The Case for Narrowband Receivers

R. Eric Phelts, Per Enge
Department of Aeronautics and Astronautics, *Stanford University*

BIOGRAPHY

R. Eric Phelts is a Ph.D. candidate in the Department of Aeronautics and Astronautics at Stanford University. He received his B.S. in Mechanical Engineering from Georgia Institute of Technology in 1995, and his M.S. in Mechanical Engineering from Stanford University in 1997. His research involves multipath mitigation and satellite signal anomalies.

Per Enge is an Associate Professor of Aeronautics and Astronautics at Stanford University, where he has been on the faculty since 1992. His research deals with differential operation of GPS for landing aircraft. Previously, he was an Associate Professor of Electrical Engineering at Worchester Polytechnic Institute.

ABSTRACT

Most high performance GPS receivers today are wideband. In most cases, they require the wider precorrelation bandwidth for enhanced multipath mitigation capability. Wideband receivers, however, are still significantly more susceptible to electromagnetic interference. In addition, they are more likely to produce unacceptably large pseudorange errors in the presence of satellite signal anomalies, or “evil waveforms.”

Conversely, narrowband receivers are less vulnerable to narrowband interference and more robust against GPS signal faults. However, they tend to have relatively poor multipath performance. Since significant rounding of the correlation peak occurs, narrow correlators and other wideband mitigation techniques at best provide little improvement in multipath performance.

In this paper, a case is made for narrowband receivers by describing their interference rejection advantages, and greater robustness against the effects of evil waveforms. Analyses for both classes of receivers under the influence of wideband and narrowband interference are provided. Evil waveforms are modeled and the expected resulting maximum differential pseudorange errors are simulated. The simulation results are given for a range of correlator spacings and receiver bandwidths.

The Tracking Error Compensator (TrEC) mitigation technique was investigated as a potential solution to the multipath problem in narrowband receivers. Theoretical bandlimited mitigation performance plots for this approach were generated and compared to those of a conventional delay-lock loop. A TrEC algorithm was developed and implemented on a Mitel Semiconductor receiver having a 2MHz precorrelation bandwidth. A GPS signal generator was used to generate multipath with known characteristics on a prescribed pseudorange. For short-delay multipath, measured pseudorange errors were compared to those estimated by the algorithm. This data suggests that it may be possible to attain both high multipath performance and reject interference in narrowband receivers.

INTRODUCTION

Precorrelation Bandwidth

The RF front-end of most conventional GPS receivers is analog. Usually, the incoming signal enters through the antenna and gets immediately amplified by the preamplifier. It is then cable-routed to a multi-stage downconversion process. A series of intermediate frequency (IF) mixers and filters, downconversion converts brings the GPS signal (nominally 20 MHz wide and centered at 1.545GHz for L1, C/A code receivers) to (or near) baseband for correlation, tracking, and subsequent processing.

The final intermediate frequency (IF) stage filter bandwidth generally sets the precorrelation bandwidth (PCB). In this paper, the term narrowband will apply to receivers having a (two-sided) PCB less than 2.5MHz wide. These narrowband receivers pass only the main lobe of the C/A code power spectrum. All others will be considered wideband.

Wideband Interference

Wideband interference (e.g., white Gaussian noise), theoretically, has constant energy over all frequencies, f, given by

$$S(f) = \frac{h}{2}, \quad -\infty \leq f \leq \infty \qquad (1)$$

where h is a constant. Although actual wideband interference sources have finite bandwidth, they effectively raise the thermal noise floor of GPS receivers of all PCB's. Because the GPS signal is below noise floor, relatively weak jammers are capable of jamming even the most sophisticated GPS receivers. Intentional wideband jammers include white noise and spread-spectrum transmitters.

Narrowband Interference

In general, narrowband interference has a bandwidth <<1MHz [1]. This type of interference consists of both intentional and unintentional continuous wave (CW) sources including transmitter harmonics from CB's and AM and FM stations. For civilian (C/A code) receivers these jammers pose a significant potential threat if they occur at specific line frequencies in the C/A power spectrum (See Figure 1.). At these frequencies the energy of the narrowband pulse is not spread but rather "leaks through" to baseband with the L1 carrier [2]. However, since the lines are 1kHz apart, a very narrowband interferer (e.g., 10Hz wide) is unlikely to cause a serious, long-term problem. A wider-bandwidth interferer (e.g, 10kHz-1MHz) could pose a more realistic threat since so little power is required to jam GPS [3]. For high-integrity systems like the Local Area Augmentation System (LAAS), this susceptibility to interference could be a critical liability.

Evil Waveforms

Satellite signal anomalies or "evil waveforms" result from a failure of the signal generating hardware on one of the GPS space vehicles (SV). These anomalies may cause severe distortions of the autocorrelation peak inside GPS receivers. In local area differential systems, undetected evil waveforms may result in large pseudorange errors, which in general do not cancel. One such failure occurred on SV19 in October of 1993. It caused differential pseudorange errors on the order of 3 to 8 meters. Undetected errors of this size are much too large for aircraft conducting a precision approach [4].

These satellite failures are rare events. However, LAAS requires that a monitoring system be placed at ground reference stations to detect these failures when they occur. The envisioned signal quality monitoring (SQM) scheme would consist of one or more (wideband) GPS receivers having several correlators configured to sample the correlation peak at multiple locations and accordingly determine its level of distortion. Rather than simply sending corrupted differential corrections to airborne users, the SV would then be flagged as unhealthy and its pseudorange subsequently removed from the users position solution.

For designing an SQM scheme, an evil waveform model was developed. This model assumes the anomalous waveform is some combination of second-order ringing (an analog failure mode) and a lead/lag (a digital failure mode) of the pseudorandom noise (PRN) code chips [4]. A good ground monitoring implementation would detect any and all waveforms that would result in large differential PRE's.

If an evil waveform is undetected by a particular monitoring scheme, it is necessary to determine the impact on the differential PRE's of airborne users. These users may have varied receiver implementations. Of course, if a user's receiver configuration matches (or nearly matches) the ground reference station receiver (16MHz bandwidth, 0.1-chip correlator spacing), the differential PRE's will be quite small. However, not all avionics are closely matched.

In general, receiver manufacturers desire the freedom to implement both narrow and wide PCB's with narrow and/or wide correlator spacings. For LAAS, the current goal for Category I precision approaches is to protect an L-shaped region of this two-dimensional user design space using a practical ground monitoring scheme (See discussion and results in Evil Waveforms Section). To meet the requirements, the maximum PRE's within these regions must be less than 3.5 meters.

Multipath

For GPS users, multipath (MP) is caused by reflections of the satellite signal from the ground or from nearby buildings or other obstacles. Multipath errors result when the receiver receives the direct or line-of-sight (LOS) satellite signal via multiple paths and processes the combined signal as if it were only the direct. These errors are particularly difficult to remove since, in general, the following is true:

1) The pseudorange measurement is derived from a code-tracking delay-lock loop (DLL). DLL's essentially attempt to derive time-of-arrival measurements from measurements of incoming signal amplitudes. This is accomplished by maximizing the signal code autocorrelation function. In the receiver, this translates to using a minimum of 2 correlators to straddle the peak (e.g., such that Early-Late=0). Since the combined LOS and MP autocorrelation function will have a distorted shape, the distortion introduces a tracking error into the DLL. An example of ideal and MP-corrupted autocorrelation functions is given in Figure 1.
2) Pseudorange errors due to multipath, in general are nonlinear functions of MP amplitude delay, phase and phase rate [1]. Accordingly, changes in any of these parameters may significantly change the tracking response of the DLL.
3) Multipath errors are not zero-mean. This is particularly true for MP signals with relatively large amplitudes. Consequently, even infinite smoothing of the pseudorange cannot guarantee unbiased position errors [5].
4) Multipath is not spatially correlated. MP signals affecting a receiver at one location will not affect a receiver at another location in the same way. Hence , differential processing is not effective against multipath.

INTERFERENCE ANALYSIS

Wideband Interference

The variance at the output of a system whose input is white gaussian noise is given by

$$s^2 = \int_{-\infty}^{\infty} |H(f)|^2 N(f) df \tag{2}$$

Where the input noise power spectral density, *N*(*f*), is the sum of the thermal noise and interference PSD's respectively given by Eqn. XX.

$$N(f) = N_0 + I_0 \tag{3}$$

However, a GPS receiver has two primary stages of filtering—the precorrelation filter (or final stage IF filter), $B_{pre}(f)$, and the correlation process itself. Hence,

$$s^2 = \int_{-\infty}^{\infty} |B_{pre}(f)|^2 |C(f)|^2 N(f) df \tag{4}$$

For simplicity, we may assume the precorrelation filter has a unity gain, rectangular PSD where

$$B_{pre}(f) = \begin{cases} 1, & -f_0 \le f \le f \\ 0 & otherwise \end{cases} \tag{5}$$

The spectrum "envelope" of the autocorrelation is approximated by

$$C(f) = \frac{\sin(\ f)}{\ f} \tag{6}$$

Figure 2 shows a plot of the squared magnitude of *C*(*f*). Applying the rectangular bandpass filter assumptions described in (4), a plot of output variance as a function of receiver front-end bandwidth is given in Figure 4. Note that there is less than a 0.5dB difference between a receiver with 2 MHz bandwidth and one with a bandwidth of 16 MHz.

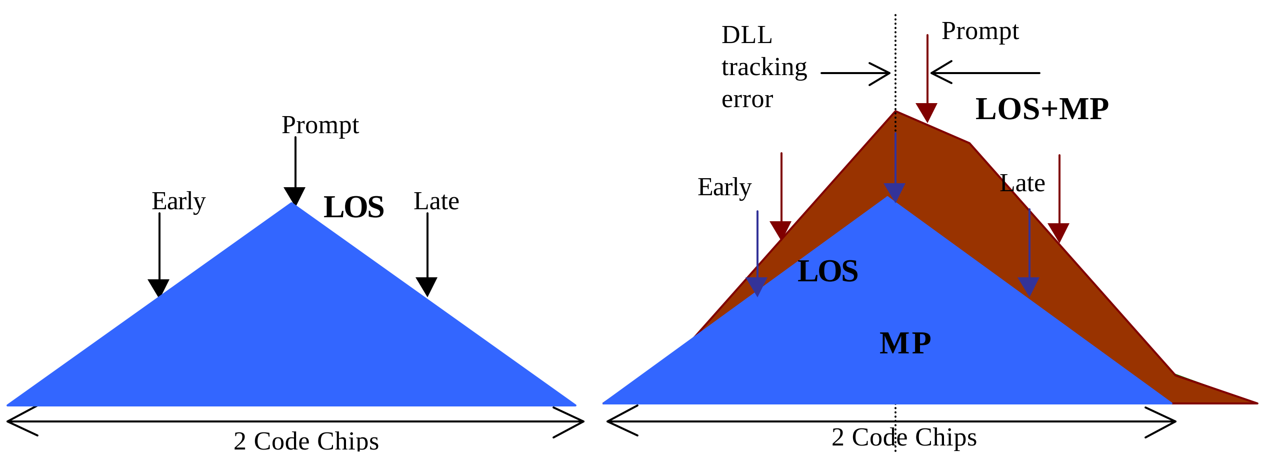

Figure 1. Ideal autocorrelation peaks with and without multipath (pictured in-phase)

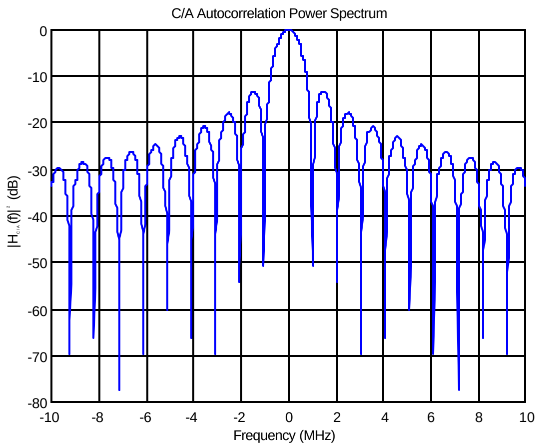

Figure 2. C/A Autocorrelation Power Spectrum Envelope

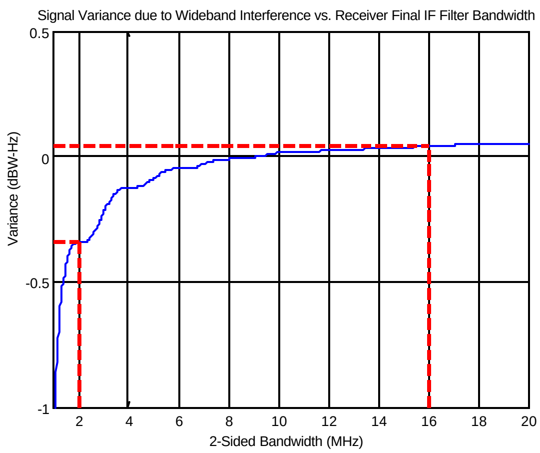

Figure 3. Noise variance vs. Precorrelation Filter Bandwidth

Narrowband Interference

Narrow-PCB receivers are, significantly less vulnerable to narrowband interference. Figure 4 summarizes the advantages of a 2MHz-PCB receiver over three wider-PCB ones in terms of the amount of attenuation each would apply to such interferers at frequency offsets greater than 1MHz. (The frequency offsets are measured relative to L1). The precorrelation filter was modeled as a 6^{th}-order Butterworth. The single-sided passband was set to 1MHz, 4MHz, 8MHz, and 10MHz.

Clearly, the larger the frequency offset of the interference, the greater the relative suppression advantage of narrowband receivers. Even for a 1MHz-wide interferer offset 2MHz or more from L1, a 2MHz receiver can provide many orders of magnitude more attenuation than the 8, 16 and 20MHz ones. At an offset of 10MHz, the 2MHz receiver provides over 100dB more suppression than the 16MHz and 20MHz PCB's and over 80dB more than the 8MHz PCB. Indeed even at a frequency offset of 1-2MHz, there is more than a 10dB difference in attenuation between the narrow PCB and the three wider ones.

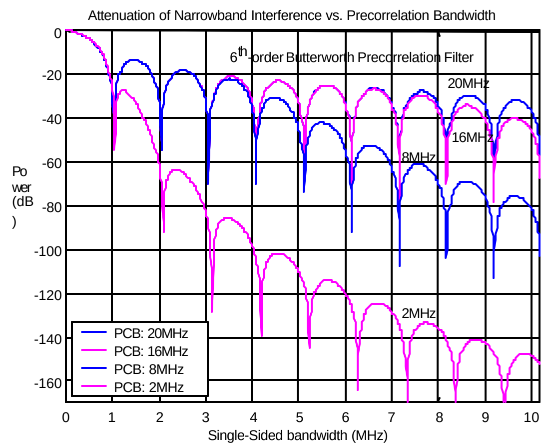

Figure 4. Relative attenuation of narrowband interference for 4 different PCB's.

EVIL WAVEFORMS

The current LAAS-approved model for satellite signal anomalies is based on the second-order step response of a linear system given by

$$e(t) = \begin{cases} 0, & t \le 0 \\ 1 - \exp(-st)\left[\cos w_d t + \dfrac{s}{w_d} \sin w_d t\right], & t \ge 0 \end{cases} \quad (11)$$

where

ω = $2\pi f_d$

and

f_d = damped natural frequency of oscillation (MHz)

σ = damping factor (nepers/second)

In addition, it uses a lead/lad (Δ, chips) factor which effectively models changes in duty cycle of the code chips. Using these equations, the following three threat models (TM A, B, and C) are used to evaluate performance of airborne user receivers.

1) Threat Model A (digital failure only):

$$f_d = 0,$$
$$-0.12 \le \Delta \le 0.12$$

2) Threat Model B (analog failure only):

$$4 \le f_d \le 17$$
$$\Delta = 0$$

3) Threat Model C (digital and analog failure):

$$4 \le f_d \le 17$$
$$0.8 \le s \le 8.8$$
$$-0.12 \le \Delta \le 0.12$$

A MATLAB simulation was developed to model the effect of evil on airborne user receivers of various PCB's and correlator spacings. The assumptions used in the simulation are as follows:

- The anomalous correlation function is accurately modeled using the closed-form equations derived in [4]
- E-L Coherent DLL is used for both ground and air receivers.
- Quantization effects are negligible.
- Ground (monitor/reference) RF front-end: 16MHz (6th-order Butterworth).
- Airborne user RF front-end: 2-20MHz bandwidth (6th-order Butterworth).
- Pseudorange corrections are based on 0.1-chip correlator spacing.
- Nominal (filtered) tracking errors removed.

Assuming all evil waveforms within each threat model are undetected, we may examine how inherently robust the airborne receivers are to these satellite failure modes. Figures 5, 6, and 8 show the user receiver performance for each of the three basic threat models. For each, the three "protected" regions are outlined. Also, whenever the maximum PRE contours approximately equal 3.5 meters, thick (clustered) contour lines are drawn.

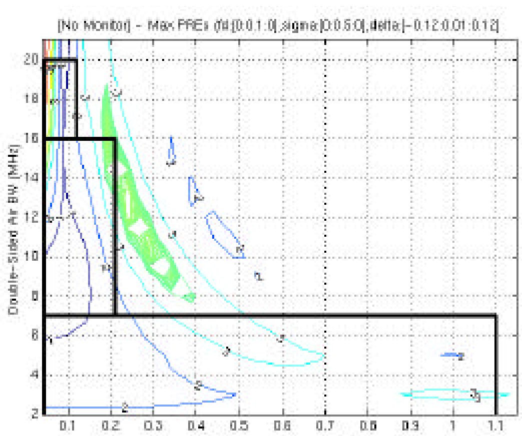

Figure 5. TM A: Maximum PRE's for airborne users

Figure 5 reveals that Region 3 (top left block) receivers are most sensitive to perturbations in lead/lag. The maximum PRE's for the narrowband user receivers consistently remain relatively small for Threat Model A.

For Threat Model B, observe that almost all of the receivers are severely affected by the ringing effects. Only the approximately "matched" receivers have relatively small PRE's (See Fig. 6). This is of course because the PRE's are differential. When the ground reference and the user receiver configurations match, the errors cancel.

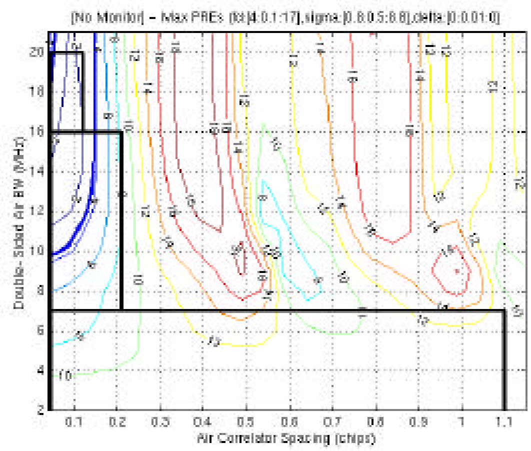

Figure 6. TM B: Maximum PRE's for airborne users

The narrowband receivers still have a small advantage even for this threat model. The largest errors only result from relatively low frequencies of ringing. In fact, the maximum PRE's remain essentially less than or equal to 3.5 meters for f_d greater than 7.3MHz—the lower limit on f_d for TM C. A 2MHz bandwidth receiver would satisfy the requirements in this case. Figure 7 clearly show this is in general not the case for the wideband receiver configurations.

The results for Threat Model C are given in Figure 8. For this combined failure mode, clearly all the receiver configurations are quite sensitive to this combined digital-analog failure mode. Still, the lower bandwidth receivers tend to have slightly smaller maximum PRE's. Again, the nearly-matched configurations have the smallest PRE's.

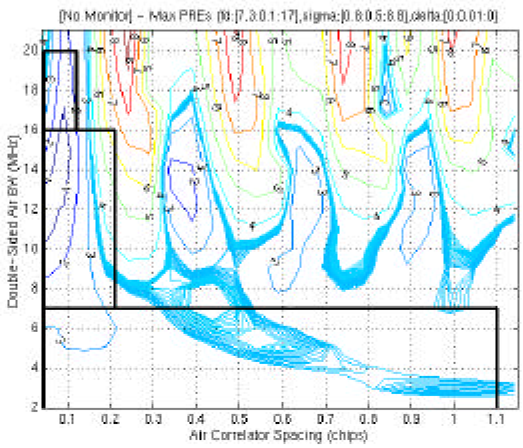

Figure 7. TM B: Maximum PRE's for airborne users. ($f_{d,min}$=7.3MHz)

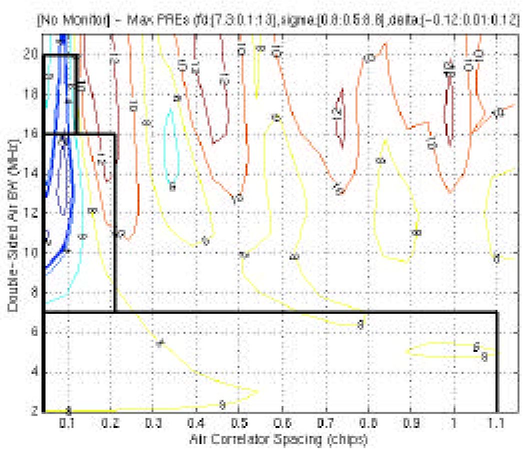

Figure 8. TM C: Maximum PRE's for airborne users

MULTIPATH

Most current MP mitigation techniques can be grouped into two major categories: *Separation* and *Estimation*. Separation techniques (e.g., the Narrow Correlator [6], Strobe Correlator [7], MET [8], etc.) essentially attempt to separate the LOS and MP signals. These approaches attempt to track only the LOS signal and thereby reduce or eliminate the effects of the multipath. Estimation techniques (e.g., MEDLL [9]) attempt to estimate the parameters of the LOS and/or MP signals, and approximate their combined effect on the tracking errors. They may also use some combination separation and estimation schemes to form a correction factor for (or an estimate of) the code tracking error [10].

Both classes of techniques rely on an ability to distinguish the multipath from the line-of-sight. This is usually accomplished by special signal processing of the autocorrelation functions and/or discriminator curves. Extra hardware (i.e. more than the usual Early, Late and sometimes Prompt correlators) and usually a wide bandwidth (e.g., at least 8 MHz) is frequently employed for this purpose. A fundamental limitation these methods must overcome is their sensitivity to the changing characteristics of the multipath. For example, the closer the MP parameters match those of the LOS, the more difficult it becomes to either separate or estimate one from the other. This explains the characteristic degradation in performance these techniques suffer when the MP relative delays are very short. The theoretical delay-lock loop (DLL) tracking performance of the best of these techniques approximates P-code tracking performance. Accordingly, most degrade to conventional C/A code tracking performance at relative MP delays below about 20 meters.

One of the most common mitigation techniques is the Narrow Correlator [6]. Theoretical performance plots for a standard 1-chip correlator, a 0.5-chip and a narrow (0.1-chip) correlator for a 16 MHz front-end bandwidth is shown Figure 9. The same plots for a 2MHz bandwidth are shown in Figure 10.

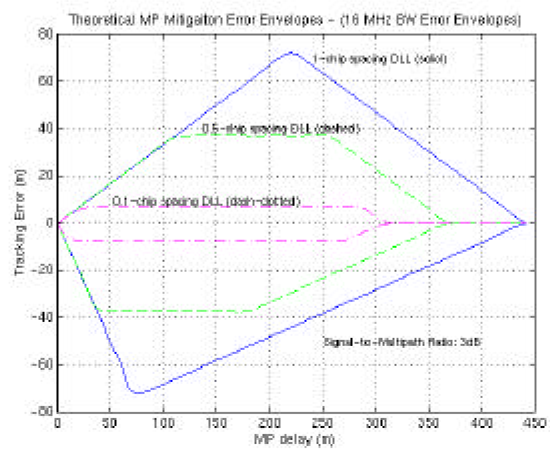

Figure 9. DLL Tracking error vs. MP relative delay for 16MHz bandwidth receivers

Observe that since many narrowband receivers use a 0.5-chip correlator spacing for code tracking, there is relatively little advantage to using a narrow correlator in a receiver with such a narrow front-end bandwidth. Hence, any wideband, narrow correlator-based WAAS receivers tracking the 2.2MHz-bandlimited geostationary (GEO) satellite will effectively have this poor multipath performance. In addition, since for many users the GEO is at low elevation angles and is essentially stationary for static users, the multipath problem could be even more significant.

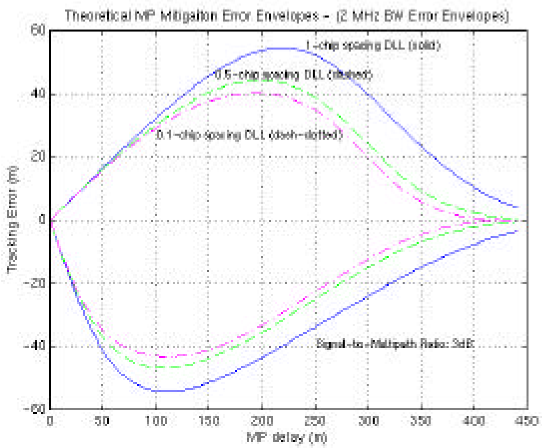

Figure 10. DLL Tracking error vs. MP relative delay for 2MHz bandwidth receivers

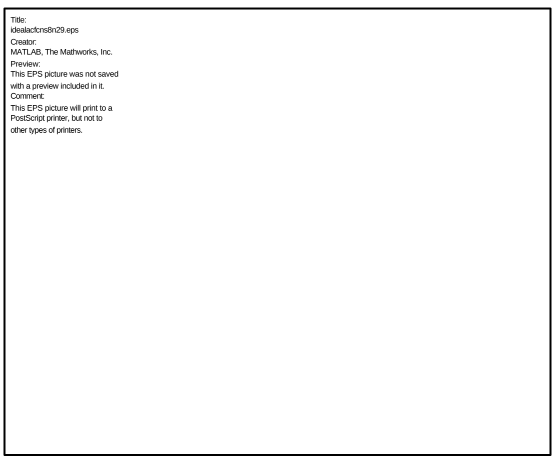

Figure 11. Normalized ideal autocorrelations

The Tracking Error Compensator (TrEC), however, is an effective mitigation technique for both narrowband and wideband receivers. It leverages the fact that there exist regions and/or properties of the autocorrelation function that do not vary as a function of the multipath parameters. These multipath *invariant* (MPI) regions and/or points are located at the plateaus of the autocorrelation function of a particular PRN code sequence. These MPI plateaus are not always located adjacent to the peak (See Figure 11).

More details on the MPI concept can be found in [11], however a general TrEC algorithm is as follows:

1) Generate (offline) the ideal autocorrelation functions corresponding to each PRN and compose a look-table corresponding to the ideal distance between the peak and the MPI point.
2) Once the receiver is tracking a satellite in a given channel, use knowledge of the ideal distance to the MPI location for the corresponding PRN and reasonable bounds on the current tracking error to bound the desired point.
3) Search and optimize within the localized region of the autocorrelation function to find the location of the MPI point with respect to the primary (tracking) correlators.
4) Once the point is identified, correct the tracking loop solution by the difference between the Measured and Ideal DLL positions. (See Figure 12.)

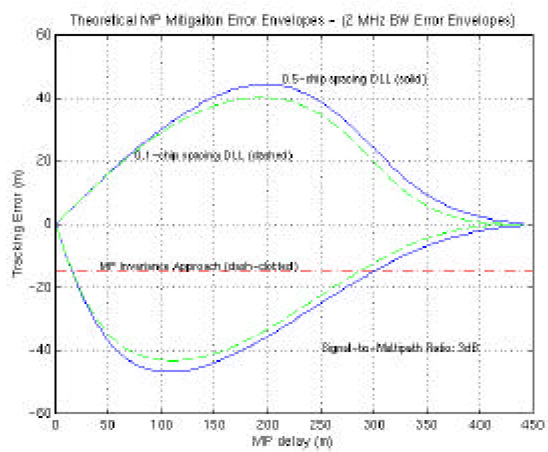

Figure 13. TrEC and conventional DLL Tracking error vs. MP relative delay for 2 MHz bandwidth

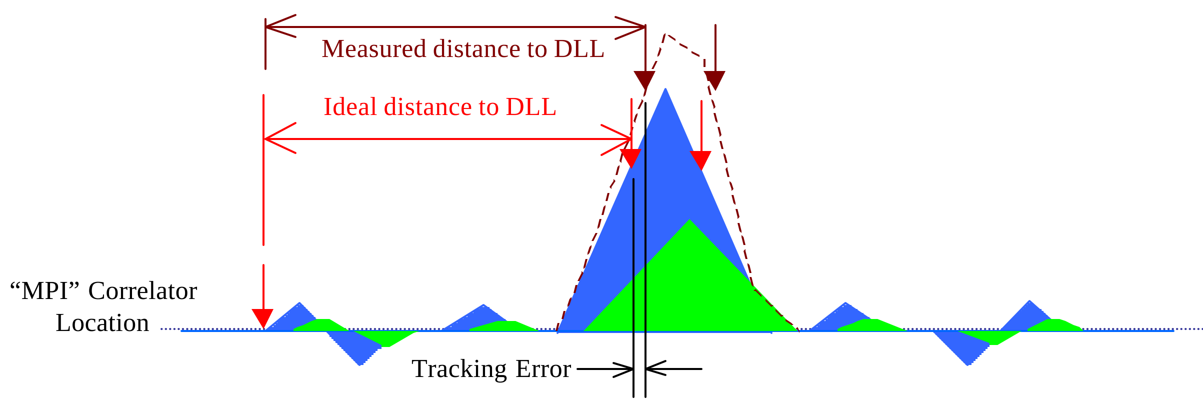

Figure 12. Tracking Error Compensator (TrEC): Updating the primary DLL tracking solution with the corrected, relative position to the MPI point.

The performance curves differ for the TrEC in that they are not really "envelopes". The theoretical MPI envelope is actually straight line, offset by bias that can be calibrated out. (If a bias remains, and the TrEC is used to correct all valid pseudoranges, it will simply become a part of the clock bias in the navigation equations and will not affect the final position solution.) The theoretical performance curves for the noise-free, 2MHz-bandlimited case is shown in Figure 13.

Assuming the multipath has a relatively short delay—the most troublesome multipath—we verified these performance curves using a Welnavigate GPS Signal Generator. It was capable of simulating the entire GPS satellite constellation, and also of generating multipath (with known parameters) on a given pseudorange. This latter feature was used to experimentally verify the short-delay multipath performance of a narrowband received implementing the TrEC. The algorithm was implemented on a Mitel Semiconductor (formerly GEC Plessey) receiver with a front-end bandwidth of 2MHz and a 0.5-chip correlator spacing.

Figure 14 illustrates the experimental setup. The signal generator was used to output the GPS satellite signals at a C/N_0 of approximately 50-53dB-Hz. A single pseudorange (PRN25) was corrupted by multipath appropriate for generating the performance curves. The measured pseudoranges were obtained from the Mitel receiver. The "true" pseudoranges (retrieved from the signal generator truth file) were then subtracted from the measured ranges. A single subsequent inter-channel difference was performed to remove the clock bias. Only variations due thermal noise and the bias due to multipath remained.

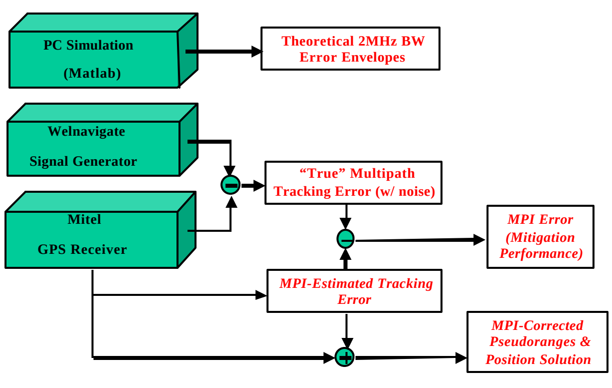

Figure 14. Setup for MP performance validation experiment.

The nominal error envelope for a 2MHz bandlimited receiver having a 0.5-chip DLL is shown in the top plot of Figure 15. The bottom plot compares the envelope and the actual tracking error due to the multipath at relatively short-delays. The multipath was programmed to slew at a rate of 3.6 meters/sec starting from a relative delay of 0 meters. The MP signal was attenuated 3dB with respect to the direct signal.

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Figure 15. Nominal and measured 2MHz multipath error envelopes.

The theory for a standard DLL (independent of correlator spacing or front-end bandwidth) is well established. As expected, with no carrier aiding the envelopes bound the tracking errors quite well. Carrier aiding caused the tracking errors to integrate the multipath errors. Nominally, however, many GPS receivers' code-tracking loops are carrier-aided. For this reason, subsequent tests using the TrEC were performed with carrier aiding turned on.

The top plot in Figure 16 shows the true multipath tracking error compared to the TrEC-measured tracking error. Using the Mitel receiver, the TrEC algorithm measured the tracking error in real-time. Note that at approximately t=10 minutes, the tracking loop temporarily lost lock on SV 25. A momentary fault in the generated signal caused this outage. It occurred at the same time for each trial and forced the TrEC to reinitialize. (Note that the outages partially affected the resulting performance curve, since about 5 minutes were required for the routine to re-converge to the proper MPI point and resume making valid pseudorange corrections.) The second plot shows both the raw and carrier-smoothed (120-sec) differences between the actual and measured tracking errors. The carrier-smoothed curve is the performance curve for the TrEC corresponding to this particular trial (See bottom plot of Fig. 16). The mean of this curve is 1.26 meters and its standard deviation is 2.13 meters

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Figure 16. TrEC-measured and actual tracking errors.

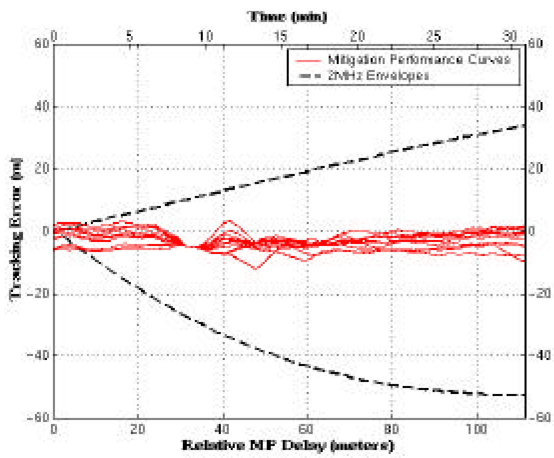

Figure 17. Short-delay TrEC performance curves (10 trials, 1-Hz data)

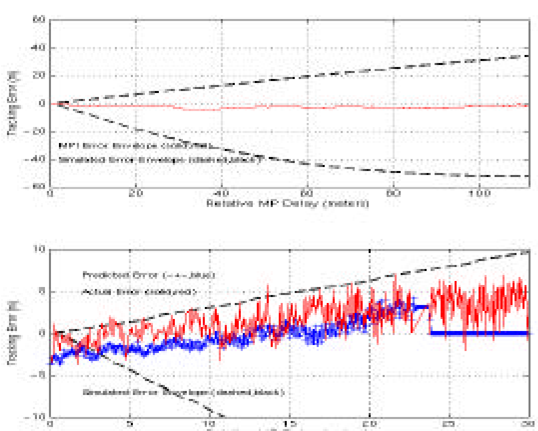

Figure 18. Short-delay MPI performance curves (mean of 10 trials, 1-Hz data)

In order to validate the theoretical performance curve (the straight line) of Figure 13, data from 10 trials was taken, and is plotted in Figure 17. The mean bias (for this TrEC implementation) is approximately -2.5m. Accounting for the increased variance due to the single-difference, the average smoothed and unsmoothed standard deviations are 1.4m and 3.2m respectively. Significant perturbations occurred, however, because of the temporary signal outage on SV25, so the true (undisturbed) standard deviation of the performance curves is actually smaller than that reported here.

The averaged results from the 10 trials—as performed in [12] for a conventional DLL—are shown in Figure 18. The top plot again compares the curves for the 0.5-chip spacing DLL and a conventional receiver implementing the TrEC. The bottom plot compares the true and measured tracking error curves at ultra-short relative MP-delays. Note that where all widely used current mitigation techniques (including wideband) have performance curves little better than a conventional DLL, the nearly identical upward slopes indicate the TrEC is still compensating for the tracking errors in this range. In other words, this technique is able to provide useful tracking corrections even for nearly zero-delay multipath. Although to date this fact has only been demonstrated on a narrowband receiver, it holds true for wideband receivers as well.

CONCLUSIONS

There are several advantages to using very narrowband GPS receivers as opposed to wideband ones. Although only a marginal reduction in wideband noise is achieved, the susceptibility to narrowband interference can be significantly reduced.

Simulations revealed that narrowband receivers are also less vulnerable to degradations in the transmitted GPS signals. Specifically, they were primarily vulnerable to low frequency ringing (evil waveforms). They suffered large differential pseudorange errors under TM B, when f_d was less than 7.3MHz, and TM C where the digital and analog effects were combined. Wideband receivers are susceptible to almost all ringing frequencies *and* lead/lag failures. Only the nearly-matched (approximately 0.1-chip spacing, 16MHz bandwidth) configurations consistently had relatively small maximum PRE's. As a result, they will at least require more-carefully designed (if not more complex) ground-based monitoring techniques if they are to be employed.

Wideband receivers are conventionally used to address the multipath problem. However, the Tracking Error Compensator was shown experimentally to be quite effective at mitigating multipath in narrowband receivers.

The results indicate comparable performance to advanced wideband mitigation techniques on the whole. They also indicate that the TrEC may even provide superior mitigation capability for the difficult short-delay multipath.

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