Approaches to Improve Advanced RAIM Protection Levels

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ABSTRACT

We propose two changes in the Advanced RAIM Horizontal Protection Level that improve upon the baseline ARAIM algorithm used for the development of the standards, for an equivalent computational load. The first is a direct HPL computation that improves upon recently proposed ones. The second one is a refinement in the treatment of the effect of temporal exposure that undoes some of the conservatism associated with temporal exposure.

INTRODUCTION

As part of the standardization of Advanced RAIM, a baseline user algorithm has been developed and has evolved to account for more accurate threat models and loss of integrity evaluation. The purpose of this algorithm is to provide a feasible means of computing Protection Levels (PLs) that is relatively simple to describe and that meets the integrity and continuity requirements. It is therefore not necessarily providing the best possible Protection Levels. However, improvements that do not increase the computational load or the complexity of the description are worth integrating in this baseline algorithm. The goal of this paper is twofold: first to propose a set of improvements for both the test statistics and the PL, and second to evaluate how close these improvements get us to the best possible performance.

Among the simplifications done in the current baseline algorithm were the following:

- 1) the use of the one-dimensional PL to derive the Horizontal Protection Level (HPL). This approach simplified the description of the algorithm and the safety case while providing adequate performance at the levels of service of interest (Horizontal Alert (HAL) Limits of 556 m and 185 m, corresponding to the RNP0.1 and RNP0.3 operations). It is known that this approach is suboptimal for the HPL and several approaches with varying degrees of computational complexity have been proposed (Yang (2015), Langel (2021), Racelis (2022)). Although these approaches do not substantially change the availability of H-ARAIM, they do lower the HPLs, therefore providing additional margin, or enabling operations with lower HALs. These approaches are especially interesting when they are not more complex or computationally intensive than the baseline algorithm (Racelis (2022)).
- 2) the treatment of the temporal exposure. As described, the integrity risk is allocated among the time steps and then a PL is computed to meet that allocation.

We propose a set of improvements in these three areas that have minimal impact on the computational load. For 1) we provide new formulas for the HPL that improve upon previous approaches without

increasing the computational complexity. For 2) we undo some of the conservatism in the current equations by evaluating the loss of integrity risk for a given class of fault modes over time. This approach makes the allocation of integrity among the time steps more efficient.

DIRECT COMPUTATION OF HPL

The HPL can be reduced further than what is proposed in ARAIM ADD v4.2 by computing it directly rather than computing an PL in each coordinate and computing the HPL as the smallest disk that contains the rectangle defined by the two PLs (and therefore defined by the radius $HPL = \sqrt{PL_1^2 + PL_2^2}$). The notations used in what follows can be found in the ARAIM ADD v4.2 or in Blanch (2022).

We want an upper bound of the probability of an HMI event given fault hypothesis k. This probability is bounded by the probability that the horizontal position error exceeds the HPL and that test statistics are within their thresholds

$$P\left(\left|\hat{x}^{(0)} - x\right| > HPL, \left|\hat{x}_{q}^{(k)} - \hat{x}_{q}^{(0)}\right| \le T_{k,q}\right)$$

Where

x is the true location in the horizontal plane

 $\hat{x}^{(k)}$ is the location on a horizontal plane of the subset solution that is fault tolerant to the fault mode k.

q is the index corresponding to the two coordinates in the horizontal plane

 $T_{k,q}$ is the detection threshold for each of the coordinates q.

We have the implication $|\hat{x}^{(0)} - x| > HPL$, $|\hat{x}_q^{(k)} - \hat{x}_q^{(0)}| \le T_{k,q} \Rightarrow |\hat{x}_q^{(k)} - x_q| > PL_q - T_{k,q}$ for any PL_1, PL_2 such that $PL_1^2 + PL_2^2 \le HPL^2$. This is because the rectangle defined by PL_1, PL_2 is included in the disk defined by HPL.

As a consequence, we have

$$P\left(\left|\hat{x}^{(0)} - x\right| > HPL, \hat{x}^{(k)} - \hat{x}^{(0)} \in D\right) \le 2Q\left(\frac{PL_1 - T_1^{(k)}}{\sigma_1^{(k)}}\right) + 2Q\left(\frac{PL_2 - T_2^{(k)}}{\sigma_2^{(k)}}\right)$$
(2)

where $T_q^{\,(k)} = T_{k,q} + b_q^{(k)}$ (the last term is the nominal bias).

This inequality is the one that is used in the proof of safety for solution separation in one coordinate. The important point here is that we can choose any pair of PL_1 , PL_2 such that $PL_1^2 + PL_2^2 \le HPL^2$. Ideally, we would choose the combination that minimizes the right-hand side of Equation (2). Since this is potentially complex, we choose instead to set PL_1 , PL_2 such that the two arguments in the *Q* function are equal.

After some algebra involving the quadratic formula, we obtain

$$\frac{PL_{1}-T_{1}^{(k)}}{\sigma_{1}^{(k)}} = \frac{PL_{2}-T_{2}^{(k)}}{\sigma_{2}^{(k)}} = \frac{\sqrt{HPL^{2}\sigma_{H}^{(k)2} + \left(T_{1}^{(k)}\sigma_{2}^{(k)} - T_{2}^{(k)}\sigma_{1}^{(k)}\right)^{2} - T_{1}^{(k)}\sigma_{1}^{(k)} - T_{2}^{(k)}\sigma_{2}^{(k)}}{\sigma_{H}^{(k)2}}}{\sigma_{H}^{(k)2}}$$
(3)

where

$$\sigma_H^{(k)} = \sqrt{\sigma_1^{(k)2} + \sigma_2^{(k)2}}$$

This leads to the upper bound (from Equation (2)):

$$P\left(\left|\hat{x}^{(0)} - x\right| > HPL, \left|\hat{x}_{q}^{(k)} - \hat{x}_{q}^{(0)}\right| \le T_{k,q}\right) \le 4Q\left(\frac{\sqrt{HPL^{2} + c^{(k)2}} - a^{(k)}}{\sigma_{H}^{(k)}}\right)$$

where we have defined

$$c^{(k)} = \frac{T_1^{(k)}\sigma_2^{(k)} - T_2^{(k)}\sigma_1^{(k)}}{\sigma_H^{(k)}}$$
$$a^{(k)} = \frac{T_1^{(k)}\sigma_1^{(k)} + T_2^{(k)}\sigma_2^{(k)}}{\sigma_H^{(k)}}$$

A valid PL equation is therefore given by

$$2\overline{Q}\left(\frac{HPL-T_{H}^{(0)}}{\sigma_{H}^{(0)}}\right) + \sum_{k=1}^{N_{fault modes}} 2p_{fault,k}\overline{Q}\left(\frac{\sqrt{HPL^{2}+c^{(k)2}}-a^{(k)}}{\sigma_{H}^{(k)}}\right)$$

$$= \frac{PHMI_{ALLOC}}{2N_{ES,INT}}$$
(3)

The Cauchy-Schwarz inequality gives

$$T_1^{(k)}\sigma_1^{(k)} + T_2^{(k)}\sigma_2^{(k)} \le T_H^{(k)}\sigma_H^{(k)}$$

which implies that

$$\frac{\sqrt{HPL^{2}\sigma_{H}^{(k)2} + \left(T_{1}^{(k)}\sigma_{2}^{(k)} - T_{2}^{(k)}\sigma_{1}^{(k)}\right)^{2}} - T_{1}^{(k)}\sigma_{1}^{(k)} - T_{2}^{(k)}\sigma_{2}^{(k)}}{\sigma_{H}^{(k)2}} \ge \frac{\sqrt{HPL^{2}\sigma_{H}^{(k)2}} - T_{H}^{(k)}\sigma_{H}^{(k)}}{\sigma_{H}^{(k)2}} = \frac{HPL - T_{H}^{(k)}}{\sigma_{H}^{(k)}}$$

$$(4)$$

The right-hand side term of the Equation (4) is the argument on of the Q function in the Equation proposed by Racelis (2022) (where we have added a factor of 2 in the terms under the sum, which is what is

supported by the proof by provided in Joerger (2022)). Since the *Q* function is a decreasing function, Equation (3) always leads to a smaller HPL than in Racelis (2022).

Further refinement

As pointed out above, the HPL defined in Equation (3) has an additional factor of 2 in the terms under the sum compared to the expression used in the baseline HPL for each of the components (East and North). This factor of 2 can in some cases cause the HPL computed by (3) be larger than the baseline one (this also applies to Racelis 2022). It is possible to further refine the bound in Equation (2) by using the inequality developed in Appendix B of Blanch (2015):

$$P\left(\left|\hat{x}^{(0)} - x\right| > HPL, \hat{x}^{(k)} - \hat{x}^{(0)} \in D\right) \le Q\left(\frac{PL_1 - T_1^{(0)}}{\sigma_1^{(0)}}\right) + Q\left(\frac{PL_2 - T_2^{(0)}}{\sigma_2^{(0)}}\right) + Q\left(\frac{PL_1 - T_1^{(k)}}{\sigma_1^{(k)}}\right) + Q\left(\frac{PL_2 - T_2^{(k)}}{\sigma_2^{(k)}}\right) + Q\left(\frac{PL$$

For this we need the expression of PL_1 and PL_2 that realize Equation (3). These are given by

$$PL_{1}^{(k)} = \sigma_{1}^{(k)} \frac{\sqrt{HPL^{2} + c^{(k)2}} - a^{(k)}}{\sigma_{H}^{(k)}} + T_{1}^{(k)}$$
$$PL_{2}^{(k)} = \sigma_{2}^{(k)} \frac{\sqrt{HPL^{2} + c^{(k)2}} - a^{(k)}}{\sigma_{H}^{(k)}} + T_{2}^{(k)}$$

The HPL can then be computed using the equation

$$4\bar{Q}\left(\frac{HPL-T_{H}^{(0)}}{\sigma_{H}^{(0)}}\right) + \sum_{k=1}^{N_{fault,k}} p_{fault,k}\left(2\bar{Q}\left(\frac{\sqrt{HPL^{2}+c^{(k)2}}-a^{(k)}}{\sigma_{H}^{(k)}}\right) + \bar{Q}\left(\frac{PL_{1}^{(k)}-T_{1}^{(0)}}{\sigma_{1}^{(0)}}\right) + \bar{Q}\left(\frac{PL_{2}^{(k)}-T_{2}^{(0)}}{\sigma_{2}^{(0)}}\right)\right)$$
$$= \frac{PHMI_{ALLOC}}{N_{ES,INT}}$$
(5)

As an alternative to using Equation (5), one can simply compute both the baseline HPL and the one given by Equation (3) and take the minimum of both.

Effect on HPL for a H-ARAIM scenario

We evaluate the direct evaluation of the HPL in an H-ARAIM scenario based on the default ISD that has been proposed for the ARAIM SARPS (Table 1). These are the values that a receiver would always be allowed to assume, even without receiving a broadcast Integrity Support Message. Because the URA in GPS is the broadcast IURA, we will assume URA = 2.4 m, because it is the most likely broadcast value in the current system (and the smallest one).

Table 1. ISD	for H-ARAIM scena	rio based on draft	ARAIM SARPS
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	GPS	Galileo
P _{const} , default	1×10 ⁻⁸	2×10 ⁻⁴
P _{sat, default}	1×10 ⁻⁵	3×10 ⁻⁵
R _{const, default}	1×10 ⁻⁸ /h	1×10⁻⁴/h
R _{sat, default}	1×10 ⁻⁵ /h	2×10⁻⁵/h
MFD _{const} , default	1 hour	ILB
MFD _{sat, default}	1 hour	ILB
$\sigma_{\text{URA,default, dual frequency}}[m]$	IAURA	6
$\sigma_{\text{URE},\text{default, dual frequency}}[m]$	Nominal URA	4
b _{nom, default} [m]	0	0



Figure 1. HPL histogram ratio for the proposed HPL to the baseline ARAIM ADD v4.2 (a) and for the proposed HPL to the HPL described in Racelis 2022 (b)

The histogram in Figure 1 shows that we can expect reductions of up to 30% in the HPL using Equation (5). Figure 1 (b) shows that Equation (3) improves up to 10% on Racelis 2022. We also see that the HPLs are always smaller (as shown by Equation (4)). Despite these reductions in HPL, Table 2 shows that the effect on coverage is quite modest. However, given that this change does not increase the computational load significantly, it is still worthwhile considering.

	HAL = 556 m	HAL = 185 m	HAL = 556 m	HAL = 185 m
	LS estimate	LS estimate	Opt. estimate	Opt. estimate
Baseline	97.94%	86.11%	99.08%	91.89%
Racelis 2022	97.94%	86.03%	99.08%	91.89%
Proposed method	97.94%	86.11%	99.08%	92.77%

Table 2. Coverage of HAL = 185 m and 556 m for direct HPL computation and comparison with baseline

REFINING TREATMENT OF TEMPORAL EXPOSURE

The PL equation proposed in the ARAIM ADD v4.2 can be written

$$2N_{ES,INT}\overline{Q}\left(\frac{PL_q - b_q^{(0)}}{\sigma_q^{(0)}}\right) + \sum_{k=1}^{N_{fault} modes} N_{ES,INT} p_{fault,k} \overline{Q}\left(\frac{PL_q - T_{k,q} - b_q^{(k)}}{\sigma_q^{(k)}}\right) = \frac{PHMI_{ALLOC}}{2}$$

(again, with the notations used in the ARAIM ADD v4.2 and related papers (e.g. Blanch 2022))

In this equation, each of the terms represents the contribution to the integrity risk over the exposure interval of each of the fault modes that is monitored. The contribution of a fault mode over the exposure interval is also bounded by the probability of the fault mode occurring during the interval in the first place (this is the bound that is used to account for the modes that are not monitored), which is noted $p_{fault,k}(T_{EXP})$ in the ADD [Milner (2020)]. We can therefore bound the contribution of one fault mode in the *q* coordinate $P(HMI, H_k)$ as follows

$$P(HMI, H_k) \leq \min\left(p_{fault,k}(T_{EXP}), p_{fault,k}N_{ES,INT}\overline{Q}\left(\frac{PL_q - T_{k,q} - b_q^{(k)}}{\sigma_q^{(k)}}\right)\right)$$

The new proposed PL equation would then be

$$2N_{ES,INT}\overline{Q}\left(\frac{PL_q - b_q^{(0)}}{\sigma_q^{(0)}}\right) + \sum_{k=1}^{N_{fault modes}} \min\left(p_{fault,k}\left(T_{EXP}\right), p_{fault,k}N_{ES,INT}\overline{Q}\left(\frac{PL_q - T_{k,q} - b_q^{(k)}}{\sigma_q^{(k)}}\right)\right) = \frac{PHMI_{ALLOC}}{2}$$
(6)

We do note that this equation does assume a stationary geometry, it might therefore be easier to justify for short exposure times (like 150 s for precision approach) than for long ones.

Effect on PL for Vertical ARAIM

We evaluate the proposed improvement above for a Vertical ARAIM scenario. The parameters of the scenario are given in Table 3 (note that these parameters are arbitrary, since there are no performance commitments for V-ARAIM).

	GPS	Galileo
P _{const} , default	1×10 ⁻⁴	2×10 ⁻⁴
P _{sat} , default	1×10 ⁻⁵	3×10 ⁻⁵
R _{const, default}	1×10 ⁻⁴ /h	1×10 ⁻⁴ /h
R _{sat, default}	1×10 ⁻⁵ /h	2×10⁻⁵/h
MFD _{const} , default	1 hour	ILB
MFD _{sat, default}	1 hour	ILB
$\sigma_{\text{URA,default, dual frequency}}[m]$	1 m	1 m
$\sigma_{\text{URE,default, dual frequency}}[m]$	0.66 m	0.66 m
b _{nom, default} [m]	0.75 m	0.75 m

 Table 3. ISD for Vertical ARAIM scenario.

The effect on the VPL can be seen in the histogram shown in Figure 1. Although for most VPLs the reduction is modest, there are a few for which the reduction is more than 40%. It turns out that the geometries for which the VPLs are reduced are quite critical, as can be seen in Figures 2 and 3, where we compare the 99.9% PL as well as the availability for each location for the baseline and the proposed modification. The coverage of 99.9% availability increases from 41% to 71%.



Figure 2. 99.9% VPL for the scenario corresponding to Table 1 for the baseline (left) and the proposed modification (right).



Figure 3. Availability map for the scenario corresponding to Table 1 for the baseline (left) and the proposed modification (right).

Although extremely simple, this modification can yield a large improvement in coverage, as evidenced by Figure 3. The improvement proposed in the first section can be easily combined with this one, using the following equation for the HPL

$$\begin{split} &4\bar{Q}\bigg(\frac{HPL-T_{H}^{(0)}}{\sigma_{H}^{(0)}}\bigg)+\\ &\sum_{k=1}^{N_{fault\,modes}}\min\bigg(p_{fault,k}(T_{EXP}), N_{ES,INT}p_{fault,k}\bigg(\bar{Q}\bigg(\frac{\sqrt{HPL^{2}+c^{(k)2}}-a^{(k)}}{\sigma_{H}^{(k)}}\bigg)+\bar{Q}\bigg(\frac{PL_{1}^{(k)}-T_{1}^{(0)}}{\sigma_{1}^{(0)}}\bigg)+\bar{Q}\bigg(\frac{PL_{2}^{(k)}-T_{2}^{(0)}}{\sigma_{2}^{(0)}}\bigg)\bigg)\bigg)\\ &=\frac{PHMI_{ALLOC}}{N_{ES,INT}}\end{split}$$

Although apparently unwieldy, this PL equation always provides smaller HPLs than the one specified in the ARAIM ADD for a similar computational load.

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