Correlation Structure of the Equatorial Ionosphere

Sriram Rajagopal, Todd Walter, Seebany Datta-Barua, Juan Blanch, *Stanford University*
Takeyasu Sakai, *Electronic Navigation Research Institute*

ABSTRACT

With the successful implementation of the Wide Area Augmentation System (WAAS) for the North America, there is interest in implementing similar systems in other parts of the world. The ionosphere over certain parts of the globe, notably equatorial regions and Brazil in particular, can have a far greater impact on the signals received from the satellites. The Ionospheric corrections form an integral component of the WAAS solution. The main requirement is to come up with a bound on the user vertical error, the first step of which is to perform a correlation analysis of the ionosphere.

The WAAS MOPS and ICAO SARPS specify a thin-shell approximation at a height of 350 km to model the ionosphere. This model is used to convert slant range delays into equivalent vertical observations. The correlation analysis of the Vertical Total Electron Count (VTEC) measurements is performed using processed data for Jan 11 2000 and Feb 19-21 2002 from a set of 12 Reference Stations in Brazil.

The data indicates that even co-located IPP's may have significantly different equivalent vertical ionospheric delay values, differences greater than 4 m at L1 on a quiet day. Thus, the thin shell model is much less accurate for the equatorial ionosphere than it is for mid-latitudes. We will discuss the correlation methodologies used in this analysis, which will bring out differences for the equatorial structure from the mid-latitude regions.

INTRODUCTION

While the user base of the Wide Area Augmentation System (WAAS) is varied, it was primarily built to provide integrity to the aviation user while improving accuracy. The architecture of WAAS [1] is based on providing differential corrections for improving accuracy. The Ionospheric delay remains the most important component of the correction and has hence received much attention.

The Ionospheric correction problem can be envisioned as a three step process [4]: estimation, transmission and prediction. A network of dual-frequency GPS receivers provide samples of the ionospheric delays. A model is used to fit these samples, constituting the estimation process. The model needs to be transmitted to the users, who will then use this information to predict the errors on the signals they receive.

It is the correlation of the Ionospheric delays that lends itself to the proper design of such a system. It allows us to predict what the ionospheric delays on certain Line of Sights (LOS) will be, knowing them for other signal paths. On the prediction side, it is also essential to know the confidence bounds on the estimates. These bounds can be derived from knowing the statistics of the ionospheric delays.

These critical components of the WAAS solution thus rely on the knowledge of the correlation structure of the ionosphere. The structure is assumed to be a correlated random field over a deterministic trend. The correlations for different fits [3] are analyzed to find the appropriate trend and to characterize the random behavior of the residues. The analysis of the correlation structure of the equatorial ionosphere will therefore lead to an evaluation of correction techniques for the equatorial region.

IONOSPHERIC MODEL

GPS uses time-stamped signals from satellites for its ranging. The ionosphere which is an intermediate medium consisting of ionized particles, introduces frequency-dependent delays in the signal. This dependency can be exploited by dual-frequency receivers to get an accurate measure of the ionospheric activity along that signal path.

A distribution of dual-frequency receivers on the ground can thus provide a good sampling of the ionospheric activity. The goal remains to predict similar delays for other Line of Sights. The ionosphere extends for a few hundred kilometers in height and three-dimensional modeling using tools like tomography [9] have been contemplated. Such methods however, require a large number of samples and are computationally expensive.

For implementation of the WAAS over CONUS, this was reduced to a two dimensional problem by considering the ionosphere as a thin shell at a height of 350 km [2] from the earth's surface. Since a two dimensional model is used, the slant delays are converted to equivalent Vertical delays at the point where the LOS pierces the shell, by using the obliquity factor:

$$Ob(el,H) = \sec\left(\sin^{-1}\left(\left(\frac{R_e}{R_e + H}\right)\cos(el)\right)\right) \tag{1}$$

This results in a model which is invariant in the vertical direction and varies only with latitude, longitude and time. Using these samples of equivalent vertical delays, a deterministic trend is fit to the ionosphere. This trend is used to predict the delays on a grid of points, called the Ionospheric Grid Points (IGP's) and these delays along with the confidence bounds on the estimation process are broadcast to the user regularly. The spatial correlation is critically important in generating these delays and the confidence bounds.

However, a vital point to remember is that the correlation analysis is dependent on the ionospheric model used. Hence, our analysis and results will be within the domain of the thin-shell ionospheric model. Despite the fundamental defect of compressing one dimension, this model works exceedingly well for the mid-latitude regions most of the time. We will use a similar approach for our analysis of the equatorial region.

DETERMINISTIC AND RANDOM FIELDS

Once the vertical delays at some pierce points on the ionospheric shell are known, we need to predict vertical delays at other locations. For this purpose, the delays are modeled as the sum of a deterministic trend and a correlated random field [8]:

$$I_{v,true}(x,y) = f(x,y) + R(x,y) \tag{2}$$

Here $f(x,y)$ is a function of the location of the pierce point under evaluation and represents the deterministic trend. The function can be 0th order, representing no trend, or a higher order function. Even after detrending, residues remain on the measurements. These residues $R(x,y)$ are modeled as random fields. We seek to characterize the behavior of these residues as they determine the deviation from the underlying trend. Treating the equivalent vertical delays at different locations as random variables, leads to the concept of correlated fields, with dependence based on distance between locations.

The random field is seen to exhibit a multivariate Gaussian distribution [3] for a quiet day over the CONUS. This Gaussian behavior makes it easier to analyze the bounds in the estimation process. It allows us to completely define the behavior of the random field by its covariance matrix. This emerges to be a well researched problem in Geo-statistics and much of our analysis will be drawn from those methodologies.

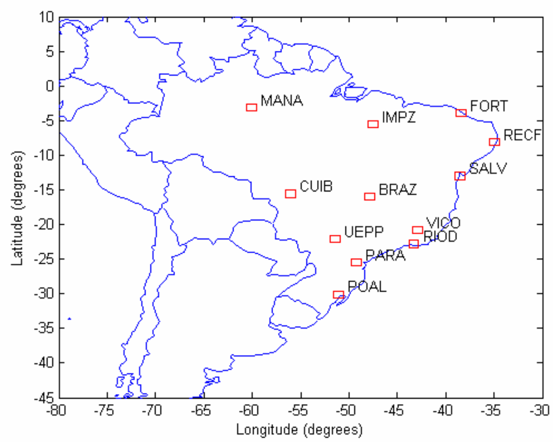

Figure 1. Map of Reference Stations in Brazil.

TREND ANALYSIS

Finding the right trend is a tricky problem. Along with the problem of selecting what order of function to fit, we also encounter the problem of which points to include in the fitting process. Inclusion of outliers in the fit will skew the fitting function and reduce the residues. For purposes of integrity, we want to see what the largest deviation from the fit can be.

There are many methods of using the available data for fitting functions. One such method could be to exclude a pierce point and perform a fit using the remaining points to find the residue at that location. We can then repeat this for all IPP's and find the residues for every point and use this for further analysis. But this will result in different fits for different points and the correlation function so generated will be incorrect.

As we shall see, we need to find the difference between all pairs of residues for correlation analysis. Hence in our trend determination process, we combine the process of fitting and correlation. In this method, we isolate a pair of IPP's and find all points within a fit radius, R_{max}, of the center of the IPP pair. All these points inside the fit radius, with the exclusion of the pair under consideration, are used to define the trend using the least squares approximation. Using these parameters we can compute the residues for the IPP pair. We now perform this process for all possible pairs of IPP's and this is repeated for every time epoch separately. This method ensures a consistent fit for the pair of IPP's under consideration, while also allowing us to find the outliers. Figure 2 gives a diagrammatic explanation of this procedure.

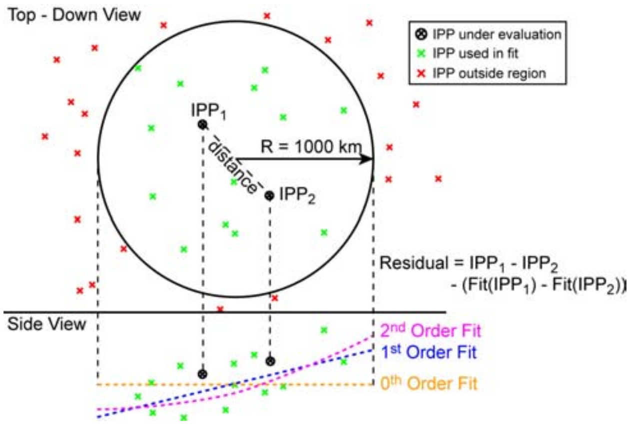

Figure 2. The estimation process.

While this method is computationally time consuming, it is an offline analysis and need not be performed in real-time. The circular shape to determine the boundary of the fit points, is chosen to be consistent with the current WAAS [1] implementation. We have performed 0^{th} order, planar and quadratic fits as a part of our analysis. As we increase the order, we will be over fitting the data, giving low residues for the points used in the fit, but potentially giving erratic residues for the IPP's under evaluation. The trend can also be skewed by the distribution of the IPP's within the fit radius. A check was performed to see if each of the fit was stable. This however, did not make much difference to the final results.

CORRELATION ANALYSIS

In order to see the dependence of the random fields on distance, we will use a variant of *scatter plots*. The scatter plots are generated by plotting the difference between the residues versus the corresponding distance between them. By intuition we would expect to see the dependence decrease with distance, depicted by increased difference in residues. Since there are a lot of points, we will bin them on both the difference in delays and distance. We will then count the number of occurrences for the ordered pair $(\Delta I_v, \text{Distance})$.

The concept above is commonly called *additive correlation*. Rather than find the mean of the product of the random variables, we seek to characterize the difference of the random variables. For our requirement, this technique is more accurate [4]. We define the correlation by $\left| I_{v_i} - I_{v_j} \right|$ for all residues i, j. The additive correlation function of the residues allows us to directly derive the confidence bound $\sigma_{decorr}(d)$ as a function of distance.

The data analyzed was from 12 reference stations in Brazil (Figure 1). The data was processed to remove cycle slips and biases. We can regard this data as having low measurement noise. We have analyzed data for 11 Jan 2000 and 19-21 Feb 2002. Looking at the scintillation plots [7], we see that 19^{th} Feb has the lowest scintillation and hence regard that as a 'quiet' ionospheric day for the equatorial region.

We have removed 0^{th} order, planar and quadratic trends from the data using the previously explained method. The corresponding correlation plots for the residues of these trends will be called the 0^{th}, 1^{st} and 2^{nd} order correlation plots. Figure 3 shows the correlation plots for the 19^{th} February. These plots depict the number of times each specific ionospheric difference in residues was encountered at each distance.

The 0^{th} order plot indicates a clear trend in the data similar to that observed over CONUS. The ionospheric shell height was fixed at 350 km. Varying the shell height was not found to affect the results by much and hence the parameter used for CONUS was adopted.

While CONUS uses a maximum fit radius of 2100 km, this gives larger residues for the sparse Brazil data. A fit radius of 1000 km was found to yield best results. We see that most of the difference in delay lies in the lower part of the plot. But for the purposes of integrity, we are more interested in the largest differences. The shape of the plots for the planar and quadratic residues is mostly similar, except for the outliers. Over larger fit radii, higher order fits (quadratic) may model the data better, although such fits become far more sensitive to the distribution of the Ionospheric Pierce Points (IPP's) and therefore require more observations. Over distances of the order of 1000 km, a planar model is sufficient and all our analysis from hereon will focus on planar fits.

While the 20^{th} and 21^{st} Feb 2002 have greater ionospheric activity, 11^{th} Jan 2000 is relatively quiet, similar to the 19^{th} Feb 2002. The correlation histograms for these days are shown in Figure 4.

To find the decorrelation bound, we use the difference in residues and the correlation histograms. Figure 5 shows such a bound for 19^{th} Feb '02. This plot is obtained by integrating out the 68^{th}, 95^{th} and 99.9^{th} percentile for each of the distance bins in the correlation histograms. These values are then normalized to obtain a 1 sigma value [3]. The normalization values are 1 for 68%, 2 for 95% and 3.29 for 99.9%. To obtain significant results for greater percentiles requires a larger dataset.

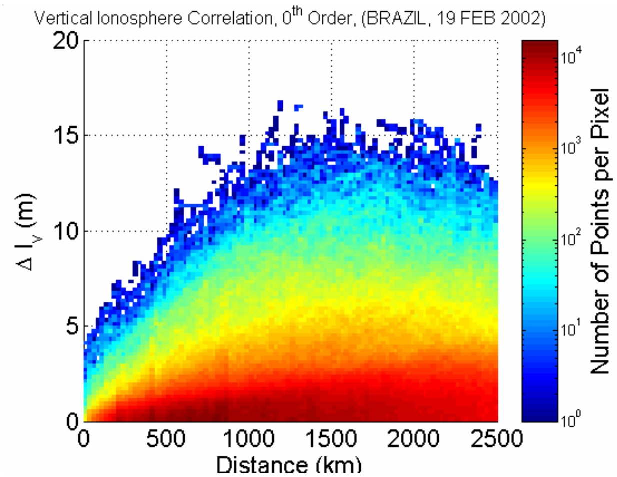

Figure 3(a)

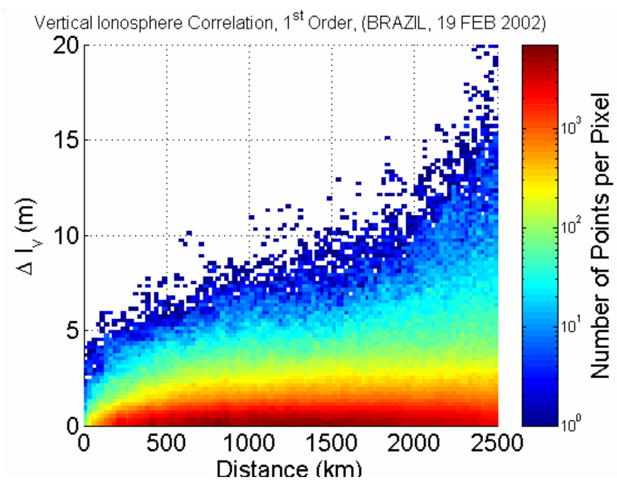

Figure 3(b)

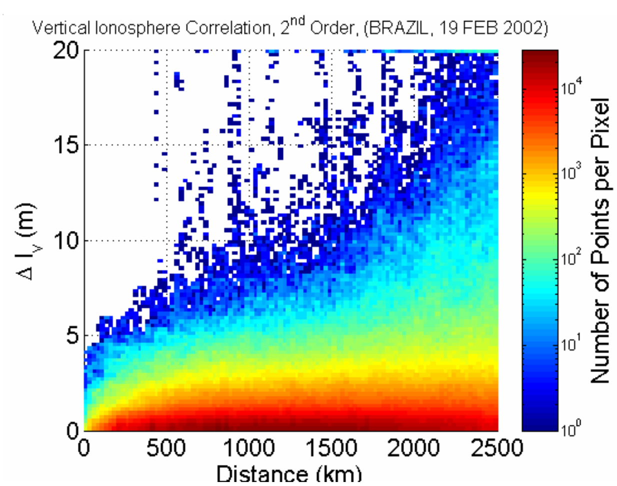

Figure 3(c)

Figure 3. The *Correlation histograms* for Brazil, 19th February 2002 showing the (a) 0th, (b) 1st and (c) 2nd order vertical ionosphere correlation.

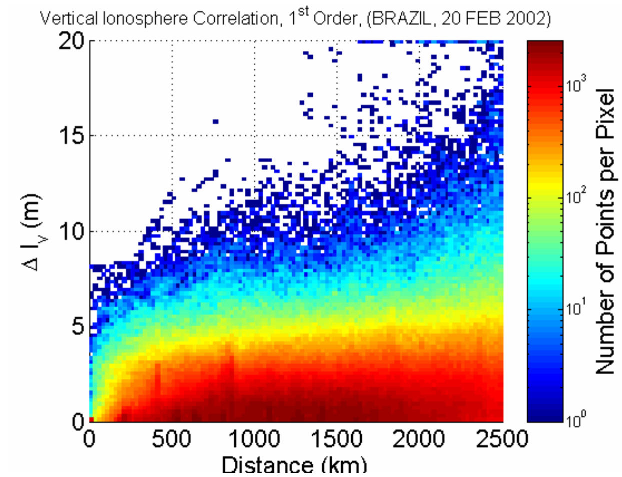

Figure 4(a)

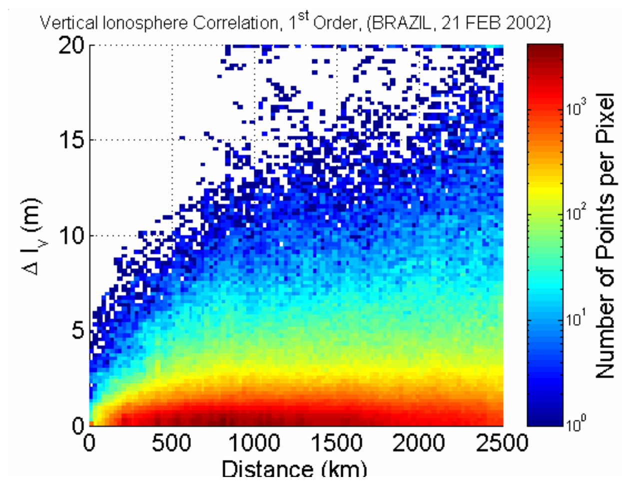

Figure 4(b)

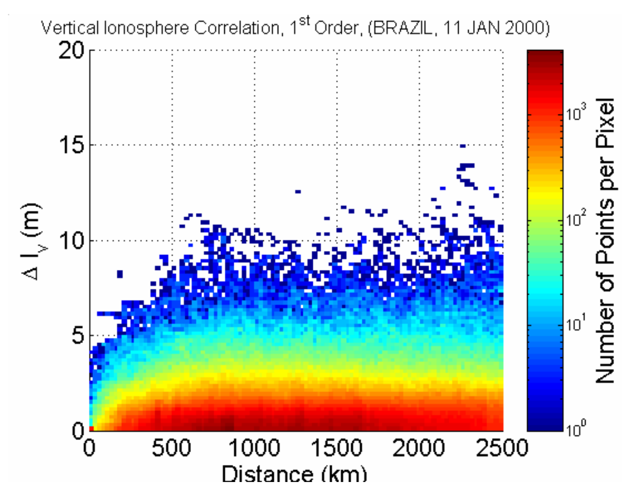

Figure 4(c)

Figure 4. The Vertical Ionosphere Correlation for residues of planar fit for (a) 20th Feb 2002, (b) 21st Feb 2002 and (c) 11th Jan 2000.

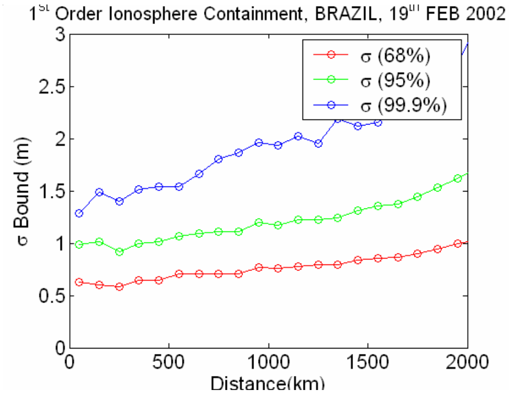

Figure 5. Ionosphere containment plot for Brazil, 19th Feb 2002.

As we see from this plot, the original Gaussian assertion seems to be violated, because the normalized sigma's do not line up on top of each other. We note that this plot was done by combining the residues of all epochs. The assertion, however, was made only for each epoch. Verification of this is difficult due to the statistical insignificance of a smaller dataset. Hence, we try to separate the different activities of the ionosphere by looking at the vertical ionospheric delay over the whole day. Figure 6 shows the vertical delays for the 19th of February, which has been grouped into three regions of different activities. We perform the same analysis again for each of the 3 regions.

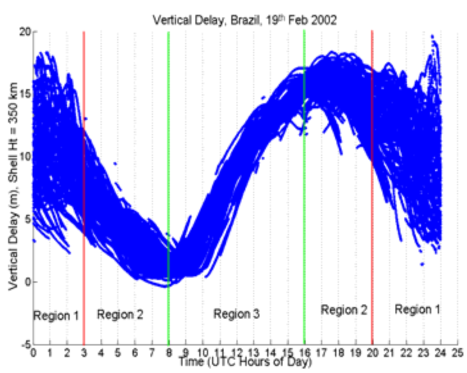

Figure 6. Vertical Delay, Brazil, 19th Feb 2002, separated into 3 regions of varying activities. The time axis is the corresponding UTC time.

Looking at Figures 7 and 8, we can see that taking smaller slices of time, we are able to get lower decorrelation values for specific times of the day and that during this time the Gaussian assumption is valid. During a 'quiet' day over CONUS, there were not many different phenomena going on in the ionosphere. Hence, despite the apparent mixing of residues over different epochs, we are able to obtain a constant decorrelation function for the whole day. However in the equatorial region, we know that different activities take place during a single day [6].

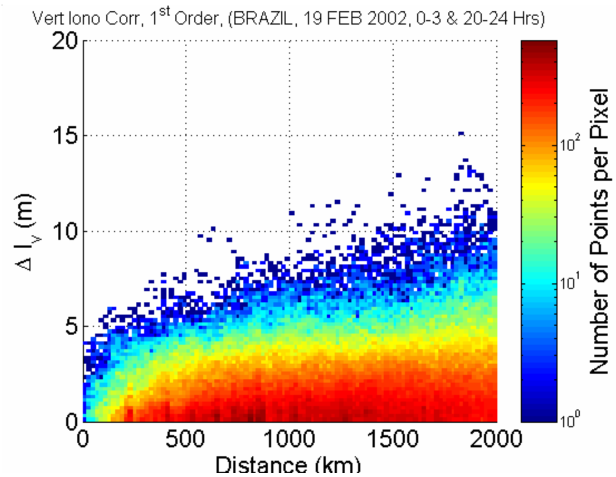

Figure 7(a)

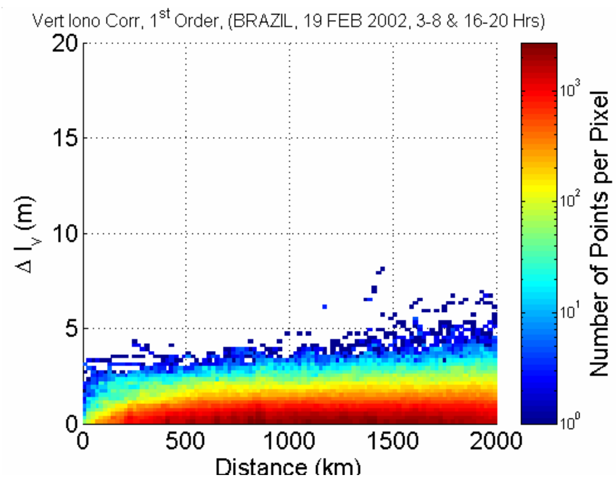

Figure 7(b)

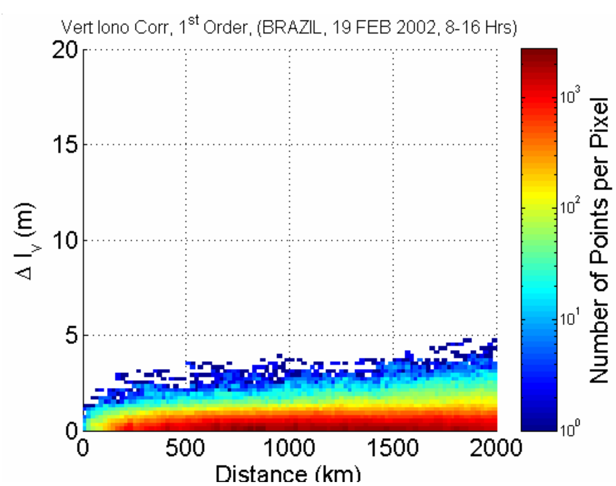

Figure 7(c)

Figure 7. Correlation histograms for 19th Feb for the separate time regions: (a) Region 1 (0-3 and 20-24 Hrs), (b) Region 2 (3-8 and 16-20 Hrs) and (c) Region 3 (8-16 Hrs).

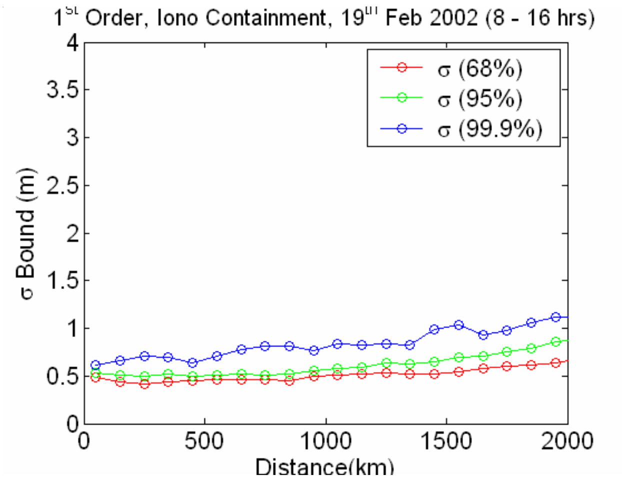

Figure 8. Sigma Containment plot for Time Region 3 for 19th Feb 2002.

This prompts us to take time slices as small as possible that still provide significant results. These results are provided for 19th and 21st Feb in Figures 12 and 13 respectively. Taking smaller slices does indeed validate the Gaussian assumption.

By looking at these plots we can see that the difference in delays of residues that are nearly co-located is far from being negligible. In fact, these values are larger than the decorrelation sigma value for CONUS. As mentioned before, the measurement noise is negligible. So we can safely point the source of the errors to the thin shell approximation of the ionosphere. This seems to suggest a non-reducible error floor in the system for the equatorial region. This also means that decreasing the grid spacing may not help as long as the thin-shell model is used and that we may have to deal with a 2m σ_{decorr} value even during nominal days.

To verify that the thin-shell model is indeed the source of these errors, the correlation analysis was performed by constraining the azimuth and elevation angles of the Line of Sights of the pair of IPP's under consideration. The difference in the look angles between the IPP's was constrained to be within 30 degrees. This will ensure that only IPP's corresponding to similar ionospheric activities are compared. The correlation histogram is shown in Figure 9. The difference in residues for closely located IPP's is now negligible. Hence, to overcome the error for co-located IPP's, alternate models for the ionosphere need to be considered. . However, after about 500 km separation between the IPP's, the maximum difference in residues becomes similar to the unconstrained case.

A comparison of the 95th quantile (Figure 10) for the time period of the highest sigma value (0-2 Hrs) is performed to see the benefit of using the constraints. While there seems to be an improvement for closely located IPP's, this advantage is lost beyond about 800 – 1000 km. Moreover, incorporating a stricter constraint did not affect the results by much.

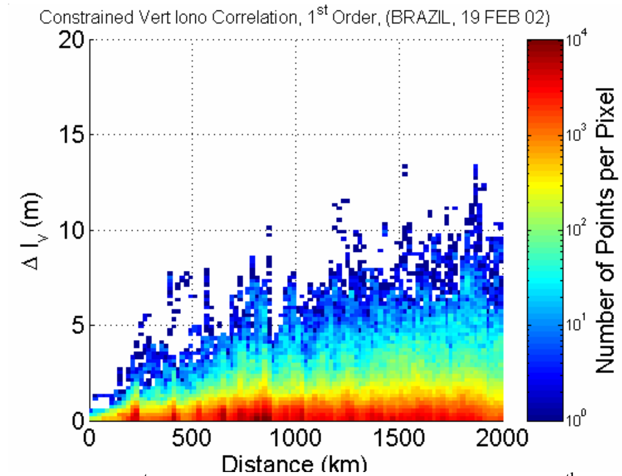

Figure 9. 1st Order Correlation Histogram for 19th Feb 2002 with constrained Look Angles.

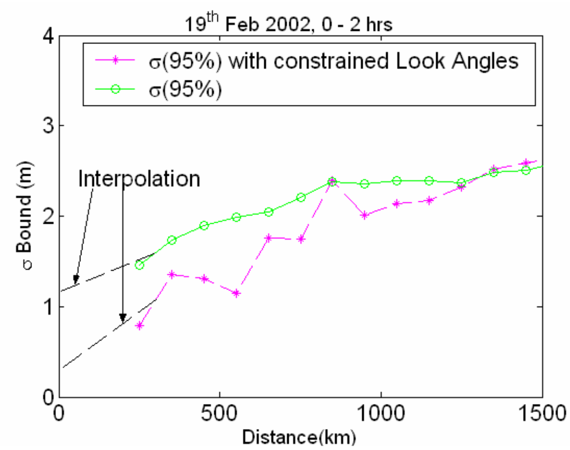

Figure 10. Comparison between the normal and constrained 95th Quantile, for 0-2 Hrs on 19th Feb 2002.

Figures 12 and 13 also indicate a varying slope for the sigma values, during the course of the day. The slope increases consistently from its lowest value during midnight-early morning to its largest activity during the local evening-early night. The greatest decorrelation occurs during the local early night time rather then the midday value over CONUS.

This can be associated with the Equatorial Anomaly [6]. The depletion associated with this phenomenon is known to have a specific structure. Directional sigma-bound plots in the North-South and East-West directions were plotted to see if we could capture this phenomena (Figure 11). The plots look nearly identical, indicating that we may not have enough sampling of the depletion region. Another explanation could be that the depletions were removed during processing. Hence for the methodology used, directional analysis does not provide us with any extra advantage.

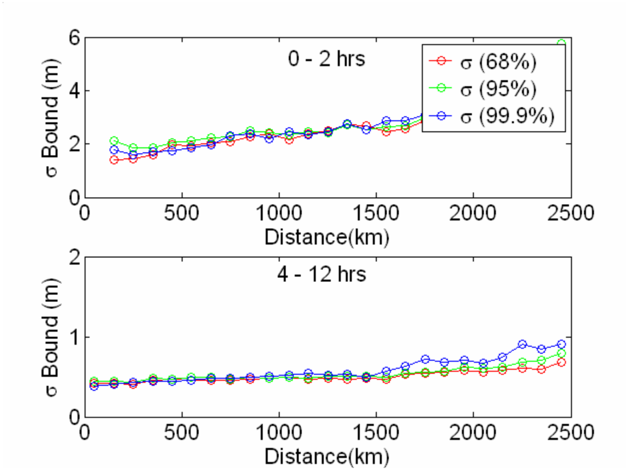

Figure 11(a).

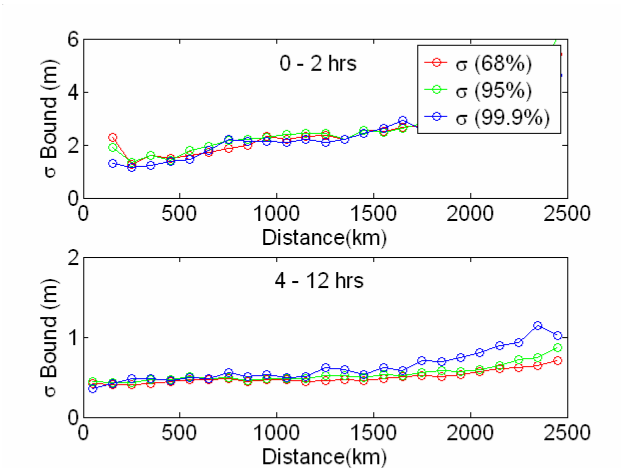

Figure 11(b).

Figure 11. Directional Sigma bound plots for 19th Feb 2002 (a) North-South and (b) East-West.

CONCLUSION

Correlation structure of the ionosphere is important for the estimation process and determining the decorrelation value. This process depends on the ionospheric model used. While the thin shell model worked well for the CONUS, it has deficiencies for the Equatorial region. The Slant to Vertical delay conversion maps different ionospheric activities to the same location and even constraining the look angles does not provide much advantage beyond 800 km separation. This indicates that the current correction method specified by the MOPs would not benefit significantly by using smaller grid spacing in these regions. Instead, we must find new methods to model the ionospheric delays. Another appreciable difference from mid-latitude is that while a single decorrelation value was used for the entire day in CONUS for WAAS, the equatorial structure appears to be much more variable with time. There is significantly higher decorrelation during the local evening and early night than during other times of day. Thus, it is desirable to use a decorrelation value that changes with the time of day.

ACKNOWLEDGEMENT

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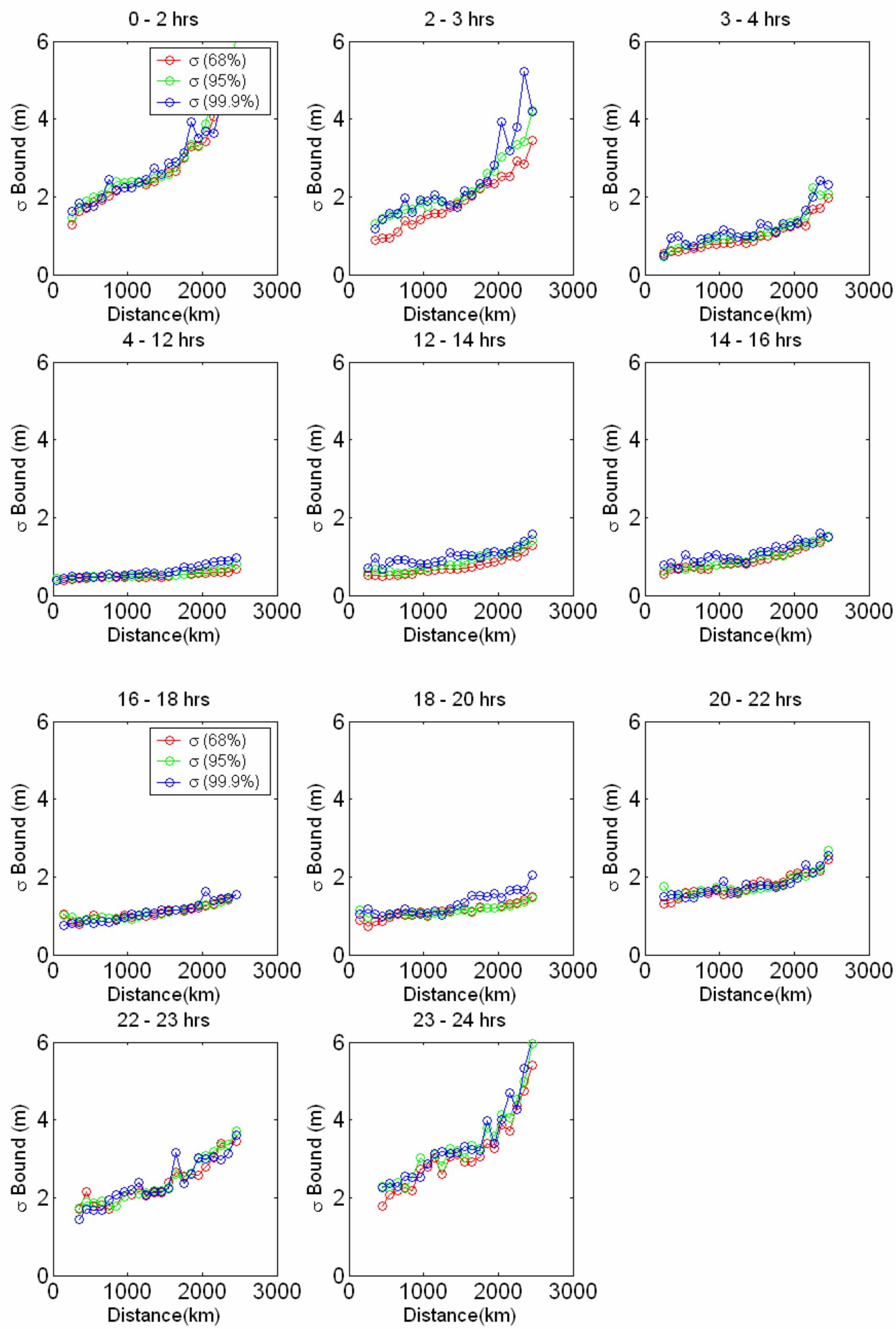

Figure 12. Sigma Containment value for 19th Feb 2002. The time references are relative to corresponding UTC.

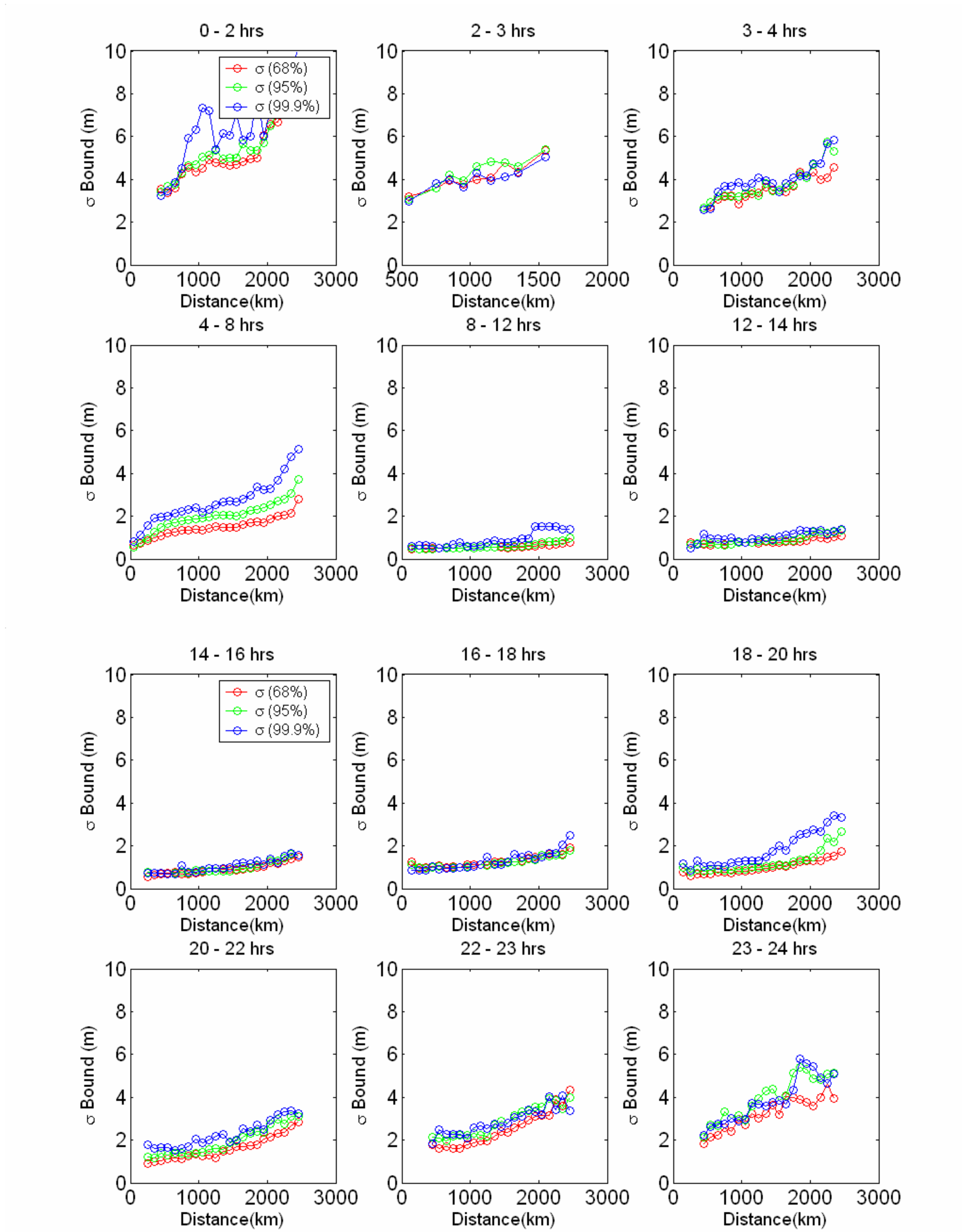

Figure 13. Sigma Containment value for 21st Feb 2002. The time references are relative to corresponding UTC.