Range Biases on Modernized GNSS Codes

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BIOGRAPHY

R. Eric Phelts is a Research Associate in the Department of Aeronautics and Astronautics at Stanford University. He received his B.S. in Mechanical Engineering from Georgia Institute of Technology in 1995, and his M.S. and Ph.D. in Mechanical Engineering from Stanford University in 1997 and 2001, respectively. His research involves signal deformation and signal quality monitoring techniques for WAAS, LAAS, JPALS, and the GPS Evolutionary Architecture Study (GEAS).

ABSTRACT

Multiple constellations and modernized GNSS signals promise to bring a wealth of ranging possibilities to navigation users. They offer improved geometry and redundancy, immunity to ionospheric errors, and reduced susceptibility to multipath. Range errors resulting from code tracking and clock bias differences between different constellations are frequently ignored assuming the navigation message from one of the constellations will contain information about this offset. Under ideal conditions, this would be sufficient; however, biases resulting from the different signal structures of different GNSS systems may require that additional measures be applied to truly correct for this error.

Range bias errors result from non-uniformity or dissimilarities between incoming signals. These conditions are frequently exacerbated by receiver-dependent properties such as discriminator type, correlator spacing, and receiver filter characteristics. While having diversity of ranging codes is beneficial in many ways, it also brings the potential for these additional errors that should be taken into account.

This paper begins to analyze the kinds of code range biases inherent in ranging from different GNSS codes. It modifies the standard WAAS geostationary satellite bias modeling tool for aviation users. It also discusses the tradeoffs associated with mitigating the errors using differential corrections, estimation, or calibration techniques. The results suggest that some receiver design changes may be needed to decrease sensitivity to them.

BACKGROUND

Range biases result form dissimilarities between received signals or from differences in the processing of those signals. There are three hey ways in which these dissimilarities may manifest themselves. Satellite signal generators can cause signal deformations due to variations between the signal generation and transmission paths of the satellites themselves. Code modulations and PRN differences couple with filter realizations to cause variations as well. Finally receiver filter non-idealities and tracking implementations may under some conditions worsen range accuracy. (Note that this paper discusses only deterministic biases; this does not include multipath, thermal noise, or atmospheric effects.)

Nominal Signal deformations, code modulations, and PRN-to-PRN variations are each described briefly in the following subsections. (Receiver design is discussed within the context of each of these.) These are followed by a discussion of the common mitigation techniques and a brief summary of the WAAS GEO bias problem. The latter is an example of how these biases can occur and potentially decrease accuracy in actual GNSS systems.

Nominal Signal Deformations

Nominal signal deformations refer to distortions of GNSS signals caused by imperfections in the signal transmission path. They are frequently created by either

- Code pulse generation rise/fall time asymmetries
- Phase nonlinearity across the full bandwidths of any amplifiers, filters, or antennas in the transmission path
- Magnitude response asymmetry across the full bandwidths of any amplifiers, filters, or antennas in the transmission path

The code generation asymmetry causes a negative or positive shift along with a flattening of the correlation peak. Filter phase and magnitude asymmetries precipitate asymmetries in the correlation peak. All three of these conditions may lead to receiver tracking (and, hence, range and position) errors that may vary widely as a function of correlator spacing. Although this section is included for completeness, these satellite-generated signal deformations are not considered in this paper. They are, however, discussed in more detain in [2] and [3].

Code Modulations

Code modulation differences between the envisioned GNSS codes are inherently dissimilar. Because their frequency domain and correlation function representations are different so they will require different filters and code tracking implementations. A picture of the modulations analyzed in this paper is given in Figure 1.

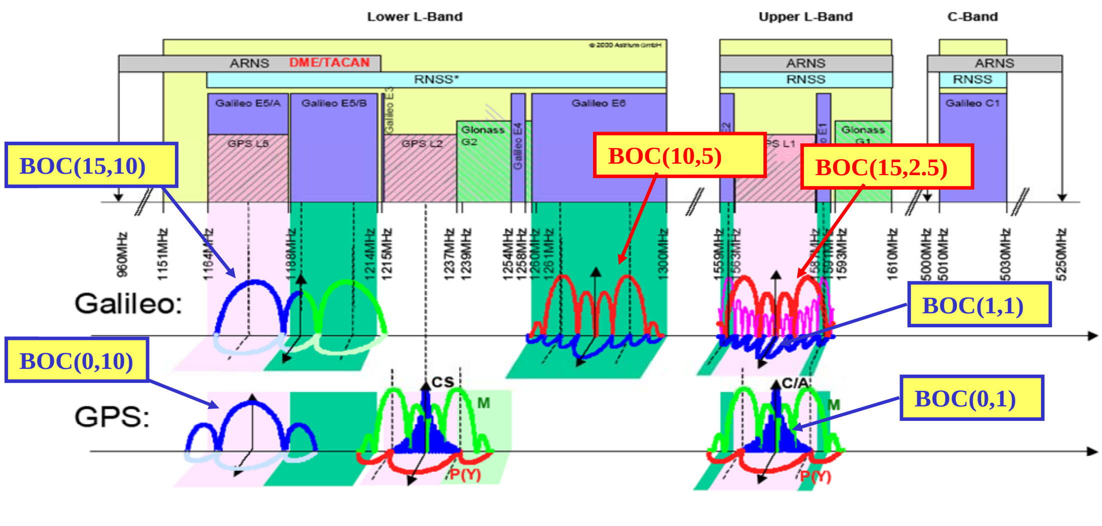

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Figure 1. Overview of PSDs for modernized GNSS code modulations. (Only six modulations indicated are evaluated in this paper.)

PRN-to-PRN Variations

Another less commonly understood mechanism for signal dissimilarities involves the use of different PRN codes. The variation of one PRN code to another itself in combination with the aforementioned filter effects can precipitate range errors. This is caused by the fact that the power spectrum of each code autocorrelation function varies slightly from one to another. That variation can be compared to a PRN-specific filter being applied to a single (ideal) power spectrum (e.g., $sync^2(f)$ for GPS C/A code).

This PRN-specific filter is symmetric and has zero phase, and alone introduces no correlation peak asymmetries. (It may however modify the peak shape in other ways. However, in combination with other antenna and filter non-idealities, including those of the receiver, the differences between the codes can become noticeably larger. So the PRN-to-PRN variations lead to increased range (and position) errors only when certain combinations of PRNs are used together in the navigation solution.

Bias Mitigation Methods

Mitigation of code range biases can occur in one or more of the following techniques

- Differential corrections
- Calibration
- Estimation
- Receiver Design
- Error budget allocation

Differential Corrections

Differential correction implies that a reference receiver is available to correct for the error. As GNSS signals generally employ a ground network of reference stations, many large biases are implicitly corrected. Often these corrections are contained within the either the satellite clock correction, or, when multiple frequencies are used, the inter-frequency bias correction. The accuracy of the correction, however, is dependent on the reference receiver filter and tracking characteristics as well as those of the user receiver. If these characteristics are not perfectly matched, errors will remain.

Calibration

Calibration methods involve making models or measurements of the net biases and applying that correction for them. Precise models—either analytical or data-based—may account for filters at the reference stations and the user receiver. However, while the correction is usually fixed, actual filter characteristics may vary slightly with time. In addition models seldom account for manufacturing variations which may make a set of corrections based off of one pair of receivers inaccurate when one or more similar receivers in that system are changed.

Estimation

Estimation methods involve adding the biases as states in the navigation solution to solving for them. This is most likely to be employed as GLONASS, Galileo and other global satellite systems become available since the clock offsets may differ from one system to the next. The cost of this is effectively the loss one range for improving positioning accuracy. For instance, if only a single Galileo satellite were added to a navigation solution containing only GPS satellites, the Galileo satellite could only be used to estimate its own clock offset in this implementation. In other words, the added equation (e.g., the Galileo pseudorange) would be used to solve for the added unknown—the clock (and filter) bias.

Receiver Design

Another way to potentially reduce or mitigate signal deformation biases is through receiver design. Careful selection of receiver filters, discriminator type, and correlator spacings may ensure biases remain as small as possible given the aforementioned signal conditions. Although limited by manufacturing tolerances, symmetric filters with linear phase can minimize PRN-to-PRN variations and correlator peak asymmetry, respectively. Discriminator implementations should, where possible, should be matched to those of a reference (or master station) receiver. For GNSS receivers, mitigation by design could mean an increase in available high-accuracy signals usable for improving positioning accuracy.

Error budget

An error budget allocation is used to account for the residual bias that cannot be otherwise removed. To some extent, this will exist despite the use of any of the aforementioned mitigation techniques. Of course, this number should be as small as possible for best performance.

WAAS GEO Bias

The geostationary satellites (GEOs) for the Wide Area Augmentation System (WAAS) is an example of signal deformation resulting from a mixed signal design. The dissimilarity of the narrowband GEOs potentially results in large range biases. In response to this potential threat an analysis tool was developed to model the tracking errors users may experience based on their specific receiver design.[1][4]

The tool uses filter models for the WAAS GEOs—two narrowband and two wideband. The two MSAS GEOs, MTSAT 1 and 2, are modeled as well. For GPS, no filters are applied; however the tool compares the four GEOs against all current C/A code PRNs in order to find the worst case GEO-GPS bias with PRN variations taken into account. For input, the tool uses a lumped model of the users receiver filter characteristics—magnitude, phase (or group delay) as a function of frequency. In addition, the tool requires the user receiver discriminator type (early-minus-late or double-delta) and correlator spacing.

The tool was designed to reduce the error allocation for this range bias. Few options were available as the system was essentially complete when the bias was discovered. The limit was set to 5m for narrowband GEOs and to 50cm for wideband GEOs. (The former were more similar to GPS satellites.) The manufacturing tolerances on the receiver filters along with potentially other mismatched receiver discriminator attributes prohibited making the allowance any smaller than 50cm. This fundamentally limits range accuracy. Such poor performance would certainly be unacceptable to future GNSS users.

ANALYSIS

Code correlation models

The analysis of this paper assumes the incoming signals have been translated to baseband and are phase locked with zero phase error. To model a BOC(*n,m*) code at a chipping rate of *m**1.023MHz, the following equation may be used

$$c_{n,m}(t) = c_{\mathrm{m}}(t) \cdot \mathrm{s}_{\mathrm{n}}(t) \tag{1}$$

where $\mathrm{s}_{\mathrm{n}}(t)$ is a square wave of frequency *n**1.023MHz. For simplicity, in this paper, the GPS PRN1 was used for $c_{\mathrm{m}}(t)$. A square wave was then modulated onto it at a frequency of *n**1.023MHz.

Code distortion may be analyzed by examining the autocorrelation functions. The ideal autocorrelation function for na arbitrary GNSS signal ${}_{n,m}R(\tau)$ is given by

$${}_{n,m}R(\tau) = \int_{-\infty}^{\infty} {}_{n,m}H(f)\,{}_{n,m}C(f)\,{}_{n,m}C_R^*(f)e^{j2\pi f}df \tag{2}$$

where, ${}_{n,m}H(f)$ represents the transfer function of the combined filter that affect the incoming signal $C(f)$. For accurate modeling, ${}_{n,m}H(f)$ should include the filters on the satellite, the antenna and LNA, and inside the receiver. $C_R^*(f)$ is the complex conjugate of the power spectrum of the replica code.

The BOC(0,1) GPS signal is modeled as an ideal incoming signal (i.e., $C(f) = C_R(f)$) and Equation 1 becomes

$${}_{0,1}R(\tau) = R_{GPS}(\tau) = \int_{-\infty}^{\infty} H_{GPS}(f)C(f)C_R^*(f)e^{j2\pi f}df = \int_{-\infty}^{\infty} C(f)C_R^*(f)e^{j2\pi f}df \tag{3}$$

For a given user receiver with front-end filter characteristics captured by $H_{USER}(f)$, this becomes

$$R_{USER,GPS}(\tau) = \int_{-\infty}^{\infty} H_{USER}(f)C(f)C_R^*(f)e^{j2\pi f}df \tag{4}$$

For a GEO signal, Equation 1 becomes

$$R_{GEO}(\tau) = \int_{-\infty}^{\infty} H_{GEO}(f)C(f)C_R^*(f)e^{j2\pi f}df \tag{5}$$

And for an arbitrary GNSS signal, Equation 1 becomes

$${}_{n,m}R(\tau) = \int_{-\infty}^{\infty} {}_{n,m}H(f)C(f)C_R^*(f)e^{j2\pi f}df \tag{6}$$

After receiver filtering, this equation becomes

$$R_{USER,GEO}(\tau) = \int_{-\infty}^{\infty} {}_{n,m}H_{USER,GNSS}(f)\,{}_{n,m}H(f)C(f)C_R^*(f)e^{j2\pi f}df \tag{7}$$

In the above equations, ${}_{n,m}H(f)$ and ${}_{n,m}H_{USER,GNSS}$ represent the transfer functions of the GNSS satellite and user receiver for the BOC(*n,m*) signal of interest, respectively.

Satellite and Receiver Filter models

For this paper only infinite bandwidth, ideal "brick wall" rectangular filters were modeled. The center frequencies (f_c) and bandwidths of these filters are found according to the respective ICD specifications of the following full-bandwidth signals: GPS C/A code, GPS-L5, and Galileo.

All of the current and envisioned GNSS signals except for the E5a/b signal were filtered using a rectangular filter of magnitude 0dB for f_c-20≤bw≤ f_c+20 and -200dB otherwise. The E5a/b signal for Galileo, however, is a BOC(15,10) code and is 90MHz wide. The first filter applied to it had a magnitude of 0dB for (${}_{n,m}f_c$)-45≤bw≤(${}_{n,m}f_c$)+45 and -200dB otherwise. To single out the E5a signal, a secondary filter was applied at a frequency offset of (${}_{15,10}f_c$)-15*1.023MHz and a bandwidth of 45MHz was also applied. The transition band attenuation for this filter was 30dB per octave. For simplicity, no group delay effects were modeled in this analysis; however, this is an added design variable that will need to be included in subsequent error analyses.

Tracking error models

Assuming coherent tracking and negligible phase error, the steady-state tracking error for an early-minus-late discriminator about the equilibrium point is given by equation 8 below.

$$_{n,m}\tau = \arg\left[{}_{n,m}R_{d(\bullet)}\left(\tau - \frac{d}{2}\right) - {}_{n.m}R_{d(\bullet)}\left(\tau + \frac{d}{2}\right) = 0 \right] \tag{8}$$

A comparison of how this discriminator compares to other implementations is not included in this paper.

Summary of Assumptions

Code Modulation and SV Filtering

- Perfect code modulation transitions (i.e., no nominal digital lead/lag distortions present)
- Ideal, non-distorting filters on the transmitted codes;.
- For simplicity, ${}_{n,m}H(f)$ from Equation EE is replaced by a "brick wall" filter having 40MHz bandwidth and zero group delay; this simulates an ideal, non-distorting (although bandlimited) SV signal. This assumption can only be modified with better knowledge of the GNSS satellite filter transfer functions.
- Ideal GNSS generalized autocorrelation peaks were formed from C/A PRN 1 and modulated with a square wave appropriate to mimic the following binary offset carrier GNSS signals.

User Receiver Code Tracking

- The tracking error bias was measured for correlator spacings relative to an ideal, undistorted correlation peak was modeled. No measurement noise or multipath errors were considered. Correlator spacings will be given in nanoseconds (ns).
- Perfect, noise-free EML code tracking of only the central peak of all correlation functions. No off-center peak tracking errors or code discriminator implementations were considered. The carrier loop was assumed to be phase-locked and have zero phase error.

User Receiver Filters

The following two user receiver filters were examined:

- 20MHz SAW filter: This is the same as example that accompanies the GEO tool itself. It is an actual filter model (obtained from a filter manufacturer) and is believed to be fairly typical of conventional wideband GPS receivers. It delivers acceptably small GEO bias results for most users.
- 20MHz Chebyshev (Type 2). This filter is a purely analytical IIR filter model. Its magnitude and phase characteristics as well as its GEO bias results indicate it would be an acceptable filter for aviation receivers. However, because it is an analytical filter, it has perfectly symmetric magnitude and group delay responses.

Each filter was equally applied to all incoming signals. Note, however, that actual multi-frequency receivers will likely employ multiple filters to reduce out-of-band noise and interference. In general, each of these filters will have slightly different characteristics.

Figures 2 and 3 show the magnitude and group delay responses of each of these filters. The asymmetries of the SAW filter contrast with the smooth regularity of the Chebyshev. Both, however, yield acceptably small GEO bias results (for appropriate discriminator spacings) and are acceptable for aviation users.

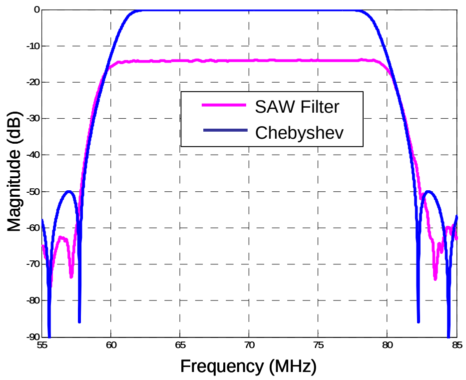

Figure 2. Magnitude response comparison of SAW and Chebyshev filter.

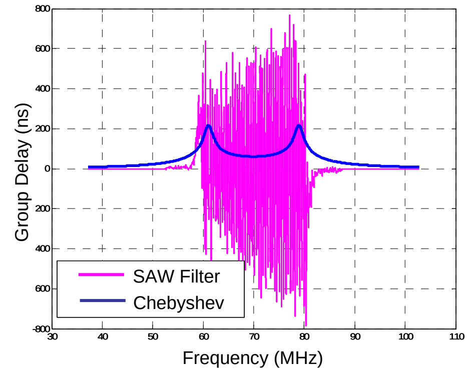

Figure 3. Magnitude response comparison of SAW and Chebyshev filter.

RESULTS

Each of the following GPS and assumed Galileo code autocorrelations were modeled for the digital-only and analog-only failure modes:

- C/A Code BPSK or BOC(0,1) centered on 1575.42MHz; 40MHz bandwidth
- L5 and P(Y) code: BPSK or BOC(0,10) centered on 1176.45 ; 40MHz bandwidth
- E5A/E5B: 2 x NPSK (10.23 Mcps) or BOC (15,10) between 1164-1215 MHz
- E6: BPSK (5.115 Mcps) & BOC (10,5) centered at ~ 1279 MHz (1260-1300 MHz)
- E2/L1/E1 centered at 1575.42 MHz (1559-1592 MHz)
  - BOC (1,1) Open Service (OS)
  - BOC (15,2.5) cosine phased Public Regulated Service (PRS)

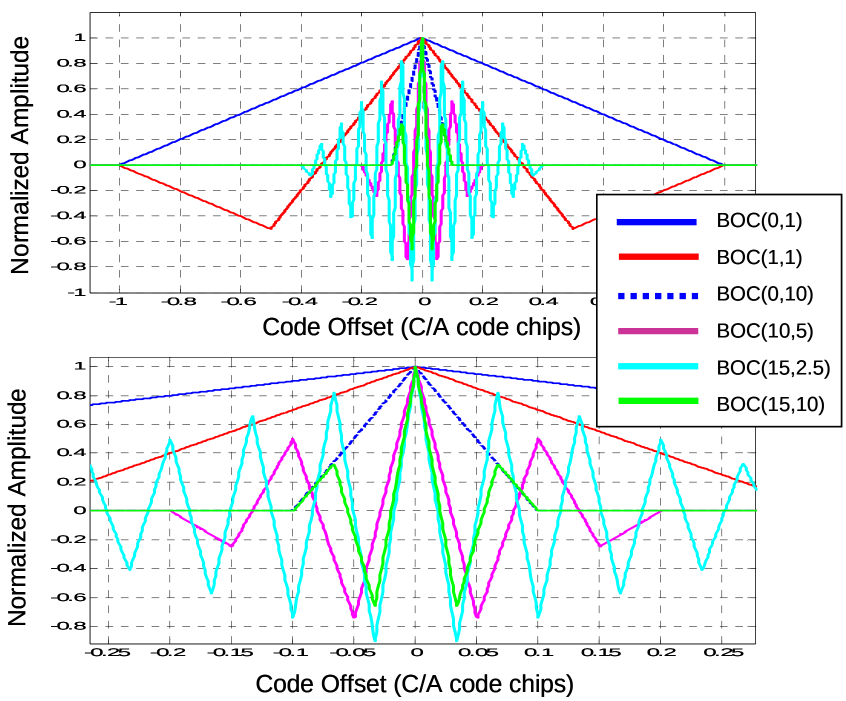

Figure 4. Six BOC(n,m) correlation peaks plotted relative to BOC(0,1)—C/A code

The nominal autocorrelation peaks for each of the six types are shown in Figure 4. For comparison, they are each plotted as a function of C/A code chip offset.

Tracking Errors

The modified GEO Bias tool was used to generate tracking error estimates. The results are all referenced to GPS C/A code and are plotted vs. EML correlator spacing in nanoseconds. Although figures 5 and 6 plot the computed, "absolute" tracking error output from the modified GEO bias analysis tool, only the relative tracking error is significant. In subsequent plots, the tracking error for GPS PRN1 at the narrowest correlator spacing computed (0.05 chips or about 49ns) is set equal to zero; all other GNSS results plotted were referenced to this.

Figure 5 plots the modified GEO bias tool output for all the 32 GPS C/A plus (9) SBAS PRN codes for a user having a 20MHz SAW filter. The output corresponding to an ideal BOC(1,1) code autocorrelation is plotted for comparison. The PRN-induced relative tracking error differences for GPS vary from as little as 10cm for narrow correlator spacings to greater than 1 meter for the widest correlator spacings.

Similar results corresponding to exact PRNs for BOC (1,1) are not shown, however the variation would be lessened if the Galileo master station corrections are each referenced to the BOC(1,1) code. (This is generally not the case for GPS where the GPS Operational Control Segment references the L1 clock corrections to the P(Y) codes. So the C/A code deformations are not generally correlated with them.) Still, the trend of the curve is similar to that for the BOC(0,1), and it is likely that narrower spacings produce smaller relative errors for that modulation as well. A future incarnation of this model tool will incorporate the specific PRNs for other modulations to see these effects more clearly.

Figure 7 shows the PRN-dependency result for a Chebyshev (Type 2) filter. There are no PRN-to-PRN variations visible here since the filter magnitude and group delay responses are symmetric. (See Figures 2 and 3.) When this filter is applied to all the signals, the relative frequency domain characteristics of the signals are preserved. In other words, although the filter makes the correlation peaks asymmetric, it makes them all similarly asymmetric. Note that SAW filters are more likely to be used in receivers, however.

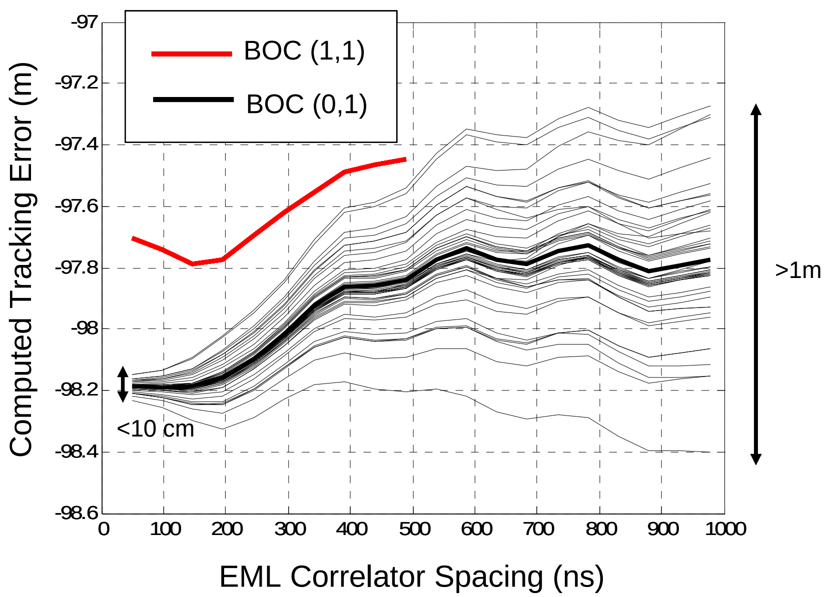

Figure 5. Computed tracking errors of an ideal BOC(1,1) various GPS C/A code PRNs for an actual 20MHz SAW receiver filter.

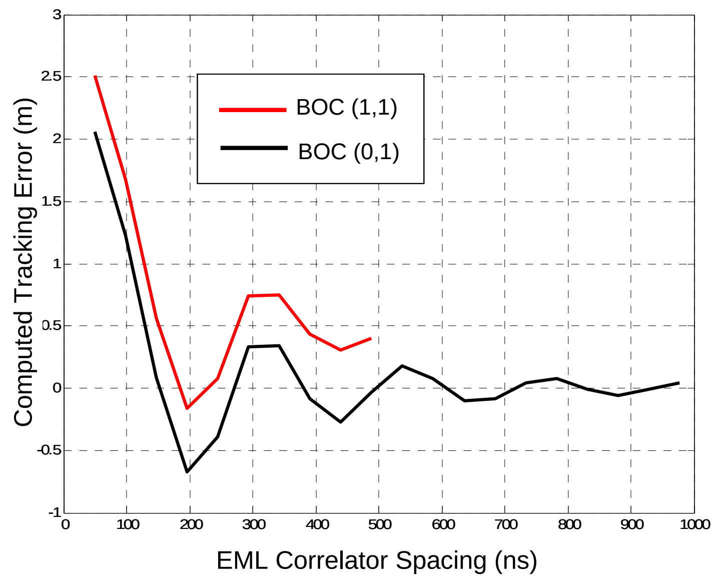

Figure 6. Computed tracking errors of an ideal BOC(1,1) various GPS C/A code PRNs for an analytical 20MHz Chebyshev receiver filter.

Figure 8 plots the tracking errors for 5 modernized GNSS codes relative to GPS PRN1 where each code autocorrelation has been filtered by the SAW filter. There are biases present that vary which correlator spacing, however there are few practical correlator spacings possible for codes with higher modulations. (As previously shown in Figure 4, the higher chipping rates make the peak extremely narrow.) Assuming that a future GNSS clock correction will apply to a single correlator spacing, it is unlikely that mismatched receiver correlator spacings will keep the total variation (on BOC(n,*m*) modulations with, $m > 1$) to less than 10cm. Note that the BOC(1,1) varies by about 30cm over its range of spacings (with a trend similar to that of GPS PRN1). Mismatched correlator spacings has a more significant impact for this modulation.

Careful selection of correlator spacings could also be used to reduce the residual biases. For example, biases between BOC(1,1) and BOC(0,10) are smallest when the BOC(0,10) spacing is 100ns and the BOC(1,1) spacing either 140ns or 200ns. Ignoring PRN variations, these spacings are also well-paired with BOC(0,1) at correlator spacings above 500 ns (i.e., wider than 0.5 chips). If the BOC(1,1) spacing is larger than 350ns, the biases to BOC(10,5), BOC(15,2.5) and BOC(15,10).

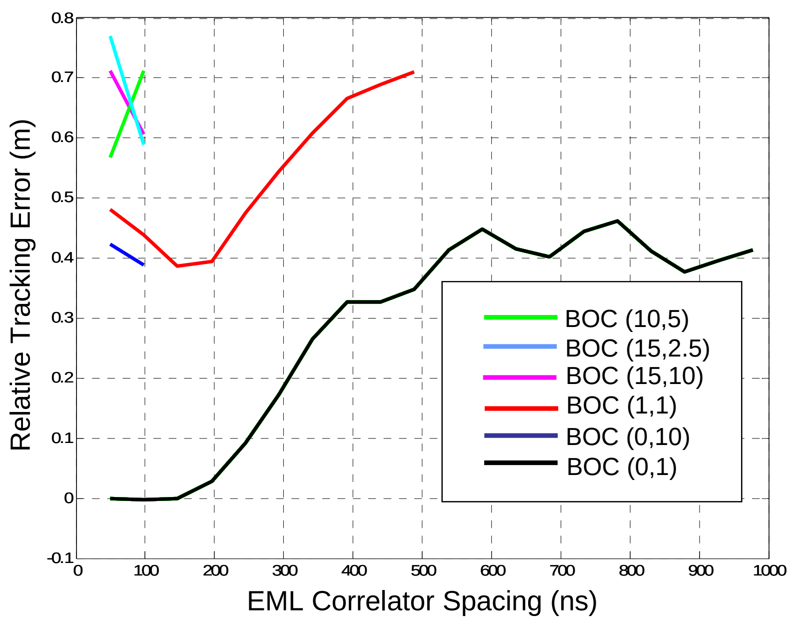

Figure 8. Tracking errors of GNSS code modulations relative to GPS PRN 1 for the SAW filter. (Only practical, central peak correlator spacings are plotted for each.)

Figure 9 plots the tracking errors for 5 modernized GNSS codes relative to GPS PRN1 where each code autocorrelation has been filtered by the analytical Chebyshev filter. Here, the biases relative to GPS PRN1 (BOC(0,1)) are representative of the other GPS PRNs as well. The tracking errors from one GNSS signal to the other are generally larger, however. Even after clock corrections are applied, the correlator spacing-dependent variations are larger than they were for the SAW filter.

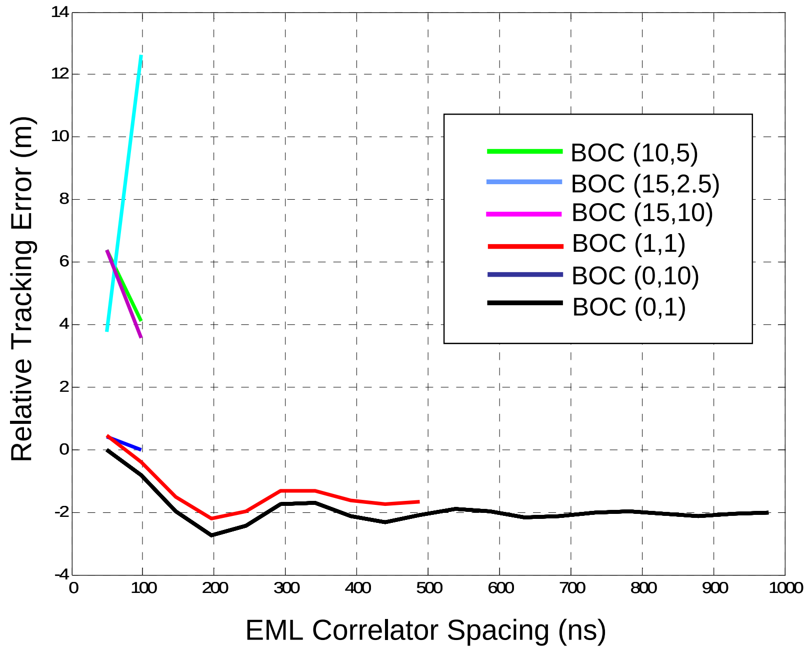

Figure 9. Computed tracking errors of an ideal BOC(1,1) various GPS C/A code PRNs for a 20MHz Chebyshev receiver filter. (Only practical, central peak correlator spacings are plotted for each.)

For this filter, there are relatively few correlator spacing combinations that minimize the biases. BOC(1,1) and BOC(0,10) are nearly matched when they are both tracked with a spacing of about 50ns. However, if BOC(0,1) is tracked at a spacing approximately 50ns less than that for BOC(1,1), the biases between those signals are minimized. No spacing selection can reduce the biases between the aforementioned modulations and BOC(10,5), BOC(15,2.5) and BOC(15, 10).

CONCLUSION

Satellite clock and inter-frequency bias corrections will mitigate a significant portion of tracking error biases on GNSS signals. Those that remain will be due to satellite signal generation variability, modulation differences, PRN codes, and receiver design implementations.

Bias estimation techniques can be used to mitigate common mode biases that cannot be removed by design but they come at the expense of a single ranging source that might otherwise be used to improve position accuracy. Also, PRN-to-PRN biases cannot be removed by estimation, and could be significant if PRN-specific corrections are not applied. Calibration is the most attractive mitigation solution in cases where the expected hardware tolerances, aging, and temperature variations are small. Because the signals are not currently, online, "intelligent" receiver design presents another alternative to reduce the biases. Although it is difficult to find a design that mitigates the biases to all signals simultaneously, for some receivers, tradeoffs can be made.

The results presented herein are compelling, but they are far from complete. They indicate a need to more rigorously analyze the effects of all of these potential error modes—particularly those of PRN and tracking implementation differences—for the proposed modernized signals. This type of analysis reveals that in order to reliably achieve the highest degree of position accuracy, a combination of all of the aforementioned mitigation methods may be required.

REFERENCES

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