Availability Benefit of Future Dual Frequency GPS Avionics under Strong Ionospheric Scintillation

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BIOGRAPHY

Jiwon Seo is a Ph.D. candidate in aeronautics and astronautics at Stanford University. He received his B.S. in mechanical engineering (division of aerospace engineering) from KAIST (Korea Advanced Institute of Science and Technology) and received M.S. degrees in aeronautics/astronautics and electrical engineering from Stanford. His current research focuses on aircraft navigation using GPS and WAAS under severe ionospheric scintillation of the equatorial region. He was a recipient of the Samsung Lee Kun Hee Graduate Fellowship for five years.

Todd Walter received his B.S. degree in physics from Rensselaer Polytechnic Institute, Troy, NY, and his Ph.D. degree from Stanford University, Stanford, CA, in 1993. He is currently a Senior Research Engineer in the Department of Aeronautics and Astronautics, Stanford University. He has long been active in the development of the Wide Area Augmentation System and its international counterparts. His early work included some of the first operational prototyping and development of many key operational algorithms and standards in use on these systems. His current research focuses on the future use of the two civil frequencies available through modernized GPS and new satellite navigation systems being implemented around the world. Dr. Walter is a Fellow of the Institute of Navigation (ION). He was a corecipient of the 2001 Early Achievement Award from the ION and winner of the 2009 Thurlow Award.

Per Enge received his Ph.D. degree in electrical engineering from the University of Illinois, Urbana–Champaign. He is the Kleiner-Perkins, Mayfield, Sequoia Capital Professor in the School of Engineering, Stanford University, Stanford, CA. He is also Director of the GPS Research Laboratory, which pioneers satellite-based navigation systems for aviation and maritime use. Two of these systems are in widespread use today. The first uses medium frequency beacons to broadcast differential GPS corrections to some 1.5 million, mostly marine, users around the globe. The second uses geostationary satellites to broadcast differential corrections and realtime error bounds to GPS users in North America. This latter system came on line for aviation in the United States in July 2003, and similar systems are being developed in Europe, Japan, and India. Prof. Enge is a Fellow of the Institute of Navigation and the IEEE, and a member of the National Academy of Engineering. He has received the Kepler, Thurlow, and Burka Awards for his work.

ABSTRACT

A Global Positioning System (GPS) receiver may lose carrier tracking lock to the GPS signal under deep and frequent fades due to ionospheric scintillation. The frequent loss of lock observed during a strong scintillation period from the past solar maximum can significantly reduce GPS aviation availability. However, the frequency diversity (L1 and L5 frequencies) from the future GPS is expected to mitigate scintillation impact on GPS aviation.

In order to assess its mitigation effectiveness, we propose a way to generate correlated fading processes based on our definition of a correlation coefficient between fading channels. Using the correlated fading process model that we propose, navigation availability of Localizer Performance with Vertical guidance (LPV)-200 during severe scintillation is parametrically studied. We then present results showing that high navigation availability is attainable if a receiver reacquires the lost channel within 1 or 2 seconds.

Based on this result, we propose a new performance requirement for the future dual frequency GPS aviation receiver performance standards to guarantee high navigation availability during strong scintillation.

Institute of Navigation, GNSS 2009
22-25 September 2009, Savannah, GA

INTRODUCTION

The Global Positioning System (GPS) with integrity information from the Wide Area Augmentation System (WAAS) [1] is currently used in the United States to guide the approach of aircraft down to 200 ft above the runway. This approach procedure, also known as Localizer Performance with Vertical guidance (LPV)-200 [2], can be expanded globally by future GPS with augmentations that provides dual frequency civilian codes in the aviation band (L1 and L5 frequencies) [3, 4]. The ionospheric delay can be directly measured by future dual frequency GPS avionics, and higher-order ionosphere errors are not problematic for LPV-200 [5]. However, deep and frequent GPS signal fades due to ionospheric scintillation [6] raises a concern about the operational availability of LPV-200 in the equatorial area during solar maxima. Deep and frequent signal fading observed during the past solar maximum (2001) led to frequent loss of the carrier tracking lock of GPS receivers, which can significantly reduce navigation availability depending on a receiver's reacquisition capability [7].

However, the frequency diversity from L1 and L5 frequencies from future GPS satellites can mitigate scintillation impact on GPS aviation availability. For example, if a receiver tracks both L1 and L5 frequencies from the same satellite, it can rely on one frequency when it briefly loses the other frequency. Hence, the correlation of signal fades across the two frequencies is important to understand in order to assess the effectiveness of mitigation from the frequency diversity. If signal fades of two frequencies are highly correlated, i.e., if the probability of the simultaneous fading of L1 and L5 channels of the same satellite is high, the actual benefit from frequency diversity would be limited.

A recent study reported very high correlation coefficients (exceeding 0.7) among signal intensities of L1 and L2 channels based on early GPS data collected at Thule, Greenland during 1989-1991 [8]. However, the conventional definition of sample correlation coefficient between two time series of signal intensity used in the study evaluates the similarity of the trends of two time series. It does not describe how often both frequencies are lost simultaneously. Hence, the conventional correlation coefficient is not a good metric for evaluating scintillation impact on GPS aviation. Since there is no appropriate metric to measure the probability of simultaneous fading of two channels, this paper suggests a new correlation coefficient from the perspective of GPS aviation.

Although strong scintillation data from the L5 signal are not yet available, correlated L1 and L5 fading processes with arbitrary correlation coefficients can be generated by the method proposed in this paper. Using this method, the availability of LPV-200 during strong scintillation can be parametrically studied with any correlation levels between frequencies, even before L5 scintillation data can be collected during the next solar maximum (around 2013). A new performance requirement for GPS aviation receivers is proposed based on the availability analysis results.

DEEP GPS SIGNAL FADES AND CORRELATION COEFFICIENTS

Electron density irregularities in the ionosphere can cause amplitude fading and phase jitter of transionospheric radio waves, which is referred to as ionospheric scintillation [6]. Most of the Carrier-to-Noise-density ratio (C/N_0) fluctuations due to scintillation in Figure 1 (bottom) are not problematic for GPS aviation while the receiver maintains its carrier tracking lock. However, the deep signal fading (marked in red) causing loss of lock is of real concern. Our primary interest is not how the GPS signal fluctuates, but when a receiver loses lock due to deep fading.

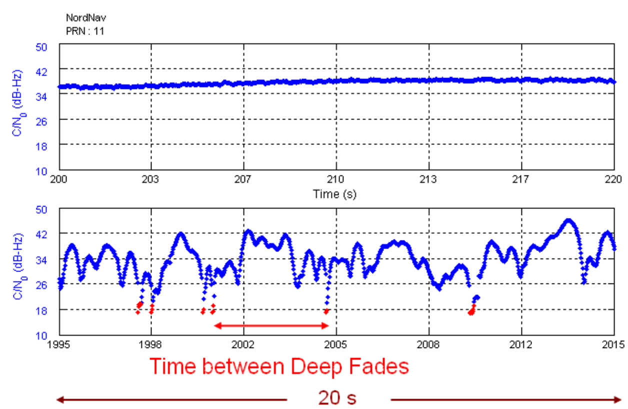

Figure 1. Comparison of 50 Hz C/N_0 outputs of nominal (top) and scintillation (bottom) cases.

Future GPS satellites will provide civilian codes at dual frequencies (L1 and L5 frequencies) for aviation applications. When a receiver loses lock on the L1 channel due to deep fading, it can still track the L5 channel to provide range measurements unless L1 and L5 channels are lost simultaneously. Hence, the correlation of signal fades between L1 and L5 frequencies should be well understood to assess mitigation effectiveness from frequency diversity.

A high correlation of signal fades in this paper means that the signal fades of different frequencies occur simultaneously with a high probability. However, the conventional definition of the sample correlation coefficient of Equation (1) does not provide any information with respect to how often the deep fades of two channels occur simultaneously.

$$\hat{\rho} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}} \quad (1)$$

Figure 2 illustrates a weakness of this definition of the sample correlation coefficient. The x_i and y_i of Equation (1) are the C/N_0 sample points in green and blue in Figure 2. The $\bar{x}$ and $\bar{y}$ are the mean values. In the top plot of Figure 2, only one channel (blue) experiences deep fading. Hence, the receiver would still be able to track the other channel (green) when the other channel (blue) is briefly lost due to deep fading. The bottom plot of Figure 2 represents a more challenging scenario. In this case, two channels sometimes fade simultaneously (red circles); as a result, the receiver does not have a backup channel during these outage periods.

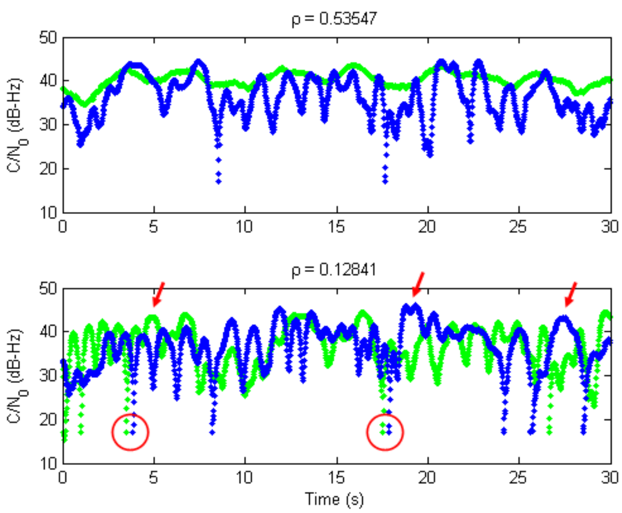

Figure 2. Weakness of the conventional definition of a sample correlation coefficient.

Obviously, the bottom plot is more problematic for GPS navigation, but it has a lower sample correlation coefficient (0.13) than the other case (0.54) when the correlation coefficient of Equation (1) is used. This is because the definition of Equation (1) merely evaluates the similarity of the trends of two time series. The green time series of Figure 2 (top) follows the general trend of the blue time series. Consequently, the correlation coefficient of the top figure is high. However, the green time series of Figure 2 (bottom) sometimes goes up while the blue time series goes down and vice versa (red arrows). This opposite trend significantly reduces its sample correlation coefficient. Again, we are interested in how often deep fades of two frequencies occur simultaneously. However, deep fades are very brief and sample points during deep fades are few in number, so they have little influence on the sample correlation coefficient obtained by Equation (1), which is dominated by the large number of samples out of deep fades, which are not of our interest.

Figure 2 clearly demonstrates the weakness of the conventional definition of sample correlation coefficient for the study of scintillation impact on GPS aviation. We suggest and validate an alternative definition of a correlation coefficient in the next section.

SUGGESTION FOR A BETTER CORRELATION COEFFICIENT

In order to suggest a better correlation coefficient of signal fades for this study, we analyzed statistics of signal fades during a strong scintillation period of the previous solar maximum. The data were collected at Ascension Island for nine days in 2001 and the worst 45 min observation was used for this study. Detailed information about the data set is in [9]. The empirical Probability Density Function (PDF) of the time between deep fades is shown in Figure 3, based on the definition of time between deep fades of Figure 1. An interesting observation is that the distribution of the time between deep fades approximately follows an Exponential distribution with a mean of 9.71 s.

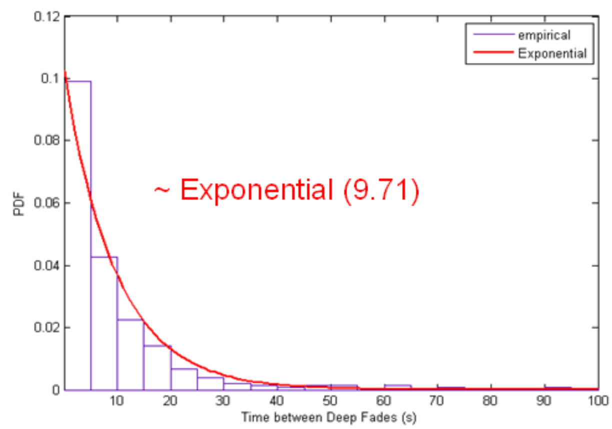

Figure 3. Empirical probability density function of time between deep fades during strong scintillation.

An important implication of the exponentially-distributed time between deep fades is that the corresponding counting process of the fading itself is a Poisson process [10]. Figure 4 illustrates this relationship. The time between deep fades corresponds to an inter-arrival time of an arrival process. Since the inter-arrival time (or the time between deep fades) follows the Exponential distribution with a mean of 9.71; the counting process of the number of arrivals (or number of deep fadings) is the Poisson process of rate 1 / 9.71.

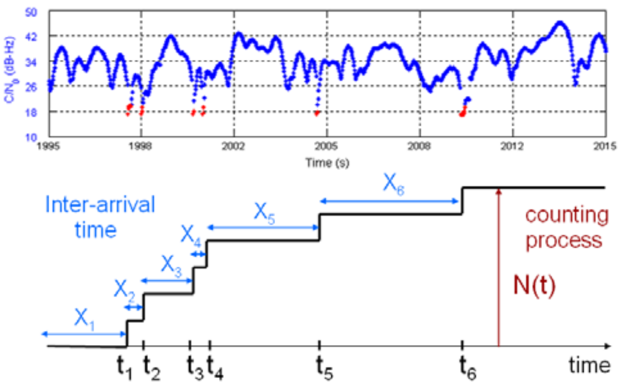

Figure 4. Modeling a fading process as a Poisson process.

The Poisson process has many interesting properties. For instance, the number of arrivals during an interval *t*, that is N(t) in Figure 4, of a Poisson process of rate λ is a Poisson random variable with a mean and variance λt. If two independent Poisson processes of rates λ_1 and λ_2 are combined together, the resulting process is still a Poisson process of added rate $\lambda_1 + \lambda_2$ [10]. Using this property, we can generate two correlated Poisson processes from three independent Poisson processes (Figure 5).

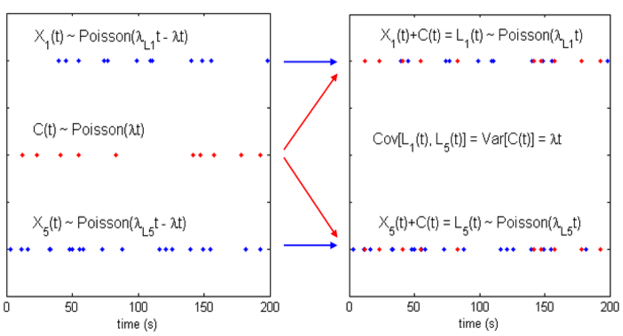

Figure 5. Generation of two correlated Poisson processes from three independent Poisson processes.

In Figure 5, the common Poisson process, C(t), of rate λ is combined with two independent Poisson processes, $X_1(t)$ and $X_5(t)$, of rates $\lambda_{L1} - \lambda$ and $\lambda_{L5} - \lambda$, respectively. The result is two correlated Poisson Processes, $L_1(t)$ and $L_5(t)$, of rates λ_{L1} and λ_{L5}, respectively. Since the covariance between $L_1(t)$ and $L_5(t)$ is the same as the variance of the common Poisson process, C(t) (See the Appendix for proof), the correlation coefficient between $L_1(t)$ and $L_5(t)$ is obtained as follows.

$$\rho = \frac{\mathrm{Cov}[L_1(t), L_5(t)]}{\sqrt{\mathrm{Var}[L_1(t)]}\sqrt{\mathrm{Var}[L_5(t)]}} = \frac{\mathrm{Var}[C(t)]}{\sqrt{\mathrm{Var}[L_1(t)]}\sqrt{\mathrm{Var}[L_5(t)]}} = \frac{\lambda t}{\sqrt{\lambda_{L1} t}\sqrt{\lambda_{L5} t}} = \frac{\lambda}{\sqrt{\lambda_{L1}}\sqrt{\lambda_{L5}}} \qquad (2)$$

The physical interpretation of λt is the expected number of arrivals of a Poisson process of rate λ during an interval *t*. Hence, when a fading process is modeled as a Poisson process, λt is interpreted as the expected number of deep fadings during an interval *t*. Since the common Poisson process C(t) in Figure 5 models the simultaneous fading of L1 and L5 channels, the correlation coefficient of Equation (2) can be interpreted as follows.

$$\rho = \frac{\lambda t}{\sqrt{\lambda_{L1} t}\sqrt{\lambda_{L5} t}} = \frac{\text{Expected number of simultaneous fadings during } t}{\sqrt{\text{Expected number of } L_1 \text{ fadings during } t}\sqrt{\text{Expected number of } L_5 \text{ fadings during } t}} \qquad (3)$$

Therefore, the correlation coefficient defined in Equation (3) describes how often simultaneous fading of two frequencies occurs. We propose this definition of a correlation coefficient as an appropriate metric to measure the correlation level between fading channels. This definition of a correlation coefficient is mathematically derived from the statistical observation and makes good physical sense.

Furthermore, Figure 5 illustrates a way to generate two correlated fading processes with an arbitrary correlation coefficient. The rate of L1 fading (number of L1 deep fadings per unit time) was obtained from the real scintillation data ($\lambda_{L1} = \frac{1}{9.71}$), but the rate of L5 fading is not yet known. Currently, only one GPS satellite is transmitting an L5 testing signal [11, 12]. According to [13], the amplitude scintillation index (S_4 index) of the L5 frequency (1.176 GHz) is expected to be greater than the S_4 index of the L1 frequency (1.575 GHz).

$$S_{4,L5} = S_{4,L1}\left(\frac{f_{L1}}{f_{L5}}\right)^{1.5} = 1.55\, S_{4,L1} \qquad (4)$$

However, Equation (4) does not relate the fading rates of L1 and L5. The fading rate refers to the number of deep fades or the number of losses of lock of the channel per unit time. Although the signal fluctuations of L5 could be worse than the L1 signal fluctuations, the higher power and the better code structure of the L5 signal [14] would reduce the chance of loss of lock of the L5 tracking channel. Hence, we may assume that the L5 fading rate is similar to the L1 fading rate, which should be validated during the next solar maximum (around 2013).

After assuming that λ_{L5} is equal to λ_{L1}, we can obtain λ for an arbitrary correlation coefficient ρ from Equation (2).

$$\lambda = \rho\sqrt{\lambda_{L1}}\sqrt{\lambda_{L5}} = \frac{\rho}{9.71} \qquad (5)$$

Once λ is obtained for a given correlation coefficient, three independent Poisson processes, $X_1(t)$, $C(t)$, $X_5(t)$, of rates $\lambda_{L1}-\lambda$, λ, $\lambda_{L5}-\lambda$, respectively, can be generated. Finally, the correlated fading processes, $L_1(t)$ and $L_5(t)$, with the given correlation coefficient ρ are obtained by combining these three Poisson processes (Figure 5). Using this method, the navigation availability of LPV-200 under strong scintillation is parametrically studied in the following section.

AVAILABILITY BENEFIT FROM DUAL FREQUENCY GPS AVIONICS

Availability Simulation Using Correlated Fading Processes

A previous study evaluated the GPS aviation availability during an observed strong scintillation period [7]. The study considered shortened carrier smoothing time due to frequent fades and presented availability results as a function of a receiver's reacquisition time. However, the expected benefit from uncorrelated L1/L5 channels was not considered. This previous study assumed 100% correlation between L1 and L5 channels, which means that whenever an L1 channel was lost due to deep fading, the L5 channel was assumed to be lost as well. It is a very conservative assumption.

We assume a 1 m User Range Accuracy (URA), the iono-free dual frequency code noise and multipath model based on the WAAS Minimum Operational Performance Standards (MOPS) [15], the troposphere model from the WAAS MOPS, and the same satellite constellation observed during the strong scintillation period in [7]. The availability of this paper is also obtained for a single user at Ascension Island where the scintillation data were collected. However, the availability simulation of this paper does not rely on the fading time series of the collected scintillation data. The collected data is L1 only and there is no L5 strong scintillation data available. For this reason, we generate correlated L1/L5 fading processes with arbitrary correlation coefficients and parametrically evaluate the expected benefit from frequency diversity.

Two parameters are considered for the availability simulation of this paper. These are the correlation coefficient between L1 and L5 channels and the reacquisition time of a receiver. The reacquisition time is defined as a time to reestablish the lock after loss of lock and reintroduce the lost channel into the position calculation. If a receiver reacquires the lost channel quickly, the satellite outage duration is reduced; consequently, scintillation impact is also reduced.

The correlated fading processes generated by the method of the previous section simulate time information of instances of deep fades for all channels (L1/L5 channels of eight satellites in view in our case). The receiver is conservatively assumed to lose its carrier lock with 100% probability at each deep fading.

The severe scintillation scenario for the availability simulation of this paper is much worse than the strong scintillation data collected at Ascension Island during the previous solar maximum. In the real data, 7 satellites out of 8 were fading for some fraction of 45 min. In the severe scintillation scenario that we generated, L1/L5 channels of all 8 satellites are fading at the maximum rate for the whole time period.

There is one more issue to consider for the availability analysis. A GPS receiver has to track both frequencies to directly measure the ionospheric delay. If a receiver briefly loses one frequency after deep fading, the ionospheric delay cannot be directly measured. Hence, we need a way to bound this unknown ionospheric delay error during brief outage periods. Three different ways to bound ionospheric delay errors are discussed in the following section, and the operational availability of LPV-200 is obtained for each case.

Availability under the Most Conservative Ionospheric Delay Estimation

The most conservative approach to bound the ionospheric delay error is that a receiver does not trust range measurements from a satellite if the receiver lost either the L1 or L5 channel of the satellite. If this approach is applied, higher correlation between L1 and L5 channels gives better availability. The higher correlation of this paper means that fading of L1 and L5 channels occurs simultaneously with high probability. In this most conservative approach, a receiver does not use range measurements for position calculations unless dual frequency measurements are available. Hence, if a receiver loses one frequency followed by the other, the total duration of satellite loss would be extended. Therefore, it is better to lose both frequencies together instead of losing one frequency after the other, if the fading rates of two frequency channels are fixed.

This intuitive explanation is validated by the availability simulation result in Figure 6. The 35 m Vertical Alert Limit (VAL) and the 40 m Horizontal Alert Limit (HAL) for LPV-200 are considered for this analysis. Since the approach to bound the ionospheric delay error is so conservative, even a short reacquisition time of 1 s and 100% correlation between L1 and L5 channels provides only about 95% availability.

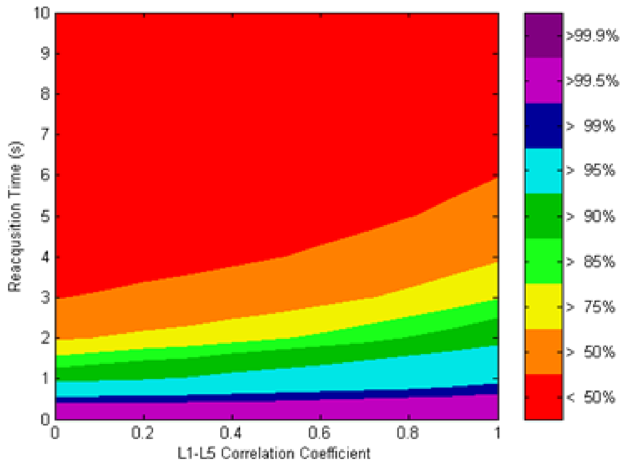

Figure 6. Availability of LPV-200 with the most conservative ionospheric delay estimation.

Availability under the Most Optimistic Ionospheric Delay Estimation

Now we consider the most optimistic approach for bounding the ionospheric delay error. Since fading duration is very brief—about 0.2 s (95%) at 20 dB-Hz [9]—the ionospheric delay error may not grow significantly during this short period. Hence, a receiver may use the most recent ionospheric delay measurement during the brief period of single frequency loss. In this case, the receiver does not lose a satellite unless both frequency channels are lost simultaneously.

The availability of LPV-200 using this approach is presented in Figure 7. A receiver fully relies on the most recent ionospheric delay measurement and the range measurement from one frequency when it briefly loses the other frequency. As expected, lower correlation between two frequencies provides better availability. In this case, a 1 s reacquisition time of a receiver gives more than 99.5% availability if a 50% correlation between L1/L5 channels is assumed. Although the actual correlation level between L1/L5 channels should be validated during the next solar maximum, Figure 7 parametrically illustrates the expected availability levels depending on a correlation coefficient and a receiver's reacquisition time under a severe scintillation scenario.

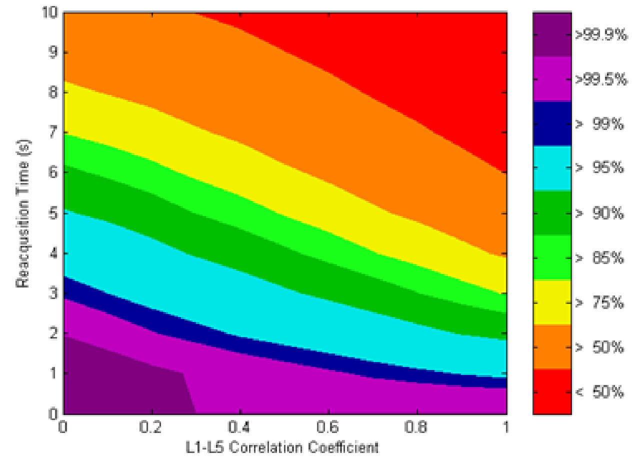

Figure 7. Availability of LPV-200 with the most optimistic ionospheric delay estimation.

Availability under the Sharpest Ionospheric Gradient Observed

Two extreme approaches for bounding the ionospheric delay error were discussed. The conservative approach is too conservative to provide high navigation availability. The optimistic approach gives high availability, but it may be too optimistic to guarantee navigation integrity. In equatorial area, scintillation often accompanies depletions, regions of steep ionospheric gradient. Thus, sudden large changes in ionospheric delay are expected to occur simultaneously with strong scintillation.

In order to guarantee the integrity, the sharpest observed ionospheric gradient and the worst-case geometry is assumed. According to [16], 425 mm/km was the largest observed gradient in slant ionospheric delay in the United States. Based on this sharpest ionospheric gradient, the worst-case combination of geometries of Figure 8 is considered. If a satellite and a plane move in the same direction and if the sharpest ionospheric gradient moves towards the plane, the ionospheric range error can grow with the rate of 0.5 m/s.

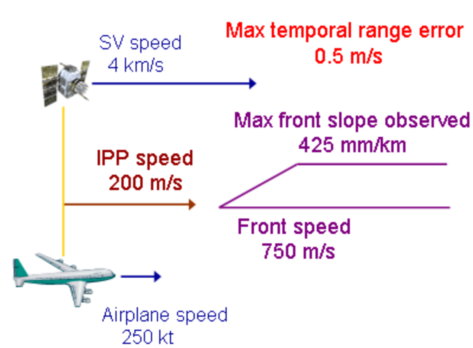

Figure 8. Maximum temporal range error under the sharpest ionospheric gradient.

This maximum temporal range error can be bounded by a new VPL equation.

$$\mathrm{VPL}_{\mathrm{New}} = \mathrm{VPL}_{\mathrm{Nominal}} + \sum_i \left| S_{vert,i}\, a\, t_i \right| \qquad (6)$$

where a is the growing rate of the ionospheric range error, and t_i is the time after losing the range measurement of one frequency of the satellite *i*. (Refer to Page J-1 of [15] for the definitions of $\mathrm{VPL}_{\mathrm{Nominal}}$ and *S* matrix.)

Figure 9 graphically illustrates this approach to bound the maximum temporal range error. Once a receiver loses one frequency of the satellite *i*, the $a\,t_i$ term of the new VPL increases with the rate of $a = 0.5$ m/s to bound the error. If both frequencies are lost, the satellite is excluded from calculating position solutions. If the receiver re-tracks both frequencies, the new VPL value drops to the nominal VPL value, since the ionospheric delay can be directly measured again. Note that this approach is still very conservative because it assumes the sharpest ionospheric gradient under the worst-case combination of geometries (Figure 8) for all satellites in view.

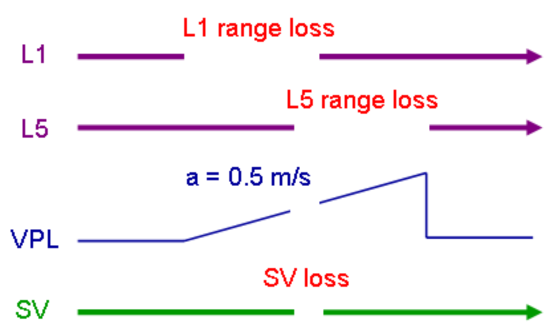

Figure 9. Vertical Protection Level (VPL) calculation under single frequency loss.

Using this approach, the availability of LPV-200 is obtained in Figure 10. Comparing to the most optimistic case of Figure 7, there is not much difference in availabilities if a receiver reacquires the lost channel within 3 s.

In order to evaluate the sensitivity of the availability to the temporal range error, a safety factor of 2 is applied to the maximum temporal range error. Even with the 1.0 m/s temporal range error, the availability for a short reacquisition time (less than 3 s) does not change much. We can still expect more than 99.5% availability, if the L1/L5 correlation is about 50% and a receiver's reacquisition time is 1 or 2 s even under the severe scintillation scenario that we generated.

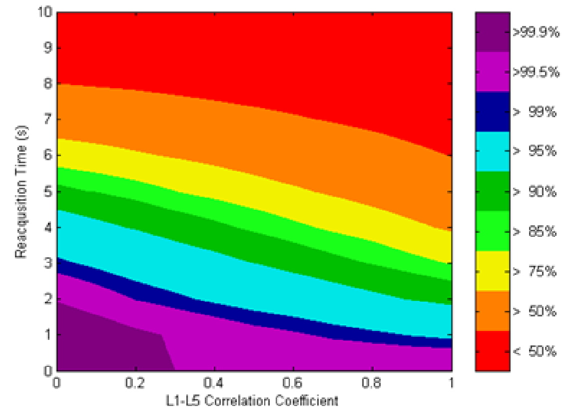

Figure 10. Availability of LPV-200 with the new VPL bounding the maximum temporal range error (0.5 m/s).

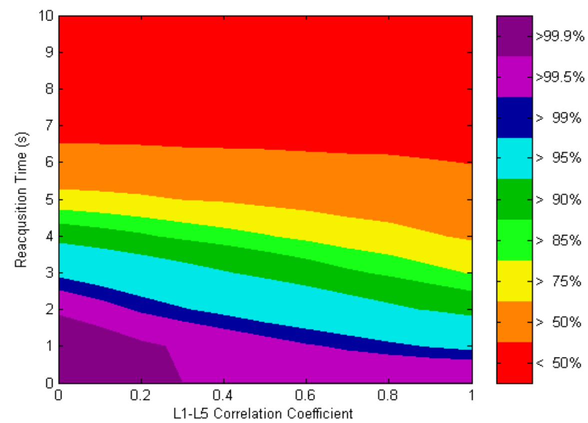

Figure 11. Availability of LPV-200 with the new VPL bounding twice of the maximum temporal range error (1.0 m/s).

New Requirement for Future Dual Frequency Aviation Receiver Performance Standards

The current WAAS MOPS includes the following note about scintillation, "There is insufficient information to characterize scintillation and define appropriate requirements and tests for inclusion in this MOPS. … New requirements may be defined when ionospheric effects can be adequately characterized [15]." Although the current WAAS MOPS does not have any specific performance requirement to mitigate scintillation impact, we observe that the current Satellite Reacquisition Time requirement is related to this mitigation. The current requirement says, "For satellite signal outages of 30 seconds or less … the equipment shall reacquire the satellite within 20 seconds from the time the signal is reintroduced [15]." This requirement considers Radio Frequency Interference (RFI), but signal outages under strong scintillation are much shorter than 30 s and are actually shorter than 1 s.

In order to provide high navigation availability under strong scintillation, we suggest an additional requirement for a future dual frequency MOPS. The new requirement that we propose is, "For satellite signal outages of 1 second or less … the equipment shall reacquire the satellite within 1 second from the time the signal is reintroduced [17]." If a receiver is propagating its tracking loop for brief outages instead of going through its full reacquisition procedures, it can immediately re-track the lost channel when the signal comes back. Hence, fast reacquisition after a brief outage is technically possible. However, if a receiver performs extensive safety checks before reintroducing the lost channel into position solutions, the reacquisition time can be much longer. Therefore, this new proposal needs to be evaluated against RFI threats as well. This new requirement was proposed at the RTCA Special Committee-159 (GPS) Working Group-2 (GPS/WAAS) Meeting on 24 June 2009.

CONCLUSION

The expected availability benefit from the frequency diversity of the future GPS is parametrically studied in this paper. For the parametric study, we propose a new sample correlation coefficient and a method to generate L1/L5 fading processes with an arbitrary correlation coefficient.

The availability result shows that high navigation availability (more than 99.5% availability) of LPV-200 is attainable even under the severe scintillation scenario that we generated, if a receiver's reacquisition time is short. Without this requirement, a certified aviation receiver may provide less than 50% availability during severe scintillation. Therefore, if a future dual frequency MOPS mandates fast reacquisition after brief loss of lock, LPV-200 can achieve high availability during strong scintillation of solar maxima in the equatorial area.

APPENDIX

This appendix proves $\mathrm{Cov}[L_1(t), L_5(t)] = \mathrm{Var}[C(t)]$, where

$L_1(t)$, $L_5(t)$, $C(t)$ are given as in Figure 5.

Since $X_1(t)$, $X_5(t)$, $C(t)$ are all independent,

$$
\begin{aligned}
E[L_1(t)\,L_5(t)] &= E[\{X_1(t)+C(t)\}\{X_5(t)+C(t)\}] \\
&= E[X_1(t)\,X_5(t) + X_1(t)\,C(t) + X_5(t)\,C(t) + C(t)^2] \\
&= E[X_1(t)]\,E[X_5(t)] + E[X_1(t)]\,E[C(t)] \\
&\quad + E[X_5(t)]\,E[C(t)] + E[C(t)^2]
\end{aligned}
$$

$$
\begin{aligned}
E[L_1(t)]\,E[L_5(t)] &= E[X_1(t)+C(t)]\,E[X_5(t)+C(t)] \\
&= \{E[X_1(t)] + E[C(t)]\}\{E[X_5(t)] + E[C(t)]\} \\
&= E[X_1(t)]\,E[X_5(t)] + E[X_1(t)]\,E[C(t)] \\
&\quad + E[X_5(t)]\,E[C(t)] + E[C(t)]^2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\mathrm{Cov}[L_1(t), L_5(t)] &= E[L_1(t)\,L_5(t)] - E[L_1(t)]\,E[L_5(t)] \\
&= E[C(t)^2] - E[C(t)]^2 \\
&= \mathrm{Var}[C(t)].
\end{aligned}
$$

ACKNOWLEDGMENTS

The authors gratefully acknowledge the Federal Aviation Administration (FAA) CRDA 08-G-007 for supporting this research and Theodore Beach, AFRL (Air Force Research Lab), for providing the data sets. The authors also appreciate the valuable comments from Sam Pullen, Stanford University, and Seebany Datta-Barua, ASTRA (Atmospheric and Space Technology Research Associates). The opinions discussed here are those of the authors and do not necessarily represent those of the FAA or other affiliated agencies.

REFERENCES

[1] Enge, P., T. Walter, S. Pullen, C. Kee, Y.-C. Chao, and Y.-J. Tsai, *Wide area augmentation of the global positioning system*, Proceedings of the IEEE, 84, 1063–1088, 1996.

[2] Cabler, H., and B. DeCleene, *LPV: New, improved WAAS instrument approach*, Proceedings of the 15th International Technical Meeting of the Satellite Division of the Institute of Navigation, Portland, OR, 24-27 September, 2002.

[3] Walter, T., P. Enge, J. Blanch, and B. Pervan, *Worldwide vertical guidance of aircraft based on modernized GPS and new integrity augmentations*, Proceedings of the IEEE, 96, 1918-1935, 2008.

[4] Hegarty, C. J., and E. Chatre, *Evolution of the Global Navigation Satellite System (GNSS)*, Proceedings of the IEEE, 96, 1902-1917, 2008.

[5] Datta-Barua, S., T. Walter, J. Blanch, and P. Enge, *Bounding higher-order ionosphere errors for the*

dual-frequency GPS user, Radio Science, 43, RS5010, 2008.

[6] Crane, R. K., *Ionospheric scintillation*, Proceedings of the IEEE, 65, 180–199, 1977.

[7] Seo, J., T. Walter, T.-Y. Chiou, J. Blanch, and P. Enge, *Evaluation of deep signal fading effects due to ionospheric scintillation on GPS aviation receivers*, Proceedings of the 21st International Technical Meeting of the Satellite Division of the Institute of Navigation, Savannah, GA, 16-19 September, 2008.

[8] El-Arini, M. B., J. Secan, J. A. Klobuchar, P. H. Doherty, G. Bishop, and K. Groves, *Ionospheric effects on GPS signals in the Arctic region using early GPS data from Thule, Greenland*, Radio Science, 44, RS0A05, 2009.

[9] Seo, J., T. Walter, T.-Y. Chiou, and P. Enge, *Characteristics of deep GPS signal fading due to ionospheric scintillation for aviation receiver design*, Radio Science, 44, RS0A16, 2009.

[10] Gallager, R. G., *Discrete Stochastic processes*, Kluwer Academic Publishers, 1995.

[11] Gao, G. X., L. Heng, D. De Lorenzo, S. Lo, D. Akos, A. Chen, T. Walter, P. Enge, and B. Parkinson, *Observing the first GPS satellite with an L5 payload*, Inside GNSS, May/June, 30-36, 2009.

[12] Gunawardena S., Z. Zhu, and M. Braasch, *Observing the GPS L5 test transmission from SVN49 using software radio processing*, Inside GNSS, May/June, 22-29, 2009.

[13] Fremouw, E. J., R. L. Leadabrand, R. C. Livingston, M. D. Cousins, C. L. Rino, B. C. Fair, and R. A. Long, *Early results from the DNA Wideband satellite experiment—Complex-signal scintillation*, Radio Science, 13(1), 167–187, 1978.

[14] Van Dierendonck, A. J., and C. Hegarty, *The new L5 civil GPS signal*, GPS World, September, 2000.

[15] RTCA, Inc., *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment*, RTCA DO-229D, December 13, 2006.

[16] Pullen, S., Y. S. Park, and P. Enge, *Impact and mitigation of ionospheric anomalies on ground-based augmentation of GNSS*, Radio Science, 44, RS0A21, 2009.

[17] Seo, J., T. Walter, and P. Enge, *Possible modification to the WAAS MOPS regarding ionospheric scintillation*, RTCA SC-159 WG-2 Meeting, 24 June, 2009.