporal and spatial variations in propagation. Bounds and statistics for all hazards causing ECD bias and envelope noise must factor into the determination of overall confidence that the correct cycle is selected. For signals with high SNR (generally 4 dB or higher), the signal is of adequate fidelity that the receiver can be confident of tracking the correct cycle to the required level. Since at least three signals are necessary for a position solution, there needs to be at least three such stations. However, there are often instances where that does not exist. Cycle resolution using an overdetermined solution and a residuals test is used to verify cycle selection. Hazard bounds and statistics must be used to determine overall confidence. Details on this technique are given in [9]. The overall process is seen in Figure 10.

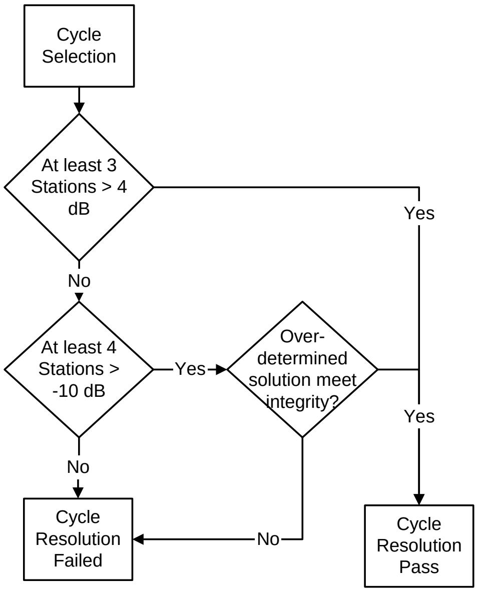

Figure 10. Cycle Selection Flow Chart

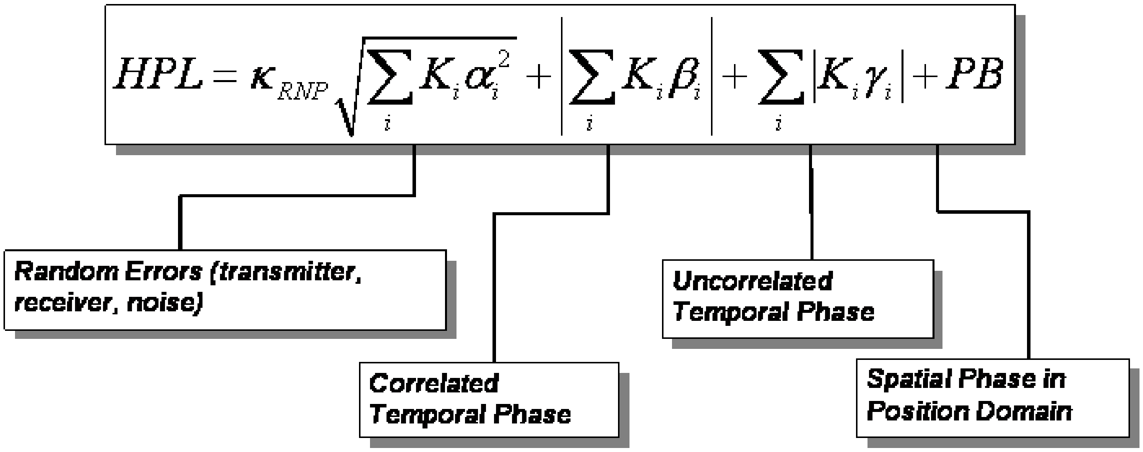
$$HPL = \kappa_{RNP}\sqrt{\sum_i K_i\alpha_i^2} + \left|\sum_i K_i\beta_i\right| + \sum_i \left|K_i\gamma_i\right| + PB$$

Figure 11. The HPL or Integrity Equation

Only after successful cycle resolution is the HPL calculation valid. The HPL equation is seen in Figure 11 and depends on factors such as SNR, phase variations due to propagation (temporal and spatial) and phase variation due to the transmitter and receiver. Analysis of these hazards generates bounds thought adequate to meeting integrity. These bounds are then used to determine the HPL. The temporal variation in phase provides an illustration of how a hazards figures into the integrity equation. It is divided in to two bound components: a correlated and uncorrelated bias. The variation is different from region to region and hence these bounds on these variations are scaled accordingly. Some details on the integrity equation and hazards are presented in [10]. The coverage tool calculates both the HPL and cycle selection confidence. Availability exists when both the HPL and cycle selection calculations meet the required level.

4.3 Station Availability, Noise Levels and Continuity

The above process demonstrates a single iteration of the availability determination. The determination needs to be made at many different noise conditions to calculate overall availability as well as continuity. In addition, there are different station availability conditions that need to be examined. First, various noise levels need to be assessed. For any given scenario, the tool assesses noise levels by first performing the calculation at the highest noise level (99.9th percentile). If the system is available at a given location, then it has 99.9% availability and no more calculations are done for that location for the given condition. If the system is not available, it will try to calculate availability at the next lower noise level until it finally is available. The process is complete once the availability at all locations has been determined. This represents one full availability calculation at a given station availability condition.

Station availability also figures into the calculation of overall availability. The tool iterates the full availability calculation over all possible one station unavailable scenarios. Station availability is high enough that two stations out is a probability that is below our calculation threshold. Thus the calculated overall availability is given by the weighted sum of the availability for all cases of one or fewer unavailable stations normalized by the total probability that one or fewer unavailable stations. Using the results from Section 5, the calculation covers 99.98% of all cases for the 25 stations used for the analysis of CONUS.

$$A = P\left(all\ tx's\ avail\right) * P\left(avail \mid all\ tx's\ avail\right) + \sum_{m=1}^{N} P\left(tx\ m\ unavail\ \ \&\ \ other\ tx's\ avail\right) * P\left(avail \mid tx\ m\ unavail\right)$$

$$A = \left(p_{tx,avail}\right)^{N} * P\left(avail \mid all\ tx's\ avail\right) + \sum_{m=1}^{N} \left(p_{tx,avail}\right)^{N-1} \left(1 - p_{tx,avail}\right) * P\left(avail \mid tx\ m\ unavail\right)$$

$$Normalization\ Factor = \left(p_{tx,avail}\right)^{N} + \sum_{m=1}^{N} \left(p_{tx,avail}\right)^{N-1} \left(1 - p_{tx,avail}\right)$$

$$Availability \approx \frac{A}{Normalization\ Factor}$$

Continuity calculation assumes that cycle selection is initially good. The probability of cycle slip is below our calculation precision (< 10^{-10}) and thus cycle should continue to be good throughout an RNP (and HEA) approach. Hence, only the HPL is of concern. Continuity is then calculated in a similar manner to availability. It is the weighted sum of the probability that HPL is good over all cases of one or fewer stations. The probability of each case is different for continuity. In availability, the long term transmitter availability was used whereas, in continuity, the probability the station is continuously available throughout an approach. These two probabilities are slightly different and will be calculated in Section 5. Again, the probability of two stations being simultaneously available will be shown to be negligible for RNP and reasonably small for HEA such that it does not need to be considered.

5 TRANSMITTER AVAILABILITY & CONTINUITY

5.1 Transmitter Availability

Volpe (Early 1990s)			***LOIS (2000-2002)***		
Type	**Number/Sta/Year**	**MTTR (min)**	**Type**	**Number/Sta/Year**	**MTTR (min)**
Mom	246.5	1.25	**Mom (blk)**	7.19	< 1
Short Term	14.0	2.29	**Mom (oa)**	139.59	< 1
Medium Term	4.2	10.86	**UUT (blk)**	14.86	7.4
Long Term	5.5	102.84	**UUT (oa)**	9.75	43.68
Total	270.2	3.5212	**Total**	171.38	3.98

Table 3. Comparison of Station Outage Statistics

Transmitter availability analysis uses historical data from the Loran-C Operational Information System (LOIS). LOIS classifies outages into multiple categories. LOIS classifies the outages into four categories: momentary blink (blk), momentary off air (oa), unusable time (UUT) blink and unusable time off air. A comparison of LOIS data statistics from 2000-2002 with data gathered by Volpe [11, 12] in the early 1990s is given in Table 3. The results show the number and duration of outages per station per year and are calculated using data from multiple stations. The results appear reasonably consistent though the number of momentaries has been reduced by about 40%.

Since the Modernized Loran system includes upgraded equipment and procedures, the effects of these changes are estimated and incorporated. A notable change is the reduction of momentary outages due to equipment switches from a minute to three seconds. Since new receivers will coast through a three-second outage, equipment switch outages will not result in a loss of continuity. The LOIS data is used to determine the effect of the upgrades in the Loran transmitter equipment and airborne/maritime user equipment. Three scenarios are examined:

1) Upgrades due to new transmitter equipment (power management, timing, etc.) and receiver ability to coast through three second outages. This represents the nominal availability scenario for RNP and HEA under modernized Loran.
2) Same as 1 but also elimination of authorized unavailability time maintenance (AUTM). This represents the best availability scenario for RNP.
3) Same as 2 but eliminate outages of less than one minute since maritime receivers should be able to coast through those outages. This represents the best availability scenario for HEA.

Scenario 1: New Transmitter Equipment & Receiver coast 3 sec			**Scenario 2: Scenario 1 without AUTM**		**Scenario 3: Scenario 1 w/o AUTM & Receiver coast 60 sec**	
Type	**Number/Sta/Year**	**MTTR (min)**	**Number/Sta/Year (eliminate AUTM)**	**MTTR (min)**	**Number/Sta/Year (eliminate AUTM)**	**MTTR (min)**
Mom (blk)	2.96	< 1	2.96	< 1	0	< 1
Mom (oa)	72.83	< 1	72.83	< 1	0	< 1
UUT (blk)	8.15	9.6	6.72	6.9	5.26	8.5
UUT (oa)	5.77	62.6	4.47	19.2	3.12	26.2
Total	89.716	5.745	86.986	2.391	8.378	15.096

Table 4. Estimated (for Upgrades) Station Outage Statistics (does not include the removal of upgrade times)

Table 4 presents the statistics calculated for each scenario using the LOIS data in a manner similar to Table 3. There is some reduction in UUT due to the addition of uninterruptible power supplies (UPS) and other power management equipment. There is significant reduction in momentaries. Since the LOIS records outages rounded to the next minute, the mean times to repair (MTTRs) are not known precisely and are slightly overestimated.

Table 5 shows a comparison of the estimates for availability with the current and new (scenario 1, 2, 3) system. The overall availability of each transmitter is calculated be 99.915 to 99.96% depending on whether AUTMs are counted.

While the analysis of LOIS data provides coarse numbers on availability, the calculation of continuity requires determining the probability that a station, currently available, will go off air some time during the approach (150 seconds for RNP 0.3, 3 hours for HEA). The LOIS statistics on the number, duration and distribution of outages can be used to calculated continuity. An estimate of this probability can be derived using a simple Markov chain model and the LOIS availability statistics shown in Table 4. The next part of the section will discuss the basic Markov model for station continuity.

Station	GRI	Current	Scenario 1	Scenario 2	Scenario 3
Malone	8970 W	99.848%	99.890%	99.943%	99.968%
Malone	7980 M	99.848%	99.901%	99.947%	99.973%
Seneca	8970 X	99.890%	99.917%	99.976%	99.990%
Seneca	9960 M	99.884%	99.918%	99.979%	99.998%
Baudette	8970 Y	99.893%	99.923%	99.944%	99.954%
Baudette	8290 W	99.903%	99.933%	99.953%	99.965%
Boise City	8970 Z	99.906%	99.933%	99.968%	99.987%
Boise City	9610 M	99.924%	99.952%	99.981%	99.997%
Gillette	9610 V	99.924%	99.964%	99.975%	99.995%
Gillette	8290 X	99.928%	99.956%	99.967%	99.995%
Las Cruces	9610 X	99.896%	99.924%	99.952%	99.973%
Raymondville	9610 Y	99.856%	99.897%	99.928%	99.939%
Raymondville	7980 X	99.845%	99.892%	99.924%	99.935%
Grangeville	9610 Z	99.827%	99.910%	99.950%	99.967%
Grangeville	7980 W	99.827%	99.905%	99.944%	99.962%
Havre	8290 M	99.947%	99.966%	99.987%	99.994%
Jupiter	7980 Y	99.842%	99.888%	99.964%	99.971%
Carolina Beach	7980 Z	99.869%	99.907%	99.982%	99.991%
Carolina Beach	9960 Y	99.877%	99.912%	99.986%	99.995%
Caribou	9960 W	99.898%	99.918%	99.980%	99.988%
Nantucket	9960 X	99.823%	99.851%	99.961%	99.971%
Total		99.879%	99.917%	99.961%	99.977%

Table 5. Estimated Transmitter Availability

5.2 Markov Model for Continuity

Transmitter availability statistics shows the average availability in the long term. For the statistics be used to analyze the continuity hazard posed transmitter availability, the probability that a transmitter, initially available, becomes unavailable during the approach period (150 seconds) needs to be calculated. A simple Markov chain model (Figure 12) is used for the analysis. Off air and on air correspond to the states that a transmitter can be in at any given period. In this model, p_{ij} represents the probability of being in state i and transitioning to state j. Since there are only two possible states, there are two independent quantities that need to be determined. The transition matrix is seen in Equation (1).

$$A = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix} = \begin{bmatrix} p_{00} & (1 - p_{11}) \\ (1 - p_{00}) & p_{11} \end{bmatrix} \qquad (1)$$

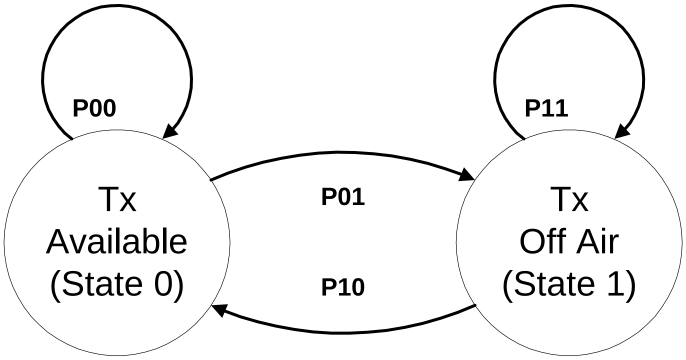

Figure 12. Basic Markov Model for Transmitter Availability and Continuity

The value of p_{00} can be determined by matching the mean time in state 0 to the mean time between failure (MTBF). Similarly, p_{11} can be determined by matching to the mean time to repair (MTTR). So for p_{00}, assume that we started in state 0. The only way to leave state 0 is to go to state 1, which has probability p_{01}. Hence the expected time in state 0, E(time in State 0| State 0), is given by Equation (2).

$$E\left(t_{s_0}\left|s_0\right.\right)=1p_{01}+2p_{00}p_{01}+3{p_{00}}^2p_{01}+\cdots=p_{01}\sum_{N=0}^{\infty}\left(N+1\right){p_{00}}^N=1+\frac{p_{00}}{\left(1-p_{00}\right)}=MTBF \qquad (2)$$

since $\sum_{k=0}^{\infty}r^k=\frac{1}{(1-r)},0\le r<1$

The infinite series is just the sum of a geometric series and Gabriel's staircase. The variance can also be calculated. Similarly, Equation (3) yields the relationship between p_{11} and MTTR.

$$E\left(t_{s_1}\left|s_1\right.\right)=p_{10}+2p_{11}p_{10}+3{p_{11}}^2p_{10}+\cdots=p_{10}\sum_{N=0}^{\infty}\left(N+1\right){p_{11}}^N=1+\frac{p_{11}}{\left(1-p_{11}\right)}=MTTR \qquad (3)$$

Knowledge of p_{00} and p_{11} permits the calculation of the probability that a station changes from available to unavailable during some portion of the approach. It is necessary to look at an entire approach since if only the beginning and end states are considered, it misses cases where a station may transition from available to unavailable and then back to available. One way to perform the calculation is to create a Markov chain that "traps" the station into a given state. To calculate the probability that a station goes unavailable during the approach, a Markov chain, seen in Figure 13, which forces the station to remain unavailable if it should enter that state is used.

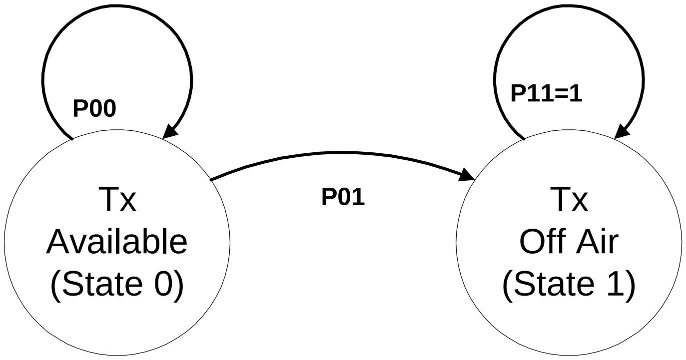

Figure 13. Trapping Markov Chain for Determining Probability of Station Goes Unavailable during Approach

	MTBF (sec)	MTTR (sec)	P_{00}	P_{11}	Station Long Term Availability	P(no loss of avail in 150 s)	P (on air after 150 s)
Scenario 1	351509	344.7			99.917		
1 sec update	351509	344.7	0.99999716	0.9970989	99.902	0.999573	0.999654
30 sec update	351509	344.7	0.99991465	0.9129678	99.890	0.999573	0.999641
Scenario 2	362541	143.5			99.961		
1 sec update	362541	143.5	0.99999724	0.9930294	99.9604	0.999586	0.99974
30 sec update	362541	143.5	0.99991725	0.7908825	99.9604	0.999586	0.99973

Table 6. Results of Markov Model for Scenario 1 & 2

Table 6 shows the results using from match MTBF and MTTR for scenario 1 and 2. The results indicate that 99.95% is a reasonable value for the probability that a station will stop transmitting during the approach given that the station is initially transmitting.

A simple binomial distribution provides a conservative bound since it assumes independence between each time interval. Our statistics are given per minute so the calculation of the probability of no loss over a 150 second (2.5 minute) period should examine the probability of no outages over that period. Hence the probability is $(99.915)^{2.5}$ = 99.788%. However, a binomial distribution for scenario 1 results in a MTTR of 1.001 minutes and a MTBF of 19.61 hours, which does not agree adequately with the data.

For N stations, the probability of m stations out during an approach is given by

$$P(m\ out \mid N) = \binom{N}{m} \left(1 - p_{tx,150\sec}\right)^{m} \left(p_{tx,150\sec}\right)^{N-m}$$

For scenario 1 and a high case of ten possible stations available, the probability of having no stations and one station or less out during an RNP approach is greater than 99.5% and 99.999% respectively. The probability of having more than one station out has a magnitude on the order of 0.001% (10^{-5}), which is beyond the level of precision of our continuity calculations. Hence, cases of two or more stations out are not considered for RNP.

5.3 Calculation for HEA

The analysis is extended for continuity calculations for HEA. Most locations can meet HEA requirements with one station off air but generally not with two. Calculation of continuity is thus related to the probability that the number of off air stations at anytime during an approach is less than two. For HEA, this is significantly different than the probability that two stations go off air during the approach. Since the MTTR is much shorter than the duration of the HEA, it is possible that two stations are off air during an approach but never concurrently. This was not a concern for aviation since the approach time is so short that if two stations go off air during the approach, it is assumed that there is a period over which they are concurrently off air.

Another Markov model is used to determine the probability that two or more stations are off air concurrently during an approach given that a total of N stations are received. At any given period, the system can be in three states: All N transmitters are on air, one station is off air and two (or more stations) are off air. The Markov chain used, shown in Figure 14, will “trap” instances where two stations go off air thereby determining the desired probability. The transition probabilities in the model are related to the probabilities p_{00} and p_{11} developed previously. Table 7 shows the probability of at most one station concurrently off air for the Volpe data, current LOIS data and the three scenarios based on the LOIS data. This probability is the converse of the probability of two or more station off air. From the results of Table 7, the probability of two stations or more being out is very small.

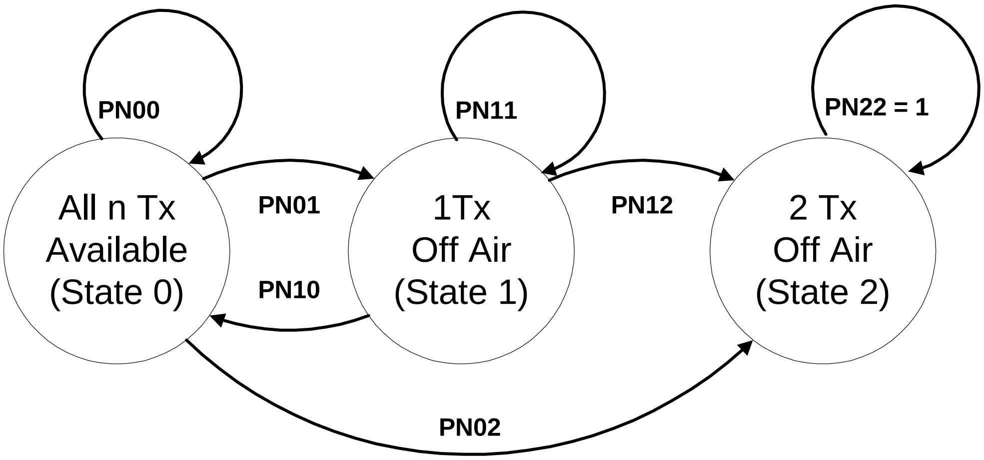

Figure 14. Trapping Markov Model for HEA

$$P_{N,00} = \left(p_{00}\right)^{N}$$

$$P_{N,01} = N\left(p_{00}\right)^{N-1}\left(p_{01}\right) = N\left(p_{00}\right)^{N-1}\left(1-p_{00}\right)$$

$$P_{N,02} = \binom{N}{2}\left(p_{00}\right)^{N-2}\left(p_{01}\right)^{2} = \binom{N}{2}\left(p_{00}\right)^{N-2}\left(1-p_{00}\right)^{2}$$

$$P_{N,10} = \left(p_{00}\right)^{N-1}\left(p_{01}\right) = \left(p_{00}\right)^{N-1}\left(1-p_{11}\right)$$

$$P_{N,11} = \left(p_{00}\right)^{N-1}\left(p_{11}\right) + \left(N-1\right)\left(p_{00}\right)^{N-2}\left(p_{01}\right)\left(p_{10}\right) = \left(p_{00}\right)^{N-1}\left(p_{11}\right) + \left(N-1\right)\left(p_{00}\right)^{N-2}\left(1-p_{00}\right)\left(1-p_{11}\right)$$

$$P_{N,12} = (N-1)(p_{00})^{N-2}(1-p_{00})(p_{11}) + \binom{N-1}{2}(p_{00})^{N-3}(1-p_{00})^{2}(1-p_{11})$$

Recall $p_{01} = (1-p_{00})$ and $p_{10} = (1-p_{11})$.

Number of Stations	Volpe	LOIS	LOIS (Modern) Scenario 1	LOIS (No AUTM) Scenario 2	LOIS (NO AUTM, coast 1 min) Scenario 3
4	99.8060	99.9116	99.9653	99.9861	99.9992
5	99.6779	99.8531	99.9423	99.9769	99.9987
6	99.5190	99.7803	99.9136	99.9654	99.9981
7	99.3295	99.6933	99.8792	99.9516	99.9974
8	99.1100	99.5923	99.8393	99.9355	99.9965
9	98.8611	99.4774	99.7938	99.9171	99.9955
10	98.5831	99.3488	99.7428	99.8965	99.9943
11	98.2767	99.2066	99.6863	99.8736	99.9931
12	97.9425	99.0509	99.6244	99.8485	99.9817

Table 7. Analysis of probability of at most one concurrent outage for HEA

The determination of the transmitter availability and continuity through an approach is a critical element in the coverage assessment. As discussed in Section 4, the loss of stations due to transmitter outage is used for assessing overall availability and in the determination of continuity. For a given noise level, other factors such as early skywave also affect station availability. However, these effects are of the order of 10^{-4} (0.01%) in CONUS and contribute little to the overall coverage.

6 COVERAGE TOOL RESULTS

Item	Model Parameter
W/O	With or without Canadian stations
HAL	Horizontal alarm limit
CCR	Credit for clipping
ENB	ECD bias
ETC	Seconds to average envelope
PTC	Seconds to average phase
SPE	Range error for spatial
SRE	Position error for spatial ASF decorrelation in HPL
KCT	Coefficient that scales correlated seasonal phase variation map
KUT	Coefficient that scales uncorrelated seasonal phase variation map
HMN	Threshold of probability for Gaussian noise contribution to HPL
HCY	Threshold of probability of undetected cycle error

Table 8. Parameter Key for Coverage Diagrams

Transmitter availability and continuity are two important parameters that are necessary for calculating overall Loran coverage. The coverage tool is a complicated and sophisticated piece of software that integrates many such parameters. As such, there are many parameters that can change the results. Parameters in the SNR and station determination portion of the software include noise level selected, processing credit and stations in the system. The cycle selection and HPL calculations greatly depend on the bound values and confidence statistics used for the biases and random errors. These values all can be varied and scaled depending on selection of parameters. The flexibility allows for the calculation of HEA availability and continuity by just adjusting the parameters to the HEA values. The same is true for accuracy calculation. As a means of record keeping, the major parameters used (Table 8) can be indicated on the plot.

6.1 Performance for RNP 0.3

Nominal condition assumes the use of year round average noise values at each percentile level. It also uses a relatively uses a relatively conservative value of 99.9% for station availability and continuity for RNP 0.3 (over 150 seconds).

6.1.1 Nominal Availability

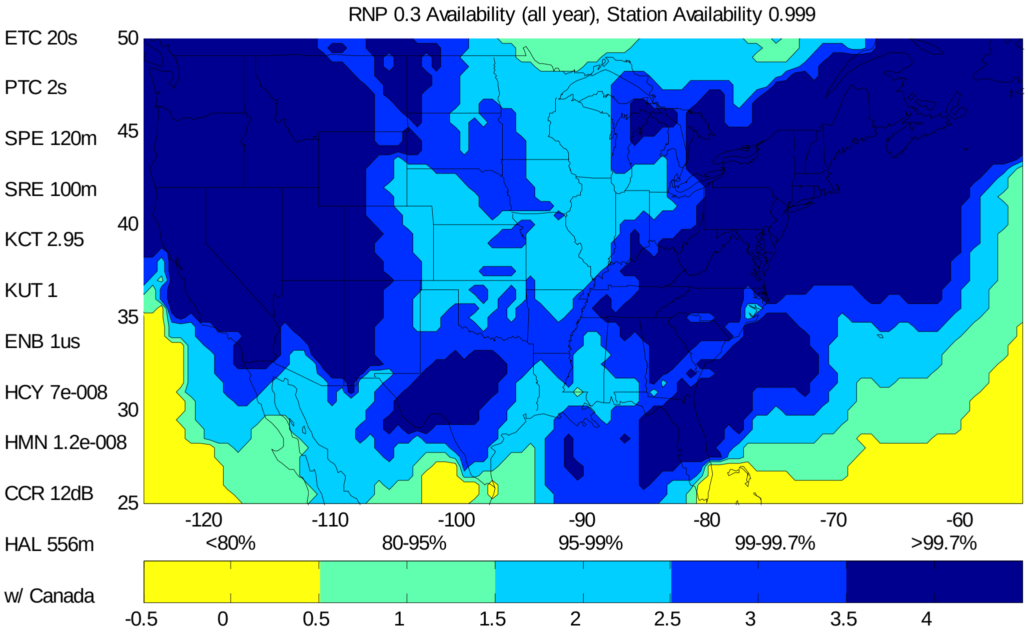

Figure 15. Expected RNP 0.3 Modernized Loran Coverage (Availability Contours in Percent) in CONUS with Existing Infrastructure

Nominal availability is shown in Figure 15. Most of CONUS attains at least 95% coverage with significant portions achieving greater than 99%. From examining the results, cycle resolution is the dominant factor in determining availability and the availability plot based on cycle resolution is very similar to the overall availability plot.

6.1.2 Nominal Continuity

Nominal continuity is shown in Figure 16. Generally, wherever there is availability at a given location, continuity is at a higher level. Hence, continuity is not the limiting factor in coverage.

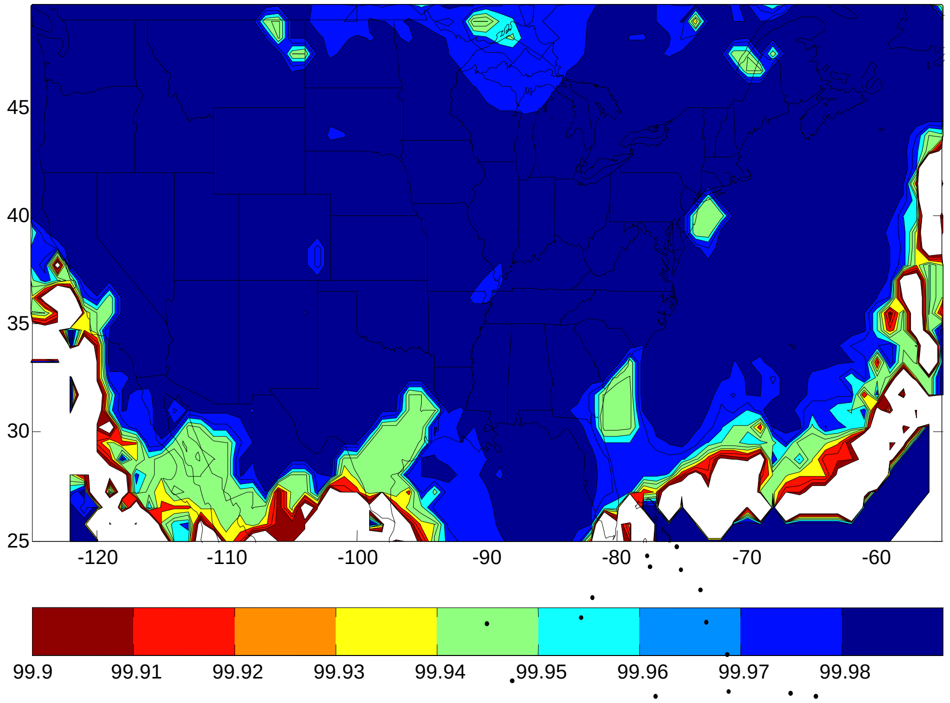

Figure 16. Expected RNP 0.3 Modernized Loran Coverage (Continuity Contours in Percent at a 0.999 Station Availability) in the CONUS with Existing Infrastructure

6.1.3 Worst Case Availability

The availability in the worst noise time block for each location is shown in Figure 17. Rather than use the noise values similar to those in Figure 6, values similar to those found in Figure 7 are used. So for each percentile level, the noise at the worst noise time block is used. The increase in noise reduces the number of stations available for a solution at a given location. This has a weak influence on the HPL but a strong influence on cycle resolution, particularly when an overdetermined solution is used. As seen from the figure, availability, in some places, is greatly reduced with areas having levels of less than 80%. This again illustrates the point that cycle resolution is the generally the determining factor in availability.

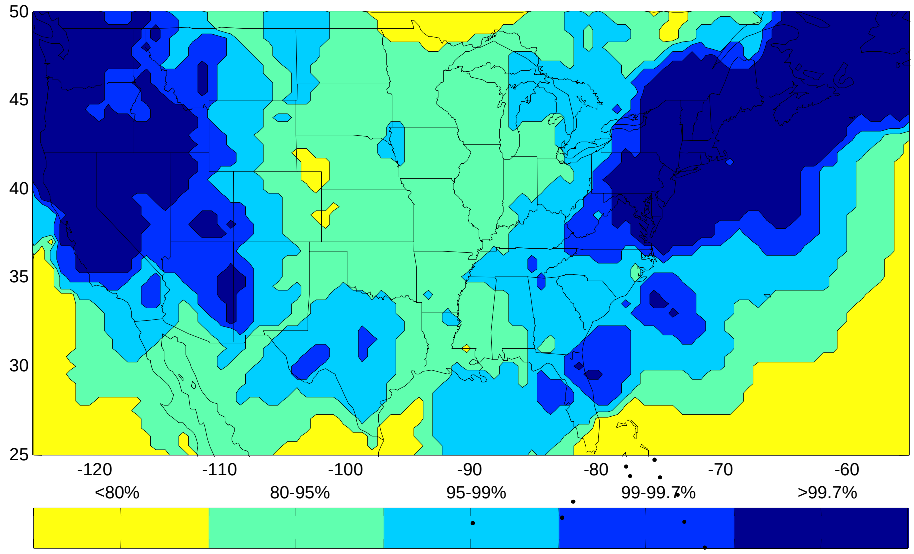

Figure 17. Expected RNP 0.3 Modernized Loran Coverage (Availability Contours in Percent) in CONUS with the Existing Infrastructure for the Worst Noise Time Block at Each Location

6.2 Performance for HEA

Availability and continuity under nominal conditions for HEA based on the analysis and coverage tool describe previously is shown in Figure 18 and Figure 19, respectively. Figure 20 shows a theoretical accuracy (2 drms) plot for HEA. The accuracy plot shows potential extension of coverage if HEA accuracy requirements were changed. Such changes may occur if target level of service (TLS) is accepted. While this paper is not primarily focused on HEA, these results demonstrate both the synergy between the RNP and HEA work as well as the power of the coverage tool. One difference in is that station continuity of 99% is used since this is now continuity over three hours. Note that this value is achieved under scenario 3, the best case scenario. However, this does not mean that the result applies only to scenario 3. In fact, the model assumes a binomial distribution for station availability. The assumption results in a probability that at most one station is not available of 99.38% when there are 12 stations. As seen from Table 7, this is achieved under all three scenarios.

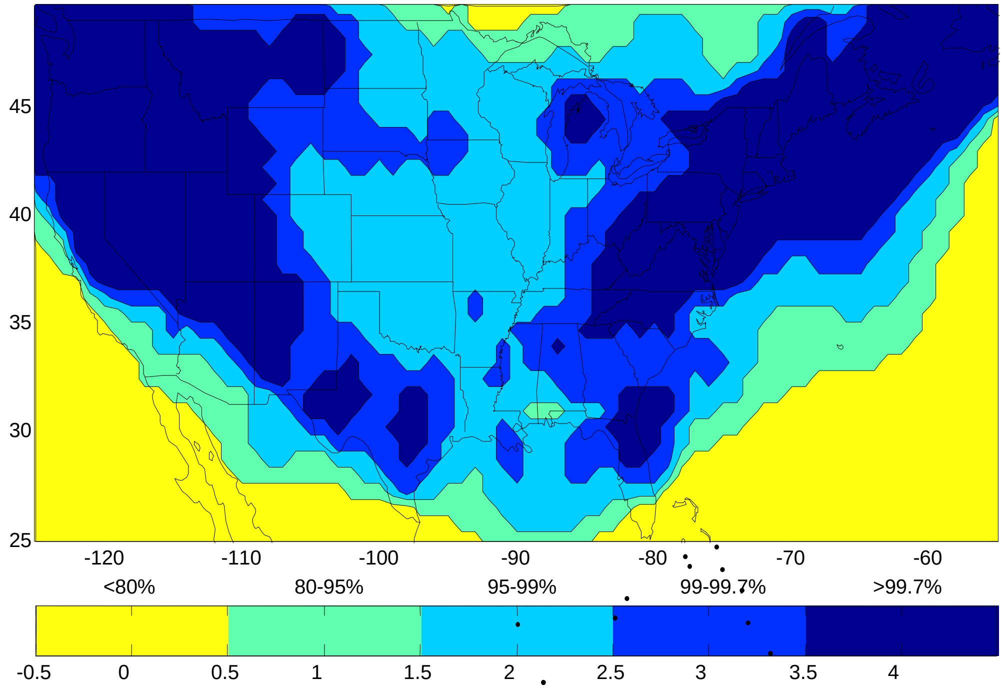

Figure 18. Expected HEA 20-Meter Accuracy Modernized Loran Coverage (Availability Contours in Percent at a Station Availability of 0.999) in CONUS with the Existing Infrastructure

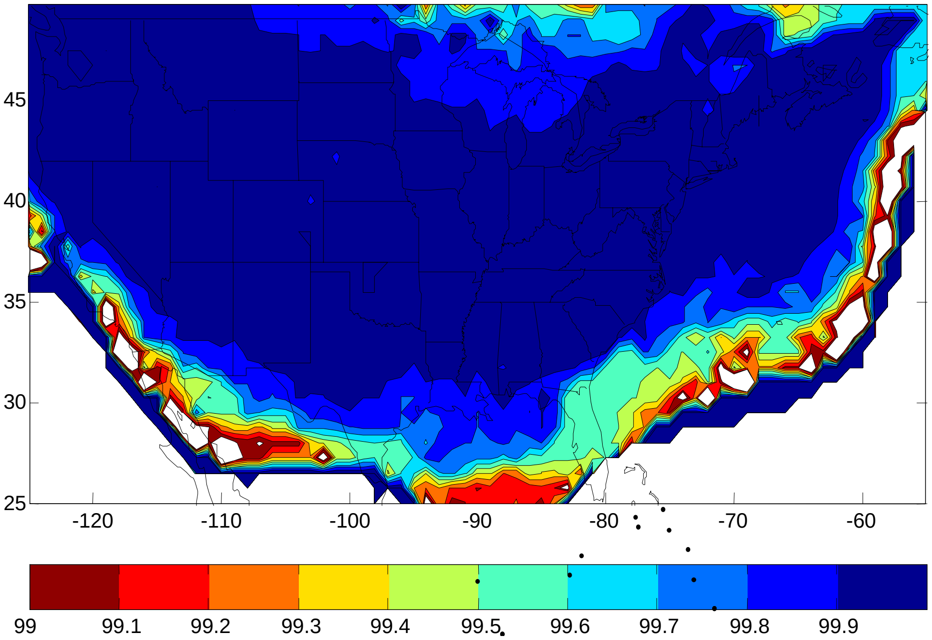

Figure 19. Expected HEA 20-Meter Modernized Loran Coverage (Continuity Contours in Percent for Station Continuity of 0.99) in CONUS with the Existing Infrastructure

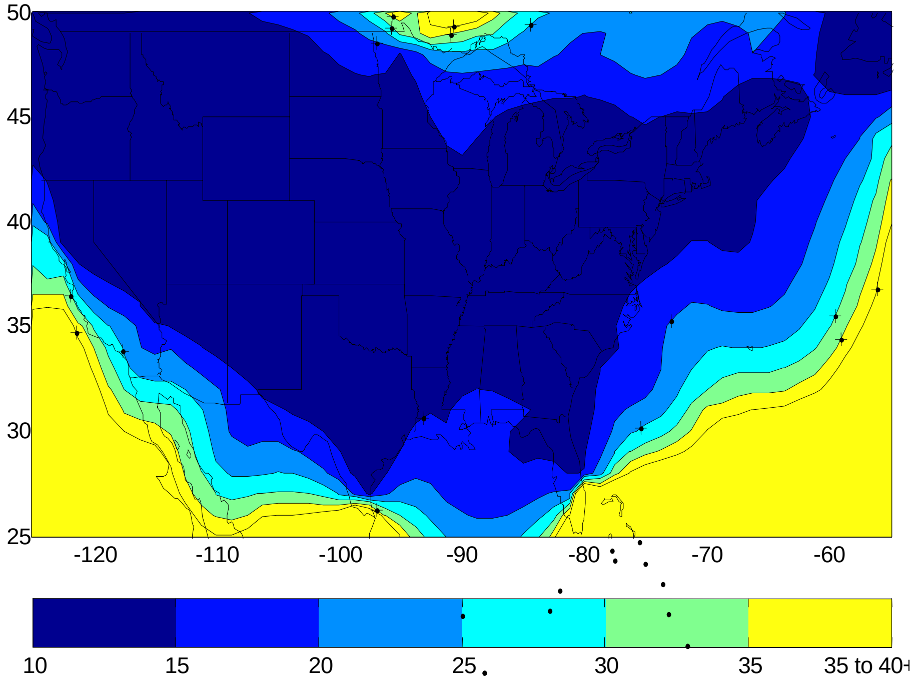

Figure 20. Expected HEA Modernized Loran Coverage (Accuracy in Contours in Meters at the 95 Percent Noise Level) in the CONUS with the Existing Infrastructure

7 CONCLUSIONS

The coverage tool integrates the results of the hazard and integrity analysis to determining availability and continuity. It utilizes transmitter availability and continuity studies and the integrity algorithms developed. These results, coupled with the integrity analysis, forms the core results of the Loran evaluation for RNP and HEA. The availability and continuity results show that a modernized Loran system can generally meet the requirements for RNP 0.3 and HEA. However, there are distinct circumstances where it does not meet these requirements – particularly in the worst noise conditions. Additional study and analysis should prove fruitful in both understanding the high noise level conditions and hopefully improving availability under those conditions.

8 REFERENCES

[1] “Vulnerability Assessment of the Transportation Infrastructure Relying on the Global Positioning System,” John A. Volpe National Transportation System Center, August 20, 2001.

[2] FAA report to FAA Vice President for Technical Operations Navigation Services Directorate, “Loran’s Capability to Mitigate the Impact of a GPS Outage on GPS Position, Navigation, and Time Applications,” March 2004.

[3] Forssell, Borje, Radionavigation Systems, Prentice Hall, Englewood Cliffs, NJ 1991.

[4] Lo, Sherman, et al., "Loran RNP 0.3 Approach: The Preliminary Conclusions of the Loran Integrity Performance Panel (LORIPP)", Proceedings of Institute of Navigation-GPS, Portland, OR, 2003

[5] “Loran-C Users Handbook,” 1992, U.S. Coast Guard COMPDTPUB P16562.6.

[6] International Radio Consultative Committee, Characteristics and Applications of Atmospheric Radio Noise, CCIR Recommendation 322-3, Geneva 1988.

[7] Boyce, Lee, et al., "A Time Domain Atmospheric Noise Level Analysis", Proceedings of the International Loran Association 32nd Annual Meeting, Boulder, CO, November 2003

[8] Enge, P. K. and Sawate, D. V., “Spread-Spectrum Multiple-Access Performance of Orthogonal Codes: Impulsive Noise,” IEEE Transactions on Communications, Vol. 36, No. 1, January 1988.

[9] Peterson, Benjamin et. al., “Hazardously Misleading Information Analysis for Loran LNAV”, Proceedings of the International Symposium on Integration of LORAN-C/Eurofix and EGNOS/Galileo, DGON, June 2002.

[10] Lo, Sherman, et al., " Loran for Required Navigation Performance 0.3: The Current Work of Loran Integrity Performance Panel (LORIPP)", Proceedings of GNSS 2003 – The European Navigation Conference, Graz, Austria, April 2003

[11] van Dyke, Karen, "Formulation of an Availability Requirement for Navigational Aids in the National Airspace System." Proceedings of the Wild Goose Association, October 1993.

[12] van Dyke, Karen, “Signal Availability Requirement for Radio Navigation Aids in the National Airspace System - Phase II”, Proceedings of the 23rd Annual Technical Symposium of the Wild Goose Association, Newport, RI, October 31 - November 4, 1994., pp. 55-67.

DISCLAIMER

-Note- The views expressed herein are those of the authors and are not to be construed as official or reflecting the views of the United States Federal Aviation Administration or Department of Transportation.