

# Assessing the Loran Cycle Confidence Algorithm

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*In 2002, Peterson, et. al. proposed using redundant signal measurements to provide the necessary confidence that the cycles are identified correctly. The proposal used a weighted sum squared error (WSSE) statistic as a metric to determine if the correct cycle on every signal used has been selected with adequate probability. The method increases availability by using redundant information to provide a better estimate of cycle confidence.*

*Estimating the probability of undetected incorrect cycle selection is necessary to ensure that the system provides the integrity necessary for its intended aviation applications. However, it was not shown that the method itself had integrity in that it would calculate a conservative estimate of the probability given unknown biases. This is necessary for the implementation of the algorithm in an avionics receiver.*

*This paper examines, assesses, and refines the use of the WSSE statistic for Loran cycle selection. It examines two weightings for generating WSSE for use in the aviation cycle selection algorithm. It outlines how to implement the algorithm so that integrity may be proven. Availability coverage results from each implementation will be shown and a discussion of implementation will be made.*

## 1.0 Introduction

Long Rang Navigation, or Loran, is one of the few position, navigation and timing (PNT) systems capable of providing back up to the Global Positioning System (GPS) in multiple modes of operation. The capability is critical as redundancy for GPS is vital to the national infrastructure. The Volpe GPS vulnerability study indicated that the current GPS is susceptible to deliberate or inadvertent interference [1]. As such, it recommended examining various alternatives to providing redundancy to GPS, particularly in safety critical applications. The 2004 FAA Loran Technical Evaluation examined the capability of Loran for providing GPS redundancy [2]. It offered several recommendations and changes, that if followed, would yield a Loran system that is capable of meeting requirements for non precision approach (NPA) for aviation, harbor entrance approach (HEA) for maritime, and highly synchronized timing and Stratum 1 frequency for timing and frequency users. The preferred NPA is Required Navigation Performance 0.3 (RNP 0.3).

### 1.1 Loran as Aviation Back Up to GPS

Aviation is an area where redundancy to GPS is highly desirable. GPS is becoming a major part of the aviation navigation infrastructure for all phases of flight operations from take off, enroute, terminal area and approach and landing. Additionally, GPS may serve as part of the surveillance infrastructure by providing the navigation input to Automatic Dependent Surveillance – Broadcast (ADS-B). ADS-B is a system by which navigation and other information on ones aircraft through is shared with other aircraft and ground surveillance via a one hertz transponder broadcast. Loran, with area navigation (RNAV) capability, conterminous United States (CONUS) coverage, and ¼ nautical mile or better accuracy has many of the qualities necessary to serve as GPS back up for aviation. Particularly, the focus is for Loran to provide approach guidance by supporting RNP 0.3 and ADS-B as well as en route capabilities by supporting RNP 1.0. However, it must be shown that Loran can meet the strict requirements of an aviation navigation system.

## 1.2 Integrity and Availability for Aviation

To serve as an aviation navigation system, Loran must demonstrate integrity while achieving acceptable availability. Integrity is the fidelity of the system – the reliability that the information provided is true and the ability to provide a timely warning should there be incorrect information. In aviation jargon, this means minimizing the occurrence of hazardously misleading information (HMI) where the horizontal protection level (HPL), the bound on horizontal position error (HPE) generated by the receiver, is exceeded by the true HPE. For RNP 0.3 and RNP 1.0, the requirement is that the probability of HMI is less than  $10^{-7}$  per hour. In Loran, HMI can result from unwittingly tracking the wrong cycle, inadequate error bounds, and unwarned anomalous events. Meeting the integrity requirement is the most challenging task in demonstrating the ability of Loran to serve either RNP procedure.

Availability represents how often the system can be used for the desired application. To use a system for a specified RNP procedure, HPL must not exceed the horizontal alert limit (HAL) of the procedure. It is desired that availability be at least 99.9%. The requirement must be met at every point in the coverage area and cannot represent a spatial average.

<b>Requirement</b>	<b>Definition (Metric)</b>	<b>Minimum Requirement</b>
Integrity	HMI is when $HPL > HPE$	Probability HMI $\leq 10^{-7}$ /hour
Availability	$HPL \leq HAL$ No HPL means not available	99.9%
Continuity	Given $HPL \leq HAL$ initially HPL must exist & $HPL \leq HAL$ for 150 seconds	99.9% over 150 seconds (RNP 0.3) 99.9% over 1 hour (RNP 1.0)

**Table 1. Primary Requirements for Aviation**

### 1.3 Cycle Selection, Availability and Integrity

A key determinant of Loran RNP 0.3 and RNP 1.0 availability is the ability to conduct cycle selection with adequate confidence. Cycle selection is the process of choosing the same cycle on the Loran pulse to track for all signals. This ensures consistency between measurements. The tracked cycle is typically the sixth zero crossing commonly termed the standard zero crossing (SZC). An incorrect cycle selection can result in a range error of three kilometers or more and hence it is important that one has confidence in our cycle selection. This paper uses the term cycle integrity or confidence algorithm to denote the method for determining the certainty of cycle selection. Cycle confidence is necessary to ensure that the calculated HPL has integrity.

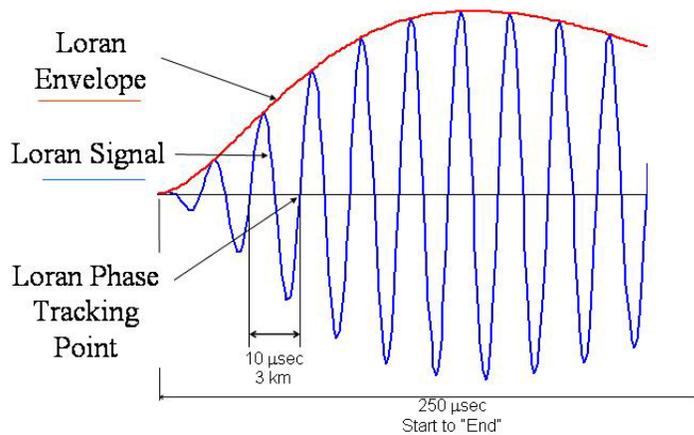


Figure 1. Loran Pulse, Envelope, and Tracking Point

Peterson et. al. developed a Loran cycle confidence algorithm based on the weighted sum squared error (WSSE) statistic in [3]. However, the demonstration of the integrity and implementation of algorithm remain open issues. Demonstrating that the algorithm has integrity is clearly a necessary requirement. It must be shown that the true probability of having an incorrect cycle is less than or equal to the estimate given by the algorithm. This establishes that the confidence given by the algorithm reflects the true confidence. However demonstrating integrity is not sufficient. Even if algorithm has integrity, it is useless if it cannot be implemented in the receiver. It may not be reasonable to implement some algorithms (e.g. due to complexity) even though they have integrity. It is important to understand how the implementation affects availability and receiver complexity.

### 1.4 Outline

The paper will discuss the integrity, availability and receiver complexity aspect of implementing the WSSE based cycle integrity algorithm. The first two sections provide background on the cycle selection and the WSSE cycle integrity algorithm proposed in [3]. A major determinant of algorithm performance and integrity is the weighting matrix used and two choices are discussed. The body of the paper examines the integrity and

performance of each choice. An outline of the means of demonstrating integrity for both choices will be provided. Finally the performance of the choices in regards to availability and receiver complexity will be discussed.

## 2.0 Cycle Selection & Integrity

This section presents a basic description of the cycle confidence algorithm. For additional information, see [3][4].

### 2.1 Cycle Selection and Integrity Overview

The cycle selection process in Loran typically involves examining the envelope of the signal and choosing the desired cycle based on the envelope slope or ratio. The determination is complicated by the presence of noise on the signal and group delay between the envelope and the carrier of the signal due to propagation. The group delay results in an effect typically referred to as the envelope to cycle difference (ECD). These measurement uncertainties can result in incorrect cycle identification. The integrity of the HPL requires high confidence that the correct cycles were selected. The cycle confidence algorithm quantifies the fidelity of the cycle selection. The algorithm estimates the probability of having a wrong cycle on any of signals used. In this paper, we denote this probability as  $P_{WC}$ . For the cycle selection to be useable for the HPL calculation, a  $P_{WC}$  of  $7 \times 10^{-8}$  or lower is required [5]. For the cycle confidence algorithm to have integrity, the estimate of this probability must be conservative. This means that the estimate must be higher than the true probability of a wrong cycle.

The cycle confidence algorithm, as assessed in the 2004 FAA Loran technical evaluation, is calculated confidence by two means, depending on station availability. When only three stations are available, there is no redundant information. In this case, cycle confidence is determined using the probability of being on the incorrect cycle ( $P_{IC}$ ) for each station.  $P_{WC}$  is one minus the probability of being on the correct cycle for all signals (see Table 2 for equation). The method is also used if there are three strong stations with which we have high confidence ( $P_{IC} > 10^{-8}$ ) available. This is known as having a “trusted triad”. The method is simple, yet there is no true attempt at detecting incorrect cycle selection using receiver measurements.

If more than three stations are available, redundant information can be used. The weighted sum squared error (WSSE) statistic provides the means of utilizing the multiplicity of signals. The WSSE is used as a statistic for deciding if an incorrect cycle selection exists. If the statistic is above a specified threshold, then the receiver assumes that there is a fault on at least one cycle. Otherwise it assumes that there are no faults - all cycles are correct. Estimates of the distribution of the statistic for the no fault and faulted cases are used to determine the confidence of the calculation. Table 2 shows the two different methodologies and when the conditions under which they are employed.

The next section discussed the demonstration of integrity for the first method. The integrity demonstration for the WSSE method is examined in Section 4.0.

Conditions	Methodology
3 stations or “trusted triad”	Product of individual cycle probabilities $P_{WC} = (1 - I)(1 - P_{IC})$
> 3 stations	Use redundant information to bound $P_{WC}$ Current method employs WSSE statistic

**Table 2. Methodology for Determining Cycle Identification Confidence**

## 2.2 Integrity of Three Station Methodology

The integrity of cycle confidence when using only three stations depends solely on the probability of being on an incorrect cycle ( $P_{IC}$ ) as seen from Table 2. The equation for determining  $P_{IC}$  is given in Equation (1). The calculation depends on the variance of the ECD due to noise. The general formula for the variance is seen in Equation (2) and can be derived either theoretically [6] or empirically [3]. The results from the cited papers are consistent with only the value of  $C$  varying (depending on receiver performance)<sup>1</sup>. Given the ECD bias, the probability is solely dependent on SNR. The equation should, from both theoretical and empirical analysis, well approximate the true  $P_{IC}$ . The methodology used for three stations should have integrity provided the value of  $C$  is well chosen.

$$P_{IC} = \text{normcdf}(-5\mu\text{sec}, ECD_{\text{bias}}, \sigma_{\text{noise}}^{ECD}) + \text{normcdf}(-5\mu\text{sec}, -ECD_{\text{bias}}, \sigma_{\text{noise}}^{ECD}) \quad (1)$$

$$\sigma_{\text{noise}}^{ECD} = \frac{C}{\sqrt{N * SNR}} \mu\text{s} \quad (2)$$

## 3.0 Using the WSSE to Determine Cycle Confidence

The WSSE derived cycle confidence algorithm is more complicated than the previous method. As such, to understand its performance and assess its integrity, the section covers how the WSSE is calculated and used to determine cycle confidence.

### 3.1 Calculating the WSSE

The WSSE statistic is the combination of the square of the residual errors of each Loran pseudorange measurement weighted by a weighting matrix,  $W$ . This is seen in Equation (3) where  $\hat{\epsilon}$  is the estimated residual error calculated from the difference of the

<sup>1</sup> Boyce derived a value of roughly  $C = 21 \mu\text{sec}$  for the theoretical best performance while Peterson estimates that  $C = 29 \mu\text{sec}$  is currently achievable.  $C = 42 \mu\text{sec}$  is the value derived by Peterson using the Austron 5000 receiver

pseudorange and the estimated position solution using those pseudoranges. The WSSE is related to the true residual error  $\varepsilon$  by the matrix  $P$  which depends on a weighting matrix,  $W$ , and the geometry matrix,  $G^2$ . This is seen in Equation (4). It should be noted that the weighting matrix for Equations (3) and (4) do not have to be the same. Typically, the weighting matrices used are the same. The equations for the full calculation of the WSSE are given in other papers [7]. The WSSE is used as its distribution is different if we have a faulted (undetected missed cycle) condition versus the no fault (all cycles correct) condition. More significantly, if certain conditions are met, the distributions of the faulted and no fault WSSE is mathematically tractable.

$$WSSE = \hat{\varepsilon}^T W \hat{\varepsilon} = \varepsilon^T W (I - P) \varepsilon \quad (3)$$

$$P = G(G^T W G)^{-1} G^T W \quad (4)$$

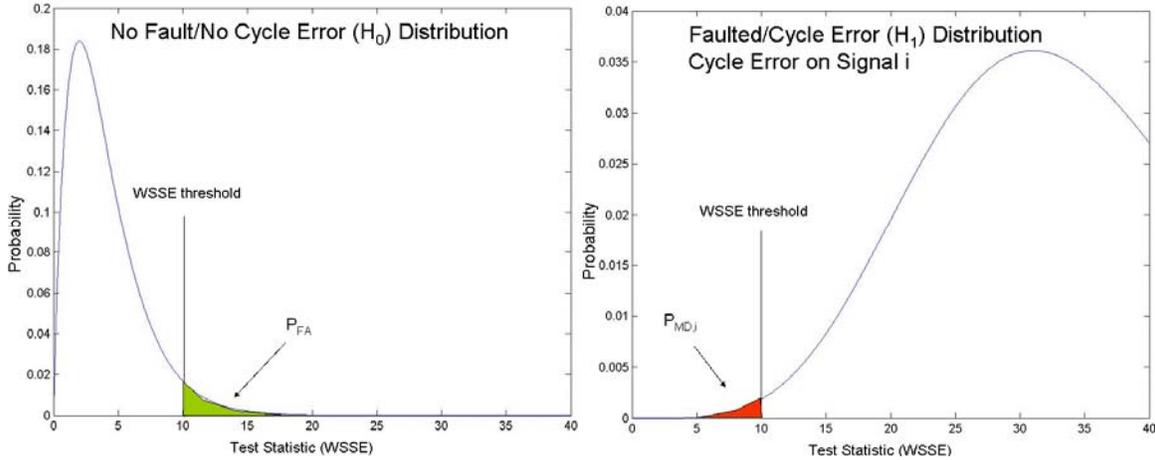
### 3.2 Hypothesis Testing

While the WSSE is used as a statistic for deciding if there exists an incorrect cycle selection, estimates of its distribution is used to determine the confidence of the decision. This is because the distribution of the WSSE depends on whether there is an incorrect cycle determination and on which signal(s) that incorrect cycle determination exists. The determination of cycle confidence is illustrated in Figure 2 where the probability density function (pdf) of the no fault or  $H_0$  (left distribution) and a given faulted or  $H_1$  distribution (right distribution) is shown. The process starts by setting a threshold to decide if an incorrect cycle selection exists. The threshold is set by the acceptable probability of false alarm ( $P_{FA}$ ). The probability depends solely on the no fault distribution and is the integral of the value of the no fault pdf above the threshold. This is the area in yellow in the Figure 2. If the WSSE is less than the threshold, the user will assume that all cycles are correct. Otherwise, it will be assumed that there is a cycle error. However, it is possible that a faulted condition can yield a WSSE value that is below the threshold. The area in red is the integral of the faulted pdf for values below the threshold. It is the conditional probability of missed detection ( $P_{MD}$ ) given that this particular faulted condition exists. The result applies to both single and multiple faults conditions. The sum of all such probability weighted by the probability of the faulted condition,  $P_{IC}$ , yields the overall probability of having an undetected wrong cycle,  $P_{WC}$ . Equation (5) shows the calculation with the first term representing the contribution from all single fault conditions, the second term representing the contribution from all two fault conditions and the third term representing the contribution from all three fault conditions. The probability of a faulted condition generally decreases as the number of faults increase and hence it is usually not necessary to calculate beyond two fault conditions to test for  $7 \times 10^{-8}$  integrity.

$$P_{WC} = \sum P_{MD_i} P_{IC_i} + \sum P_{MD_{ij}} P_{IC_i} P_{IC_j} + \sum P_{MD_{jki}} P_{IC_i} P_{IC_j} P_{IC_k} + \dots \quad (5)$$

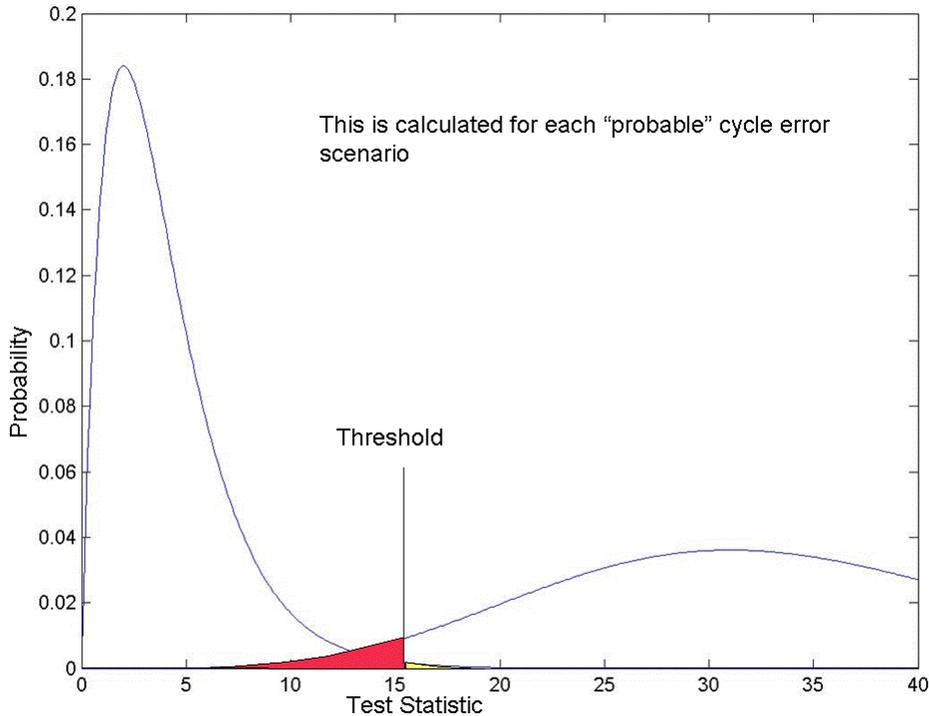
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<sup>2</sup>  $G$  is a  $N \times 3$  matrix, where  $N$  is the number of transmitters used in the position solution. Each row consists of the  $[\cos(A_z) \sin(A_z) 1]$  where  $A_z$  is the azimuth from the transmitter to the user



**Figure 2. No Fault (Left) and Faulted (Right) Distribution and the calculation of probability of false alarm ( $P_{FA}$ ) and missed detection ( $P_{MD}$ )**

While Equation (5) illustrates how to determine our cycle confidence, as expressed by  $P_{WC}$ , we still need to know how to calculate the terms in the equation.  $P_{IC}$  can be estimated from parameters, such as signal to noise ratio (SNR) and ECD bias, available to the receiver (see Equation (1)). The calculation of  $P_{MD}$  requires knowledge of the distribution of the WSSE statistic for the no faulted and faulted cases.



**Figure 3. PDF of No Fault (Right) and Faulted (Left) Distribution for Determining  $P_{MD}$**

### 3.3 Central and Noncentral $\chi^2$ Distribution

Knowing the no fault ( $H_0$ ) and faulted ( $H_1$ ) distributions allows for the determination of the threshold for a specified  $P_{FA}$  and for the calculation of  $P_{MD}$ . The calculation is only tractable if the distribution can be described mathematically. A common description for the distribution is the  $\chi^2$  distribution and under certain conditions, the WSSE will follow this distribution.

The  $\chi^2$  distribution is formed from the sum of independent identically distributed (iid) normal random variables (rv) with variance of one. If the random variables are zero mean, the resulting distribution is central  $\chi^2$  otherwise the distribution is noncentral  $\chi^2$ . All  $\chi^2$  distributions are characterized by the degree of freedom which represents the number of iid distributions used to form the sum. Because of the nonzero, noncentral  $\chi^2$  are characterized by an additional parameter known as the noncentrality parameter (ncp). Figure 4 presents a flowchart on how central and noncentral  $\chi^2$  distributions are formed.

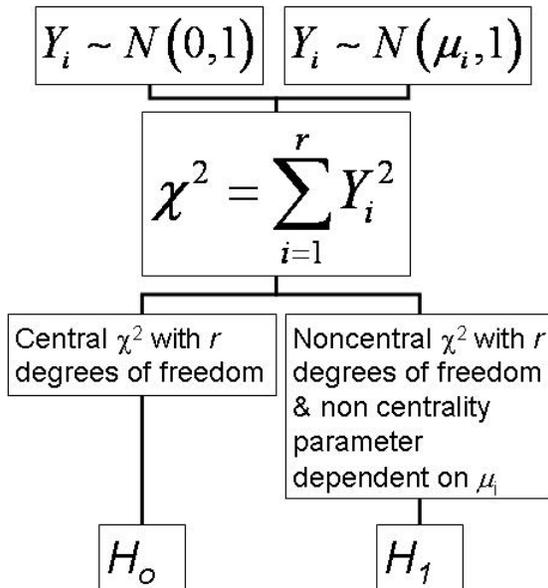


Figure 4. Central and Noncentral  $\chi^2$  Distribution

For WSSE statistic used in cycle confidence to be  $\chi^2$ , two conditions must hold. First, the true residual errors must be normally distributed. Second, the weighting matrix ( $W$ ) used must be the inverse of the error covariance matrix ( $R$ ) where  $R$  depends on the random components of the residual errors. The proof is given in another paper [7].

### 3.4 Weighting Matrix

As mentioned in the previous section, the weighting used for the WSSE determines the distribution of the statistic. In this paper, we examine two different weightings. One is

based on random errors only (denote by  $\sigma$  weighting) and one is based on random plus bias errors (denote by  $\sigma+b$  weighting). Random errors on Loran result from transmitter jitter and noise while bias errors result from propagation delays such as additional secondary factor (ASF) and temporal phase variations. The basic definition of the weighting matrix is given in Equation (6)

$$W = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N1} & s_{N2} & \cdots & s_{NN} \end{bmatrix}^{-1} \quad (6)$$

For the  $\sigma$  weighting, the terms of the matrix is given by

$$s_{ii} = \sigma_{random,i}^2 \quad \text{and} \quad s_{ij} = 0 \quad \text{where} \quad \sigma_{random,i} = \sqrt{\sigma_{noise,i}^2 + \sigma_{tx,i}^2} \quad \text{and}$$

$$b_i = b_{cor\_tempASF,i} + b_{uncor\_tempASF,i} + b_{spatialASF,i}$$

The residual error, given no cycle errors is expected to be distributed as

$$\varepsilon_i \sim N(b_i, \sigma_{random,i})$$

For the  $\sigma+b$  weighting, the terms of the matrix is given by

$$s_{ii} = \sigma_{random,i}^2 + b_{cor\_tempASF,i}^2 + b_{uncor\_tempASF,i}^2 + b_{spatialASF,i}^2 \quad \text{and} \quad s_{ij} = b_{cor\_tempASF,i} * b_{cor\_tempASF,j}$$

The distribution of the residual errors, given no cycle errors, is modeled as  $\varepsilon_i \sim N(0, \sqrt{s_{ii}})$ . It turns out that the model is not accurate as the  $\sigma+b$  weighting does not result in a distribution that is well approximated by  $\chi^2$ . Unfortunately, the true distribution is not mathematically tractable.

The  $\sigma$  weighting results in distributions that are more mathematically tractable and hence easier to demonstrate integrity. However it comes at the cost of receiver complexity. The  $\sigma+b$  weighting is simpler to implement but is more difficult to demonstrate integrity. It was the weighting initially used for the 2004 evaluation [2]. Details of the advantages and disadvantages of these weightings are given by [4].

## 4.0 Integrity

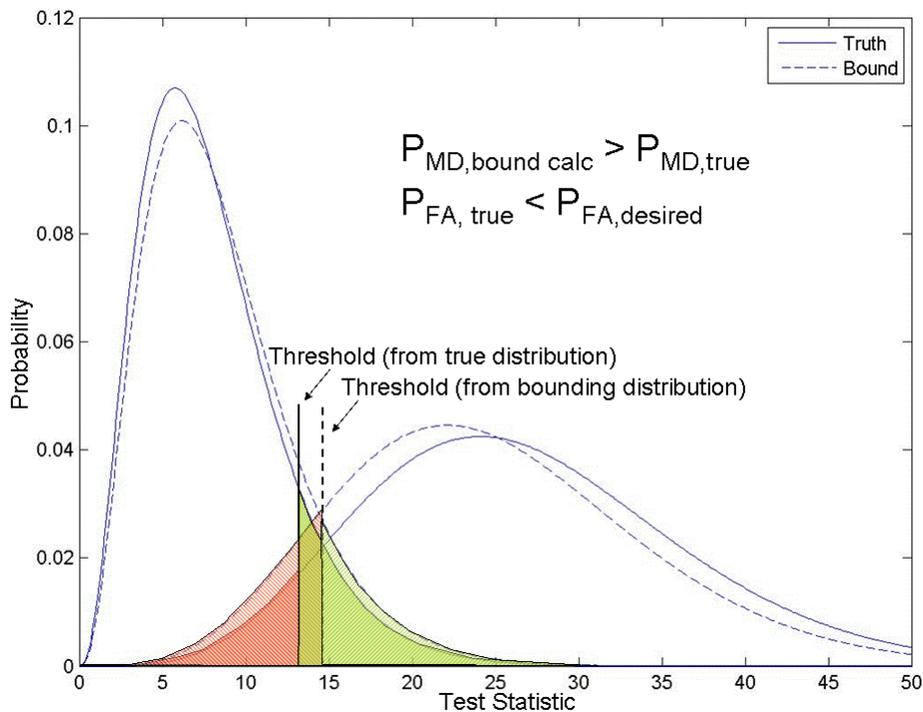
Integrity must be demonstrated for the cycle confidence algorithm if Loran is to be certified for aviation. For cycle confidence to have integrity means that the calculated or estimated  $P_{WC}$  is larger than the true  $P_{WC}$ . A sufficient condition for integrity is to demonstrate that the individual components of  $P_{WC}$ ,  $P_{MD}$ , are overbounded by the estimated  $P_{MD}$ .

### 4.1 Demonstrating Integrity

The methodology used for demonstrating that the cycle selection has integrity is to show that the calculated  $P_{MD}$  is larger than the true  $P_{MD}$  for each possible missed cycle situation. As the true distributions are not known, bounding distributions have to be used. To get this conservative estimate of  $P_{MD}$ , an overbound concept similar to that presented in [8] is useful. For integrity, it is only necessary to overbound the lower (left) tail of  $H_I$  distribution to get a conservative estimate for  $P_{MD}$ . If the lower tail of the faulted distribution is overbounded, the estimated  $P_{MD}$  will be greater than the true  $P_{MD}$ . This conservatism helps insure that the integrity requirement is met. The estimated  $H_I$  distribution only needs to overbound the true distribution up to a cumulative distribution function (cdf) of  $7 \times 10^{-8}$  divided by the probability of the fault occurring. This translates to a contribution to  $P_{WC}$  of  $7 \times 10^{-8}$ . Since any probability of undetected wrong cycle ( $P_{WC}$ ) greater than  $7 \times 10^{-8}$  will be regarded as unacceptable, regardless of how much it exceeds that level, the overbounding of these higher probabilities is unnecessary. This because any contribution that exceeds  $7 \times 10^{-8}$  that does not affect the cycle decision.

Overbounding the upper (right) tail of the no fault distribution is useful as it results in a higher threshold than the one that would be derived from the true no fault distribution. The significance of this is that the higher threshold results a true  $P_{FA} < P_{FA}$  calculated.

Hence, it is desirable to overbound both the upper (right) tail and lower (left) tail of the no fault and faulted distribution, respectively. This is seen in Figure 5. This provides both availability at or higher than specified by the  $P_{FA}$  and integrity to the estimate of cycle confidence.



**Figure 5. Overbounding to Achieve Conservative Estimate of False Alarm & Missed Detection**

## 4.2 Achieving Bounds on the Distribution

The question that needs to be answered is “how can the bounds on the distribution be determined?” Generally, the receiver should have a good estimate of random errors since these depend on parameters that can be measured (i.e. SNR) or known in advance. However, the biases are not well known. Instead, the receiver will have bounds on the value of the biases. These may be used to create bounding  $\chi^2$  distributions.

With good estimates of the random errors and bounds on the bias values, the no fault distribution can be bounded at the desired false alarm level. The demonstration of this will be left for a later paper. The result is provable for all cases of  $\sigma$  weighting. The basic idea is that the maximum (over all possible permutations of signs for the bias bounds) estimated noncentrality parameter (ncp) generated using the bias bounds will be larger than the true ncp. Having the estimated ncp larger than the true ncp results in overbounding the true no fault distribution for the  $\sigma$  case. A similar result is provable for the  $\sigma+b$  weighting case. While true WSSE is not  $\chi^2$  distributed, it can be shown that the distribution, for CONUS geometries, is overbounded by the central  $\chi^2$  at the desired false alarm levels. Bounding the no fault distribution allows for conservative estimate of the probability of false alarm resulting in  $P_{FA,true} < P_{FA,est}$ .

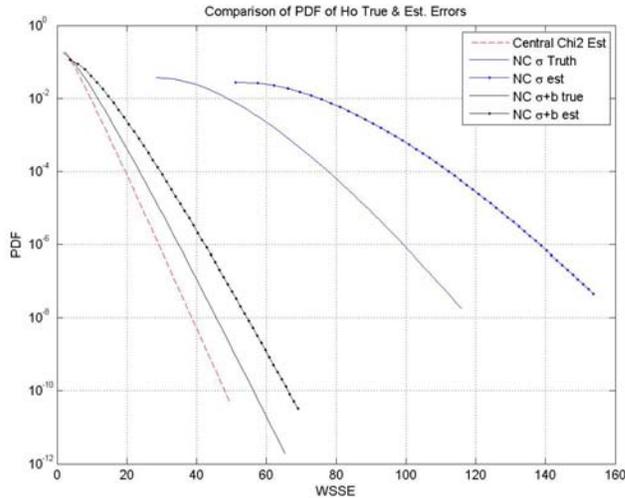
For integrity, the critical factor is to bound the faulted distribution. This can be achieved in a couple of ways. First, it is possible to calculate a value for the  $H_I$  ncp that is lower than the true  $H_I$  purely by examining all variations of signs for the bias bounds. The result is demonstrable for all cases of  $\sigma$  weighting and is presented in another paper. A second method is to conduct a multi-dimensional search over the space of all possible values of the bias for the lowest ncp. The resulting ncp is less conservative than the first method and should yield better availability. However, it is computationally more costly than the first method. Since approximately 60 to 100 fault cases need to be examined for cycle confidence, the computational burden may be too much. Unfortunately, we currently cannot show that it is possible to bound the faulted distribution generated by the  $\sigma+b$  weighting at the levels desired ( $10^{-7}$ ).

## 4.3 Comparing Bounding and True Distributions

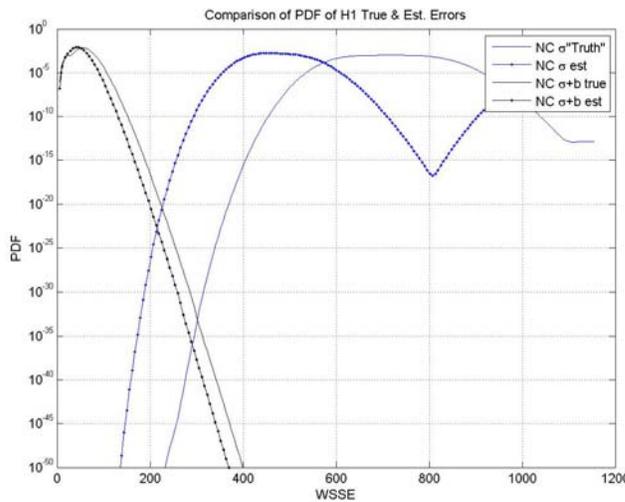
Simulations can be used to demonstrate and visualize the conclusions of the bounding argument. A geometry with  $N$  stations at varying distances and azimuths was chosen. The true biases are known and used to calculate the true distributions. The estimated distributions are generated without any knowledge of the true bias. They are generated only using bias bounds.

Figure 6 shows the true and estimated no fault distributions for both weightings. The plots show the estimated distribution calculated using the median ncp. As mentioned previously, for the  $\sigma$  weighting, it can be shown that the worst case (maximum) ncp for the no fault distribution is guaranteed to overbound the true no fault distribution. In this

case, as in most cases, the estimated no fault distribution using the median ncp overbounds the true no fault distribution. Also note that the central  $\chi^2$  bounds the true  $\sigma+b$  weighting.



**Figure 6. Comparison of the PDF of the No Fault True & Estimated WSSE Distribution (Estimated distrib. chooses median ncp combination of bias), NC = Noncentral**



**Figure 7. Comparison of the PDF of the Faulted True & Estimated WSSE Distribution (Estimated distributions use ncp from no fault case), NC = Noncentral**

Figure 7 shows the true and estimated fault distributions for both weightings. Again, the plots show the estimated distributions calculated using the median ncp. This plot is a bit different than Figure 6 as it represents the average distribution from all faulted cases weighted by fault probability of each case. It can be seen that even with the use of a median ncp, the  $\sigma$  weighting, the lower tail is overbounded in this case. In the case of the  $\sigma+b$  weighting, it is difficult to tell whether the tail is overbounded. However, since the faulted distribution is reasonably close to that of the no fault distribution, the resulting estimated  $P_{WC}$  will indicate inadequate cycle confidence.

## 5.0 Availability

Another important performance consideration is the availability achievable using each method. A method may provide integrity but be so conservative as to result in low availability. As a result, the method, while safe, is not operationally useful. In this section, we examine the performance of each method for aviation approach (RNP 0.3) and enroute (RNP 1.0). The specifics of each operational mode and the tool used to conduct the availability analysis are discussed more in other papers [4][9].

Cycle confidence is necessary for both operations. The same high level of cycle confidence,  $P_{WC} < 7 \times 10^{-8}$ , is required for both operations to have Loran availability. The difference between the two operations lies in the quality of information available to the receiver. Under the current Loran architecture for aviation, an enroute user will generally have larger errors in estimating parameters such as ASF and ECD. Since both these parameters affect cycle selection, cycle availability will not necessarily be the same for RNP 0.3 and RNP 1.0 even though the fundamental algorithm is the same.

### 5.1 RNP 0.3

Figure 8 shows the RNP 0.3 availability for Loran when  $\sigma+b$  (left) and  $\sigma$  (right) weighting is used for the cycle confidence algorithm. The availability is calculated under the worst case noise condition. The availability is not at the desired level though it is generally reasonable (above 90%). The two availability plots look comparable. Under RNP 0.3 conditions, the bias values, while significant, does not dominate the weighting distribution. The result is that the WSSE statistic and  $H_0$  distribution for the  $\sigma+b$  weighting will be reasonably similar to that from the  $\sigma$  weighting in many instances provided that the biases are not large. One caveat about the plot is that the noncentrality parameters for no fault and faulted WSSE distribution used for the  $\sigma$  case are not the worst case but a nominal case. Currently, the worst case ncp calculation has not been implemented into the coverage model. If the worst case were used, the availability should be worse.

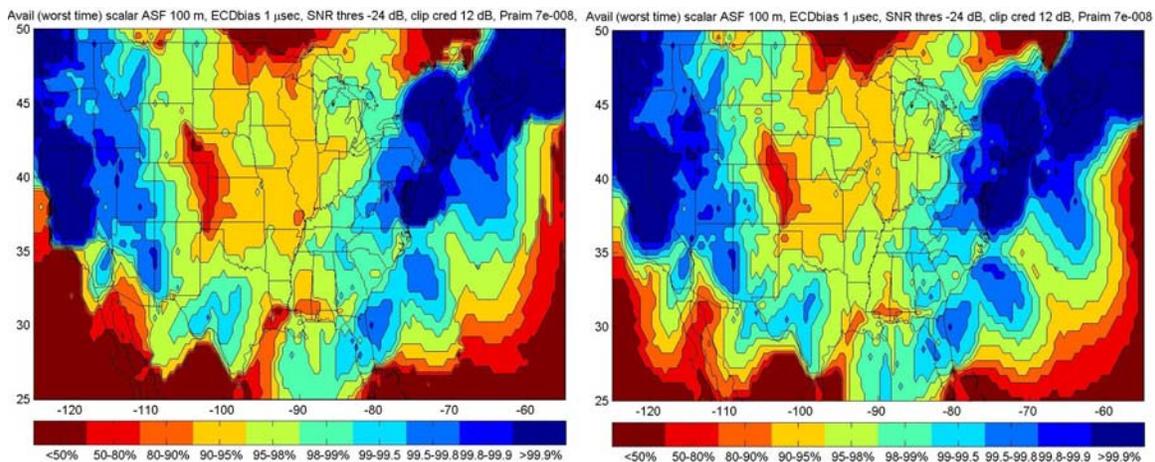
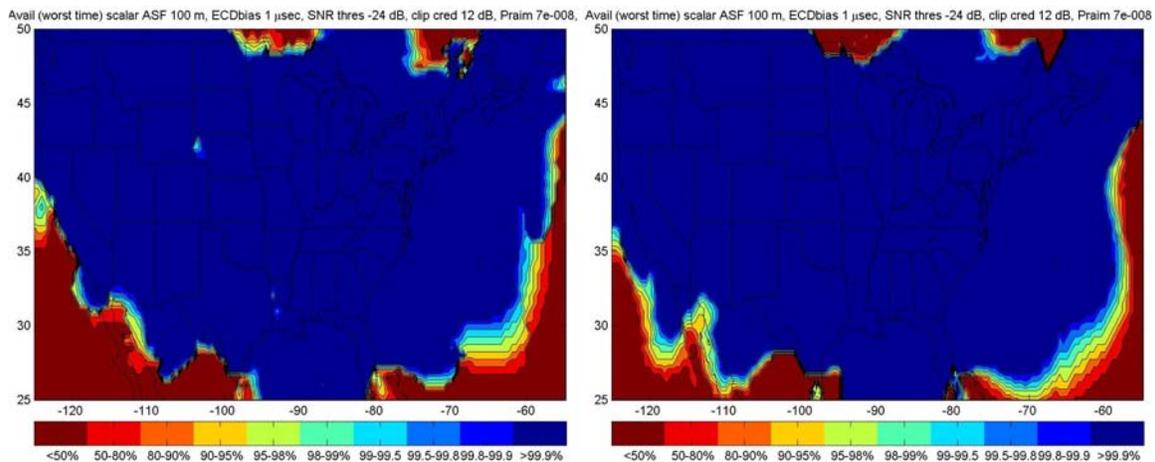


Figure 8. RNP 0.3 Availability at Worst Case Noise Conditions for  $\sigma+b$  (L) and  $\sigma$  (R) Weighting

The 2004 evaluation report used what was considered to be a conservative model for noise processing. Since that report, further study of noise has identified potential improvements to our noise model that better reflect the nature of atmospheric noise. These are described in [10]. The conclusion of [10] is that higher noise levels are positively correlated to higher levels of impulsive noise. Higher impulsivity results in larger noise processing gain. This is because noise suppression through processing is positively correlated with impulsivity. As a result, more processing gain is generally available than assumed in the 2004 evaluation report. Effectively, the noise is reduced. Figure 9 shows the RNP 0.3 availability calculation as in Figure 8 with the exception that this revised noise processing model is used. Given the model, the availability throughout most of CONUS for both weightings is at the desired level. Again, there is little difference between the two weightings. While the effective noise has generally decreased (in the worst locations), it is still significant and dominates the weighting matrix.

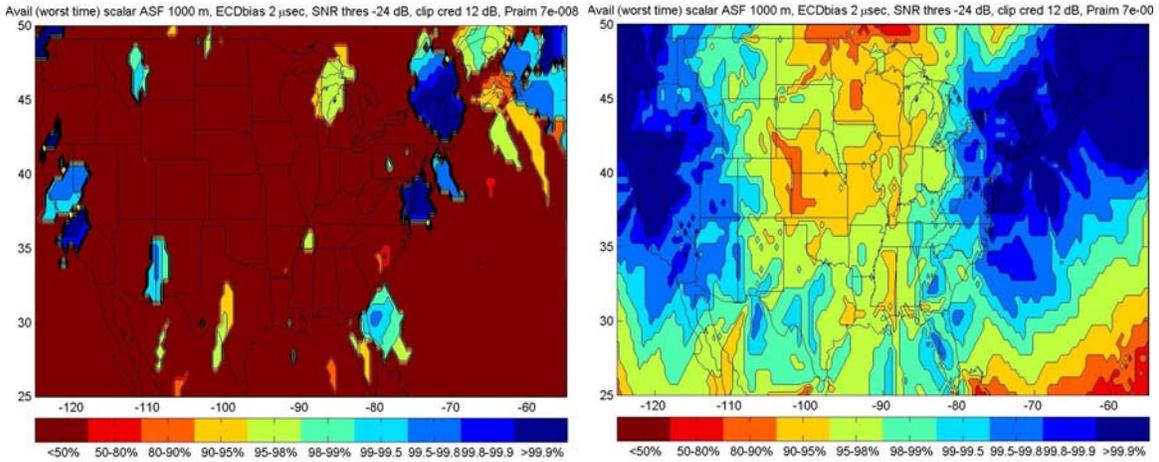


**Figure 9. RNP 0.3 Availability at Worst Case Noise Conditions for  $\sigma+b$  (L) and  $\sigma$  (R) Weighting with Revised Noise Model**

## 5.2 RNP 1.0

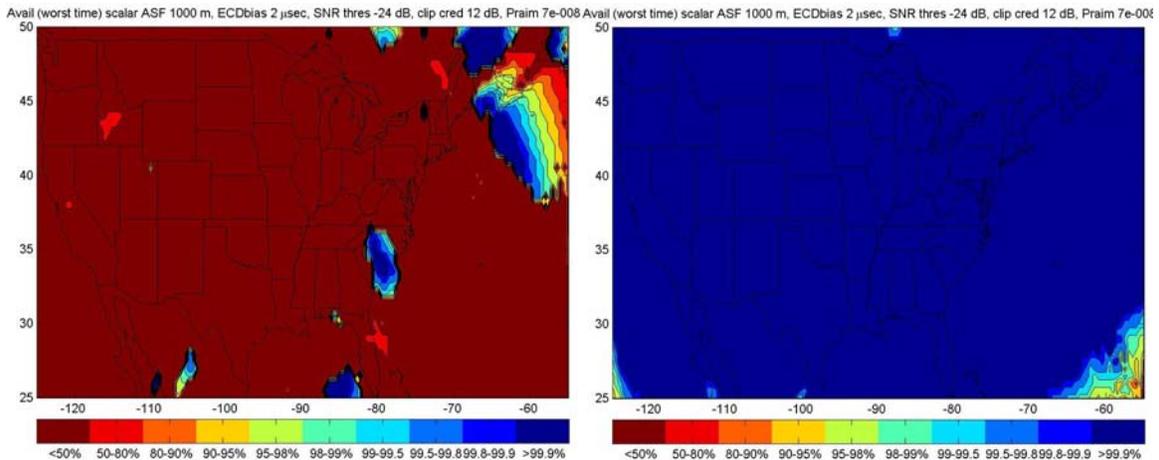
Figure 8 shows the RNP 1.0 availability for Loran when  $\sigma+b$  (left) and  $\sigma$  (right) weighting is used for the cycle confidence algorithm. Again, the availability is calculated under the worst case noise condition. The availability using  $\sigma+b$  weighting is almost non-existent while the availability using  $\sigma$  weighting is similar to its RNP 0.3 availability. The reason for this is that under RNP 1.0, it was assumed that residual phase errors such as spatial ASF can be much larger than that encountered under RNP 0.3 (1000 m vice 100 m for spatial ASF). This results in the bias values being the dominant term in the  $\sigma+b$  weighting matrix. Since the weighting matrix is inversely related to the magnitude of the bias values, the resulting true WSSE is relatively small. As such, the central  $\chi^2$  distribution used to overbound the  $\sigma+b$  no fault distribution greatly overbounds it, resulting in the poor availability. This is a case of overconservatism. The  $\sigma$  weighting,

on the other hand, has an overbound model that is closer to the true distribution. The closer approximation of model to truth is part of the design of the  $\sigma$  weighting algorithm.



**Figure 10. RNP 1.0 Availability at Worst Case Noise Conditions for  $\sigma+b$  (L) and  $\sigma$  (R) Weighting**

Figure 8 shows the RNP 1.0 availability for Loran for both weightings when the revised noise model is used. Again, the  $\sigma+b$  weighting results in almost zero availability while using  $\sigma$  weighting achieves the desired availability throughout CONUS. The availability using  $\sigma$  weighting is similar to its RNP 0.3 availability under the refined noise processing model.



**Figure 11. RNP 1.0 Availability at Worst Case Noise Conditions for  $\sigma+b$  (L) and  $\sigma$  (R) Weighting with Revised Noise Model**

## 6.0 Implementation

Another important consideration is receiver implementation. Having a mathematically expressible distribution helps enable the implementation of the cycle integrity algorithm.

However, it does not ensure that the algorithm can reside reasonably within the receiver. This section examines the implementation of the algorithm and the potential hardware requirements of implementation.

The determination of  $P_{WC}$  and cycle confidence is a five step process as seen in Figure 12. First, the receiver must calculate the parameters defining the no fault and faulted distributions. This is a reasonably non intensive process as only the degree of freedom (dof), which is just the number of stations minus three, and the noncentrality parameter need be calculated. Second, the receiver must calculate the threshold based on the no fault distribution. If no fault distribution is bounded by a central  $\chi^2$  as is the case for the  $\sigma+b$  weighting, the threshold depends solely on the dof and can be stored in a small look up table ( $\sim 10$  elements). If a noncentral  $\chi^2$  distribution is used, calculation of the threshold depends on both the ncp and dof. If a look up table is used, approximately 100 to 10000 values need to be stored. These values are the thresholds resulting from different combinations of dof and ncp. Step three examines the threshold with the calculated WSSE value. If the WSSE is above the threshold, it is likely that there is a fault and the condition is flagged. If the WSSE is below the threshold, it is likely that there is no fault, though we do not know the confidence of this determination. The next step determines the confidence and the last step checks to see if the confidence is adequate.

Step four is to calculate the probability of incorrect cycle ( $P_{IC}$ ) for each station and then probability of missed detection ( $P_{MD}$ ) for every probable faulted (missed cycle) case. Calculating  $P_{MD}$  requires the determination of the cumulative density function (cdf) of noncentral  $\chi^2$  distributions at the threshold value. If a look up table is used, this calculation will also require approximately 200 or more values need to be stored per threshold value. This result implies that for the  $\sigma+b$  weighting, the number of values that need to be stored is of order  $10^3$  to  $10^4$ . For the  $\sigma$  weighting, that number becomes roughly order  $10^5$  to  $10^7$ . Hence,  $\sigma$  weighting receiver requires more memory and computational resources. The result of the calculations is used to compute the overall probability of having an undetected wrong cycle,  $P_{WC}$ . The final step checks if there is adequate confidence in the cycle selection ( $P_{WC} < 7 \times 10^{-8}$ ).

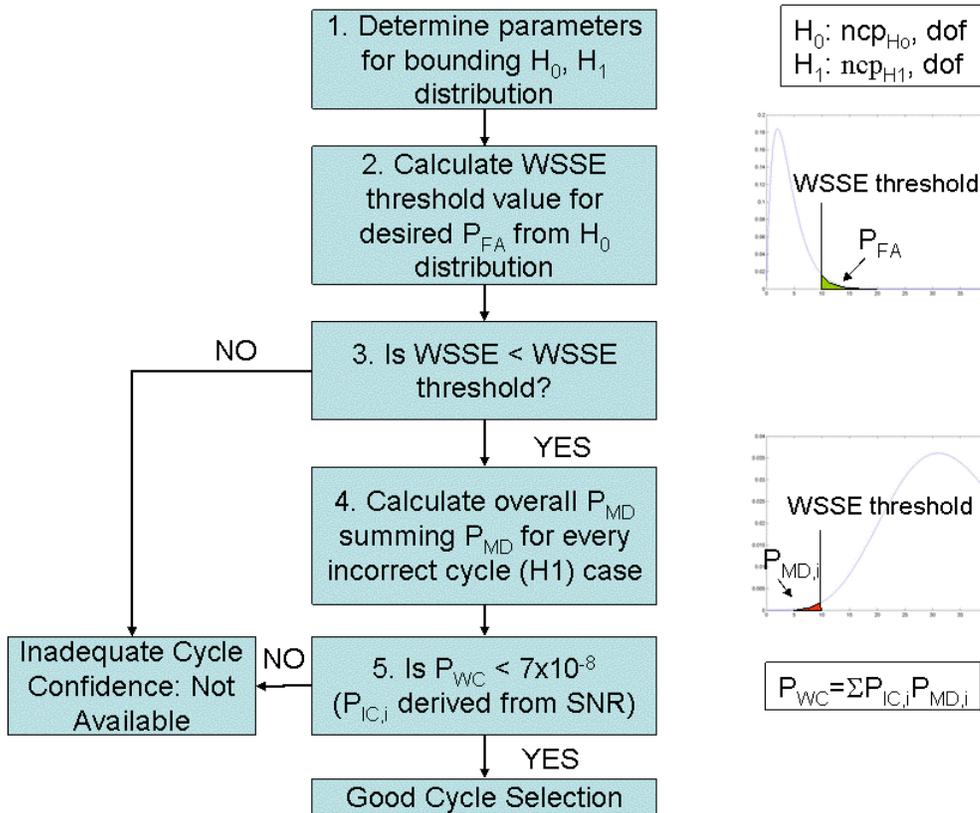


Figure 12. Steps to Determining Cycle Confidence and Decide if Cycle Selection is Valid

## 7.0 Conclusion

The primary purpose of the assessment of the Loran cycle confidence algorithm was to determine how to provide integrity to the algorithm while achieving acceptable levels availability. The assessment focused on the choice of weighting matrix used and the resulting impact on the algorithm. Two weighting matrices were examined: one based on random plus bias errors ( $\sigma+b$ ) and one based on random errors ( $\sigma$ ). From an integrity standpoint, the  $\sigma$  weighting seems to be better as the integrity of the algorithm using the weighting is more provable. From an availability standpoint,  $\sigma+b$  work reasonably for RNP 0.3 but not for RNP 1.0. Availability using  $\sigma$  weighting is reasonable for both RNP 0.3 and RNP 1.0. Using a revised noise processing model,  $\sigma$  achieves target levels of availability in both procedures while  $\sigma+b$  still is not usable for RNP 1.0. From an implementation standpoint,  $\sigma+b$  is more easily implemented as it requires less processing and storage. However,  $\sigma$  weighting can be implemented in most modern receiver and given that it has better integrity and availability characteristics, it should be the weighting used for aviation Loran receivers.

## **8.0 Disclaimer**

The views expressed herein are those of the primary author and are not to be construed as official or reflecting the views of the U.S. Coast Guard, Federal Aviation Administration, Department of Transportation or Department of Homeland Security.

## **9.0 Acknowledgments**

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## **10.0 Bibliography**

[1] "Vulnerability Assessment of the Transportation Infrastructure Relying on the Global Positioning System," John A. Volpe National Transportation System Center, August 20, 2001.

[2] FAA report to FAA Vice President for Technical Operations Navigation Services Directorate, "Loran's Capability to Mitigate the Impact of a GPS Outage on GPS Position, Navigation, and Time Applications," March 2004.

[3] Benjamin Peterson, Lee Boyce, et al., "Hazardously Misleading Information Analysis for Loran LNAV", Proceedings of the 2nd International Symposium on Integration of LORAN-C/Eurofix and EGNOS/Galileo, Munich, Germany, June 2002

[4] Sherman Lo, Per Enge, "Preliminary Assessment of Loran for Required Navigation Performance 1.0", Proceedings of the International Loran Association 35th Annual Meeting, Groton, CT, October 2006.

[5] Sherman Lo, et al., "Loran Integrity Analysis for Required Navigation Performance 0.3", Proceedings of the 5th International Symposium on Integration of LORAN-C/Eurofix and EGNOS/Galileo, Munich, Germany, June 2004

[6] Lee Boyce, Sherman Lo, J.D. Powell, Per Enge, "Analysis of Noise and Cycle Selection in a Loran Receiver", Proceedings of the International Loran Association 35th Annual Meeting, Groton, CT, October 2006

[7] Sherman Lo, Per Enge, "Implementing the Loran Weighted Sum Square Error (WSSE) Statistic Cycle Selection Algorithm for Integrity", Proceedings of the Institute of Navigation National Technical Meeting, San Diego, CA, January 2007

[8] DeCleene, Bruce, "Defining Pseudorange Integrity - Overbounding," Proceedings of the Institute of Navigation GPS, 2000, Salt Lake City, UT, pp. 1916-1924

[9] Sherman Lo, et al., "Loran Availability and Continuity Analysis for Required Navigation Performance 0.3" Proceedings of GNSS 2004 – The European Navigation Conference, Rotterdam, The Netherlands, May 2004

[10] Lee Boyce, Sherman Lo, J.D. Powell, Per Enge, "Mitigating Atmospheric Noise for Loran", Proceedings of the Institute of Navigation GNSS Conference, Fort Worth, TX, September 2006