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L5 Satellite Based Augmentation Systems Protection Level Equations

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ABSTRACT

The current L1 Space Based Augmentation System (SBAS) protection level equations were agreed upon nearly a decade ago. These equations are provided for L1-only users and are based upon covariance propagation of zero-mean gaussian errors. While this description is reasonably accurate for some nominal error sources, it is not always a good model for actual error characteristics. When departures from the zero-mean gaussian model are significant, the broadcast confidence terms must be inflated in order to provide protection for all user geometries. This leads to a loss of availability even for users who do not observe the satellite with the problematic errors. Since these equations were first adopted, a significant amount of work has gone into accurate characterization of the error sources and the treatment of non-zero and non-gaussian errors. New SBAS signals planned for the L5-frequency offer an opportunity to revisit these decisions and make changes to improve integrity and availability.

KEYWORDS: SBAS, integrity, protection level, bias

1. INTRODUCTION

An additional civil frequency at L5 will be incorporated into future Global Navigational Satellite System (GNSS) design (Van Dierendonck, 2005). As part of this modernization, Space Based Augmentation Systems (SBASs) (Walter and El-Arini, 1999) will also be upgraded to incorporate the new civil frequency (Walter and Enge, 2004). This offers a unique opportunity to design a new messaging structure that improves upon the existing L1-only SBAS design. Signals transmitted on L5 can contain different information and formats than those on L1. It is possible to now use our experience with L1-only SBAS to suggest improvements for the L5 design. This paper will focus specifically on the Protection Level (PL) equations.

When the L1 SBAS Minimum Operational Performance Standards (MOPS) (recently updated in RTCA, 2006) were originally developed, it was envisioned that any knowable biases would be estimated and incorporated into the broadcast corrections. The user would only be left with the error on the correction, which was expected to be random over time. The only identified unknowable biases at that time were fault modes. Deformed signals, such as those observed on space vehicle 19 in 1993 (Mitelman, 2004), are an example of an unknowable bias. Separate analyses were created to account for these biases during their onset time before they were fully visible to the system. Additionally, there is a severe bandwidth constraint for SBAS. It only has available 250 bits per second to transmit corrections and confidences for all satellites in view plus the full ionospheric grid over the service area. It also has to meet a six second time-to-alarm requirement. Further, at the time the MOPS were initiated, selective availability was active, necessitating frequent clock updates for each satellite. Therefore, at that time it was decided not to broadcast separate bias terms.

Since that time, several unknowable biases have been characterized and are now included in the safety design of the Wide Area Augmentation System (WAAS) (Walter and Enge, 2006) and other SBASs. These bias terms include nominal deformations on the signals (Phelts, 2004) and group delay biases in the antennas (Shallberg and Grabowski, 2002). These sources create small repeatable biases in the system. For nominal deformation, the bias is dependent upon user hardware and cannot be removed by the ground infrastructure. Antenna biases could theoretically be calibrated and removed, but the actual process is not practical for WAAS. Further, there is no mechanism to eliminate small bias drifts over time.

Another limitation of the L1 protection level formulation is that it is specifically formulated for gaussian errors. Although many of the error sources do have errors that have nearly gaussian shapes, the formula does not explicitly account for even small deviations. Several mathematical approaches have been formulated to address this shortcoming, but they each introduce other limitations. All limit the number of biased error distributions that can be convolved together. All require that additional margin be left in the broadcast sigma terms to account for the non-gaussian behaviour (Rife et al. 2004a), (Rife et al., 2004b), (Schempp and Rubin, 2002), (Shively, 2000), (Van Graas, et al., 2004).

The protection level equations for L5 SBAS should be updated to explicitly account for non-zero means and non-gaussian behaviour. By broadcasting bias magnitude terms in addition to overbounding sigma terms, non-zero means can be explicitly handled. Smaller sigma values may then be broadcast so as not to adversely penalize all users. Further, the bias term allows a technique called paired bounding (Rife et al. 2004a) to be applied to handle non-gaussian

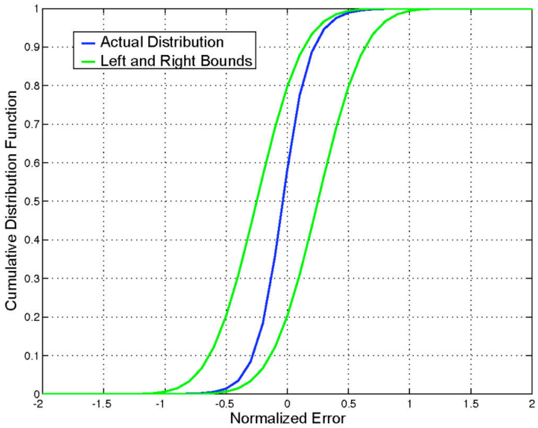

Figure 1. CDF of WAAS normalized UDRE data

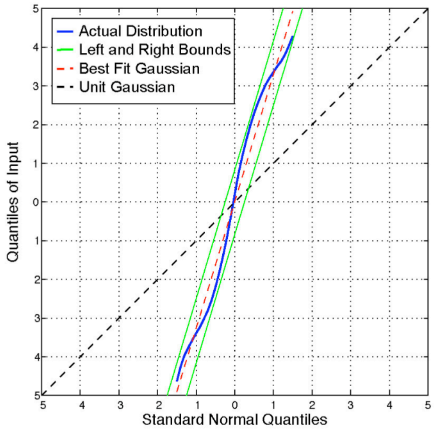

Figure 2. QQ plot of WAAS normalized UDRE data

error sources.

The broadcast of a bias term requires that additional information be transmitted to the user. However, the additional bandwidth needed may be quite small if the same UDREI index is used to indicate both the sigma and bias term. Flexibility can be maintained by occasionally broadcasting a definition table for the bias magnitude terms. Each individual service provider would optimise this table for their own error characteristics. Including a bias magnitude term in the L5 protection level equations will allow for both a simpler certification process for the ground system and higher availability for the user.

2. AVAILABILITY ANALYSIS

2.1 Modelling of Errors

We begin by examining the error distribution that we are trying to protect. As part of the formal proof of safety of WAAS, extensive data was collected (Raytheon, 2007). In particular, the term bounding the clock and ephemeris error, termed the User Differential Range Error or UDRE, was analysed. Here we show data for when satellites are best observed, that is, when the UDRE is at its lowest. This minimum UDRE value for WAAS is 3 m which is treated as a 3.29-sigma value. Figure 1 shows the Cumulative Density Function (CDF) for 30 days worth of pseudorange residual data when the UDRE was 3 m. This data has been normalized by the broadcast σ_{UDRE} term having a value of 0.912 m. The data is extremely well behaved and appears to be very close to Gaussian. We have also plotted some potential bounding distributions. These are normal distributions with sigma values of 0.3 and biases of plus and minus 0.25. They appear to surround the actual data to the left and to the right, but it is not clear on this scale.

Figure 2 is a so-called QQ plot where the quantiles of the actual distribution are plotted versus the quantiles of the normal distribution. On this plot, it is much easier to see the departure of the actual data from normality. It is also much clearer that the left and right bounds actually surround the real data. This data is extremely well behaved, but still is not perfectly Gaussian, nor perfectly symmetric. The original overbounding proof (DeCleene, 2000) requires symmetry, and does not offer a means to handle even a slight departure. This restriction led to the creation of alternate methods for bounding real data (Rife et al. 2004a), (Rife et al., 2004b) (Schempp and Rubin, 2002). Using these other methods, it was determined that a Gaussian with a variance as small as 0.4 could by itself bound the distribution provided in Figure 1. Therefore, it is very well overbounded by the broadcast value.

2.2 Protection Through Broadcast of Biases

The desired VPL when incorporating biases in addition to Gaussian error is

$$VPL_{Bias} = \sum_{i=1}^{N} \left| S_{3,i}\, \mu_i \right| + K_{V,PA} \sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_i^2} \tag{1}$$

where ***S*** is the projection matrix that takes errors from the pseudorange domain to the position domain (see Appendix J of RTCA, 2006), μ is an upper bound on the bias and σ^2 is an overbounding variance. The first term accounts for the biases and the second term covers the Gaussian errors. The L1-only SBAS MOPS VPL equation consists of only the second term. We will use a subscript *a* on the above bias and variance to denote the actual values we would send if we could use (1) as the VPL equation. Currently, SBAS must overbound the above equation with the equation defined in the SBAS MOPS. That is, it requires that

$$K_{V,PA} \sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{B,i}^2} \geq \sum_{i=1}^{N} \left| S_{3,i}\, \mu_{a,i} \right| + K_{V,PA} \sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{a,i}^2} \tag{2}$$

for all possible values of $S_{3,i}$. Here σ_B^2 is the broadcast variance inflated to cover the bias term and overbound the random errors. Because this relationship must hold true for any user geometry, the inflation must be made larger than necessary for the average user. Hence, we can see that the ability to broadcast a bias term would in fact lead to a smaller VPL for the majority of users.

First, we will investigate how to formulate σ_B^2 given a μ_a and a σ_a^2. We will impose the simplifying requirement that for all lines-of-sight, the ratio of the actual values to the broadcast values be below some maximum value

$$\frac{\mu_{a,i}}{\sigma_{B,i}} \leq \gamma \quad and \quad \frac{\sigma_{a,i}}{\sigma_{B,i}} \leq \alpha \quad \forall_i \tag{3}$$

From (2) we can find

$$1 \geq \frac{\sum_{i=1}^{N} \left| S_{3,i}\, \mu_{a,i} \right|}{K_{V,PA} \sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{B,i}^2}} + \frac{\sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{a,i}^2}}{\sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{B,i}^2}} \tag{4}$$

By dividing through by the left-hand side we obtain

$$1 \geq \frac{\sum_{i=1}^{N} \left| S_{3,i} \right|}{K_{V,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2}} \gamma + \alpha \tag{5}$$

Both sides are multiplied by $\sigma_{B,i}$ to yield

$$\sigma_{B,i} \geq \frac{\sum_{i=1}^{N} \left| S_{3,i} \right|}{K_{V,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2}} \mu_{a,i} + \sigma_{a,i} \tag{6}$$

We can use the Cauchy-Schwarz inequality to place an upper bound on the right-hand side

$$\sigma_{B,i} \geq \frac{\sqrt{N}}{K_{V,PA}} \mu_{a,i} + \sigma_{a,i} \tag{7}$$

Here we can see the inflation required by the presence of a bias. First, $\sigma_{a,i}$ must overbound the random error. Thus, it is often much larger than the sample standard deviation, as it protects against rare errors and fault modes whose magnitude is not easily detectable. The bias term is linearly added to this and scaled by the ratio of the square root of the maximum possible number of measurements to the MOPS constant, $K_{V,PA}$. Assuming a maximum of 12 ranging signals, this ratio will equal 0.65. Further, the worst ratios for the bias and the sigma terms must be used for all lines-of sight regardless of whether they actually contain a bias.

It is evident that there can be a significant penalty in using the current SBAS MOPS VPL equation in the presence of unknowable biases. If we allowed the direct broadcast of bounding bias magnitudes, these penalties could be avoided through the use of (1). In the next section, we investigate the reduction offered by transmitting the biases directly.

2.3 Availability Improvement

The improvement offered by (1) can be characterized by dividing that equation by the WAAS MOPS VPL equation

$$\frac{VPL_{Bias}}{VPL_{RSS}} = \frac{\sum_{i=1}^{N} \left| S_{3,i}\, \mu_{a,i} \right| + K_{V,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{a,i}^2}}{K_{V,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2\, \sigma_{B,i}^2}} \tag{8}$$

When the same ratios are applied on each line of sight, the above expression reduces to

$$\frac{VPL_{Bias}}{VPL_{RSS}} = \frac{\sum_{i=1}^{N} \left| S_{3,i} \right|}{K_{V,PA}\sqrt{\sum_{i=1}^{N} S_{3,i}^2}} \gamma + \alpha \tag{9}$$

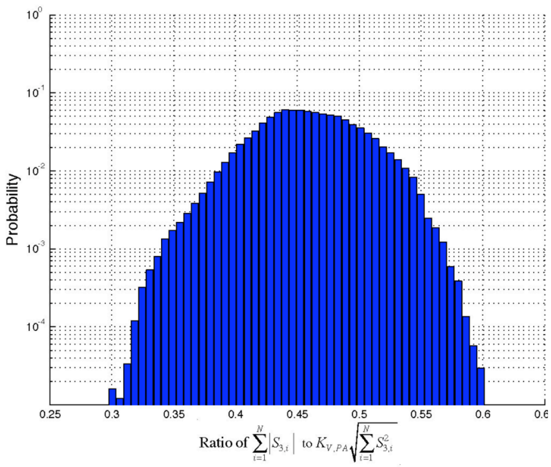

Figure 3. The geometrical ratio in (9) for the 24 satellite constellation

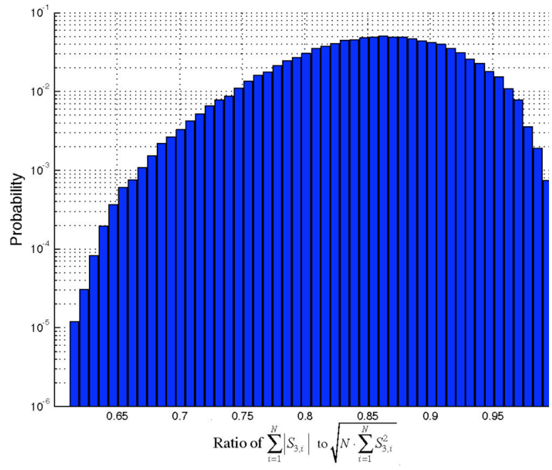

Figure 4. The geometrical ratio in (10) for the 24 satellite constellation

In order to estimate the performance benefit, we need to determine values for the geometry ratio, γ and α.

As discussed above, well-observed satellites have a minimum UDRE value of 3 m under WAAS. In the analysis of the Probability of Hazardously Misleading Information (PHMI) for the system, it was discovered that a minimum value for α was 0.4 (Raytheon, 2006). Further, from nominal signal deformation analysis, biases on user equipment could be in the range of 0.5 - 0.75 m. Taking an upper limit, this provides an upper bound of γ = 0.75 m / 0.912 m or approximately 0.8225.

To find values for the geometry ratio in (9) we conducted a simulation using our Matlab Availability Analysis Simulation Tool (MAAST) (Jan, et al., 2001). We simulated users across the world on a five-degree by five-degree grid. We calculated a position every 5 minutes using one of two GPS constellations. The first was the optimised 24-satellite constellation described in Appendix B of the WAAS MOPS. The second constellation is the same except that the most critical satellite (A2) is removed. Figure 3 shows the ratio in (9) for the 24-satellite constellation. In Figure 3, we can see that the ratio has an average value of about 0.46 and is largely bounded between 0.3 and 0.6. In Figure 4, we examine the ratio

$$\frac{\sum_{i=1}^{N}\left|S_{3,i}\,\sigma_{B,i}\right|}{\sqrt{N\cdot\sum_{i=1}^{N}S_{3,i}^{2}\sigma_{B,i}}}\leq 1 \tag{10}$$

which we know is bounded by the Cauchy-Schwarz inequality. We see that indeed the ratio does reach the upper bound of 1 indicating that the upper bound in (7) is required by some users although Figure 3 shows that the maximum value of 0.65 from the example following (7) was not quite reached.

When we select α and γ to evaluate we will choose them such that the RSS version of the VPL and the bias version provide the same protection for the worst-case hypothetical user. In this case the selection of α and γ are not independent. The relationship is governed by (7) and can be expressed as

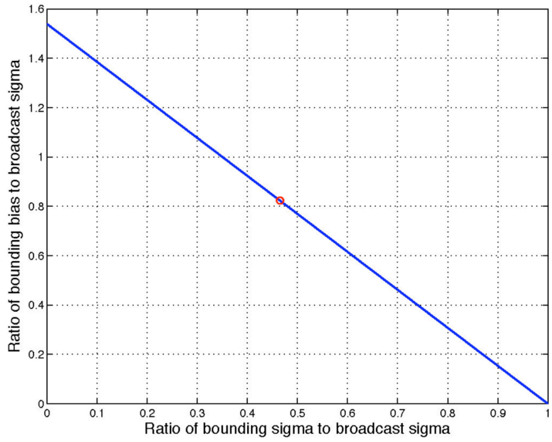

Figure 5. The relationship between α and γ to match the σ_B overbound. The red circle indicates the selected value for simulation

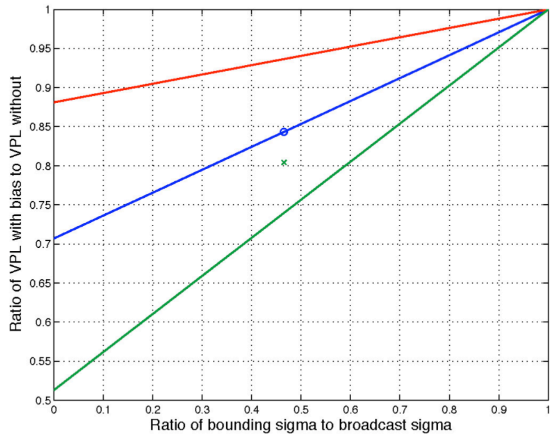

Figure 6. The ratio of the VPL with biases to the WAAS MOPS VPL as a function of α.

$$\gamma = \frac{\sqrt{N}\,(1-\alpha)}{K_{V,PA}} \tag{11}$$

This relationship is plotted in Figure 5. By selecting a value of $\gamma = 0.8225$ above, we require that $\alpha = 0.4654$ to exactly match the bounding provided by σ_B for the assumed worst-case user. The red circle in Figure 5 indicates this selection.

The reduction in VPL (9) is then determined by the geometric ratio multiplying the γ term. We have seen that this ratio ranges from 0.3 to 0.6 and has an average value of 0.46. If we look at the reduction in VPL for constant values of this geometric ratio, we can determine the benefit as a function of α. Figure 6 shows these values for the higher geometric ratio value (red), mean value (blue) and lower value (green). As would be expected, the benefit is a function of α. When $\alpha = 1$, there is no bias and therefore no benefit. When $\alpha = 0$, there is only bias and no variance. For this situation, the benefit is the greatest. The benefit varies linearly with α between these two extremes.

For the values selected in the simulation, the ratio between the two VPLs is about 0.84 as shown by the blue circle in Figure 6. This indicates more than a 15% reduction in VPL on average by broadcasting separate bias and sigma terms. Even better, the reduction is largest for the worst geometries: those with fewer satellites. If we look only at cases where the WAAS MOPS VPL is greater than 35 m, the average ratio is just above 0.8 as indicated by the green x. Thus, there is nearly a 20% average reduction of the largest VPLs.

Figure 7 shows histograms of the distribution of VPLs for the two VPL formulations. The blue bars indicate the distribution for the WAAS MOPS VPL and the red for the VPL with the bias term. It is obvious that there is a shift to lower VPLs when explicitly using the biases in the VPL. For this case, all of the VPLs originally above 35 m were shifted to below this value, creating 100% availability of LPV-200 service (Cabler and DeCleene, 2002). When the constellation is made a little weaker by removing a valuable satellite (A2), the performance is noticeably worse. Figure 8 shows the histograms for this 23-satellite constellation. Although there is still substantial improvement by broadcasting the biases, not

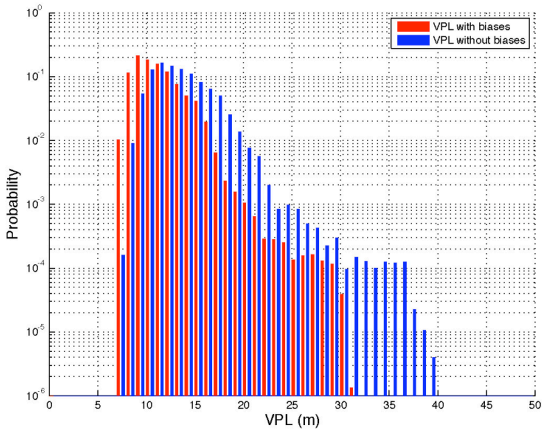

Figure 7. Histograms of VPL without biases (blue) and with biases (red) for a 24 satellite constellation.

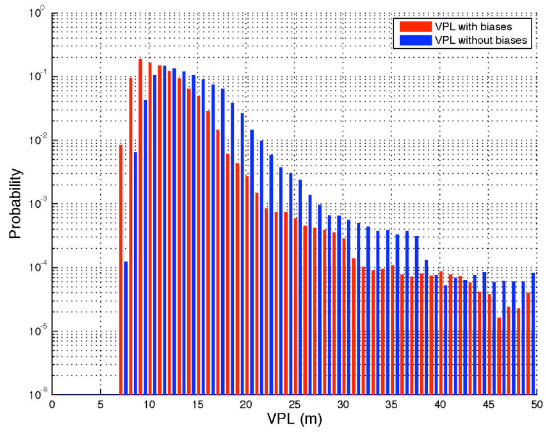

Figure 8. Histograms of VPL without biases (blue) and with biases (red) for a 23 satellite constellation.

all users have their VPLs improved to below 35 m. However, the average availability is improved from 99.72% to 99.88% and the fraction of the Earth that achieves 99% availability goes from 87% to 92%. Thus, there still is a significant improvement also under the worse constellation.

3. CONCLUSIONS

The upcoming civil signals on L5 offer us a unique opportunity to define new methods to broadcast information from the ground to the user. One possible improvement is to broadcast bias parameters in addition to sigma terms to overbound the user error positions. By making this information directly available to the user, we can avoid applying a worst-case inflation term on all users. As was demonstrated here VPLs can be lowered by 15-20% for the current GPS constellation. The reduction should be even larger when more signals are incorporated into the solution. With the advent of Galileo and Compass, the number of measurements could increase significantly. Under the existing protection scheme, UDREs would need to be increased to handle the additional sources even though each individual error distribution does not change. The inflation would be needed to protect against the greater possible number of convolved errors.

The addition of broadcast bias terms simplifies the ground integrity analysis and allows for better interoperability between constellations. Currently each system must inflate the broadcast sigma terms in order to protect against biases and non-Gaussian behaviour. The inflation factors are not transparent outside of the system. However, every sigma is required to be inflated by a minimum amount. This makes compatibility more difficult. By broadcasting biases, the inflation factors are no longer necessary, thus users can combine corrections and confidences from two different systems. Thus, the broadcast of bias terms can reduce complexity while increasing flexibility and availability.

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