Hazardously Misleading Information Analysis for Loran LNAV

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1 INTRODUCTION

The U. S. Department of Transportation (DOT) has accepted the findings and recommendations of the Volpe National Transportation Systems Center (VNTSC) report on GPS Vulnerability [1] in general and specifically the need for a backup to GPS in safety critical applications. The various branches of the DOT will be making recommendations to DOT Secretary Mineta on how they propose to meet this requirement. As part of the Federal Aviation Administration (FAA) process to generate its recommendations to the Secretary, a Loran Murder Board was convened on March 19th of this year. As a result of that meeting, the main issue associated with an FAA recommendation on Loran is whether it can support non-precision approach. This preferably would mean Required Navigation Performance (RNP) 0.3 performance and if not RNP 0.3, then RNP 0.5 as a fall back. Our effort since that date has focused almost exclusively on the development of tools to analyze this ability. This effort is expected to last for 18 to 24 months. This paper is summary of our progress since March 19th and an outline of where we are headed over that period.

Our focus has not been so much to come up with a binary answer, but more on sensitivity analysis to determine what are the most critical criteria that will have to be met to satisfy the requirement. For example, do we need better ASF models, better user receiver performance, tighter specifications on the signal in space, more transmitters, etc.

2 ACCURACY/INTEGRITY/HMI

Since Loran only provides horizontal positioning, we are concerned with the Horizontal Protection Level (HPL). The HPL is an error bound on the horizontal position error such that the probability of the true error being within this bound is greater than 99.99999%. In other words, there is at most is a one in ten million chance that the true system error is greater than the HPL. Should this be the case, the system would be generating Hazardous Misleading Information (HMI) that could lead to catastrophic results for the aircraft. In our case, in order to meet the requirements of RNP 0.3 or RNP 0.5, the receiver must be able to calculate an HPL that is less than the Horizontal Alarm Limit (HAL) of 0.3 NM and 0.5 NM, respectively.

3 KEY ASSUMPTIONS IN CURRENT ANALYSIS

In the context of our analysis, the assumption is that an all-in-view receiver combining times of arrival (TOAs) as pseudoranges is used. It is also assumed that adjacent chains are synchronized to each other within some tolerance. Implicit in this assumption is that all cross chain lane ambiguities have been correctly resolved. In North America, all Loran signals have periods that are integral multiples of 200 μsec meaning that if cross chain TDs are measured or calculated, they are repeatable only when taken modulo 200 μsec. This results in 30 km lanes along baselines. There are several ways to resolve these lane ambiguities, including but not limited to, a preliminary fix based on intra-chain time differences (TDs), transmission of time parameters via modulation, an external source of approximate time, etc. At this point in the analysis we are assuming this has been done correctly. It should be noted that this lane ambiguity is different from that noted in Table 3.3 in [1] which is based on existing triad based receivers. There is also fix ambiguity associated with a single chain triad fix which, as noted in [1], is easily resolved.

We are assuming for now that the timing of secondaries is controlled via Time of Emission (TOE) control vice controlling TDs measured in the far field at a System Area Monitor (SAM) as is presently the case in North America [3]. Or alternatively, if SAM control is retained, secondary TOE offsets are communicated to the user receivers. Lacking either change, the analysis would need to be done for a SAM controlled system. This would complicate the analysis and add a number more noise and bias terms associated with the SAM receivers and the propagation paths from transmitters to SAMs. Due to the method described below of combining the effects of bias terms by adding them with the worst combination of signs, it is likely the calculated HPL will be worse even if the actual 2-D RMS accuracy performance does not significantly degrade.

We assume the receiver uses a crossed magnetic loop antenna where each RF channel is digitized. This assumption is mainly for the mitigation of precipitation static problems inherent in electric field antennas, but we are also claiming credit for the antenna gain due to the directionality of the steered antenna response.

We are assuming that the effects of cross rate interference have been mitigated via either blanking or canceling. As the number of pulses averaged is a critical parameter in our analysis, as we get farther along in that analysis it is quite possible we may conclude it is impossible to simultaneously have enough pulses to average and still have the necessary dynamic performance using cross rate blanking vice canceling.

We are assuming that any data modulation on the Loran signal has not degraded navigation performance. While data modulation does in fact degrade the performance of existing legacy receivers, it is assumed that if modulation is implemented, future integrated data and navigation receivers will first demodulate and then wipe off the affects of modulation prior to processing the signal for navigation. This further implies the capability to wipe off modulation on signals too weak to demodulate. For example, the same data comes from all stations. If forward error correction via block encoding is used, we assume the blocks are of short enough duration such that they can be decoded to recover the raw data without introducing unacceptable time lag in the navigation solution.

Our analysis also assumes the additional secondary factor (ASF) bias error is some percentage of the predicted value. ASFs are defined as the additional phase shift relative to an all seawater path caused by the overland portion of the propagation path [3]. The most likely way to implement ASF corrections is by performing a calibration of each airport. In the early stages of development, airports will need more intense calibration. This is necessary to validate the ASF and to determine the seasonal and local spatial variations. With experience, later airports will need no more than a one-time calibration that will be based on a yearly average of ASF. These values will be subject to the standard periodic validation by non precision approach (NPA) flight inspection that other flight systems experience. As we are analyzing the potential for dispatch reliability in the event of extended loss of GPS/WAAS capability, we assume no real time estimation of ASFs within the receiver using GPS/WAAS position data.

4 RECEIVER INTEGRITY CYCLE CHECK (RICC)

Due to the nature of the errors typically encountered in Loran, there are two parts to ensuring its integrity. The process is seen in Figure 1. The first part is a receiver integrity cycle check

(RICC) that will guarantee the correct cycle of the Loran signal is being tracked. Should the signal pass RICC, then it is used, along with signals from other Loran stations, to determine position. Then the HPL for the position solution is calculated. If the HPL meets the requirements for RNP 0.3 or 0.5, then the solution may be used for those approaches. The next two sections will analyze and detail the methods for performing the RICC and HPL calculations, respectively. Furthermore, error models will also be examined.

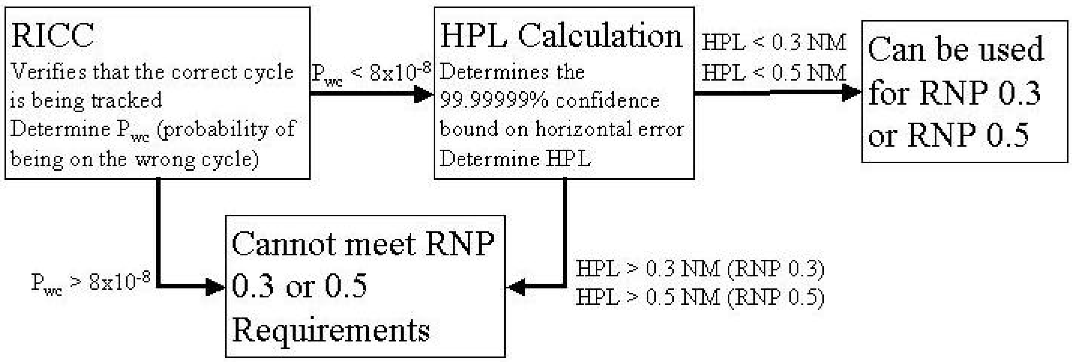

Figure 1. Block Diagram of the Integrity Process

RICC is an integral part of guaranteeing the integrity of the Loran signal. The cycle needs to be verified since the cycle width is 10 μs or 3000 m. An error in tracking the Loran signal will clearly exceed the desired system integrity requirement.

If the number of stations used in the position (N) is three, the probability of any one of them being on the wrong cycle (P_{wc}) must be less than 1 x 10^{-7} less some allowance for the probability the signal in space is out of tolerance and either it failed to warn the user or the user receiver did not detect the warning. For the time being, we are using a threshold of 8 x 10^{-8} as our P_{wc}. Since attaining this level of confidence requires signal to noise ratios (SNRs) that are not generally available for the three strongest signals, we also need to use the redundant information in an overdetermined position solution, when available, in order to calculate cycle integrity. In this case, RICC is analogous to RAIM in GPS receivers [4]. Once the correct cycle selection can be guaranteed to a sufficient level of integrity, the analysis proceeds to calculate a Horizontal Protection Level based on the type and magnitude of errors.

To begin the RICC analysis, we will take time of arrival information from each station. Thus the equations that relate the observable pseudorange measurements, y, and the position of the observer, x, will be related by a matrix of directions cosines with ones in the last row for the clock term.

$$y = Gx$$

Since Loran is a two dimensional system, G has three columns vice four as in GPS. The weighting is inversely proportional to the variance of the measurement errors on the pseudoranges. The measurement errors include errors due to receiver signal tracking, transmission jitter, and ASFs. These errors will be discussed in more detail later. Therefore the weighted least squares solution is:

$$x_{wls} = (G^T W G)^{-1} G^T W y = Ky$$

where W is the weighting matrix given by $W = R^{-1}$, R is the covariance matrix of the pseudorange (or TOA) errors and includes both noise and bias terms, y is the pseudorange measurements, and $K \equiv (G^T W G)^{-1} G^T W$. The predicted value of y based on the solution is $\hat{y} = Gx_{wls}$ and the predicted error is

$$\hat{w} = y - \hat{y} = [I - GK]y$$

As shown in [6], the result is equivalent to

$$\hat{w} = [I - GK]e$$

where ε is the vector of pseudorange errors.

Walter suggested using the positive definite Weighted Sum of the Squared Errors (WSSE) as the test statistic for determining whether a cycle slip may be detected in GPS [5]. We will use the same method for Loran.

The WSSE is given by:

$$WSSE = w^T W w = e^T [I - GK]^T W [I - GK] e = e^T W [I - GK] e = e^T M e$$

Where,

$$M \equiv W[I - GK]$$

Since W has units of 1/distance^2, WSSE becomes a dimensionless quantity. If the correct cycles are being tracked, the n ranging errors follow a zero-mean Gaussian distribution. Additionally, if we have correctly estimated their variances in the R matrix, WSSE then has a chi-square distribution with n – 3 degrees of freedom [6].

An example of the no-fault distribution is shown in Figure 3 in green, which shows the probability density function of WSSE.

In the faulted case, a cycle slip produces a bias in pseudorange. This changes the error distribution to Gaussian but with a non-zero mean, m, equal to a multiple of 3000 m. The resulting WSSE distribution is non-central chi-square distribution, with a non-centrality parameter l.

l is given by [6]:

$$l = m^T M m$$

Where m is the vector of all pseudorange means.

With l, the mean and variance of the non-central distribution become

$$m_{X^2} = l + (n - 3)$$

$$s^2_{X^2} = 2(2l + (n - 3))$$

Since m is on the order of 3000 m, (or $m^T m$ is on the order of 3000^2) and M is only of the order 10^2, then $l >> n$-3 term. This results in the following approximations

$$m_{X^2} \approx l$$

$$s_{X^2} \approx 2\sqrt{l}$$

This fault case takes on a non-central distribution shifted to the left of the no-fault case as shown in Figure 3 in red. The overlapped regions to the left and right of the threshold depict the probability of miss detection and false alarms, respectively. Now establishing a decision rule becomes simple as in the GPS case.

To determine the probability of miss detection, P_{MD}, you first assume a cycle slip on a subset of stations. Then for all possible combinations of signs, generate m with zeros for the no-fault case and ± 3000 m for the stations that have been slipped. With m and M, calculate l and the corresponding non-central distribution for the faulted case, p_{Fault}. Then, p_{Fault} is integrated up to the threshold set by the false alarm limit of 99.9% to yield P_{MD}.

That is

$$P_{MD} = \int_{-\infty}^{threshold} p_{Fault}(x)dx$$

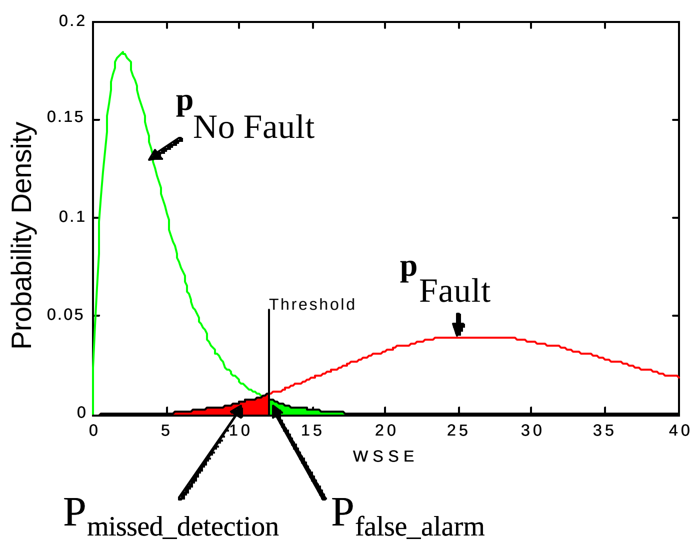

Figure 3. Probability Densities for No Fault and Fault Cases

The probability of being on the wrong cycle, P_{WC}, is then,

$$P_{WC} = \sum P_{MD_i} P_{Slip_i} + \sum P_{MD_{ij}} P_{Slip_i} P_{Slip_j} + \sum P_{MD_{jki}} P_{Slip_i} P_{Slip_j} P_{Slip_k}$$

if we consider all combinations of one, two, and three cycle slips.

Since our overall goal is a 10^{7} probability of HMI, we allocated $8x10^{-8}$ as an acceptable chance that we do not detect the correct cycle. Therefore, if $P_{WC} < 8x10^{-8}$ then we may proceed with the calculation of HPL. If this is not achievable, then integrity cannot be guaranteed and the system must flag the user that it is unavailable.

5 ERROR MODELS

The previous derivations show that RICC requires the use of the probability distributions of Loran measurement error. This section will detail how the distributions are generated and what assumptions are warranted.

To better define measurement errors, we divided up them into two general categories: Random Errors that are zero-mean Gaussian variables, and Biases that are constant offsets. First there are errors described by Gaussian zero-mean finite variance random variables that we will call Random Error. The causes of Random Error stem from receiver effects due to signal to noise ratio (SNR), and transmission jitter.

SNR values are determined by first calculating the signal strength at the receiver location by Millington's method [8]. Millington's method is a means of accounting for the attenuation in signal strength based on the propagation of the Loran signal across a round heterogeneous Earth.

Next, noise values are calculated. For Loran, the primary source of noise is atmospheric noise stemming from lightning strikes. Since lightning produces a wide band of low frequency noise and low frequency propagate long distances, noise from up to several thousand kilometers away may affect the signal.

The International Telecommunications Union (ITU) has a database of noise values [9] based on nine years of collected data that covers the world over four seasons and over all hours of the day. With these noise maps, we calculated the 95% and 99% noise levels and the corresponding SNR values. Given the SNR values, standard deviations of receiver phase noise may be calculated. Given the SNR and assuming an integration time of some given number of pulses, we can determine the 1 σ value for errors in measuring the envelope to cycle difference (ECD).

Assuming ECD follows a Gaussian distribution, we can integrate the tails of the distribution on both sides up to an ECD of 5 μs to get P_{Slip}. This limit is taken since errors of greater than 5 μs will result in a cycle slip.

The second source of phase noise is related to the transmission jitter of the signal at the Loran towers. The jitter yields a 6 meter, 1 σ error.

The second class of error is Bias Error or a simple constant that describes the upper bound on an offset that cannot be calibrated out of the system. The two sources for Bias Errors are transmission offset and ASF prediction errors. Transmission offsets are errors in the time that the pulse is actually transmitted relative to its proscribed time in UTC [3].

We model ASF prediction errors as having both correlated and uncorrelated components. For the correlated components, we divide the coverage area in regions, each with a different maximum change from winter to summer in propagation velocity. The uncorrelated ASF biases are modeled as a percentage of the whole delay.

6 CALCULATION OF HORIZONTAL PROTECTION LEVEL (HPL)

Now that we have laid out the types of errors, we will use our knowledge of their distributions and some key assumptions to calculate a HPL. Given that, for stations with P_{WC} adequately small, we may calculate an HPL average.

Figure 5 graphically depicts the values used in the HPL calculation.

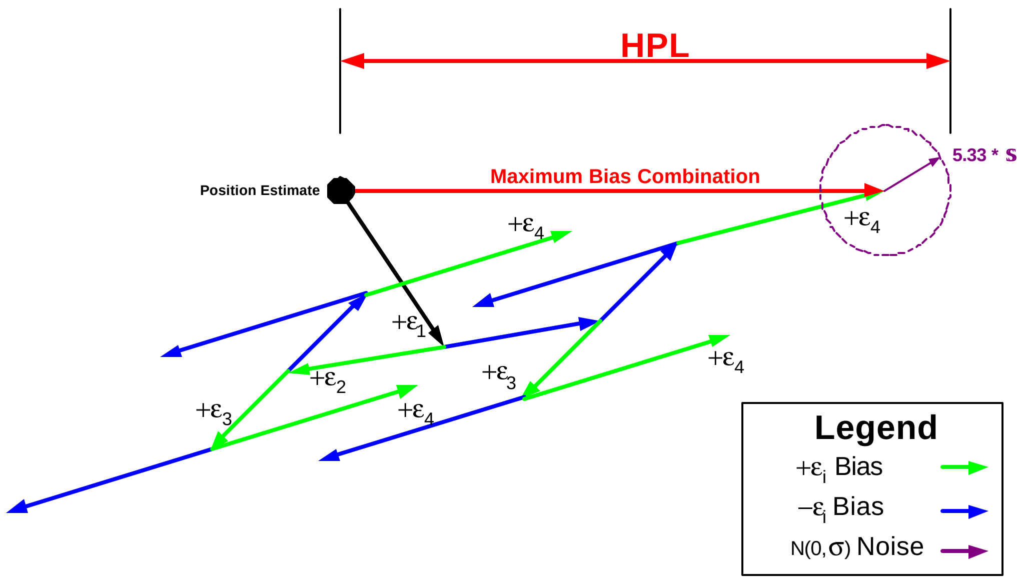

Figure 5. Calculation of HPL

Begin with the position estimate and the first bias vector taken in the positive direction. Then add all bias other vectors with all combinations of signs, or 2^{n-1} combinations. In Figure 5, there are four bias vectors being evaluated, where green represents the positive sense of the vector and blue the negative. As you can see, by adding ε_2 there are two vectors to compare, with ε_3 there are now four, and finally by adding ε_4 there are eight possible combinations. By combining these vectors you can find the combination that yields the maximum bias combination, which is shown as the maximum bias combination. We then calculate an overall σ of the Random Errors added in a root-sum-squared sense. Then in the direction of the maximum bias combination we add 5.33 σ to get the HPL.

Weighted versus Unweighted Test Statistics

There is a fundamental difference between what we are trying to accomplish in Loran RICC and GPS RAIM even if GPS is doing weighted least squares. In GPS, we are trying to detect a ranging error large enough to cause a significant position error. Pseudoranges with larger error (distributions) carry less weight under a weighted test statistics. As a result, if a particular pseudorange has very large errors, it is weighted out of the solution.

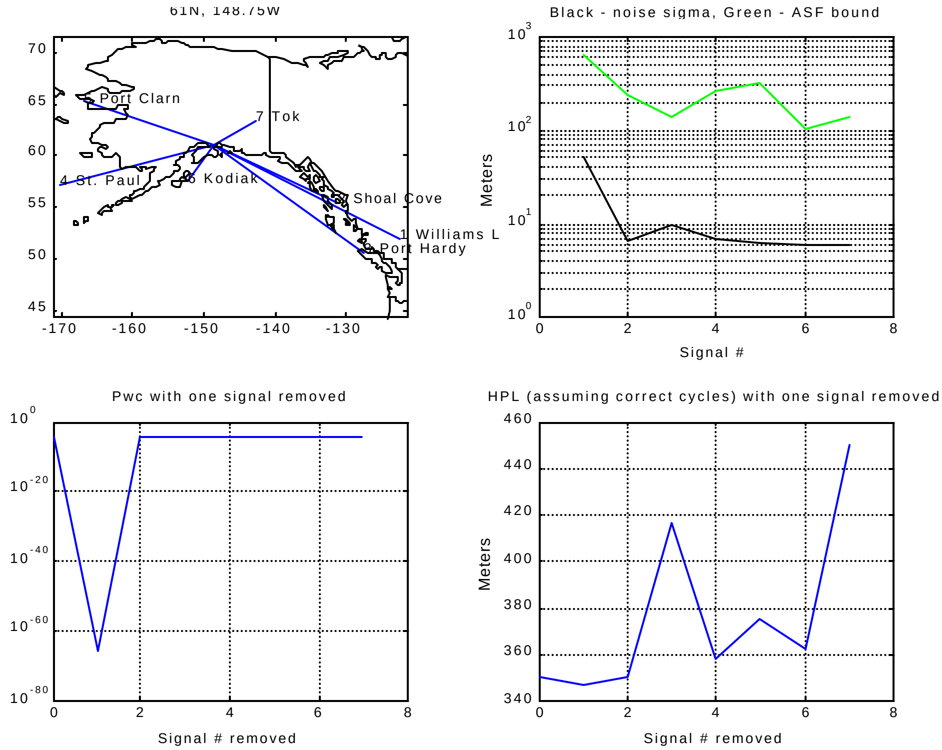

Figure 7. Example where Removing Single Station Helps Integrity Calculation

In Loran RICC, we are trying to detect cycle errors of 3000 m. Using a weighted test statistic means that cycle errors on extremely weak stations do no show up as strongly in the test statistic as do those on stronger stations. This leads to examples like that in Figure 7 where we cannot detect a cycle error with the required integrity because we do not weigh that ranging residual very much in our test statistic even though by eliminating that station we have the required integrity and accuracy.

In Figure 8 and Figure 9, we compare the distributions of the test statistics for both the no error and the cycle error case and for both a weighted and unweighted test statistic. For the unweighted case, we scaled the plot to force the threshold for 1×10^{-3} probability of false alarm to be the same as for the weighted case.

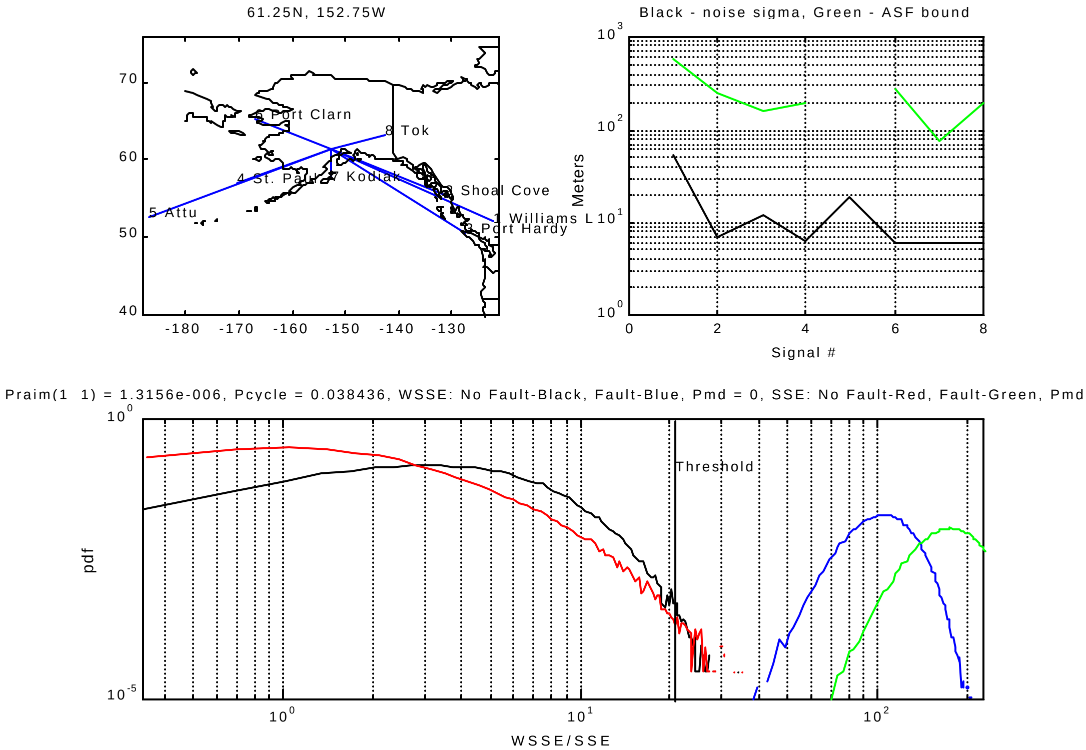

Figure 8. Example where Cycle Error Detectability Enhanced with Unweighted Test Statistic

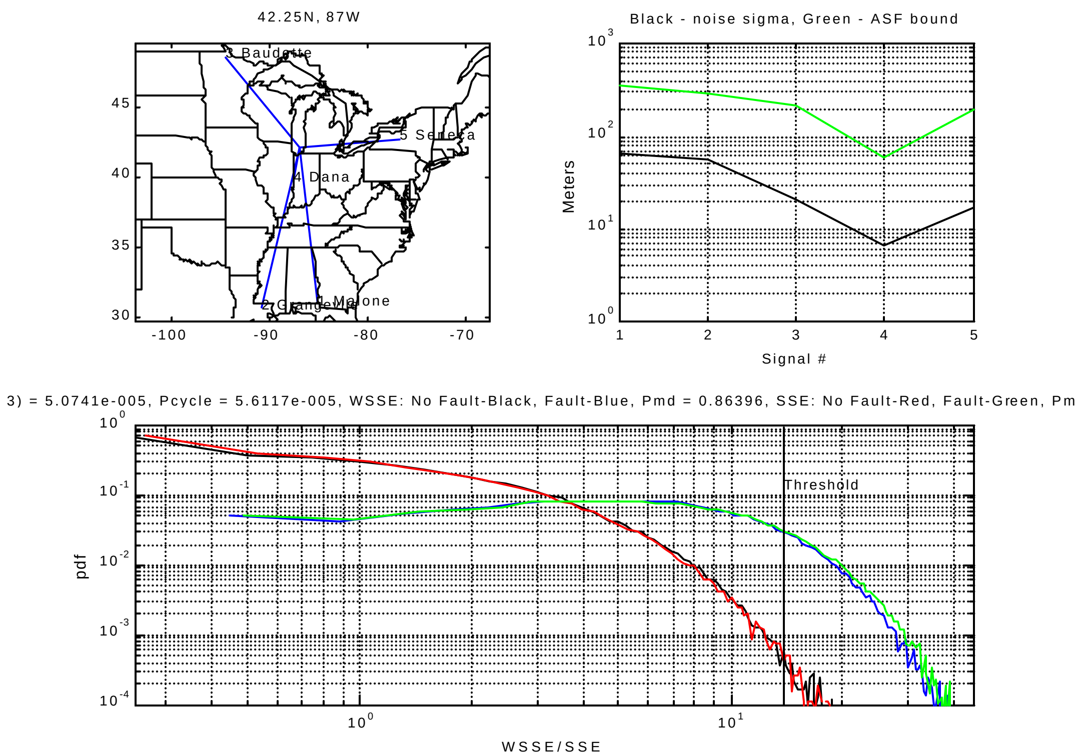

Figure 9. Example of Bad Detection Geometry

In Figure 8, we see that using an unweighted test statistic gives much better detectability of a cycle error on the signal from Williams Lake than a weighted statistic. This is because there are stronger signals that are broadcast from the same direction as Williams Lake. These signals, which are from Shoal Cove and Port Hardy, mean that Williams Lake is effectively weighted out of the solution. In the second example, Figure 9, a cycle error on the signal from Baudette is not detectable independent of whether we use a weighted or an unweighted test statistic. The reason is that there are no stronger signals from the same direction resulting in poor detection geometry suing either type statistic.

7 CONCLUSIONS AND FUTURE DIRECTIONS

The first cut analysis has been quite successful. We feel quite confident that receivers can be designed to enable Loran to satisfy RNP 0.3 requirements and we have made significant progress in estimating the necessary signal-in-space and receiver parameters. The next step is to validate and revise each part of the analysis, assumption, parameter, etc. In addition, there is credit that we can take for the impulse nature of noise by using a clipper in the receiver. Similarly, the directionality of antenna will also yield a benefit. By researching past data, we will try to determine realistic bounds on ASF estimates, transmitter timing offsets, ECD predictions, and transmitted ECD errors. In addition we will measure the bounds on probability of signal out of tolerance without blink, and the probability of missed blink detection.

The averaging time constant in receivers determines the number of samples, N. The SNR value increases by $\sqrt{N}$ and has a significant impact on the RICC and HPL calculations. Through our analysis we found there were some counter-intuitive examples where the elimination of a signal from the solution improved the probability of being on the correct cycle or the HPL. We will explore different criteria for the elimination of signals. Another avenue of exploration will be in the use of an unweighted test statistic that may yield better detectability of cycle errors and thus better availability.

As an on going effort, we will use the analysis software to see where we need to allocate effort to get the availability we need. Such software allows us to perform sensitivity studies as to the bounds on ASF errors, the need for more stations, receiver performance parameters, or the effect of increasing power from existing stations. All of these studies will guide the Loran Integrity Performance Panel (LORIPP) in establishing a work plan and maintaining a list of new Loran station and receiver requirements.

8. ACKNOWLEDGEMENT

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