The Multipath Invariance Approach for Code Multipath Mitigation

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BIOGRAPHY

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ABSTRACT

The concept of multipath invariance asserts that there exist properties and/or regions of the code correlation function that do not change as a function of the multipath amplitude, delay, phase and phase rate. These Multipath Invariant (MPI) points exist at the plateaus of the correlation function. Previous experiments have demonstrated that despite low correlation power, it is possible to implement the Tracking Error Compensator (TrEC) algorithm, which uses these MPI points to mitigate multipath. In addition, it has been previously asserted that TrEC not adversely affected by finite front-end bandwidth considerations and is potentially very useful for mitigating multipath in extremely narrowband receivers.

To evaluate the performance of the MPI approach, the TrEC algorithm was implemented in a narrowband receiver. Using code double-differences over a short baseline between two rooftop antennas, real-time position errors were recorded using live satellites. Three different multipath scenarios were tested. The results were compared with and without multipath compensation and also with and without carrier smoothing applied to the pseudoranges. For the last case—a large multipath scenario—(wideband) narrow correlator receiver performance was compared to (narrowband) TrEC performance. Significant reductions in multipath errors were attained through using this novel approach. Specifically, in all cases TrEC was significantly more effective at reducing the mean bias errors due to multipath than the other techniques.

BACKGROUND

Conventional approaches to multipath mitigation attempt either to estimate the parameters of the multipath and correct for it (e.g., MEDLL [1], MRDLL [2]) or to separate the desired (line-of-sight) from the combined signal (e.g., the Narrow Correlator [3], Strobe Correlator [4], MET [5], etc.).

These multipath mitigation approaches utilize a plurality of correlators to oversample the peak of the distorted received correlation function. However, since the peak is corrupted (distorted) by multipath and the received signals are always bandlimited, the best theoretical multipath performance of these techniques is fundamentally limited.

Few current techniques are well suited for mitigating multipath in so-called “narrowband” receivers. The high-resolution correlator techniques (e.g., Strobe Correlator, etc.) may boast good theoretical performance curves for narrowband receivers [6]. However, even these techniques are most frequently implemented on receivers with wide precorrelation bandwidths. (They likely require larger discriminator gains and better noise performance than what may be afforded by narrowband receivers.)

There are only very marginal multipath performance advantages to using narrow correlator spacings inside narrowband receivers [7]. Still, this implementation does not degrade the receiver noise performance [8]. The multipath tracking error envelopes for this situation (and for the “best” wideband tracking conditions) are summarized in Figure 1 below.

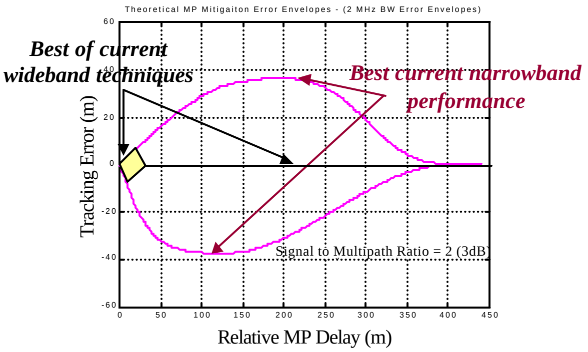

Figure 1. Wideband and narrowband theoretical multipath performance bounds (maximum tracking errors).

BACKGROUND

The multipath invariance approach asserts that the plateaus of the correlation functions do not change as a function of the multipath parameters. Although the peak and sidelobes of the correlation function do become distorted due to unknown, varying multipath conditions, the plateaus do not.

Multipath Invariant (MPI) points are defined as those locations on the correlation functions that at the edges of (on the late side) the correlation plateaus. In most cases these points also lie immediately adjacent to (on the early side of) the peak and/or sidelobes of the correlation function. Since multipath signals always arrive later than the line-of-sight, these locations remain virtually unchanged by the incident multipath. (More details about the MPI assumptions can be found in [9].)

Because the correlation functions for all the GPS satellites are known a priori, so are the ideal offsets of these MPI points relative to the main lobe. Assuming the receiver is tracking a given satellite, by locating the MPI points using one or more additional correlator pairs, the measured distance to the code tracking loop (DLL) correlators may be found. The Tracking Error Compensator (TrEC) corrects for the code tracking error (due to code multipath and thermal noise), by adding the difference between the measured and ideal distances from the MPI point to the DLL to the nominal pseudorange measurement. (See Figure 2 below.)

Despite the relatively low correlation power at these plateaus, it is already known that these MPI points can be located in real time with fidelity [7,10]. In addition, the theoretical performance of TrEC (see Figure 3) was experimentally validated using a GPS signal generator. However, it was revealed that a potentially substantial initialization time was required to locate the MPI points [10]. Also, previously little data was taken using satellites at low signal powers.

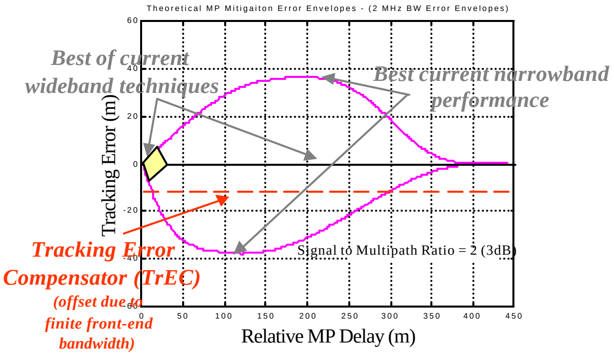

Figure 3. TrEC theoretical multipath performance bounds (maximum multipath pseudorange errors).

VALIDATION METHODOLOGY

Pseudorange performance plots may not necessarily reveal the multipath mitigation performance of a given technique. Though validation of these plots is possible, it may sometimes be challenging to completely remove the effects of other pseudorange errors (e.g., atmospheric effects). Consequently, in many cases a signal generator or variable-length delay-line is used to validate the tracking error envelopes [10,11]. This, however, may call

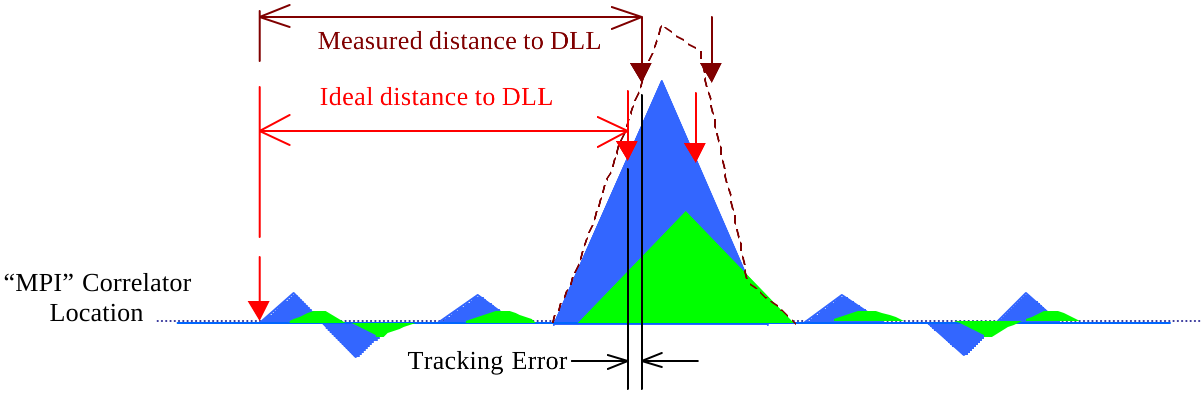

Figure 2. Tracking Error Compensator (TrEC): Updating the primary DLL tracking solution with the corrected, relative position to the MPI point.

into question the capability of a particular method to mitigate multipath on actual satellites in real time.

The theoretical performance plots for TrEC contain biases that only (potentially) manifest themselves as errors in the position domain [7,10]. This may be the case for many other techniques as well. A better assessment of the capabilities of many MP mitigation approaches can be made by examining position error results from live satellites.

Position error (reduction) comparisons permit a "bottom-line" assessment of the relative performance of various MP mitigation techniques. Unlike for pseudorange error analyses, position "truth" measurements can be relatively easily determined unambiguously to sub-centimeter accuracy. In addition, by using results from live GPS satellites real-time tracking, low signal power performance, and initialization time issues may be readily examined and compared.

EXPERIMENTAL SETUP

The TrEC algorithm was implemented on a 12-channel, narrowband (2MHz precorrelation bandwidth, 0.5-chip correlator spacing) receiver. Six channels were used to simultaneously track a total of six GPS satellites. The other available correlator pairs (from the remaining six channels) were used to locate the MPI points of each of those satellites [9]. Real-time tracking errors were subsequently estimated for each pseudorange.

Code phase double-differences were performed for the measurements taken at two (surveyed) antennas spaced 154.08 meters apart. Because the two antennas had a relatively small spatial separation, any residual atmospheric errors (along with the satellite and receiver clock biases) were negligibly small [12]. The only significant position errors that remained in the 3-dimentional double-difference position solutions were due to multipath and thermal noise.

For comparison, the following four different "modes" of receiver multipath mitigation performance were compared:

1) "Code Only" – nominal DLL code tracking performance, 0.5-chip spacing
2) "Carrier Smoothing Only" – carrier smoothed (120-second time constant), "Code-Only" measurements
3) "TrEC Only" – TrEC corrections applied to nominal Code Only pseudoranges
4) "TrEC Smoothed" – Carrier smoothing (120-second time constant) applied to TrEC-corrected pseudoranges

Note that the mean bias is frequently the most difficult multipath error component to remove. Time averaging methods (e.g., carrier smoothing operations) tend to have little effect on this error component. To make the carrier smoothing process more effective against the error variations, carrier aiding of the code tracking loops was disabled for all experiments. This permitted the nominal (unsmoothed) code tracking errors to remain as close to zero-mean as possible [7].

The four MP mitigation modes were evaluated under the following three different multipath scenarios (cases):

1) Case 1: Small amplitude, short relative delay (0~20m) "nominal" multipath
2) Case 2: Large amplitude, short relative delay (0~20m) multipath
3) Case 3: Large amplitude, Medium-long relative delay (0~120m) multipath

Each of these cases is described in more detail below.

Case 1: Small amplitude, short relative delay (0~20m) "nominal" multipath

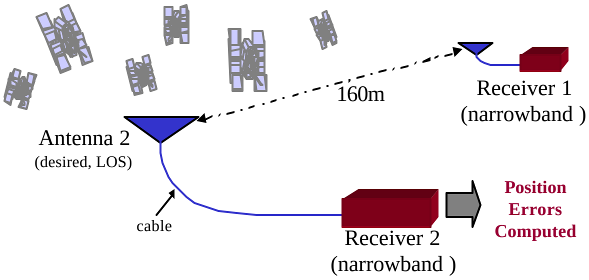

Figure 4. Experimental setup for Case 1.

The first scenario involved so-called "nominal" multipath that was normally incident on two rooftop antennas atop two different buildings on the campus of Stanford University. It was impossible to completely characterize the multipath at both of these locations under nominal conditions. Still, given the locations of most of the rooftop reflectors relative to each antenna, the majority of the incident multipath signals were presumed to have relatively short delay (less than 20 meters). The amplitudes of these reflections also were small.

These characterizations are consistent with the care taken during the siting of both of these antennas. The antenna connected to Receiver 1 also serves as the Stanford National Satellite Testbed (NSTB, or WAAS–testbed) reference station antenna. Antenna 2 (or 2a and 2b from Figures 5 and 6) normally serves as the Local Area Augmentation System (LAAS) testbed reference station antenna at Stanford. For signal quality monitoring (SQM) investigations, the distortion of the received correlation peak due to nominal multipath (and thermal noise) at both of these sites has previously been characterized [13].

Case 2: Large amplitude, short relative delay (0~20m) multipath

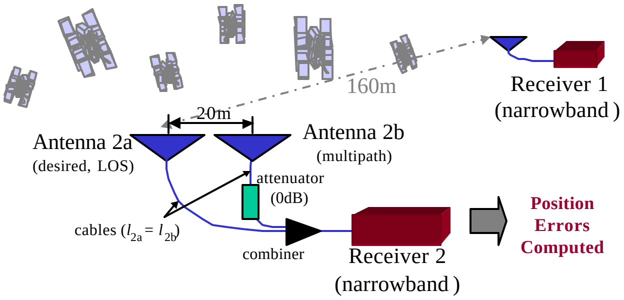

Figure 5. Experimental setup for Case 2.

To ensure the multipath had short delay relative to the line-of-sight (LOS), the signals from two nearby (LAAS testbed) antennas were combined. (See Figure 5.) The antennas were 20 meters apart, and the cables had approximately equal length. The "multipath" signal (Antenna 2b in Figure 5) was not attenuated with respect to the line-of-sight signal.

For satellite at 90° elevation and/or in the vertical plane equidistant from each antenna, the code tracking loops of the receiver perceived the received signal as a LOS signal combined with an equivalent reflection at a relative delay of approximately 0 meters. The effective relative delay (magnitude) could reach as large as 20 meters (for a satellite both in the plane of and along the collinear azimuth direction of both antennas). The fading frequencies of each of the combined signals varied for each of the satellites as they traversed the sky.

Note that the "nominal" multipath—characteristic of Scenario 1—was also present for this case (and for Case 3). Since the multipath between Antenna 2a and Antenna 2b (as well as Antenna 1) was independent, there was effectively *more than one multipath reflection* on each pseudorange. Of course, the dominant multipath source was still the single "multipath" signal entering Antenna 2b.

Case 3: Large amplitude, Medium-long relative delay (0~120m) multipath

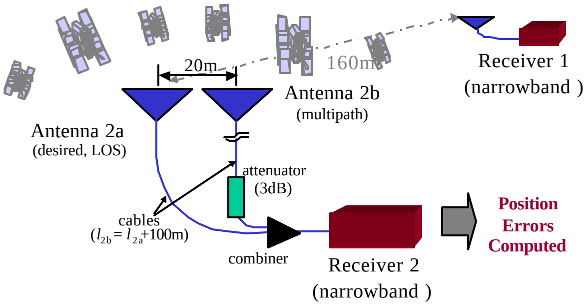

Figure 6. Experimental setup for Case 3.

To evaluate the ability of the TrEC algorithm to compensate for extremely large tracking errors, a 100-meter length of cable was added to the Antenna 2b signal path. (See Figure 6.) As in Case 2, the fading frequencies and elevation angles of each of the satellite signals varied independently over time. To prevent frequent loss of lock, the signal-to-multipath ratio for this case was set to 3dB. All other setup parameters remained the same as in Case 2.

An additional receiver "mode" comparison was made for Case 3. A pair of wideband, narrow correlator receivers was evaluated in parallel with the TrEC-enabled narrowband ones. (This will henceforth be referred to as the "Narrow Correlator" mode.) These wideband receivers were identical; each had 12 available channels. Although all 12 channels were enabled, for comparison the code double-difference position errors for these receivers were computed using only the same six satellites tracked simultaneously by their narrowband counterparts. Also, although was impossible to completely disable carrier aiding for these receivers, the carrier aiding time constant for these receivers was minimized.

Figure 7 illustrates the "best" tracking errors to be expected for a conventional (0.1-chip) code DLL subjected to half-power multipath at the maximum relative delays evaluated by all three cases. For Case 3 the maximum MP tracking error bounds for each pseudorange were nearly maximized for the out-of-phase multipath.

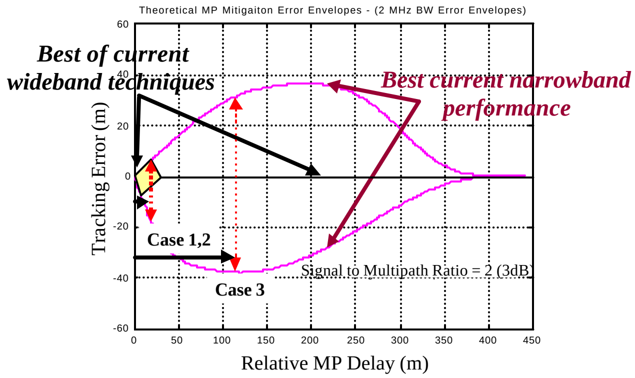

Figure 7. Theoretical maximum tracking error bounds for experimental Cases 1, 2 and 3.

Note that for Cases 2 and 3 the conventional DLL theoretical bounds were likely somewhat worse than those predicted by Figure 7. For Case 2, the signal-to-multipath ratio (SMR) was approximately 0dB. Consequently the maximum tracking error for that case may have exceeded the bounds of Figure 7. For Case 3, since the correlator spacing of the narrowband receiver under test was actually 0.5 chips (instead of 0.1-chips), the actual maximum code tracking error per channel for that case was slightly larger as well [7].

EXPERIMENTAL RESULTS

For each of the three multipath scenarios, the following (six) plots were generated:

- Signal Power (C/N_0, dB-Hz) vs. Time (minutes) for all 6 satellites tracked
- Elevation Angle (degrees) vs. Time (minutes) for all 6 satellites tracked
- TrEC-estimated DLL Corrections (meters)—applied to all 6 pseudoranges for both receivers 1 and 2—vs. Time (minutes)
- Position Error (Magnitude, meters) vs. Time (minutes) for the Carrier Smoothed Only, TrEC Only, and TrEC Smoothed receiver modes only.
- Error Statistics: Mean, Standard Deviation (1-σ), and Root-Mean-Square (RMS) for Code Only, Carrier Smoothed Only, TrEC Only, and TrEC Smoothed receiver modes.

Since code-double differences were taken using only 6 satellites (or five effective measurements), the position error computation was fairly sensitive to satellite dropouts. Consequently a few brief outages in the data can be seen where one of the receivers lost lock on a satellite. In all cases, loss of lock occurred due to low satellite elevation angle and/or excessive signal power variations caused by multipath. No changes were made to affect the normal tracking performance of the receivers.

The error traces are shown only for the most continuous set of data. Statistics were computed, however, for the length of the entire steady-state data set. Accordingly, the error statistics presented may include some position error data that are not shown in the respective preceding plots.

Case 1: Small amplitude, short relative delay (0~20m) "nominal" multipath

The plots of the signal powers and respective elevation angles of the six satellites tracked are given below in Figure 8. Two satellites dropped out of view during the time period shown. Two other satellites were reacquired shortly thereafter. The increase in C/N_0 variations for the descending (and rising) satellites illustrates the increased multipath on these signals at the lower elevation angles.

The TrEC-estimated DLL corrections for each pseudorange as measured by both receivers are shown in Figure 9. The peak-to-peak magnitude of these corrections range from about 3-20 meters (for high and low-elevation satellites repectively). The offset of the six traces corresponds to the common-mode filter bias TrEC on each pseudorange. (Refer to Figure 3.)

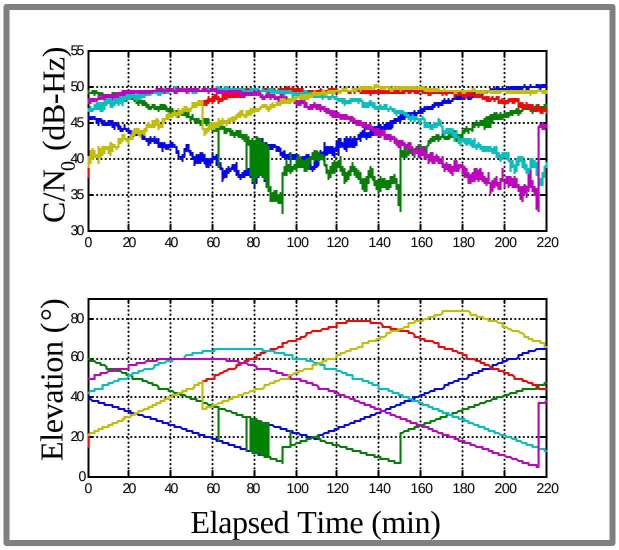

Figure 8. Satellite signal powers and elevatiion angles for Case 1.

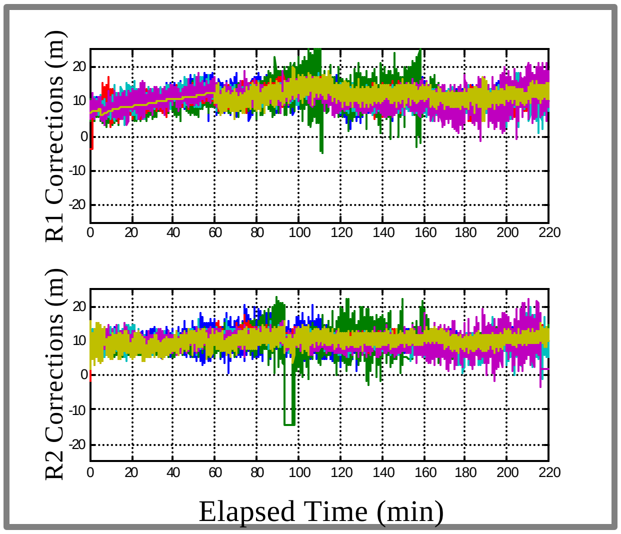

Figure 9. TrEC-estimated DLL corrections for Case 1.

The position error results for the Carrier Smoothing Only, TrEC Only, and TrEC Smoothed modes are plotted below in Figure 10. (The Code Only position errors were not plotted since they would have obscured the other error traces.) The plot reveals that the TrEC Only and TrEC Smoothed traces had a maximum error greater than that for the Carrier Smoothed Only mode. This occured as TrEC reinitialized after a new satellite came into view and was acquired by the two receivers [10].

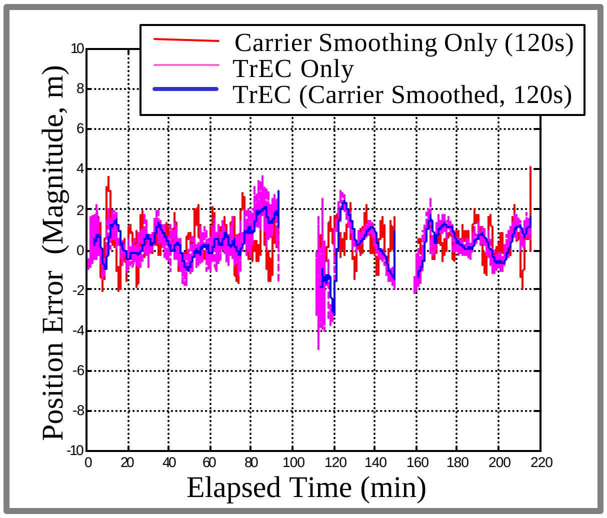

Figure 10. Position error comparison for Case 1.

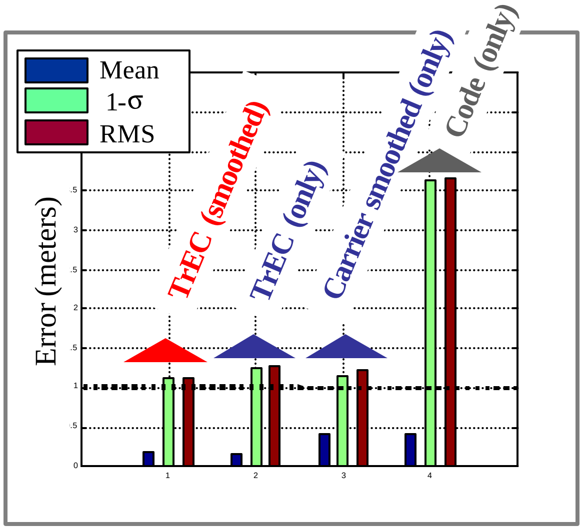

Figure 11. Position error statistics for Case 1.

The Case 1 summary statistics are provided in Figure 11. Clearly the Code Only receiver mode resulted in the worst 1-σ and RMS errors. Carrier smoothing dramatically reduced these two error statistics, but did little to reduce the mean bias. The two TrEC modes did not significantly change the 1-σ and RMS errors from the Carrier Smoothing Only results. The TrEC corrections—even without carrier smothing—did, however, *significantly reduce the mean position errors* due to multipath by more than a factor of two. Also note that since TrEC was actually correcting for code noise and multipath, the TrEC Smoothed statistics are compatable to those for the TrEC Only mode.

Case 2: Large amplitude, short relative delay (0~20m) multipath

The signal powers (for Receiver 2) and corresponding elevation angles of the tracked satellites are plotted below in Figure 12. The large-amplitude "multipath" signals combined to produce 3-4dB oscillations in the received C/N_0 of each signal. (Figure 13 illustrates the corresponding increased activity of the TrEC corrections for Receiver 2.) The frequencies of these oscillations varied as a function of the satellite trajectories through the sky. One satellite dropped from view of both receivers—due to low elevation angle and high multipath amplitude variations—at the end of the (approximately) 100-minute time interval shown.

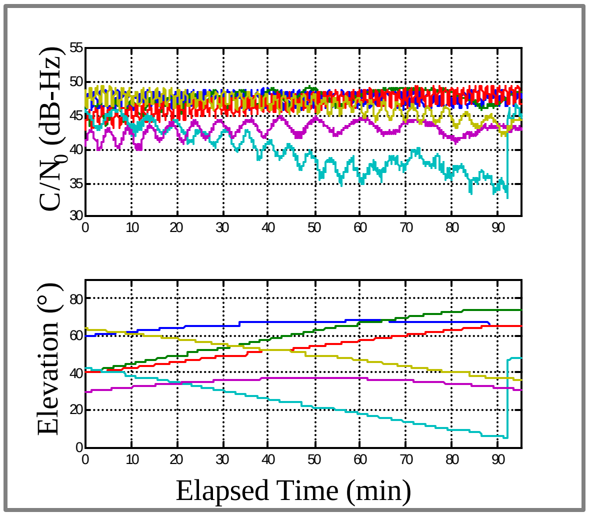

Figure 12. Satellite signal powers and elevation angles for Case 2.

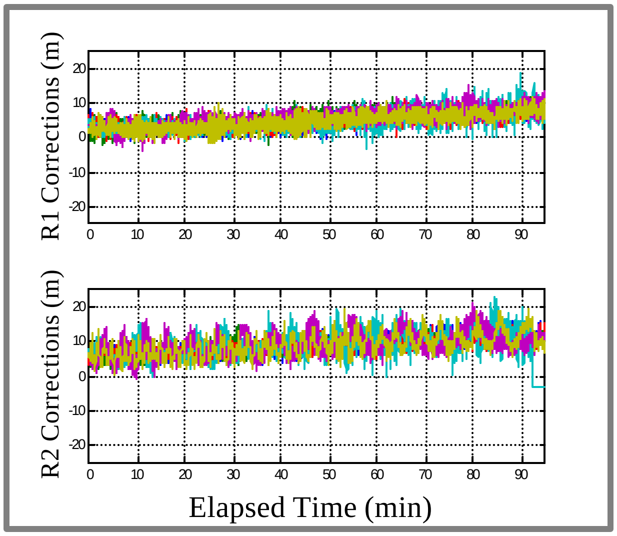

Figure 13. TrEC-estimated DLL corrections for Case 2.

The posirtion errors shown in Figure 14 illustrate that carrier smoothng alone was not capable of mitigating the low-frequency multipath tracking errors. The maximum errors for the Carrier Smoothing Only mode reached as high as 7-8 meters.

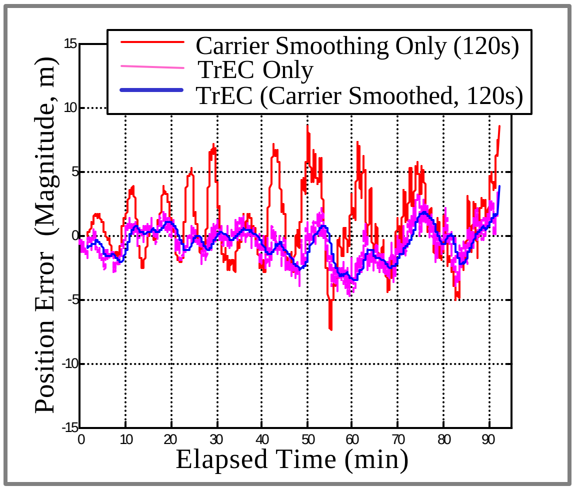

Figure 14. Position error comparison for Case 2.

Again, the Code Only mode resulted in the worst position errors. (See Figure 15.) Carrier Smoothing Only reduced those 1-σ and RMS errors from over 4.5 meters to approximately 2 meters, but it did little to affect the mean error. The TrEC Only implementation reduced the 1-σ and RMS slightly more than did the Carrier Smoothing Only mode. More significantly, the TrEC reduced the mean bias by almost a factor of 5 (as compared to the Code Only or Carrier Smoothing Only modes). In fact, the TrEC mean errors for this case are comparable to those from Case 1.

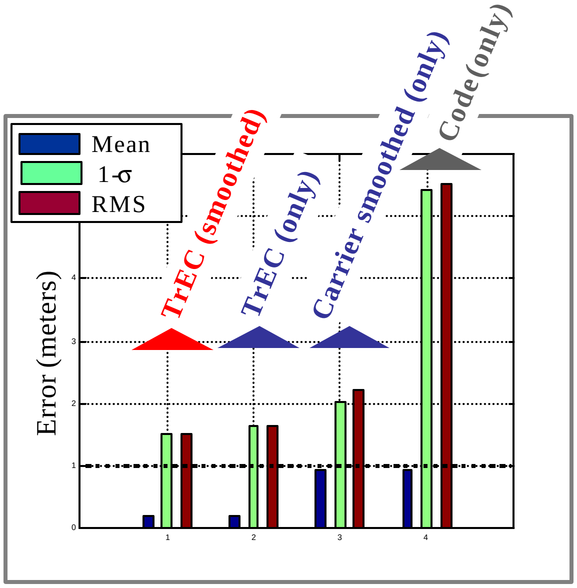

Figure 15. Positrion error statistics for Case 2.

Case 3: Large amplitude, Medium-long relative delay (0~120m) multipath

As shown in Figure 16, the long-delay multipath signal caused signal power variations ranging as much as 7-8dB on every received signal. Wide fading frequency variations were present on every channel. One satellite dropped from view (and another was subsequently reacquired) during the time period shown. The TrEC-estimated corrections (shown in Figure 17) for every pseudorange varied between 60-80 meters for Receiver 2.

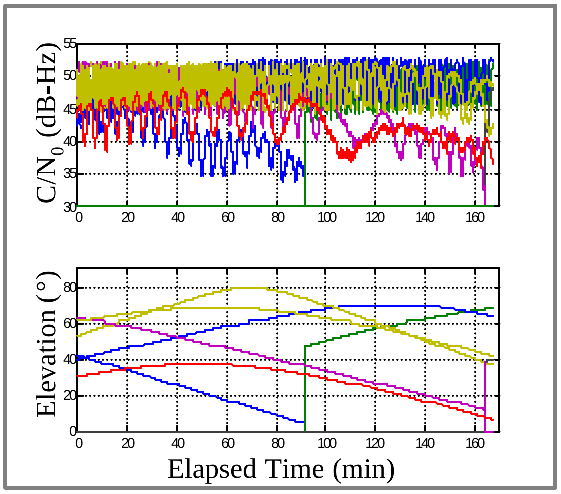

Figure 16. Satellite signal powers and elevation angles for Case 3.

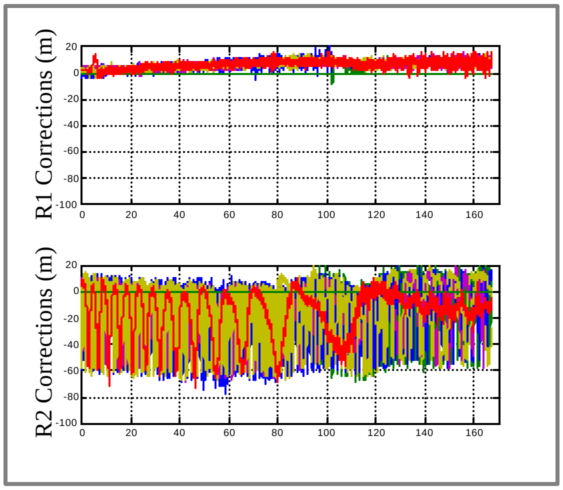

Figure 17. TrEC-estimated DLL corections for Case 3.

The position errors for the Carrirer Smoothing Only mode became as large as 250 meters. (See Figure 18.) That maximum error was reduced to less than 30 meters for the TrEC modes. Even the maximum position errors for the Narrow Correlator mode—which contained a significant negative bias—were noticeably larger than for either of the TrEC modes.

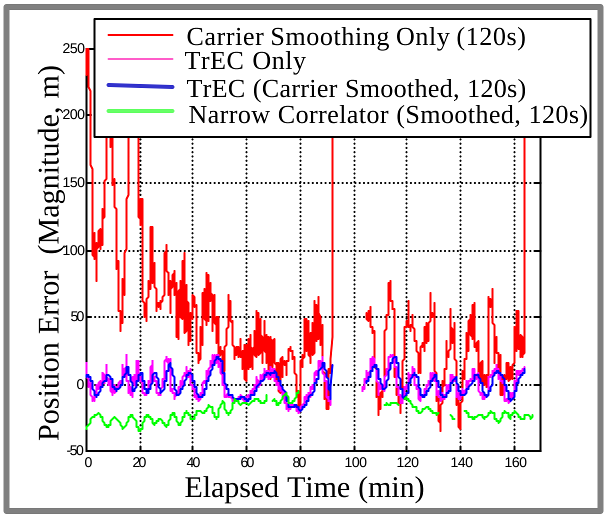

Figure 18. Position error comparison for Case 3. (Narrow Correlator results also shown.)

The statistical error diffferences (shown in Figure 19.) were even more pronounced. Code Only 1-σ and RMS errors were as large as 70-80 meters. Carrier Smoothing Only 1-σ and RMS errors, although somewhat smaller, still ranged between 40-55 meters. Once again, carrier smoothing did little to affect the mean positon error.

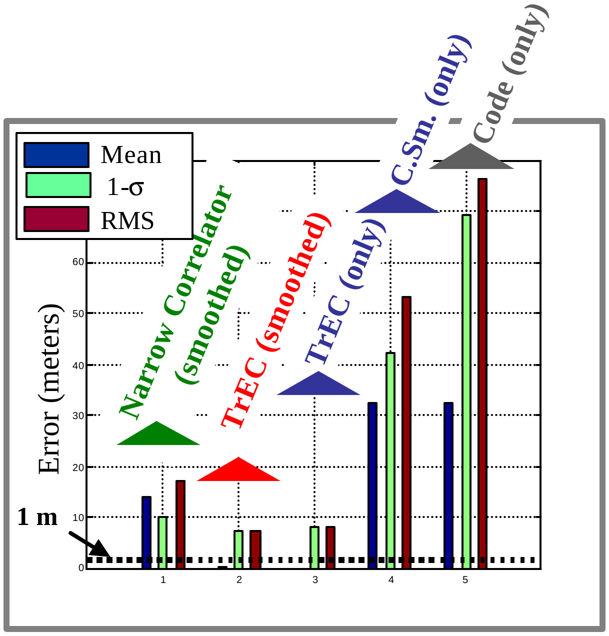

Figure 19. Position error statistics for Case 3. (Narrow Correlator results shown.)

The TrEC Only and TrEC Smoothed modes both had 1-σ and RMS errors less than 10 meters and a had mean well below the 1 meter reference line. (Note that this extremely small mean is at least partly due to the sinusoudal nature of the position errors for this case. Still, the relative advanges of TrEC are clear.) The Narrow Correlator receivers easily outperformed the Code Only and Carrier Smoothing Only modes of the narrowband receiver. Still, this wideband technique had significantly larger errors for all statistics than either of the TrEC modes.

Summary

The statistical error reduction factors for all modes evaluated (as compared to the Code Only mode) are plotted in Figure 20 for all three cases. The smallest multipath error reduction factor achieved by the TrEC implementations occurred for Case 1. (This is intuitive since the least multipath was present to mitigate in that case.) For that scenario, the error mean was reduced by a factor of 2.4. The narrow correlator was achieved a comparable mean error reduction factor for Case 3—under significantly more severe (large amplitude and delay) multipath conditions. In that case, however, the TrEC Only and TrEC Smoothed reduction factors were many times larger.

The minimum TrEC reductions in 1-σ and RMS errors also occurred for Case 1. They were each reduced by a factor of 3.2. However, unlike mean error performance, significant reductions in these error statistics are relatively easy to achieve in any receiver. TrEC 1-σ and RMS error reductions were nearly matched by the Carrier Smoothing Only mode for the first two cases. Also, for Case 3 the Narrow Correlator achieved as much 70-80% of the 1-σ reduction factor of the TrEC modes.

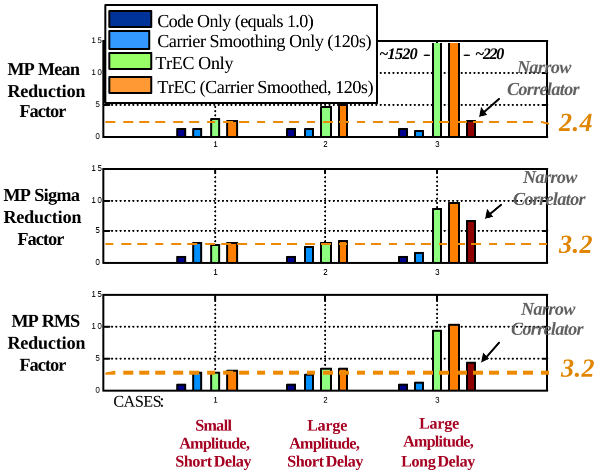

Figure 20. Summary error statisics: multipath error reduction factors.

CONCLUSIONS

Leveraging the Multuipath Invarance approach, the TrEC algorithm was implremented on a narrowband receiver. Real-time TrEC multipath mitigaiton performance was evaluated in the position domain—under three different multipath conditions—using live GPS satellites.

From the results the following conclusions can be drawn:

- Multipath invariant properties of received correlation funcitons exist and can be used to mitigate multipath.
- TrEC is extremely effective at reducing the mean bias errors due to multipath.
- TrEC is capable of providing corrections for multipath in any regime—including short-delay multipath.
- TrEC is also able to reduce tracking (pseudorange) errors due to code noise.
- In many instances, the initialization time required to find MPI points, may not significantly impact overall TrEC performance.
- A narrowband receiver using TrEC may achieve multipath mitigation performance at least comparable to that of some existing wideband receivers.
- Theoretical multipath mitigaton performance curves (in the pseudorange domain) may not reveal the true performance of multipath mitigaton techniques.

REFERENCES

[1] Townsend, B., et al, *Performance Evaluation of the Multipath Estimating Delay Lock Loop*, NAVIGATION, Journal of Navigation, Vol. 42, No. 3, pp. 503-14, 1995.

[2] Cahn, C. R., Chansarkar, M. M., "Multipath Corrections for a GPS Receiver," *Proceedings of the 10th International Technical Meeting of the Satellite Division of the Institute of Navigation, ION-GPS-97*, Part 1(of 2), Proceedings of ION GPS-97, Vol. 1, pp. 551-57, 1997.

[3] Dierendonck, A. J., Fenton, P., Ford, T., "Theory and Performance of Narrow Correlator pacing in a GPS Receiver," *NAVIGATION, Journal of the Institute of Navigation*, Vol. 39, No. 3, pp. 265-83, 1992.

[4] Garin, L., Rousseau, J.-M., "Enhanced Strobe Correlator Multipath Mitigation for Code and Carrier," *Proceedings of the 10th International Technical Meeting of the Satellite Division of the Institute of Navigation, ION-GPS-97*, Part 1(of 2), Proceedings of ION-GPS, Vol. 1, pp. 559-68, 1997.

[5] Townsend, B., Fenton, P., *A Practical Approach to the Reduction of Pseudorange Multipath Errors in a L1 GPS Receiver*, Proceedings of the 7th International Technical Meeting of the Satellite Division of the Institute of Navigation, Part 1 (of 2), Proceedings of ION GPS, Vol. 1, pp. 143-48, 1994.

[6] Braasch, M. S., Van Dierendonck, A.J. "GPS Multipath Model Validation," *Position Location and Navigation Symposium*, IEEE PLANS 96, pp. 672-8, 1996.

[7] Phelts, R. E., Enge, P. K., "The Case For Narrowband Receivers," *Proceedings of the 2000 National Technical Meeting*, Institute of Navigation, San Diego, CA, January 2000.

[8] Betz, J. W., Fine, P. B., "Effect of Narrow Correlator Spacing on Code Tracking Accuracy," *Proceedings of the 2000 National Technical Meeting*, Institute of Navigation, San Diego, CA, pp. 716-23, January 2000.

[9] Phelts, R. E., Stone, J. M., Enge, P. K., Powell, J. D., "Software-based Multipath Mitigation: Sampling for Multipath Invariance," *Program and Proceedings of the 4th International Symposium on Satellite Navigation Technology and Applications*, Session 3, 1999.

[10] Phelts, R. E., Enge, P. K., "Multipath Mitigation for Narrowband Receivers," *Position Location and Navigation Symposium*, IEEE PLANS, pp. 30-6 2000.

[11] McGraw, G. A., Braasch, M. S., "GNSS Multipath Mitigation and High Resolution Correlator Concepts," ION GPS-99 *Proceedings of the 12th-Internatinal Technical Meeting of the Satellite Division of the Institute of Navigation*, pp. 333-42, 1999.

[12] Kaplan, E. D., *Understanding GPS Principles and Applications*, pp. 209-36, Artech, Boston, Massachusetts, 1996.

[13] Akos, D. M., Phelts, R. E., Mitelman, A., Pullen S. Enge, P., "GPS-SPS Signal Quality Monitoring (SQM)," *Proceedings of Position Location and Navigation Symposium*, Conference Proceedings Addendum, IEEE PLANS, 2000.