HIGH INTEGRITY CARRIER PHASE NAVIGATION USING MULTIPLE CIVIL GPS SIGNALS

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I certify that I have read this dissertation and that in my opinion it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Abstract

A navigation system should guide users to their destinations accurately and reliably. Among the many available navigation aids, the Global Positioning System stands out due to its unique capabilities. It is a satellite-based navigation system which covers the entire Earth with horizontal accuracy of 20 meters for stand alone civil users.

Today, the GPS provides only one civil signal, but two more signals will be available in the near future. GPS will provide a second signal at 1227.60 MHz (L2) and a third signal at 1176.45 MHz (Lc), in addition to the current signal at 1575.42 MHz (L1). The focus of this thesis is exploring the possibility of using beat frequencies of these signals to provide navigation aid to users with high accuracy and integrity.

To achieve high accuracy, the carrier phase differential GPS is used. The integer ambiguity is resolved using the Cascade Integer Resolution (CIR), which is defined in this thesis. The CIR is an instantaneous, geometry-free integer resolution method utilizing beat frequencies of GPS signals. To insure high integrity, the probability of incorrect integer ambiguity resolution using the CIR is analyzed. The CIR can resolve the Lc integer ambiguity up to 2.4 km from the reference receiver, the Widelane (L1-L2) integer ambiguity up to 22 km, and the Extra Widelane (L2-Lc) integer ambiguity from there on, with probability of incorrect integer resolution of $10^{-4}$.

The optimal use of algebraic combinations of multiple GPS signals is also investigated in this thesis. Finally, the gradient of residual differential ionospheric error is estimated to increase performance of the CIR.
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1 Introduction

1.1 The Global Positioning System

1.1.1 System Overview

The Global Positioning System is a satellite based radio navigation system which reached Full Operational Capability (FOC) on July 17, 1995. The GPS constellation has 24 operational satellites in six circular orbits with 55 degree inclination at 20,200 kilometers altitude, and is designed to have more than four satellites in view from anywhere in the world for 99.9 percent of the time. The GPS is designed to provide civil users worldwide with 100 meters of horizontal accuracy, 156 meters of vertical accuracy and less than 340 nanoseconds of time transfer error for 95 percent of the time, with Selective Availability (SA) (Standard Positioning Service, SPS) [71]. SA is an intentional degradation of the GPS satellite clock accuracy, which was discontinued on May 2, 2000 [82]. Without SA, horizontal accuracy is improved to 20 meters or better, and time transfer error is decreased to less then 40 nanoseconds [81].

In the present configuration, the GPS satellite broadcasts three signals: one with Coarse/Acquisition code (C/A) modulation, and the other two with Precision code (P, or Y, when encrypted) modulation. A unique Pseudo Random Noise (PRN) is assigned to each satellite, which is used to generate the C/A code. The PRN number is used to identify each GPS satellite. The P code, with a chipping rate of 10.23 MHz, modulates its carrier at 1575.42
MHz (L1) and at 1227.60 MHz (L2). The precision code is usually encrypted, and are restricted to U.S. Armed Forces, U.S. Federal agencies, and selected allied armed forces and governments. These restrictions are based on national security considerations [75].

Civilians are limited to using the C/A code and its carrier frequency for their applications. The C/A code, with chipping rate of 1.023 MHz, modulates a carrier at the L1 frequency with a 90 degree phase quadrature shift from the P code.

1.1.2 GPS Observables

A GPS receiver estimates its location by measuring ranges to the GPS satellites in view and receiving navigation data from them. For a civil user, the range is measured by using the C/A code and its carrier. As mentioned earlier, each GPS satellite is assigned a unique C/A code, which it generates and transmits. The GPS receiver generates a replica of the C/A code, and can calculate range by multiplying the time delay between the two codes with the speed of light. Figure 1-1 describes this process.

![Figure 1-1. Pseudorange Measurement Using the C/A Code](image-url)
For this time delay to represent the true range, both the GPS satellite and receiver must be synchronized with the GPS system time. The GPS satellites carry two rubidium and one cesium atomic clock to maintain accuracy of 6 nanoseconds with respect to GPS time [3]. However, a typical GPS receiver uses a far less accurate clock, such as a crystal oscillator, set approximately to the GPS system time. Thus, the range calculated from the time delay contains clock inaccuracies, both from the satellite and the receiver, and is biased from the true range. Furthermore, the receiver clock bias is unknown and must be estimated. Because of this uncertainty in the range due to the clock bias, it is commonly called ‘pseudorange’. A simplified measurement equation (no measurement error) for the C/A code pseudorange is shown in Equation (1-1).

\[ \rho = R + c\Delta \delta \ (m) \]  

- \( \rho \) is measured pseudorange between a receiver and a GPS satellite in meters
- \( R \) is the true range between the receiver and the satellite in meters
- \( c \) is the speed of light in meters per second
- \( \Delta \delta \) is the combined clock bias from the receiver and the satellite in seconds

The carrier of the C/A code can also be used to measure the pseudorange. When a GPS receiver is turned on, the phase of the carrier wave is measured. The initial integer number of cycles, \( N \), between the receiver and the satellite is unknown, and is commonly called the ‘integer ambiguity.’ If the receiver tracks the carrier without any interruptions, the integer ambiguity remains constant. Integer ambiguity resolution is a very important issue, and is
detailed in later chapters. For the current discussion, assume the ambiguity is resolved. Next, the carrier phase pseudorange is calculated by multiplying the number of waves between the receiver and satellite by an appropriate wavelength. Figure 1-2 shows how the carrier phase pseudorange is measured. It should be noted that both the L1 and L2 carriers can be reconstructed from the C/A code. The carrier phase pseudorange measurement can be obtained from both carriers. A dual frequency civil GPS receiver uses the C/A code to reconstruct the L1 carrier, and uses a code-less technique to reconstruct the L2 carrier. This latter technique was first developed by Counselman [13] and also by MacDoran [47]. Simplified carrier phase measurement equations for L1 and L2 are shown in Equation (1-2) and Equation (1-3).

\[
\phi_{L1} = \frac{1}{\lambda_{L1}} R + \frac{c}{\lambda_{L1}} \Delta \delta + N_{L1}
\]  \hspace{1cm} (1-2)

\[
\phi_{L2} = \frac{1}{\lambda_{L2}} R + \frac{c}{\lambda_{L2}} \Delta \delta + N_{L2}
\]  \hspace{1cm} (1-3)

- \( \phi \) is measured carrier phase
- \( \lambda \) is wavelength in meters
• $R$ is true range between a receiver and a satellite in meters

• $c$ is speed of light in meters per second

• $\Delta \delta$ combined clock bias from the receiver and the satellite in seconds

• $N$ is initial unknown integer number of waves between the receiver and the satellite

The GPS satellites also transmit navigation data, which includes satellite ephemeris, satellite clock error models, an almanac of the entire GPS constellation, etc. The navigation data modulates the C/A code at 50 Hz, and is decoded by the receiver. To estimate the three-

Figure 1-3. How GPS Works
dimensional location of the receiver and its clock bias, a GPS receiver needs a minimum of four satellites in view (Figure 1-3).

1.1.3 GPS Measurement Error

In an ideal environment, a GPS receiver will receive information from GPS satellite, including the true location and time of broadcast. It would also make a perfect measurement of the signal travel time between the satellites and the receiver. Using these ideal pseudorange measurements, a very accurate receiver position and clock bias can be estimated. In real life however, there are many error sources which cause inaccuracies in pseudorange measurements. Therefore, error in the receiver position is caused when inaccurate pseudoranges are used. The source of pseudorange error can be grouped into the following six classes [51].

1. Ephemeris data

Ephemeris error is caused when the transmitted navigation data contains an incorrect locations of the GPS satellite. The root mean square (rms) ranging error due to incorrect ephemeris data is 2.1 m.

2. Satellite Clock

GPS satellites carry two rubidium and one cesium atomic clock to maintain accuracy of 6 ns with respect to GPS time. The rms ranging error due to the satellite clock bias is 2.1 m (without SA).
3. Ionosphere

When signals from the GPS satellites pass through the ionosphere, they do not travel at the speed of light in a vacuum due to free electrons. The delay in modulation of the signal is proportional to the number of free electrons in the signal path, and is also inversely proportional to the square of the carrier frequency. The phase of the carrier of the signal is actually advanced by the same amount as the code phase delay. The ionosphere is stable in the temperate zone, but it changes rapidly near the magnetic poles or the equator. For receivers in the temperate zone, the rms ranging error due to ionospheric delay is 2 to 5 meters. Large part of the ionospheric error is mitigated by using an ionosphere model, which is included in the GPS navigation data.

The rms ranging error can further be reduced to approximately 1 meter if the ionospheric delay is directly measured by using dual frequency receivers. Since the ionospheric delay is inversely proportional to the square of the carrier frequency, the difference between arrival times of the L1 and L2 signals can be used to observe the delay.

4. Troposphere

Signals from the GPS satellites are also delayed when they pass through the troposphere. Changes in humidity, temperature and pressure all affect the travel speed of the code and carrier by the same amount. The rms ranging error due to tropospheric delay is 1 m or better when a simple troposphere model is used.
5. Multipath

Multipath error is caused when reflected signals from the GPS satellites reach a receiver’s antenna. Multipath error can be reduced if the antenna is placed away from reflecting surfaces, or the antenna is designed to reduce reflected signals. If not treated carefully, multipath error can be as large as 15 meters or more for a static receiver. Moving users are less affected by multipath since it becomes more amenable to averaging. The rms ranging error due to the multipath effect is less than 1 meter for a moving user.

6. Receiver

Receiver error is caused by thermal noise, inter-channel bias and software accuracy. With modern microprocessors and chip technology, the rms ranging error due to these receiver errors should be less than 0.5 m.

1.2 Differential GPS

1.2.1 System Overview

By using the Differential GPS (DGPS) technique, shown in Figure 1-4, most of the GPS measurement errors can be reduced. DGPS relies on the fact that receivers in the same general area having the same GPS satellites in sight at the same time share common measurement errors. For example, if there is a GPS receiver at a known location (reference), it can calculate the difference between the expected and measured pseudorange, then send the difference to the other GPS receivers (users) by any suitable communication system (VHF link, FM broadcast, link through communication satellites, etc.). The broadcast information
Figure 1-4. DGPS Configuration

in the corrections for the common measurement error between the reference and user receivers. Since the reference and user receivers are looking at the same GPS satellite at the same time, ranging error caused by the satellite clock is eliminated. Pseudorange error due to satellite ephemeris data is nearly eliminated. However, since each receiver has its own line of sight to the satellite, residual differential ephemeris error will grow as the distance between the receivers increases. The residual differential GPS errors for satellite ephemeris is less than 0.05 meter when the receivers are placed 100 km from each other [52].

DGPS also reduces the pseudorange error due to ionospheric and tropospheric effects. However, because both atmospheric conditions decorrelate as the distance between reference and user receivers increases, the correction for both atmospheric contributions becomes less effective for the longer distances. Therefore, DGPS has a limited service volume. Within that volume, the pseudorange error due to satellite ephemeris, satellite clock,
ionospheric and tropospheric effects are the common errors between the reference and user receivers. DGPS can correct these errors very effectively.

Although DGPS can correct the majority of the pseudorange error, inaccuracy due to multipath delay and the receiver itself is not corrected. These two phenomena constitute the “noise floor” of DGPS. Even though both the reference and user receivers employ information from the same GPS satellite, error due to multipath delay is dependant on the antenna location for each receiver and therefore cannot be treated as common error. The pseudorange error caused by the receiver itself also cannot be considered a common error. This error is caused by thermal noise, internal software and channel bias, and is unique for each receiver. Therefore, most of the user position error in short range DGPS is caused by multipath delay and receiver error. DGPS actually increases multipath effect and receiver noise because both measurements from the reference and user receivers are combined to form the DGPS measurement. Typically, multipath delay of the C/A code measurement is on the order of meters, whereas the receiver error is on the order of centimeters.

1.2.2 Performance of DGPS

Performance of navigation systems, including DGPS, can be described by using the following system parameters, as defined by the U.S. Federal Radionavigation Plan [75].

- **Accuracy**: The accuracy of an estimated position at a given time is the degree of conformance of that position with the true position at that time.

- **Integrity**: Integrity is the ability of a system to provide timely warnings to users when the system should not be used for navigation.
- **Availability:** The availability of a navigation system is the percentage of time that the services of the system are usable by the navigator.

- **Continuity:** The continuity is the likelihood that the navigation system supports accuracy and integrity requirement for the duration of intended operation.

- **Coverage:** The coverage provided by a radionavigation system is that surface area or space volume in which the signals are adequate to permit the navigator to determine position to a specific level of accuracy.

For example, performance of a DGPS navigation system, such as the Local Area Augmentation System (LAAS), for Category I precision approach is shown in Table 1-1.

LAAS is a system developed to support precision approach and landing operations as well as other navigation and surveillance applications within a local area including and surrounding an airport [57]. Figure 1-5 describes how LAAS operates.

![Figure 1-5. Local Area Augmentation System](image-url)
<table>
<thead>
<tr>
<th>Performance Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>lateral accuracy 16.0 m, 95%</td>
</tr>
<tr>
<td></td>
<td>vertical accuracy 40 m, 95%</td>
</tr>
<tr>
<td>Integrity</td>
<td>$P_{HMI} = 1 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>Time to Alarm 6 sec.</td>
</tr>
<tr>
<td>Availability</td>
<td>between 0.999-0.99999</td>
</tr>
<tr>
<td>Coverage</td>
<td>20 nm from an airport</td>
</tr>
</tbody>
</table>

Table 1-1. LAAS Performance Parameter ($P_{HMI}$ : Probability of Hazardously Misleading Information)

1.2.3 Carrier Phase Differential GPS

The range error from multipath delay, which is not corrected by the C/A code differential GPS, is reduced significantly if one uses the carrier of the C/A code as a source of pseudorange measurements. As mentioned earlier, the C/A code pseudorange measurement is based on the time offset between the satellite broadcast and a replica C/A code generated in the receiver. The receiver also generates and tracks the L1 carrier. However, unlike the C/A code, the carrier measurement contains an ambiguity, because it is a sine wave with a wavelength of 19 centimeters for the L1, and 24 centimeter for the L2 frequency. Therefore, the number of waves between the receiver and the GPS satellite as the receiver acquires the GPS signal is unknown. Since the unknown number of the waves is an integer, resolving it is also known as resolving the integer ambiguity.

Once the carrier is acquired, the change in number of wave is tracked. If the integer ambiguity is resolved, the pseudorange is then calculated by multiplying the sum of the initial and change in number of waves by wavelength. The integer ambiguity in carrier phase DGPS represents the number of waves from the user and reference pseudorange difference, as shown in Figure 1-6. A pseudorange measurement obtained using the carrier wave is subjected to the same atmospheric error source as the C/A code measurement since they
travel through the same medium. However, due to its smaller wavelength, multipath delay in the carrier phase measurement is on the order of centimeters, an order of magnitude smaller than the multipath delay found in the C/A code measurement [8]. Thus, compared to C/A code DGPS, carrier phase DGPS (CDGPS) enjoys far greater accuracy, since the majority of DGPS error is caused by multipath delay. However, for CDPGS to work, the initial number of waves, or integer ambiguity, must first be resolved.

One way to resolve the integer ambiguity is by averaging the difference between the carrier phase and C/A code measurements (Equation (1-4)), which has been developed by a number of researchers, such as Goad [20], Hatch [27], and Hwang and Brown [35].

$$\lambda \phi - \rho = \lambda N + \nu \ (m)$$  \hspace{1cm} (1-4)

- $\lambda \phi$ is the carrier phase measurement, $\phi$, multiplied by a wavelength, $\lambda$
• $\rho$ is the C/A code measurement

• $\lambda N$ is the integer ambiguity, $N$, multiplied by a wavelength, $\lambda$

• $\nu$ is the combined measurement error from both the C/A code and the carrier phase measurements

The carrier phase and code measurement difference, $\lambda \phi - \rho$, represents the integer ambiguity and combined measurement error from both the C/A code and carrier phase measurements. With the DGPS correction, common errors are eliminated and the C/A code multipath delay dominates the measurement error. It can be time averaged (smoothed) to reduce the white noise portion. If the error can be reduced to less than one-half of the wavelength, the integer ambiguity is resolved by rounding the measurement. Once the integer ambiguity is resolved, CDGPS performs with a measurement error on the order of centimeters, as compared to meters in the C/A code DGPS.

The range error from multipath delay and receiver noise, combined with unintentional or intentional interference on the GPS signal degrade performance of the CDGPS. A change in atmospheric condition between the reference and user receivers also affects performance of CDGPS. The residual differential error exists when signal from a GPS satellite travels to separate receivers through different atmospheric conditions. In fact, since the atmospheric delay decorrelates as the distance between receivers grows, it is a limiting factor on the service volume of the CDGPS.

1.2.4 CDGPS with L1-L2 Beat Frequency Measurement

Even though the GPS signal from the L2 frequency only contains P or Y code, its carrier can be recovered, by using technique such as “code-less” L2 tracking [2]. Like the carrier
of the C/A code at the L1, this reconstructed L2 carrier, with its wavelength of 24 centimeters, can also be used as a source of pseudorange measurements. However, the initial number of waves between a receiver and the satellite is still unknown.

Instead of using the carrier phase measurements from the L1 or L2 separately, one can use the beat frequency measurements of the two. As shown in Figure 1-7, the carrier waves from the L1 and the L2 can be combined to generate a beat frequency signal with a frequency of 347.82 MHz. This signal is commonly called a widelane measurement (WL), since its wavelength of 86 centimeters is greater than wavelength of both the L1 and L2 carrier waves. By increasing wavelength of the carrier phase measurements, finding the number of waves between a receiver and the satellite becomes simpler as compared to using measurements from the L1 or L2 without forming the widelane. It will take far less time to search for the right integer if the widelane measurement is used.

The performance of carrier phase DGPS using the widelane is quite good, if one is willing to wait. It takes a few minutes to resolve the widelane integer, which yields a position accuracy equal to some fraction of 86 centimeters. Once the widelane integer is resolved, the L1 and L2 integers can also be resolved by using the widelane measurement after some averaging. It gives the user even better position accuracy, equal to some fraction of 19 or 24.
centimeters. However, since it takes a few minutes to complete the process, the availability of this system becomes a problem for a moving user, requiring a real-time position solution.

1.3 Cascade Integer Resolution

1.3.1 GPS Modernization Effort

On January 25, 1999, Vice President Gore announced a new GPS modernization initiative. GPS will provide a second civil signal at the L2 frequency along with the current military signal, and a third civil signal at 1176.45 MHz (Lc). Figure 1-8 shows the details of the proposed civil signals. This frequency diversity will alleviate concerns over accidental interference, and constitutes a major step toward achieving robustness of service. Also, the

![Graph showing current and future GPS signal configurations.](image)

*Figure 1-8. GPS Civil Signal Configuration, Present and Future*
multiple-frequency signaling would allow civil users to estimate the delay in the GPS signal due to the ionosphere. Since the delay is a function of carrier frequency, it can be estimated by measuring the difference between two pseudorange measurements with separate carrier frequencies. The civil signal at the L1 frequency will remain unchanged, thereby ensuring that all of the fielded GPS receivers continue to operate.

1.3.2 Multiple Beat Frequency Generation

With three civil frequencies, a user can generate three beat frequency signals. The L1 and L2 measurements is processed to create the Widelane (WL) with wavelength of 86 centimeters. The combination of the L1 and Lc carrier frequencies yields the second beat frequency with 75 centimeters in wavelength (Mediumlane, ML). The combination of the L2 and Lc carrier frequencies yields the third beat frequency with 5.9 meters in wavelength (Extra Widelane, EWL). Table 1-2 summarizes these combinations.

<table>
<thead>
<tr>
<th>Component</th>
<th>Frequency (MHz)</th>
<th>Wavelength (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 Carrier</td>
<td>1575.42</td>
<td>.190</td>
</tr>
<tr>
<td>L2 Carrier</td>
<td>1227.60</td>
<td>.244</td>
</tr>
<tr>
<td>Lc Carrier</td>
<td>1176.45</td>
<td>.255</td>
</tr>
<tr>
<td>L1-L2 Beat Frequency, Widelane (WL)</td>
<td>347.82</td>
<td>.862</td>
</tr>
<tr>
<td>L1-Lc Beat Frequency, Mediumlane (ML)</td>
<td>398.97</td>
<td>.751</td>
</tr>
<tr>
<td>L2-Lc Beat frequency, Extra Widelane (EWL)</td>
<td>51.15</td>
<td>5.86</td>
</tr>
<tr>
<td>L1 Code</td>
<td>1.023</td>
<td>293</td>
</tr>
<tr>
<td>L2 Code</td>
<td>1.023</td>
<td>293</td>
</tr>
<tr>
<td>Lc Code</td>
<td>10.23</td>
<td>29.3</td>
</tr>
</tbody>
</table>

Table 1-2. Civil GPS Signal Components, Near Future
1.3.3 Cascade Integer Resolution (CIR): Carrier phase DGPS with Multiple Beat Frequencies

One way to utilize the multiple available beat frequency measurements of carrier phase DGPS in the future is to resolve the integer using the following technique. The main difference between the C/A code pseudorange and carrier phase measurement is that the latter has an extra unknown, namely the initial integer number of waves between the receiver and satellite. Therefore, the integer ambiguity in the EWL measurement can be estimated as a real number by subtracting the Lc code measurement from the L2-Lc (EWL) beat frequency measurement. If the subtracted measurement has an error that is less than one-half of wavelength of the EWL, it can be rounded to a correct integer and the ambiguity is resolved. With the correct EWL integer, the EWL measurement can be used as a pseudorange. Then the L1-L2 (WL) integer can be estimated by subtracting the EWL measurement from the WL measurement. It can also be rounded to a correct integer if the subtracted measurement has error that is less than one-half of the WL wavelength. The same procedure can be used for finding the L1-Lc (ML) integer, and finally the L1 integer. Since this technique resolves the integer ambiguities from the longest to the shortest wavelength successively using the prior measurement, it is defined the Cascade Integer Resolution (CIR). Figure 1-9 provides a definition of the cascade integer resolution.

This thesis focuses on the performance analysis and optimization of the CIR. First, the integrity of each cascading rounding step is evaluated to ensure the integrity and availability of the CIR as a whole. Then a trade-off study among the integrity, accuracy and service volume is carried out by investigating algebraic combinations of the three carrier phase measurements. A performance analysis with a disabled GPS signal due to interference is
carried out to investigate robustness of the CIR. Also, the gradient of residual differential ionospheric delay is estimated to increase service volume of the CIR.

1.4 Previous work

Before the GPS, the OMEGA navigation system used multiple frequencies to resolve navigational lane ambiguity, which is similar in concept to the carrier phase integer ambiguity. The OMEGA was a very long range, Very Low Frequency (VLF) radio navigation system which provided global coverage for ships and airplanes from eight transmitters around the globe [16], [66]. The OMEGA navigation system was terminated on September 30, 1997, in accordance with the 1996 U.S. Federal Radio Navigation Plan [74].

The European Space Agency has been studying feasibility of the carrier phase navigation with the next generation of Global Navigation Satellite Systems (GNSS-2) by using the three-carrier (TCAR) method [19]. The TCAR method is an extension of the widelane tech-
nique. It forms a widelane with two carrier frequencies with the largest frequency separation, and a "super widelane" with the closest two. Preliminary test results of the TCAR method using a GNSS-2 signal simulation have been completed by Dr. Vollath, et al. [84]. Three proposed GNSS-2 signals at 1589.742 MHz, 1561.098 MHz, and 1256.244 MHz are used to form a super widelane (10.47 m) and a widelane (0.90 m). The original TCAR formulation did not achieve instantaneous ambiguity resolution mainly due to a bad error propagation in the widelane integer ambiguity fixing step along with a differential residual error from the ionosphere. Performance of the TCAR method in presence of multipath and RFI is examined by Werner, et al. [85]. This study concludes that the TCAR method will work if the code measurement noise of at least one frequency is minimized. The concept used in the TCAR method, using multiple beat frequencies to resolve the integer ambiguity, is utilized in the Cascade Integer Resolution. The concept is further enhanced in this thesis, where the optimal use of algebraic combinations of the three carrier phase measurements on forming multiple beat frequencies with arbitrary wavelength is studied.

Before the spectral location of the third civil frequency for the GPS had been selected, Mr. Ron Hatch and Dr. Per Enge investigated the optimal spectral location of the third frequency and potential benefits of that location[29]. The study reports that if the third frequency is chosen to form a second widelane with significantly different sensitivity to ionospheric refraction, the dual-widelane measurements can be linearly combined to obtain ionospheric error-free measurement. The GPS Independent Review Team (IRT) also searched for the optimal third frequency. The IRT recommended the frequency range between 970 MHz - 980 MHz to place the third frequency if the L2 is suitable for aviation
(safety-of-life) use, and 5.0 GHz - 5.150 GHz if the L2 is not suitable for aviation use [22]. The optimal location of the third frequency is also studied by Ericson by examining characteristics of linear phase combinations [17]. This study searched for frequencies yielding reasonably large wavelength to help ambiguity fixing, low ionospheric error and limited observation noise when combined with the L1 and L2 frequencies. Ericson concludes that the selected third frequency at 1176.45 MHZ is not optimally separated from the L2, which makes efficient handling of ionospheric error over long baseline distance difficult. The optimal spectral location of the third frequency and its potential benefits is also explored within this thesis.

There have been many studies done on CDGPS with the L1-L2 widelane. Landau [43] developed a centimeter level real time kinematic positioning system using the widelane. He reports that the developed On The Fly (OTF) integer ambiguity resolution system resolved the integer ambiguity within 8 to 14 seconds, in very short baseline. OTF integer ambiguity resolution using the widelane is utilized in marine application by Weisenburger [86]. The widelane is also used in real-time centimeter level GPS positioning of cutting blade and earth moving equipment [14]. In a surveying application, Wu [87] reports 1.6 cm accuracy in height within 5 km using widelane. Wu latter developed CDGPS with two types of widelaning that involves two different combinations of the L1 and L2 carrier phase measurements [88]. Their preliminary results suggest that it is possible to resolve the carrier phase integer ambiguity with as high as 95 percent success rate for a 19.6 kilometer baseline [88]. Linear combinations of the L1 and L2 carrier phase measurements is also used by Han [23]. He used two linear combinations of carrier phase observables and the P-code
pseudorange measurement to detect and remove cycle slips from long baseline data (up to 295 km).

1.5 Contributions

1.5.1 Performance Analysis of the Cascade Integer Resolution (CIR)

In this thesis, the Cascade Integer Resolution (CIR) is defined. Performance of the CIR is examined by analyzing the integrity and availability of each integer ambiguity resolution step. The integrity and availability are investigated by first finding the conditional probability of estimating the right integer with a given measurement distribution. Then covariance analysis with five states, pseudorange, ionosphere delay, integer ambiguity for the L1, L2 and Lc, plus six measurements, three C/A code and three carrier phase measurements, is then carried out to calculate the probability of getting the right EWL, WL and Lc integers.

For both analyses, the C/A code and carrier phase measurements distributions are assumed to be bounded by normal distributions. This model has been verified by analyzing the actual GPS data collection (L1 and L2).

1.5.2 Analysis of the Optimal Third Frequency

In this thesis, the optimal spectral location of the third frequency is examined. Assuming the L1 and L2 frequencies are fixed, one can change wavelength of the available beat frequencies by moving the third civil frequency. The effect of changes in wavelength on the CIR is studied, and the optimal third civil frequency for the CIR is determined.
1.5.3 Optimization of the CIR by Using Generalized Widelane

In this thesis, trade-offs among accuracy, integrity and service volume is examined by using flexibility in the selection of beat frequency measurements. By using algebraic combinations of the three carrier phase measurements, three linearly independent beat frequency measurements with arbitrary wavelengths are generated. Selection criteria for beat frequencies for the CIR with varying accuracy and integrity requirements have been developed to maximize the service volume. The increase in the measurement error due to algebraic combinations of the carrier phase measurements is also investigated using a covariance analysis.

1.5.4 Increasing Service Volume of the CIR with Estimating Gradient of the Residual Differential Ionosphere Delay

In this thesis, the gradient of the differential ionospheric delay is estimated by using measurements from two different locations. The difference in atmospheric conditions between a reference and user receivers is the limiting factor on the service volume of the CIR. Performance analysis shows an increase in service volume of the CIR when the gradient is estimated and incorporated into the model.

1.5.5 The Effect of Interference From Ultra-wideband Transmitters on GPS Receiver

In this thesis, effect of RFI from an UWB transmitter on GPS is examined. Among the many potential sources of RFI which could compromise integrity of the GPS signal, UWB technology stands out due to its capacity to disable all three civil frequencies simultaneously. A preliminary study shows that certain configurations of the UWB signal can
cause a receiver located 100 meters from the UWB device to lose lock on a GPS satellite. Since the standard for the UWB signal is not yet defined, and because the possible RFI on GPS from UWB technology is significant, determining the exact nature of the UWB signal interference on GPS requires further investigation.

1.5.6 Analysis of the CIR Performance with Interference (Loss of a GPS Signal)

In this thesis, performance of the CIR with one lost GPS signal is analyzed. Even though a GPS signal is lost, the remaining two carrier phase measurements can form two linearly independent beat frequency measurements by using algebraic combinations. Performance of the CIR with two beat frequencies is examined.
2

DGPS Measurement Error

2.1 Measurement Equations

2.1.1 Overview

A civil GPS receiver measures range to a GPS satellite by using the C/A code or its carrier. For direct ranging, clocks in both the satellite and the receiver must be synchronized with GPS system time; in this case, the measured range is the true range. Since 1 nanosecond (ns) of clock error is equal to 30 centimeters of error in the range measurement, synchronization between the receiver clock and the satellite clock is essential.

In GPS, the measured range is biased from the true range due to both clock errors. This measured range is commonly called the ‘pseudorange.’ Although a GPS satellite carries three atomic clocks (two rubidium and one cesium) to maintain clock error within 6 ns of system time, synchronization with GPS system time is not perfect. Moreover, receiver clocks are far less accurate than atomic clocks in the satellites. Thus, the receiver clock bias is considered unknown. To estimate the receiver clock bias along with the three dimensional location of the receiver, at least four GPS satellites must be in view.

Pseudorange measurement also suffers degradation from the true range due to numerous error sources. As discussed in Chapter 1, measurement errors, including satellite clock
errors, are categorized into the following six classes: Ephemeris Data, Satellite Clock, Ionosphere, Troposphere, Multipath, and Receiver.

GPS measurement equations for both the C/A code and carrier phase pseudorange are expressed in detail in Section 2.1.2 by treating each error class separately. Single and double difference GPS measurement equations along with the effect of the differencing on measurement errors are shown in Sections 2.1.3 and 2.1.4, respectively. Measurement equations for beat frequency are shown in Section 2.1.5.

2.1.2 GPS Measurement

The C/A code GPS measurement equation for an epoch $t$ is shown in Equation (2-1).

$$\rho_a^i(t) = R_a^i(t) + c(\tau_a(t) + \tau^i(t)) + I_a^i(t) + T_a^i(t) + \mu_a^i(t) \quad (m)$$ (2-1)

- $\rho_a^i(t)$ is pseudorange between receiver $a$ and satellite $i$
- $R_a^i(t)$ is geometric range between receiver $a$ and satellite $i$
- $c$ is the speed of light
- $\tau_a(t)$ is clock offset of the receiver clock from GPS system time
- $\tau^i(t)$ is clock offset of the satellite clock from GPS system time, including Selective Availability
- $I_a^i(t)$ is pseudorange error due to ionosphere effects
- $T_a^i(t)$ is pseudorange error due to troposphere effects
- $\mu_a^i(t)$ is pseudorange error due to the C/A code multipath and receiver error

The carrier phase GPS measurement ($\lambda \phi_a^i$) for an epoch $t$ is
\[ \lambda \Phi_a(t) = R_a^i(t) + \lambda N_a^i + c(\tau_a(t) + \tau^i(t)) - I_a^i(t) + T_a^i(t) + v_a^i(t) \quad (m) \]  

(2-2)

The carrier phase measurement equation is identical to that of the C/A code, except for the following. First, \( \lambda \) is wavelength of the carrier frequency in meters, and \( N_a^i \) is the time independent initial phase ambiguity (integer value). Also, \( v_a^i(t) \) is pseudorange error due to carrier phase multipath and receiver error in meters. The ambiguity term, \( N_a^i \), represents the initial phase ambiguity when the receiver \( a \) acquires the signal from satellite \( i \). It is constant as long as the receiver maintains lock on the carrier. If the lock is lost, a different value for the ambiguity term is set when the receiver re-acquires the carrier. The change in the integer ambiguity due to loss of lock on the carrier is called 'Cycle Slip,' since the carrier phase measurement suddenly changes by an unknown number of wavelengths as the receiver re-acquires the signal. Detection and elimination of cycle slip can be accomplished through use of the Receiver Autonomous Integrity Monitor (RAIM), which compares position solutions from carrier phase measurements from different sets of GPS satellites to normal error conditions [53].

Ionosphere is a dispersive medium for radio signals, including GPS. Due to the high number of free electrons in the ionosphere, radio signal propagation is affected such that its phase velocity is advanced by \( \Delta c \), and its group velocity (modulation) is delayed by the same \( \Delta c \). The amount of \( \Delta c \) is determined by the phase refractive index, which is directly tied to the number of free electrons along the path of the signal. The sign of pseudorange error due to ionosphere effect, \( I_a^i(t) \), in the carrier phase measurement equation is therefore opposite of that found in the C/A code. Also, measurement error due to ionospheric effect is a function of frequency inverse squared.
2.1.3 Single Difference GPS Measurement

If there are two GPS receivers in communication with each other, most pseudorange errors are eliminated by differencing measurements between the two. Figure 2-1 shows a conceptual Differential GPS (DGPS) setup.

![Diagram of DGPS Setup](Image)

Figure 2-1. DGPS Setup

The C/A code and carrier phase single difference measurement equations between receivers $a$ and $b$ are

\[
\rho_{ab}^i(t) = \rho_a^i(t) - \rho_b^i(t) \quad (m)
\]

\[
\rho_{ab}^i(t) = R_{ab}^i(t) + c\tau_{ab}(t) + I_{ab}^i(t) + T_{ab}^i(t) + \mu_{ab}^i(t) \quad (m)
\] (2-3)

\[
\phi_{ab}^i(t) = \phi_a^i(t) - \phi_b^i(t)
\]

\[
\lambda\phi_{ab}^i(t) = R_{ab}^i(t) + \lambda N_{ab} + c\tau_{ab}(t) - I_{ab}^i(t) + T_{ab}^i(t) + \nu_{ab}^i(t) \quad (m)
\] (2-4)
Note that the satellite clock error term has been eliminated in Equations (2-3) and (2-4) since it is common error between the two measurements. Pseudorange error due to satellite ephemeris data is nearly eliminated since both receivers are tracking the same satellite, \( i \). However, since each receiver has its own line of sight to the satellite, residual differential ephemeris will grow as the distance between the receivers increases. With SA, residual differential GPS error for satellite ephemeris is less than 0.5 meters when the receivers are placed 100 km from each other [52].

Pseudorange error due to ionospheric and tropospheric effects will be nearly eliminated by differencing if the receivers are within tens of kilometers (exact distance depends on the atmospheric conditions). Since both atmospheric conditions decorrelate as distance between the receivers increases, differential correction for both atmospheric effects becomes ineffective for longer distances. Currently assumed values of spatial decorrelation for the ionosphere and troposphere are, respectively, 2 parts per million (ppm) and 1 ppm [51]. For example, the residual differential error caused by spatial decorrelation of the ionospheric condition is 0 meters when the receivers are next to each other, but 2 meters when they are 1000 kilometers apart.

The single difference measurement actually increases pseudorange error due to multipath and receiver error since both are unique to each receiver, yet they are combined using this method. For an example, if multipath and receiver errors are randomly distributed and the error variance of both receivers is identical, the error in the single difference measurement is greater than that of an uncorrected measurement by a factor of \( \sqrt{2} \).
2.1.4 Double Difference Measurement

If single difference measurements from two satellites, $i$ and $j$, are differenced again, the receiver clock error term is eliminated. This process is called the double difference. Figure 2-2 shows a conceptual double difference GPS setup. The double difference measurement equations for the code and carrier phase are derived in Equations (2-5) and (2-6), respectively.

\[
p^i_{ab}(t) = p^i_{ab}(t) - p^j_{ab}(t) \quad \text{(m)}
\]

\[
p^i_{ab}(t) = R^i_{ab}(t) + I^i_{ab}(t) + T^i_{ab}(t) + \mu^i_{ab}(t) \quad \text{(m)} \tag{2-5}
\]

\[
\phi^i_{ab}(t) = \phi^i_{ab}(t) - \phi^j_{ab}(t)
\]

\[
\lambda \phi^i_{ab}(t) = R^i_{ab}(t) + \lambda N^i_{ab} - I^i_{ab}(t) + T^i_{ab}(t) + \nu^i_{ab}(t) \quad \text{(m)} \tag{2-6}
\]

Figure 2-2. Double Difference GPS Setup

30
Figure 2-3. Effect of Double Difference on Single Difference Measurement

Figure 2-3 shows the effect of double differencing on the C/A code and carrier phase measurement. The C/A code measurements from two receivers for a GPS satellite with PRN 7 are used to form a single difference measurement. Measurements from a GPS satellite with PRN 26 are used to form another single difference measurement. These values are combined to form the double difference measurement. The same process is used to generate the carrier phase double difference measurement. The receivers used to generate Figure 2-3 are separated by 163 meters. The combined residual differential error due to decorrelation of ionospheric and tropospheric conditions at 163 meters is less than 1 millimeter. Therefore,
most of the pseudorange error in Figure 2-3, or in any double difference measurement with a short receiver separation distance, is from multipath and receiver error. This conclusion is valid for both the C/A code and carrier phase.

Double differencing also eliminates the inter frequency bias (IFB) between the L1 and L2 measurements. The IFB exists because the L1 and L2 phase paths within the hardware, both a satellite and a receiver, are different. Equations (2-7) and (2-8) describes the IFB in the measurement from receivers a, b to a satellite i, and Equations (2-9) and (2-10) from receivers a, b, to a satellite j. \( \beta^i_{(L1, L2)} \) represent the satellite IFB, and \( \beta_{a(L1, L2)} \) represents the receiver IFB.

\[
IFB^i_a = \beta^i_{(L1, L2)} + \beta_{a(L1, L2)} \tag{2-7}
\]

\[
IFB^i_b = \beta^i_{(L1, L2)} + \beta_{b(L1, L2)} \tag{2-8}
\]

\[
IFB^j_a = \beta^j_{(L1, L2)} + \beta_{a(L1, L2)} \tag{2-9}
\]

\[
IFB^j_a = \beta^j_{(L1, L2)} + \beta_{b(L1, L2)} \tag{2-10}
\]

With the single difference, the satellite IFB is eliminated.

\[
IFB^i_{a,b} = \beta_{a, b(L1, L2)} \tag{2-11}
\]

\[
IFB^j_{a,b} = \beta_{a, b(L1, L2)} \tag{2-12}
\]

With the double difference, the receiver IFB is eliminated, therefore eliminating the IFB altogether.

\[
IFB^i^j_{a,b} = 0 \tag{2-13}
\]
As in the single difference, the double difference process increases pseudorange error due to multipath and receiver error. Again, this is due to unique error values from each receiver and satellite.

By removing the indices \((a, b, i, j, \text{ and } t)\) the double difference equations are simplified as shown in Equations (2-14) and (2-15). For the rest of this dissertation, all of the C/A code and carrier phase pseudorange measurement equations are double difference unless noted otherwise, though the simplified notations are used.

\[
\rho = R + I + T + \mu \quad \text{(m)} \tag{2-14}
\]

\[
\lambda \phi = R + \lambda N - I + T + \upsilon \quad \text{(m)} \tag{2-15}
\]

### 2.1.5 Beat Frequency Measurement

Beat frequency measurement is formed when carrier phase GPS measurements with different frequencies are combined. For example, double difference beat frequency measurements from the L1 and L2 frequencies form the following beat frequency measurement.

\[
\phi_{WL} = \phi_{L1} - \phi_{L2}
\]

\[
\phi_{L1} = \frac{R}{\lambda_{L1}} + N_{L1} - \frac{I_{L1}}{\lambda_{L1}} + \frac{T}{\lambda_{L1}} + \frac{\upsilon_{L1}}{\lambda_{L1}}
\]

\[
\phi_{L2} = \frac{R}{\lambda_{L2}} + N_{L2} - \frac{I_{L1} \gamma_{12}}{\lambda_{L2}} + \frac{T}{\lambda_{L2}} + \frac{\upsilon_{L2}}{\lambda_{L2}}
\]

\[
\gamma_{12} = \frac{f_{L1}^2}{f_{L2}^2} = 1.65 \tag{2-16}
\]

\[
\lambda_{WL} \phi_{WL} = R + \lambda_{WL} N_{WL} - \gamma_{WL} I_{L1} + T + \upsilon_{WL} \quad \text{(m)} \tag{2-17}
\]
\[ \lambda_{WL} = \frac{c}{(f_{L1} - f_{L2})} = 86.2\text{cm} \]
\[ N_{WL} = N_{L1} - N_{L2} \]
\[ \gamma_{WL} = \lambda_{WL} \left( \frac{1}{\lambda_{L1}} - \frac{\gamma_{12}}{\lambda_{L2}} \right) \]  
(2-18)

The L1-L2 beat frequency Equation (2-17) is commonly called a widelane (WL) equation, since its wavelength of 86 centimeters is greater than that of both the L1 and L2 carrier waves. Since pseudorange error due to ionospheric effect is a function of frequency squared, ionospheric error in the L2 measurement is described in terms of the error in the L1 measurement. Ionospheric error between the L1 and L2 frequencies, \( \gamma_{12} \), is shown in Equation (2-16). This same relation is shown for the L1 and WL frequencies, \( \gamma_{WL} \), in Equation (2-18). \( f \) represents frequency in both equations.

### 2.1.6 Available Measurement Equations with Three Civil GPS Signals

In the near future, there will be three civil GPS signals at the L1 (1575.42 MHz), L2 (1227.60 MHz) and Lc frequencies (1176.45 MHz). With these signals, six measurement equations, three code and three carrier, are available. Double differences of the six measurement equations are described in Equations (2-19) through (2-26), using the simplified notation.

\[ \rho_{L1} = R + I_{L1} + T + \mu_{L1} \text{ (m)} \]  
(2-19)

\[ \rho_{L2} = R + \gamma_{12} I_{L1} + T + \mu_{L2} \text{ (m)} \]  
(2-20)

\[ \rho_{Lc} = R + \gamma_{1c} I_{L1} + T + \mu_{Lc} \text{ (m)} \]  
(2-21)
\[ \lambda_{L1} \Phi_{L1} = R + \lambda_{L1} N_{L1} - I_{L1} + T + \nu_{L1} \text{ (m)} \]  
(2-22)

\[ \lambda_{L2} \Phi_{L2} = R + \lambda_{L2} N_{L2} - \gamma_{12} I_{L1} + T + \nu_{L2} \text{ (m)} \]  
(2-23)

\[ \lambda_{Lc} \Phi_{Lc} = R + \lambda_{Lc} N_{Lc} - \gamma_{1c} I_{L1} + T + \nu_{Lc} \text{ (m)} \]  
(2-24)

\[ \gamma_{12} = \frac{f_{L1}^2}{f_{L2}^2} = 1.65 \]  
(2-25)

\[ \gamma_{1c} = \frac{f_{L1}^2}{f_{Lc}^2} = 1.79 \]  
(2-26)

With three carrier frequencies, three beat frequency measurements are formed. The combination of the L2 and Lc frequencies yields a beat frequency 5.9 meters in wavelength that is designated as Extra Widelane (EWL). As mentioned, the L1-L2 beat frequency equation yields a wavelength of 86 cm and is dubbed Widelane (WL). Finally, the combination of the L1 and Lc carrier frequencies yields a beat frequency with 75 centimeters in wavelength, and is designated as Mediumlane (ML). These beat frequency measurement equations are shown in Equations (2-27) through (2-32).

\[ \Phi_{EWL} = \Phi_{L2} - \Phi_{Lc} \]

\[ \lambda_{EWL} \Phi_{EWL} = \rho + \lambda_{EWL} N_{EWL} - \gamma_{EWL} I_{L1} + T + \nu_{EWL} \text{ (m)} \]

(2-27)

\[ \gamma_{EWL} = \lambda_{EWL} \left( \frac{\gamma_{12}}{\lambda_{L2}} - \frac{\gamma_{1c}}{\lambda_{Lc}} \right) = -1.72 \]

(2-28)

\[ \Phi_{WL} = \Phi_{L1} - \Phi_{L2} \]

\[ \lambda_{WL} \Phi_{WL} = \rho + \lambda_{WL} N_{WL} - \gamma_{WL} I_{L1} + T + \nu_{WL} \text{ (m)} \]

(2-29)

\[ \gamma_{WL} = \lambda_{WL} \left( \frac{1}{\lambda_{L1}} - \frac{\gamma_{12}}{\lambda_{L2}} \right) = -1.28 \]

(2-30)
\[ \phi_{ML} = \phi_{L1} - \phi_{Lc} \]

\[ \lambda_{ML}\phi_{ML} = \rho + \lambda_{ML}N_{ML} - \gamma_{ML}I_{L1} + T + \nu_{ML} \quad (m) \]  
(2-31)

\[ \gamma_{ML} = \lambda_{ML}\left(\frac{1}{\lambda_{L1}} - \frac{\gamma_{1c}}{\lambda_{Lc}}\right) = -1.34 \]  
(2-32)

The wavelengths of each code, carrier and beat frequency measurement are shown in Table 1-2.

### 2.2 Double Difference Measurement Error Model

#### 2.2.1 Definition

As discussed earlier, the single difference process nearly eliminates satellite clock error from the pseudorange measurement. It also reduces error from atmospheric effects, but residual differential error from it grows as the distance between receivers increases. Residual differential error from ephemeris data also grows with distance, but at very small rate (0.05 ppm). Pseudorange error due to multipath and receiver errors is not corrected by single difference. In fact, because both errors are unique to each receiver, pseudorange error increases in the single difference measurement. By first using the single difference measurement from two satellites, the double difference measurement can be formed. Since two measurements with the same receiver clock error are differenced, the clock error is eliminated in the double difference measurement. However, pseudorange error due to the residual differential error from ionosphere and troposphere, multipath and receiver error increases in the double difference measurement. This occurs because each unique error from the single difference measurement is combined to form the double difference.
Pseudorange error sources in the double difference measurement include residual differential ionosphere and troposphere error, multipath and receiver error. These sources are divided into two groups. One group, $\sigma_{mpr}$, includes multipath and receiver error. $\sigma_{mpr}$ depends on receiver type and antenna location, but is independent of distance between the receivers. The other group, $\sigma_{atm}$, includes residual differential error from ionosphere and troposphere effect, and decorrelates as the distance between receivers increases. Because these error groups are independent, the following equation is used to model the double difference measurement error, $\sigma$.

$$\sigma^2 = \sigma^2_{mpr} + \sigma^2_{atm}$$ (2-33)

2.2.2 Experimental Setup

The double difference measurement error model, Equation (2-33), is used in analysis of integrity and continuity of the Cascade Integer Resolution (CIR). To simplify analysis, a normal distribution of errors is assumed. In order to validate this assumption, a series of experiments was carried out. The experiments are divided into two configurations. The first uses one antenna only, splitting the signal it receives into two receiver setups. This configuration represents zero distance between the receivers in differential GPS, and is referred to as the ‘null baseline’ setup. The split GPS signal is received by two NovAtel receivers. The data log from each receiver, including the C/A code, carrier phase pseudorange, and decoded navigation data, is stored in the desktop PC.

The other configuration uses two receiver setups separated at a distance of $N$ meters. One receiver setup is considered a ‘reference,’ and uses an antenna at a known position. The
Figure 2-4. Experimental Configurations

other receiver setup is considered a ‘user,’ with its location initially unknown. Data logs from the reference and user NovAtel receivers are stored in the desktop PC and the laptop PC, respectively. In both configurations, single and double difference measurements are post processed after data gathering. The baseline distance between antenna of the latter configuration is determined during post processing, by using two Trimble receivers and associated survey software. Figure 2-4 shows the experimental setup.

2.2.3 Single Difference with Receiver Clock Error Correction

Pseudorange measurement error due to atmosphere effect and multipath is strongly affected by the elevation angle of GPS satellites [50]. Since the double difference measurement combines single difference measurements from two satellites, the effect of elevation of
both satellites enters into the double difference measurement. Thus, examining the effect of elevation angle on measurement error using the double difference measurement is difficult. Therefore, the single difference measurement with receiver clock bias correction is used to validate a normal distribution of the double difference error.

Receiver clock bias in the single difference measurement is corrected by first estimating the bias, then removing it. It is estimated by averaging the single difference measurements from \( n \) available satellites. Standard deviation of the single difference measurement with receiver clock bias correction, shown in Equation (2-34), is then scaled to be treated as the
standard deviation of the double difference measurement. The scale factor is shown in Equation (2-35).

\[
R_{ab}(t)_{\text{ClockRemoved}} = R_{ab}(t) - \frac{\sum_{i=1}^{n} R_{ab}^i(t)}{n}
\]  

(2-34)

Where \( R \) represents either the code pseudorange, or carrier phase measurement.

\[
\sigma(R_{ab}^i) = \frac{\sqrt{2}}{\sqrt{n-1}} \sigma(R_{ab}^i_{\text{ClockRemoved}})
\]

(2-35)

\( n = \text{number of satellites} \)

Figure 2-5 shows an example of receiver clock bias correction from the single difference measurement using an average of 5 single difference measurements. The top figure shows the single difference code measurement from PRN 7 as a line, and calculated receiver clock bias as dots. The bottom figure shows the double difference measurement of PRN 7 and PRN 26 as a line and the receiver clock removed single difference \((R_{ab}^i(t)_{\text{ClockRemoved}})\) measurement of PRN 7, which is treated as the double difference measurement, as a dot. In each figure, line and dots coincide, indicating that the receiver clock bias calculated from 5 single difference measurements closely represents the actual receiver clock bias. Therefore, the single difference measurement with receiver clock removed is used to examine the effect of elevation angle on the double difference measurements.
2.2.4 Multipath and Receiver Error

2.2.4.1 Validation of Normal Distribution

Data from the 163 meter base line configuration is used to examine the distribution of the double difference multipath and receiver error. Since the combined residual differential error due to decorrelation of ionosphere and troposphere conditions at 163 meters is less than 1 millimeter, it is ignored. The C/A code and carrier phase pseudorange from the reference and the user receiver are combined to form the single difference measurement. Receiver clock bias is removed by using an average of single difference measurements from n satellites. An appropriate scale factor is multiplied to treat the receiver clock bias corrected single difference measurement as the double difference value. A histogram and normal probability plot of the double difference measurement are used to see if the measurement error can be bounded by a normal distribution with a certain standard deviation.

In a normal probability plot, a straight line which starts at the lower left corner and ends at the upper right hand corner represents a normal distribution. The slope of a line represents standard deviation. The higher the slope, the smaller the standard deviation of the data. The data distribution probability (y-axis,) can be represented by the area under the line. The numerical precision of the y-axis represents the order of magnitude of data points used. For example, one thousand epochs of the C/A code pseudorange measurement will yield numerical precision of $10^{-3}$ on the y-axis.

Figure 2-6 shows a histogram and a normal probability plot of the C/A code double difference measurement distribution from a GPS satellite with PRN 27. The data is logged for an hour, at 1 Hz update rate. Both plots suggest that it is not normally distributed. If measure-
Figure 2-6. Histogram and Normal Probability Plot of the C/A Code DD Measurements are normally distributed, the ‘+’ marks in the plot, representing the measurements, should be aligned on a straight line. Instead of a straight line, the ‘+’ mark curves toward the center of the plot. It indicates that the measurement distribution has ‘fatter’ tails than a normal distribution of the same standard distribution. However, the measurement distribution is bounded by a normal distribution with standard deviation of 1.0 meters (dash-dot line in Figure 2-6). Therefore, the C/A code double difference multipath and receiver error can be treated as a normal distribution, as long as its standard deviation is set to 1.0 meters.

The same analysis is done on the carrier phase double difference measurement. Figure 2-7 shows a histogram and a normal probability plot of the carrier phase double difference measurement distribution from a GPS satellite with PRN 27. Both plots suggest that it is not
Figure 2-7. Histogram and Normal Probability Plot of the Carrier Phase DD Measurement
normally distributed. However, the measurement distribution is bounded by a normal dis-
tribution with standard deviation of 0.02 cycle, or 2 percent of the wavelength of the L1
frequency. As in the C/A code case, carrier double difference multipath and receiver error
can be treated as a normal distribution, as long as its standard deviation is set to 0.02 cycle.
However, multipath and receiver error are related to elevation angle of a GPS satellite. To
choose the correct bounding standard deviation of a normal distribution for the C/A code,
and for carrier phase double difference multipath and receiver error, the effect of elevation
angle must be investigated.
2.2.4.2 Effect of Elevation Angle

Pseudorange measurement error due to multipath and receiver error is related to signal to noise ratio ($S/N_0$) of the GPS signal. The $S/N_0$ is a function of GPS satellite elevation angle [83]. The GPS signal from a satellite with low elevation angle is weaker than that of a higher elevation angle since the former has to travel through more atmosphere than the latter. Reflected GPS signals from nearby objects cause greater multipath error than those coming from afar. The likelihood of receiving these reflected signals is higher with low elevation satellites [21].

To examine the effect of satellite elevation angle on double difference multipath and receiver error, receiver clock bias corrected single difference measurements are grouped at each 5 degree elevation angle, and the standard deviation of each group is calculated. An appropriate scale factor is multiplied by the standard deviation to represent the double difference measurement. Figure 2-8 shows a bar chart of the C/A code double difference multipath and receiver error at different satellite elevation angles. According to the figure, it is

![Figure 2-8. Effect of Elevation Angle on the C/A Code DD Measurement](image)
Figure 2-9. Effect of Elevation Angle on the Carrier Phase DD Measurement

possible to bound multipath and receiver error in the C/A code double difference measurement with a normal distribution with standard deviation of 1 meter, as long as a GPS satellite has elevation angle of 30 degrees or higher. Figure 2-9 shows a bar chart of carrier phase double difference multipath and receiver error at different satellite elevation angles. It can be seen from the figure that multipath and receiver error in the carrier phase double difference measurement is bounded by a normal distribution with standard deviation of 0.02 cycle, or 2 percent of the wavelength, as long as a GPS satellite has elevation angle of 30 degree or higher.

2.2.4.3 Effect of Time Averaging

The white noise portion of the double difference measurement error is reduced by using time averaging. Multipath error in measurements from a static receiver tend to contain daily repeating patterns, since the GPS satellite repeats the same orbit in one sidereal day. Multipath error becomes more random, or white-noise-like if the antenna is located on a moving platform. Receiver errors from noise in a delay lock loop and a phase lock loop,
Figure 2-10. Effect of Time Averaging

which are used to track the C/A code and carrier phase, are randomly distributed. Therefore, for the most part, time averaging reduces receiver error due to white noise. Figure 2-10 shows the change in standard deviation of the double difference measurements, both the C/A code and carrier phase, when different time constants are used for the averaging. For example, if one uses 200 seconds to average the measurements, the C/A code multipath and receiver error can be bounded by a normal distribution with standard deviation of 30 centimeters or less for high elevation satellites. Carrier phase measurement does not benefit from time averaging as much as the C/A code measurement, since white-noise-like receiver error from tracking the carrier phase is small compared to carrier phase multipath error, which is mostly biased.
2.2.5 Residual Differential Error from Decorrelation of Ionospheric and Tropospheric Condition

Pseudorange error due to ionosphere and troposphere effect is eliminated by the single difference process if atmospheric conditions over the reference and user receivers are identical. However, both atmospheric conditions decorrelate as distance between the receivers increases. Therefore, differential corrections for both atmospheric effects become less useful for the longer distance case. In the study by Christie, et al., statistical bounds for the effect of residual differential ionosphere and troposphere on a Local Area Augmentation System (LAAS) are examined [10]. LAAS is a C/A code DGPS system, which is being developed by the Federal Aviation Administration for precision approach and landing of aircraft. The Christie study concludes that the decorrelation errors caused by residual differential troposphere and ionosphere are between 2 and 5 mm/km, which is fairly small relative to the precision of a code based DGPS system measured in meters. However, for a carrier phase DGPS system, which is designed for centimeter level precision, the decorrelation error becomes significant as the baseline distance between the reference and user receivers increases. Although there are some extreme cases of reported spatial decorrelation of atmospheric effects, such as 55 mm/km in Antarctica, currently assumed values of spatial decorrelation of ionosphere and troposphere are, respectively, 2 ppm and 1 ppm [51]. These values are used in this dissertation. Assuming the spatial decorrelations are independent of each other, the following error model can be used.

\[ \sigma_{\text{iono}} = 2\text{ppm} \times \text{baseline} \]

\[ \sigma_{\text{tropo}} = 1\text{ppm} \times \text{baseline} \]
\[
\sigma^2_{\text{atm}} = \sigma^2_{\text{iono}} + \sigma^2_{\text{tropo}}
\]

(2-36)

Distributions of residual differential error from decorrelation of both atmospheric effects are assumed to be normal. This assumption has not been validated through experiment, however.

### 2.2.6 Summary

Distribution of the double difference multipath and receiver error is bounded by a normal distribution, with standard deviation shown in Table 2-1. Linear spatial gradient is assumed for residual differential atmospheric error, and distribution of the error is assumed to be normal, with standard deviation shown in Table 2-3. Satellite elevation angle of 30 degrees or higher is assumed in all cases.

\[
\sigma^2 = \sigma^2_{\text{mpr}} + \sigma^2_{\text{atm}}
\]

<table>
<thead>
<tr>
<th>(\sigma_{\text{mpr}})</th>
<th>C/A code</th>
<th>C/A code w/ tc=200 sec.</th>
<th>Carrier phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m</td>
<td>30 cm</td>
<td>0.02 cycle</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-1. DD Error Model, Multipath and Receiver Error (Elevation Angle \(>30^\circ\)).

<table>
<thead>
<tr>
<th>(\sigma_{\text{atm}})</th>
<th>Ionosphere</th>
<th>Troposphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>2PPM*baseline</td>
<td>1PPM*baseline</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2. DD Error Model, Ionosphere and Troposphere
3

The Cascade Integer Resolution

3.1 Integer Ambiguity Resolution

3.1.1 Geometry-Dependent Integer Ambiguity Resolution

As discussed in Chapter 1, carrier phases of the GPS signals in the L1 and L2 frequencies can be used to measure pseudorange between a GPS receiver and a GPS satellite. Carrier phase pseudorange measurement is subjected to the same error source as the C/A code measurement, except multipath and receiver error, and ionospheric effect. Due to their shorter wavelengths, 19 cm for L1 and 24 cm for L2, compared to 300 m for the C/A code, carrier phase measurement suffers smaller error due to multipath and receiver error than the C/A code measurement. Therefore, carrier phase yields a more accurate pseudorange measurement than the C/A code, especially in DGPS where most of the measurement errors are eliminated by single or double differencing. However, in order to use carrier phase pseudorange, the initial unknown integer number of waves, commonly called ‘integer ambiguity,’ between the receiver and the satellite at the acquisition of the signal must be resolved. The integer ambiguity remains constant if the receiver tracks the carrier without any cycle slips.

The time invariant nature of the integer ambiguity is utilized in research by Hwang [35] and Loomis [46]. These authors independently discovered that the change in satellite location can provide necessary observability to resolve the integer ambiguity. To illustrate how this
works, a theoretical representation of 2-dimensional carrier phase DGPS is used, with constraints on receiver location. A reference receiver is assumed to be at the origin of the coordinate system, and a user receiver is restricted to only move along the x-axis. Satellites are free to move along the x- and the y-axes without any constraints. Figure 3-1 describes this configuration. With the constraints on receiver location, the double difference carrier phase measurement, Equation (2-6), is revised in a vector form, as shown in Equation (3-1).

\[ \lambda \phi_{ab}^{ij}(t) = \delta \lambda \phi_{ab}^{ij}(t) \cdot \hat{x}(t) + \lambda N_{ab}^{ij} + \lambda \delta \phi_{ab}^{ij}(t) \]  

(3-1)

- \( \delta \lambda \phi_{ab}^{ij}(t) \) is a sum of x components of unit line of sight vectors to satellites i and j at an epoch t
- \( \hat{x}(t) \) is a baseline vector from the reference to the user receiver
- \( \delta \phi_{ab}^{ij}(t) \) is a measurement error, which includes residual differential atmospheric effect, multipath and receiver error at an epoch t
Unknowns in Equation (3-1) are baseline distance, $x$, and double difference integer ambiguity, $N_{i+1}^{ij}$. Since there are two unknowns for a measurement at an epoch, $t$, Equation (3-1) cannot be solved in one epoch. For each $x$, there is a unique combination of integers to the satellite $i$, $N_i^{ij}$ and $j$, $N_j^{ij}$, which represents a double difference integer, $N_{ij}^{ij}$, when combined. In a 2-dimensional integer space, as shown in Figure 3-2, a line represents all possible combinations of $N_i^{ij}$ and $N_j^{ij}$ for the $x$ value. Dots in the figure represent the effect of measurement error, $\delta_{ij}^{ij}(t)$. Width of the dotted area is related to the magnitude of measurement error.

![Graph showing the effect of change in satellite geometry](image)

*Figure 3-2. The Effect of Change in Satellite Geometry*
The top plot in Figure 3-2 represents all possible combinations of \( N^i_{ab} \) and \( N^j_{ab} \) for the x value at time \( t_1 \), whereas the bottom plot in Figure 3-2 shows the effect of change in locations of satellites \( i \) and \( j \) in the 2-dimensional integer space. The two lines in the bottom plot represent the locus of combinations of \( N^i_{ab} \) and \( N^j_{ab} \) for the x value at \( t_1 \) and \( t_2 \), respectively. As the satellite moves over time, a correct integer ambiguity combination is observed, since this value is constant. The observability gained by change in the satellite geometry is illustrated in the bottom plot in Figure 3-2. Using the carrier phase measurements from the satellites \( i \) and \( j \) at two different times, a matrix equation Equation (3-2) is formed.

\[
\lambda \begin{bmatrix} \phi_{ab}^i(t_1) \\
\phi_{ab}^j(t_2) \end{bmatrix} = \begin{bmatrix} \delta \phi_{ab}(t_1) \\
\delta \phi_{ab}(t_2) \end{bmatrix} x + \lambda N_{ab}^i + \lambda N_{ab}^j + \lambda \begin{bmatrix} \delta \phi_{ab}(t_1) \\
\delta \phi_{ab}(t_2) \end{bmatrix} (m)
\]

Equation (3-2)

For a static user receiver, two unknown constants, \( x \) and \( N_{ab}^{ij} \), are estimated by using two measurements, \( \phi_{ab}^i(t_1) \) and \( \phi_{ab}^j(t_2) \). For a moving user receiver, measurements from an additional satellite are required as baseline distance, \( x \), changes with time. Due to the vector nature of Equation (3-1), this 2-dimensional analysis is easily expanded into 3-dimensions.

If a time interval, \( t_1 \) to \( t_2 \) is not long enough, the observability gained due to change in satellite geometry, represented by an intersecting angle in the bottom plot in Figure 3-2, is not enough to resolve the integer ambiguity due to measurement error. The value of an estimated integer will become close to a correct integer as the time interval increases.

Satellite geometry-dependent integer ambiguity is used in many applications, such as surveying. However, it is not suitable for applications where a solution for the integer ambi-
guity is required in a limited time. For example, the observability gained by change in satellite geometry during the time span of a typical aircraft approach to an airport is not sufficient for resolving the integer ambiguity with high confidence. Therefore, geometry-dependent integer resolution without augmentation is not a suitable CDGPS solution for aircraft landing [45]. However, a pseudolite, which is a ground-based GPS satellite, can be utilized to induce the necessary change in geometry in a short period of time. The Integrity Beacon Landing System (IBLS) developed at the Stanford University GPS laboratory uses precisely this setup [12].

3.1.2 Geometry-Free Integer Ambiguity Resolution: Single Frequency

![Graph: Code and Carrier Pseudorange vs. Time](image)

**Figure 3-3.** The C/A Code and Carrier Phase Measurement

As its name implies, geometry-free integer ambiguity resolution does not rely on changes in satellite geometry to resolve the integer ambiguity. Instead, this method utilizes the C/A code pseudorange and integer nature of the cycle ambiguity for the ambiguity resolution. In fact, because carrier phase measurement is biased from the C/A code pseudorange by an integer number of wavelengths as shown in Figure 3-3, the unknown states, pseudorange, $\rho$, and integer ambiguity, $N$, can be readily estimated by
solving Equation (3-3). The simplified double difference C/A code (Equation (2-14)) and carrier phase (Equation (2-15)) equations are used in Equation (3-3).

\[
\begin{bmatrix}
\rho \\
\lambda \phi \\
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \lambda \
\end{bmatrix} \begin{bmatrix} R \\ N \\
\end{bmatrix} + \begin{bmatrix} \delta \rho \\ \lambda \delta \phi \\
\end{bmatrix} \quad \text{(m)}
\]

- \( \delta \rho \) includes residual differential atmospheric error and the C/A code multipath and receiver error
- \( \lambda \delta \phi \) includes residual differential atmospheric error and carrier phase multipath and receiver error

However, the C/A code measurement error is included in the estimated real-valued integer ambiguity from Equation (3-3). In Chapter 2, the double difference of the C/A code and carrier phase multipath and receiver error was bounded by a normal distribution with standard deviation of 1m and 0.02 cycle, respectively. Effect of the C/A code measurement error on the estimated integer ambiguity is pronounced due to the difference in magnitude of the code and carrier measurement errors, and is shown in Figure 3-4. In this figure, the integer ambiguity is estimated by subtracting the C/A code measurement from that of the carrier phase. Since a standard deviation of the estimated integer ambiguity is 3.75 times a wavelength, obtaining a correct integer value, -15 in this case, is quite difficult.

The C/A code measurement error is reduced when time averaging is used. In Chapter 2, it is shown that when time averaging with a constant of 200 seconds is applied to the C/A code double difference measurement, multipath and receiver error is bounded by a normal distribution with standard deviation of 30 cm. However, to resolve the integer ambiguity, roughly 19 cm for the L1 frequency, the measurement error is still too large.
Figure 3-4. Integer Ambiguity Estimation using the C/A Code Measurement

As briefly introduced in Section 1.2.4, a beat frequency of the L1 and L2, having a wavelength of 86 cm, is utilized in some CDGPS applications to resolve the integer ambiguity. Because of its longer wavelength, the time averaged C/A code measurement with a suitable time constant can be used to resolve the integer ambiguity of the L1-L2 beat frequency, known as a widelane (WL). This process is part of the Cascade Integer Resolution.

3.2 The Cascade Integer Resolution (CIR)

3.2.1 Definition

In the near future, there will be two additional civil GPS signals broadcasting at the L2 (1227.60 MHz) and the Lc (1176.45 MHz) frequencies, in addition to the currently avail-
able signal at the L1 (1575.42 MHz). With three civil frequencies, three beat frequencies are produced (Table 3-1). The available beat frequencies will now be used to resolve the integer ambiguity.

<table>
<thead>
<tr>
<th>Beat Frequency Component</th>
<th>Frequency (MHz)</th>
<th>Wavelength (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra Widelane (EWL)</td>
<td>51.15</td>
<td>5.86</td>
</tr>
<tr>
<td>L2-Lc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Widelane (WL)</td>
<td>347.82</td>
<td>.862</td>
</tr>
<tr>
<td>L1-L2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mediumlane (ML)</td>
<td>398.97</td>
<td>.751</td>
</tr>
<tr>
<td>L1-Lc</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3-1. Three Beat Frequencies

A real-valued solution for the extra widelane integer, $N_{EWL}$, is estimated by subtracting the Lc code pseudorange measurement from the EWL measurement. The Lc code is used in the CIR as the principle pseudorange measurement, as it will have improved multipath performance due to its higher clock rate of 10.23 MHz [63].

$$\lambda_{EWL} \phi_{EWL} = R + \lambda_{EWL} N_{EWL} - \gamma_{EWL} I_{L1} + T + v_{EWL} \quad (m)$$

$$\rho_{Lc} = R + \gamma_{Lc} I_{L1} + T + \mu_{Lc} \quad (m)$$

$$\phi_{EWL} - \frac{\rho_{Lc}}{\lambda_{EWL}} = N_{EWL} - \frac{(\gamma_{EWL} - \gamma_{Lc})}{\lambda_{EWL}} I_{L1} + \frac{(v_{EWL} - \mu_{Lc})}{\lambda_{EWL}} \quad (3-4)$$

If the combined error from both the Lc code and the EWL measurements is smaller than one-half of an EWL wavelength (5.86 m), the goal is to obtain the correct EWL integer by rounding the real-valued solution. Note that the residual differential tropospheric term, which is independent of frequency, is eliminated when the Lc code and the EWL measurements are differenced.
With a correct integer, the EWL measurement is used as the pseudorange measurement, and is subtracted from the WL measurement to get a real-valued solution for the WL integer ambiguity.

\[
\lambda_{WL}\phi_{WL} = R + \lambda_{WL}N_{WL} - \gamma_{WL}I_{L1} + T + \nu_{WL} \quad \text{ (m)}
\]

\[
\phi_{WL} - \frac{\lambda_{EWL}}{\lambda_{WL}}(\phi_{EWL} - N_{EWL}) = N_{WL} - \frac{(\gamma_{WL} - \gamma_{EWL})}{\lambda_{WL}}I_{L1} + \frac{(\nu_{WL} - \nu_{EWL})}{\lambda_{WL}} \quad \text{(3-5)}
\]

Again, if the combined error from the EWL and WL measurements is smaller than one-half of a WL wavelength (86 cm), a correct WL integer is obtained by rounding the real-valued solution.

With a correct integer, the WL measurement is used as the pseudorange measurement, and is subtracted from the ML measurement to get a real-valued solution for the ML integer ambiguity.

\[
\lambda_{ML}\phi_{ML} = R + \lambda_{ML}N_{ML} - \gamma_{ML}I_{L1} + T + \nu_{ML} \quad \text{ (m)}
\]

\[
\phi_{ML} - \frac{\lambda_{WL}}{\lambda_{ML}}(\phi_{WL} - N_{WL}) = N_{ML} - \frac{(\gamma_{ML} - \gamma_{WL})}{\lambda_{ML}}I_{L1} + \frac{(\nu_{ML} - \nu_{WL})}{\lambda_{ML}} \quad \text{(3-6)}
\]

Finally, the same process is used to resolve the L1, L2 or Lc integer ambiguity. The Lc case is shown as an example.

\[
\phi_{Lc} - \frac{\lambda_{ML}}{\lambda_{Lc}}(\phi_{ML} - N_{ML}) = N_{Lc} - \frac{(\gamma_{Lc} - \gamma_{ML})}{\lambda_{Lc}}I_{L1} + \frac{(\nu_{Lc} - \nu_{ML})}{\lambda_{Lc}} \quad \text{(3-7)}
\]

Since this method resolves the integer ambiguities from the longest to the shortest wavelength successively using the previous measurement, it is defined as the Cascade Integer Resolution (CIR).
3.2.2 Using the CIR with the Present Civil GPS Signal

Presently, civilians are authorized to use the GPS signal on the L1 frequency, both the C/A code and its carrier, and carrier only on the L2 frequency. Broadcast P code in the L2 frequency is usually encrypted (Y code), and use of a correctly decrypted P code is restricted to U.S Armed Forces, U.S. Federal Agencies, and selected allied armed forces and governments. However, there are number of receivers which can produce pseudorange measurements from the broadcast signal in the L2 by using the following techniques: squaring, cross correlation, code correlation plus squaring, and Z-tracking [31]. Depending on the technique used, the L2 C/A code and carrier phase measurements could contain more error than their L1 counterparts due to processing error. Also, the L2 measurements could be correlated with the L1 measurement. The details of how these techniques work are beyond the scope of this dissertation, and the reader is referred to work by MacDoran [47], Allison[2] and Ashjaee[4].

With the C/A code on the L1 and carrier phase measurement from the L1 and L2 signals, the CIR is possible by first resolving the WL integer using the C/A code measurement, then resolving the L1 or L2 integer ambiguity using the WL measurement with a correct integer.

The CIR is applied to data from two NovAtel dual frequency MiLLennium GPS cards with a 163 m baseline configuration as described in Section 2.2.2. The top plot in Figure 3-5 describes the first step in the CIR process, which is resolving the WL integer ambiguity using the L1 C/A code measurement.

\[
\phi_{WL} - \frac{\rho_{L1}}{\lambda_{WL}} = \hat{N}_{WL}
\]
Due to the C/A code measurement error, instantaneous resolution of the WL integer ambiguity is not always possible. The frequency of incorrect resolution of the WL integer by rounding the real-valued solution, $\hat{N}_{WL}$, is 52% out of 3887 test cases.

$$P_{wrongWL} = 0.52$$

The bottom plot in Figure 3-5 describes the second step in the CIR, which is resolving the L1 integer ambiguity by using the WL measurement with a correct WL integer.

$$\phi_{L1} - \frac{\lambda_{WL}}{\lambda_{L1}} (\phi_{WL} - N_{WL}) = \hat{N}_{L1}$$  \hspace{1cm} (3-9)
Since all 3887 of the real-valued solutions, $\hat{N}_{L_1}$, are rounded into a correct L1 integer, probability of incorrect resolution of the L1 integer is less then $2.6 \times 10^{-4}$.

$$P_{\text{wrong}L_1} \leq 2.6 \times 10^{-4}$$

To improve the performance of the first step, time averaging is applied to the L1 C/A code measurement to reduce error. Figure 3-6 describes the WL and L1 integer resolution steps in the CIR with time constants (tc) of 60 and 300 seconds, respectively. The probability of incorrect resolution of the WL integer by rounding the real-valued solution, $\hat{N}_{WL}$, is 19% when the time constant is 60 seconds, and 2% when the time constant is 300 seconds. Probability of incorrect resolution of the L1 integer ambiguity is equal to the instantaneous case for both time averaged cases.

### 3.2.3 Applying the CIR with Three Civil GPS Frequencies

Probability of incorrect resolution of the integer ambiguity is directly related to integrity of CDGPS as users should be warned if a wrong integer is resolved. It is also related to the
availability of CDGPS, since the system is usable only when the correct integer is resolved. Successful resolution of the integer ambiguity of each step in the CIR is also related to the accuracy of CDGPS. A centimeter level accuracy for a short baseline distance, where residual differential ionospheric effect is minimal, can only be acquired if the CIR is successful in all steps, down to resolving the L1, L2 or Lc integer ambiguity.

As examined in the previous section, using current GPS signals yields a probability of incorrect integer ambiguity resolution equal to 2% out of 3887 test cases. This probability is also called the integrity risk of the CIR. The 2% value is achieved by using the time average of the code measurement with a time constant of 300 seconds to reduce measurement error. For applications with low integrity risk requirement, such as aircraft landing, 2% integrity risk is too high. For example, the Local Area Augmentation System (LAAS) integrity requirement for Category I precision approach is a probability of Hazardously Misleading Information ($P_{HMI}$) equal to $10^{-7}$. Also, HMI must be reported to users within 6 seconds [57].

However, if three civil GPS signals with three beat frequencies are used in the CIR, integrity risk should decrease considerably. Unlike the CIR with present GPS signals, where the code measurement is used to resolve the WL integer ambiguity, the CIR with three GPS signals resolves the EWL integer ambiguity with the code measurement. Since the EWL wavelength is considerably longer than the WL wavelength, 5.9 m versus 86 cm, error in the code measurement will have less effect on the EWL integer ambiguity resolution. Also, the proposed code in the Lc frequency has a wavelength of 30 m, which is 10 times smaller than a wavelength of the C/A code, and therefore will produce less multipath and receiver
Figure 3-7. The CIR with Two and Three Available GPS Signals

error. This also will decrease the probability of resolving incorrect EWL integer ambiguity when the Lc code measurement is used. Figure 3-7 describes the benefits of three GPS signals on the CIR, in comparison with the present available signals.

Because of its cascading nature, each integer resolution step must be correct for the CIR to be successful. Probability of incorrect integer resolution, or integrity risk of each step is therefore examined by using conditional probability along with a covariance analysis. Since the Lc code and carrier phase measurements are not yet available, the double difference error model (Chapter 2) is used in both of the subsequent analyses.
3.3 Integrity Analysis Using Conditional Probability

3.3.1 Analysis Setup

The CIR with three civil GPS signals uses the code measurement to resolve the EWL integer, the EWL measurement to resolve the WL integer, and finally uses the WL measurement to resolve the ML integer, and finally uses the WL measurement to resolve the L1, L2 or Lc integer. Each measurement is used to calculate a real-valued solution of the integer ambiguity for the next beat or carrier frequency in a cascading manner. Therefore, integrity of the integer solution in each step must be verified. To do so, an integer rounding criteria based on measurement error, \( \sigma \), and wavelength, \( \lambda \), is developed. The double difference error distribution is assumed to be normal, as discussed in Section 2.2.

Assume that a real-valued solution is \( \delta \) away from the true integer, shown at 0 in Figure 3-8, and \( \delta \) is positive. Given a distribution of \( \delta \), with standard deviation, \( \sigma \), conditional probability of rounding a real-valued solution to a correct integer with the given conditions is then the following.

\[
P(0|\delta, \sigma) \equiv \frac{\exp\left(-\frac{1}{2\sigma^2}\delta^2\right)}{\exp\left(-\frac{1}{2\sigma^2}\delta^2\right) + \exp\left(-\frac{1}{2\sigma^2}(\lambda - \delta)^2\right)}
\]

(3-10)
\[ P_{\text{wrongInt}} = 1 - P(0 \mid \delta, \sigma) \equiv \frac{\exp\left(\frac{-1}{2\sigma^2}(\lambda - \delta)^2\right)}{\exp\left(\frac{-1}{2\sigma^2}\delta^2\right)} \]

\[ P_{\text{wrongInt}} = \exp\left(\frac{-1}{2\sigma^2}(\lambda^2 - 2\lambda\delta)\right) \]

(3-11)

The probability of rounding to an incorrect integer, \( P_{\text{wrongInt}} \), is shown in Equation (3-11). Note that Equation (3-11) only holds when \( P_{\text{wrongInt}} \ll 1 \). It is rearranged below to find the requirement for a desired level of integrity.

\[ \exp\left(\frac{-1}{2\sigma^2}(\lambda^2 - 2\lambda\delta)\right) < (P_{\text{wrongInt}})_{\text{required}} \]

\[ \frac{\delta}{\sigma} < \frac{1}{2\left(\frac{\lambda}{\sigma}\right)} + \frac{2\log(P_{\text{wrongInt}})_{\text{required}}}{\lambda/\sigma} \]

(3-12)

Given a desired integrity risk, \( (P_{\text{wrongInt}})_{\text{required}} \), the \( \delta/\sigma \) ratio, which is a design parameter, is set to meet a suitable level of availability. Then, the required \( \lambda/\sigma \) ratio is calculated by using Equation (3-12). The \( \delta/\sigma \) ratio represents how far a real-valued solution can be from the true integer and still be accepted as a correct candidate for integer rounding. A higher ratio yields fewer rejected real-valued solutions and increased availability. The ratio between wavelength and standard deviation of measurement error, \( \lambda/\sigma \), is an integrity parameter. \( \lambda \) represents a wavelength of carrier phase or beat frequency measurement with the integer ambiguity, and \( \sigma \) represents the standard deviation of measurement error used to resolve the ambiguity. For a given measurement error, resolving the integer ambiguity of a carrier phase or beat frequency measurement with longer wavelength is less difficult than shorter wavelength resolution. Therefore, a higher \( \lambda/\sigma \) ratio brings about higher integrity of the integer ambiguity resolution, or rounding step. Table 3-2 shows a
few sample calculations of parameters for the rounding criteria for different levels of integrity and availability requirements.

<table>
<thead>
<tr>
<th>Integrity</th>
<th>Availability</th>
<th>$\delta/\sigma$</th>
<th>$\lambda/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-9}$</td>
<td>$10^{-8}$</td>
<td>5.7</td>
<td>14</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>$10^{-5}$</td>
<td>4.4</td>
<td>11</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$10^{-2}$</td>
<td>2.5</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 3-2. Examples of Integrity and Availability Parameters

Performance of the CIR using the present GPS signal, which was empirically examined in Section 3.2.2, is easily explained by examining the $\lambda/\sigma$ ratio.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{WL} = 0.86\bar{m}$</th>
<th>$\lambda_{L1} = 0.19\bar{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{L1,Code}$</td>
<td>0.30m</td>
<td>2.9</td>
</tr>
<tr>
<td>$\sigma_{WL} = 0.017m$</td>
<td></td>
<td>0.63</td>
</tr>
</tbody>
</table>

Table 3-3. The $\lambda/\sigma$ Ratio of the CIR with Present GPS Signal

In Table 3-3, the $\frac{\lambda_{WL}}{\sigma_{L1\,Code}}$ is 2.9, which is very low compared to the required $\lambda/\sigma$ ratio of 7 for probability of incorrect integer resolution of $10^{-3}$ (Table 3-2). This explains the difficulty encountered when resolving the WL integer using the L1 C/A code in Section 3.2.2. Using time averaging with a time constant of 300 seconds, the probability of wrong integer resolution was 2% out of 3887 test cases. However, once the WL integer is correctly resolved, probability of wrong resolution of the L1 integer ambiguity was less then $2.6 \times 10^{-4}$. In Table 3-3, the $\frac{\lambda_{L1}}{\sigma_{WL}}$ is 11, indicating that the probability of wrong L1 integer resolution using the WL measurement with a correct integer is $10^{-6}$ (Table 3-2).
3.3.2 Integrity of the CIR over Short Baseline Distances

The integrity parameter, $\lambda/\sigma$, of each integer ambiguity resolving step of the CIR with three civil signals is calculated in Table 3-4. A short baseline distance between the reference and user GPS receivers is assumed, and residual differential ionospheric effect is therefore ignored. Residual differential tropospheric effect is eliminated in the CIR process, as discussed in Section 3.2.1. Standard deviation of the Lc code measurement error,

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{EWL}(5.9m)$</th>
<th>$\lambda_{WL}(0.86m)$</th>
<th>$\lambda_{ML}(0.75m)$</th>
<th>$\lambda_{Lc}(0.25m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{LcCode}(0.15m)$</td>
<td>39</td>
<td>5.7</td>
<td>5.0</td>
<td>1.7</td>
</tr>
<tr>
<td>$\sigma_{EWL}(0.12m)$</td>
<td></td>
<td>7.4</td>
<td>6.4</td>
<td>2.2</td>
</tr>
<tr>
<td>$\sigma_{WL}(0.017m)$</td>
<td></td>
<td></td>
<td>44</td>
<td>15</td>
</tr>
<tr>
<td>$\sigma_{ML}(0.015m)$</td>
<td></td>
<td></td>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3-4. $\lambda/\sigma$ Ratio for the CIR with Three Civil GPS Signals

$\sigma_{LcCode}$ is assumed to be 15 cm, which is one-half of the $\sigma_{L1Code}$ value when time averaging is used with a time constant of 200 seconds. The decrease in $\sigma_{LcCode}$ comes from the smaller wavelength of the proposed Lc code (30 m), as compared to that of the L1 C/A code (300 m). Double difference carrier phase multipath and receiver error is bounded by a normal distribution with a standard deviation of 2% of its wavelength, as discussed in Chapter 2.

Integrity of the CIR at a short baseline distance can be interpreted using the integrity parameter, $\lambda/\sigma$, in Table 3-4. In the table, the integrity parameter for resolving the EWL integer ambiguity using the Lc code measurement, $\frac{\lambda_{EWL}}{\sigma_{LcCode}}$, is 39. Compared to the required
\( \lambda / \sigma \) value of 14 for \( 10^{-9} \) integrity risk (Table 3-2), this is much higher value. Therefore, the CIR can be used to resolve the EWL integer ambiguity using the Lc code measurement with integrity risk less than \( 10^{-9} \). Also, the integrity parameter for resolving the WL integer ambiguity using the EWL measurement, \( \frac{\lambda_{WL}}{\sigma_{EWL}} \), is 7.4 (Table 3-4). Per the integer rounding criteria, the WL integer is wrong once in a thousand resolutions using the EWL measurement (Table 3-2). Resolving the ML integer ambiguity using the WL measurement is a very low integrity risk process since \( \frac{\lambda_{ML}}{\sigma_{WL}} \) is 44. Its value is much higher than the aforementioned value of 14 for \( 10^{-9} \) integrity risk. Finally, the Lc integer ambiguity is resolved using the ML measurement with lower than \( 10^{-9} \) integrity risk, where \( \frac{\lambda_{LC}}{\sigma_{ML}} \) is 17 (Table 3-4).

3.3.3 Integrity of the CIR over Long Baseline Distances

As a baseline distance between the reference and user receivers increases, residual differential ionospheric effect grows. In Figure 3-9, the double difference of the L1 C/A code and carrier phase measurement error due to multipath and receiver error, along with residual differential ionospheric effect are plotted over distance. Standard deviation of multipath and receiver error is 30 cm while that of the L1 C/A code and carrier phase error is 2% of a wavelength. A linear gradient of 2 ppm is assumed for the standard deviation of residual differential ionospheric error (\( \sigma_{Ionograd} \)). As seen in the upper plot of Figure 3-9, multipath and receiver error dominates the C/A code measurement error up to 5 km. In fact, since a linear gradient is used, the baseline distance must be longer than 150 km for the residual differential ionospheric error to be greater than multipath and receiver error in the C/A code measurement. However, carrier phase measurement is affected by the residual differential ionospheric error at a much shorter baseline distance. In fact, the lower plot of Figure 3-9
Figure 3-9. Effect of Residual Differential Ionosphere Error over Baseline Distance

indicates that the carrier phase measurement error is dominated by residual differential ionospheric error when a baseline distance is longer than 2 km. Therefore, to examine integrity of the CIR over increasing baseline distance, standard deviation of measurement error is calculated at each distance, then the $\lambda/\sigma$ ratio of each integer ambiguity resolution step is calculated. Since ionospheric effect is a function of frequency squared, appropriate scale factors, calculated in Chapter 2, are used for carrier frequencies other than the L1.

Integrity of each CIR integer ambiguity resolution step over distance, from 0 to 20 km, is plotted in Figure 3-10. As expected, the integrity of each step, $(\lambda/\sigma)$, decreases over distance, as standard deviation of the measurement error increases over distance due to the
Figure 3-10. Integrity of the CIR Steps Over Distance

change in residual differential ionospheric error. Integrity of the CIR with a short baseline distance, summarized in Table 3-4, is shown at 0 km in Figure 3-10.

Figure 3-11. Available CIR Steps Over Distance
Integrity of the CIR over distance can be interpreted in several ways by using the integrity parameter \( \lambda / \sigma \) of each integer ambiguity resolution step plotted in Figure 3-10. The three horizontal lines in the figure represent the minimum value of \( \lambda / \sigma \) parameter for a desired level of integrity (Table 3-2). The ML integer resolution step using the WL measurement is skipped since the Lc integer ambiguity is resolved by using either measurement with a similar level of integrity (Table 3-4).

For an application requiring integrity risk on the order of \( 10^{-9} \), the EWL integer ambiguity is resolved using the Lc measurement for all the tested distances, since the line representing \( \frac{\lambda_{EWL}}{\sigma_{LcCode}} \) ratio (x in the figure) is higher than the minimum \( \lambda / \sigma \) value (14) for \( 10^{-9} \) integrity risk (Table 3-2). However, the WL integer ambiguity cannot be resolved using the EWL measurement since the line representing \( \frac{\lambda_{WL}}{\sigma_{EWL}} \) (o in the figure) is lower than the minimum \( \lambda / \sigma \) value at all tested distances. Due to the aforementioned cascading nature, the Lc integer ambiguity cannot be resolved without resolving the WL integer ambiguity.

For an application requiring integrity risk of on the order of \( 10^{-3} \), the EWL integer ambiguity is resolved using the Lc measurement for all the tested distances as in the \( 10^{-9} \) case. The WL integer is resolved using the EWL measurement up to 8.5 km, where the line representing \( \frac{\lambda_{WL}}{\sigma_{EWL}} \) ratio (o in the figure) goes below the minimum \( \lambda / \sigma \) value (7) for \( 10^{-3} \) integrity risk. The Lc integer is resolved using the WL measurement up to 8.0 km, where the line representing \( \frac{\lambda_{Lc}}{\sigma_{WL}} \) ratio (triangle in the figure) goes below the minimum \( \lambda / \sigma \) value (7) for \( 10^{-3} \) integrity risk. Figure 3-11 summarizes the above results.
3.4 Integrity Analysis Using a Covariance Propagation

3.4.1 Analysis Setup

The CIR with three civil GPS signals uses one-on-one combination of three carrier frequencies to form three beat frequencies. However, the generated beat frequencies are not linearly independent. For example, in the first and second rows of Table 3-5, the EWL and WL frequencies can be added together to produce the ML frequency. The effect of linearly dependent beat frequency measurements in the CIR process is examined by a covariance analysis of measurement error. The state vector, \( \mathbf{u} \), observation matrix, \( \mathbf{H} \), and measurement vector, \( \mathbf{z} \) used in the analysis are described in Equations (3-13) and (3-14). The analysis is also used to calculate the probability of incorrect integer resolution of each CIR step, verifying integrity of the CIR in an ensemble sense, in contrast to the earlier conditional probability analysis, which was a Bayesian result.

\[
\mathbf{z} = \mathbf{H} \mathbf{u} + \delta \mathbf{z} \tag{3-13}
\]
\[
\begin{bmatrix}
\rho_1 \\
\rho_2 \\
\rho_c \\
\phi_1 \\
\phi_2 \\
\phi_c \\
\tilde{I}_1
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 \\
1 & Y_{12} & 0 & 0 & 0 \\
1 & Y_{1c} & 0 & 0 & 0 \\
1/\lambda_1 & -1/\lambda_1 & 1 & 0 & 0 \\
1/\lambda_2 & -Y_{12}/\lambda_2 & 0 & 1 & 0 \\
1/\lambda_c & -Y_{1c}/\lambda_c & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
R \\
I_1 \\
N_1 \\
N_2 \\
N_c \\
\delta I_1
\end{bmatrix} +
\begin{bmatrix}
\delta \rho_1 \\
\delta \rho_2 \\
\delta \rho_c \\
\delta \phi_1 \\
\delta \phi_2 \\
\delta \phi_c \\
\delta I_1
\end{bmatrix}
\quad (3-14)
\]

- \( z \) is a measurement vector, including six double difference measurements, three code and three carrier phase. An \textit{a priori} knowledge of linear gradient of residual differential ionospheric delay is also included.

- \( H \) is an observation matrix. Scale factors for ionospheric effect on frequencies other than the L1 are included.

- \( u \) is a state vector, containing pseudorange, \( R \), residual differential ionospheric effect at the L1 frequency, \( I_1 \), and three integer ambiguities for the L1, L2 and Lc carrier phase measurements, \( N_1 \), \( N_2 \), and \( N_c \).

- \( \delta z \) is a measurement error vector.

A covariance matrix of the state estimation error, \( P_u \), is calculated from a covariance matrix of the measurement error, \( P_z \). Calculation of the \( P_z \) and the \( P_u \) matrices are shown in Equations (3-15) and (3-16). The six double difference measurements, three code and three carrier phase, are assumed to be independent, and measurement error models developed in Chapter 2 are used to form the measurement error vector, \( \delta z \). The linear gradient of standard deviation of residual differential ionospheric error is 2 ppm, and therefore the standard deviation of the residual differential ionosphere error is calculated by multiplying its gradient with the baseline distance.
\[ P_z = E[\delta z \delta z] \quad (3-15) \]

\[
P_z = \begin{bmatrix}
E[\delta \rho_1 \delta \rho_1] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E[\delta \rho_2 \delta \rho_2] & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E[\delta \rho_c \delta \rho_c] & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E[\delta \phi_1 \delta \phi_1] & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & E[\delta \phi_2 \delta \phi_2] & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E[\delta \phi_c \delta \phi_c] & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E[\delta I_1 \delta I_1] & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E[\delta I_1 \delta I_1]
\end{bmatrix}
\]

\[ P_u = (H^T (P_z)^{-1} H)^{-1} \quad (3-16) \]

\[
P_i = \begin{bmatrix}
E[\delta N_1 \delta N_1] & E[\delta N_1 \delta N_2] & E[\delta N_1 \delta N_c] \\
E[\delta N_2 \delta N_1] & E[\delta N_2 \delta N_2] & E[\delta N_2 \delta N_c] \\
E[\delta N_c \delta N_1] & E[\delta N_c \delta N_2] & E[\delta N_c \delta N_c]
\end{bmatrix} \quad (3-17)
\]

The covariance matrix of the estimation error of three double difference integers, \( P_i \), is a subset of a covariance matrix of states, \( P_u \). Using the \( P_i \) matrix (Equation (3-17)), standard deviation of the estimation error for the integer ambiguity solution for each carrier frequency maybe calculated by taking the square root of its diagonal components.

\[ \sigma_{N1} = \sqrt{E[\delta N_1 \delta N_1]} \]

\[ \sigma_{N2} = \sqrt{E[\delta N_2 \delta N_2]} \]

\[ \sigma_{Nc} = \sqrt{E[\delta N_c \delta N_c]} \]

The standard deviation of the estimation error for the integer ambiguity solution for each beat frequency is also calculated by multiplying the \( P_i \) matrix with a vector containing the multiplication factor for the involved frequencies. This step forms the beat frequency.
Taking the square root of this value yields the standard deviation of the estimation error. In Equation (3-18), this process is completed for the EWL integer ambiguity solution.

\[ \sigma_{N_{EWL}} = \left( \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} P I \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}^T \right)^{1/2} \]  

(3-18)

\[ \phi_{EWL} = 0 \times \phi_1 + 1 \times \phi_2 - 1 \times \phi_c \Rightarrow \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \]

The estimated solution of the integer ambiguity is real-valued, and it can be rounded into a correct integer if the standard deviation of the estimation error of the integer ambiguity solution is much less than 1/2. Therefore, a desired level of integrity can be specified by multiplying the standard deviation of the integer ambiguity estimation error with a factor, \( K_{MD} \). Then, to round the real-valued solution into a correct integer with a desired level of integrity, the following relation must hold true.

\[ K_{MD} \sigma_{N_{EWL}} < \frac{1}{2} \]  

(3-19)

Assuming a normal distribution of estimation error, the probability of wrong integer rounding, is then the following.

\[ P_{wrongN_{EWL}} = 1 - \int_{-\infty}^{K_{MD}} f(x)dx \]  

(3-20)

\[ K_{MD} = \frac{1}{2\sigma_{N_{EWL}}} \]  

and \( f(x) \) is a normal distribution function

Example: \( P = 10^{-9} \) when \( \frac{1}{2\sigma_{N_{EWL}}} \equiv 6.2 \) (or \( K_{MD} = 6.2 \) ).
3.4.2 Applying the Covariance Analysis to the CIR

The CIR uses the code measurements to resolve the EWL integer ambiguity, then uses the EWL measurement to resolve the WL integer ambiguity, and so on. Due to this cascading nature, the measurement vector, \( z \), and the observation matrix, \( H \), in the covariance analysis are updated after a successful rounding of real-valued integer ambiguity estimation to a correct integer for each beat frequency. For example, once a correct EWL integer is rounded with a desired integrity, as specified in Equation (3-19), probability of incorrect rounding of the estimated WL integer ambiguity solution is calculated after the following update.

\[
\begin{bmatrix}
  z \\
  N_{EWL}
\end{bmatrix} = \begin{bmatrix}
  H \\
  0 & 0 & 0 & 1 & -1
\end{bmatrix} \begin{bmatrix}
  \delta z \\
  \delta N_{EWL}
\end{bmatrix} + \begin{bmatrix}
  \delta z \\
  \delta N_{EWL}
\end{bmatrix}
\]  

(3-21)

where \( z' = H'u + \delta z' \)

\[
P_{z'} = E[\delta z'\delta z']
\]

\[
P_u' = (H'^T(P_{z'}^{-1})H')^{-1}
\]

and \( P_{z'} \) is a subset of \( P_{z'} \).

\[
\sigma_{N_{WL}} = \left( \begin{bmatrix}
  1 & 0 \\
  0 & 1
\end{bmatrix} P_i \begin{bmatrix}
  1 & 1 \\
  0 & 0
\end{bmatrix}^T \right)^{1/2}
\]  

(3-22)

\[
\phi_{WL} = 1 \times \phi_1 - 1 \times \phi_2 + 0 \times \phi_c \Rightarrow \begin{bmatrix}
  1 & 1
\end{bmatrix}
\]

Since \( N_{EWL} \) is rounded to a correct integer, \( \delta N_{EWL} \) is 0. Then probability of the wrong WL integer rounding is the following.

\[
P_{\text{wrong}N_{WL}} = 1 - \int_{-\infty}^{K_{MD}} f(x)dx
\]  

(3-23)
The updated measurement vector, \( z' \), and observation matrix, \( H' \), is updated once again with a correct WL integer, which is rounded with a desired integrity to calculate the probability of wrong Lc integer rounding. Figure 3-12 describes the update process.

```
Calculate \( \sigma_{N_{EWL}} \)

if \( K_{MD} \sigma_{N_{EWL}} < 1/2 \) no use only Lc code pseudorange

yes

Update z and H with \( N_{EWL} \)

Calculate \( \sigma_{N_{WL}} \)

if \( K_{MD} \sigma_{N_{WL}} < 1/2 \) no can use EWL pseudorange

yes

Update z and H with \( N_{WL} \)

Calculate \( \sigma_{N_{Lc}} \)

if \( K_{MD} \sigma_{N_{Lc}} < 1/2 \) no can use WL pseudorange

yes

can use Lc carrier pseudorange
```

Figure 3-12. Applying the Covariance Analysis to the CIR

### 3.4.3 Integrity of the CIR Over Short Baseline Distances

Integrity of the CIR is examined by using a covariance analysis of the measurement error. The double difference measurement error model developed in Chapter 2 is used in the anal-

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ysis. A short baseline distance between the reference and user receivers is assumed, and residual differential ionospheric effect is ignored, as in Section 3.3.2. Probability of wrong integer rounding of each step in the CIR process is calculated. The results are shown in Table 3-6.

<table>
<thead>
<tr>
<th>Rounded Integer Ambiguity</th>
<th>Probability of Wrong Integer Rounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>EWL</td>
<td>less than $10^{-15}$</td>
</tr>
<tr>
<td>WL</td>
<td>$6.43 \times 10^{-5}$</td>
</tr>
<tr>
<td>ML</td>
<td>0</td>
</tr>
<tr>
<td>Lc</td>
<td>$6.38 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

Table 3-6. Integrity of the CIR Steps

The analysis results show that resolution of the EWL integer in the CIR is highly successful, with probability of wrong integer rounding less than $10^{-15}$. Resolving the WL integer has probability of wrong integer rounding equal to $6.43 \times 10^{-5}$. While the ML integer resolution has probability of wrong integer rounding of 0. This means that once the EWL and WL integer ambiguities are resolved, the ML integer is already known as it is a linear combination of the previous two terms. Therefore, the linearly dependent beat frequency measurement, the ML measurement in this case, does not affect integrity of the CIR process, as resolution of this term is trivial and unnecessary. The Lc integer ambiguity is resolved by using the WL measurement, and probability of wrong integer rounding is $6.38 \times 10^{-9}$.

Integrity of the entire CIR process with $n$ steps is then defined as follows.
\[
\prod_{i=1}^{n} P_{\text{correct Integer}}(\text{step}(i))
\]

(3-24)

For example, integrity of the entire CIR process over a short distance is calculated in Equation (3-25).

\[
P_{\text{correct CIR}} = P_{\text{correct } N_{\text{EWL}}} \times P_{\text{correct } N_{\text{WL}}} \times P_{\text{correct } N_{\text{ML}}} \times P_{\text{correct } N_{\text{Le}}}
\]

(3-25)

\[
P_{\text{correct CIR}} = (1 - 10^{-15}) \times (1 - 6.43 \times 10^{-5}) \times (1) \times (1 - 6.38 \times 10^{-9}) = 0.99994
\]

Integrity of the CIR at a short baseline distance can be interpreted in several ways by using the results from the covariance analysis. For an accuracy requirement on the order of tens of centimeters and integrity requirement on the order of \(10^{-8}\), the CIR can be used to resolve the EWL integer ambiguity, since probability of wrong integer rounding is less than \(10^{-15}\). However, the CIR cannot be used to resolve the WL integer ambiguity for this application, since the probability of wrong integer rounding is \(10^{-5}\). Since the WL measurement cannot be used in this application, resolving the Lc integer ambiguity is not possible due to the aforementioned cascading nature of the CIR. For an accuracy requirement on the order of centimeters and integrity requirement on the order of \(10^{-4}\), the CIR can be used to resolve the EWL, WL, and Lc integer ambiguity, since integrity of the entire CIR process is 99.994% (per Equation (3-25)).

3.4.4 Integrity of the CIR Over Long Baseline Distance

As baseline distance between the reference and user receivers increases, residual differential ionospheric error grows. Probability of wrong integer rounding at each step of the CIR
Figure 3-13. Integrity of the CIR Steps Over Distance

Figure 3-14. Level of Accuracy of the CIR Over Distance

is calculated over distance, from 0 to 20 km. The results are plotted in Figure 3-13. Note that the probability of wrong EWL integer rounding is not shown on the plot as this value is lower than $10^{-9}$ even at the 20 km baseline. As expected, the probability of wrong inte-
ger rounding, or integrity risk of each step, increases over distance. As mentioned, this occurs because the standard deviation of measurement error increases over distance due to the increase in residual differential ionospheric error. Integrity of the CIR with a short baseline distance, is represented at 0 km in Figure 3-13.

The probability results in Figure 3-13 can be used to interpret integrity of the CIR over distance. For an application requiring integrity risk on the order of $10^{-8}$, the EWL integer ambiguity is resolved since the probability of incorrect EWL integer resolution is less than $10^{-9}$ for all the tested distances. The WL integer ambiguity cannot be resolved because the probability of incorrect WL integer ambiguity is higher than $10^{-8}$ for all the tested distances (o in the figure). Due to the aforementioned cascading nature, the Lc integer ambiguity cannot be resolved without resolving the WL integer ambiguity.

For an application requiring integrity risk on the order of $10^{-4}$, the EWL integer ambiguity is resolved since the probability of incorrect EWL integer ambiguity resolution is less than $10^{-9}$ for all the tested distances. The WL integer ambiguity is resolved up to 22 km, where probability of incorrect WL integer ambiguity resolution becomes higher than $10^{-4}$. The Lc integer ambiguity is resolved up to 2.4 km, where probability of incorrect Lc integer ambiguity resolution becomes higher than $10^{-4}$. Figure 3-14 summarizes the above results.

3.4.5 Effect of Multipath and Receiver Error Reduction on Integrity of the CIR

As discussed in Section 2.2.4.3, the white noise portion of the double difference measurement error is reduced by using time averaging. Figure 3-15 shows the effect of time averaging on the double difference of the L1 carrier phase measurement. The upper figures
Figure 3-15. Effect of Time Averaging on Carrier Phase Measurement

represent double difference carrier phase measurements without time averaging, in time and frequency domain (power spectral density), from left to right, respectively. The lower figures represents the same measurements with the time averaging applied. A time constant of 200 seconds is used. The time averaged double difference carrier phase measurements show decrease in measurement error in the time domain, and decrease in power of high frequency components. These results indicate that the white noise portion of measurement error, such as receiver error due to thermal noise and fast varying component of multipath,
is reduced. However, they also indicate biased noise, such as the slow varying component of multipath is not reduced.

For a short baseline distance, carrier phase multipath is the limiting error source for performance of CDGPS. Multipath mitigation is an ongoing research topic. For example, research by Axelrad [5] uses the signal to noise ratio to eliminate the slow varying component of multipath. Work by Ray [56] uses multiple antennas to reduce multipath effect for a static receiver; he reports 73% overall reduction.

If multipath and receiver error could be bounded by a normal distribution with standard deviation of 1% of a wavelength by using multipath mitigation techniques, along with time averaging, considerable improvement on integrity of the CIR can be achieved. Figure 3-16 shows probability of wrong integer rounding at each step in the CIR process over distance when the 1% value is used. Note that the probability of wrong EWL integer rounding is not shown on the plot as this value is lower than $10^{-9}$ even at the 20 km baseline.

The probability results in Figure 3-16 can be used to interpret integrity of the CIR over distance. For an application requiring integrity risk on the order of $10^{-8}$, the EWL integer ambiguity is resolved since the probability of incorrect EWL integer resolution is less than $10^{-9}$ for all the tested distances. The WL integer ambiguity is resolved up to 6.9 km, where probability of incorrect WL integer ambiguity resolution becomes higher than $10^{-8}$. The Lc integer ambiguity is resolved up to 2.2 km, where probability of incorrect Lc integer ambiguity resolution becomes higher than $10^{-8}$.
For an application requiring integrity risk on the order of $10^{-6}$, both the EWL and WL integer ambiguities are resolved since the probability of incorrect EWL and WL integer ambiguity resolution is less than $10^{-6}$ for all the tested distances. The Lc integer ambiguity is resolved up to 2.6 km, where probability of incorrect Lc integer ambiguity resolution becomes higher than $10^{-6}$. Figure 3-17 shows the level of accuracy achieved by the CIR over distance for varying integrity requirements.

![Figure 3-16. Integrity of the CIR Steps Over Distance. Carrier Phase Multipath and Receiver Error is 1% of wavelength.](image-url)
Figure 3-17. Level of Accuracy of the CIR Over Distance. Carrier Phase Multipath and Receiver Error is 1% of Wavelength.
4

Optimizing the Location of the Third Frequency

4.1 Selecting a Different Third frequency

Although the third civil frequency is now selected at 1176.45 MHz [75], performance of the CIR would be affected if a different third frequency were chosen. The third frequency will generate a set of beat frequencies, each with its own wavelength and measurement error, when combined with the L1 and L2 frequencies. Therefore, the effect of a different spectral location of the third frequency on CIR performance is examined by applying the conditional probability analysis developed in Chapter 3.

The theoretical maximum civil GPS signal power level from the current GPS satellite is -153 dBW (extremely weak), which is the signal power when the satellite is at zenith (Table 4-1) [62]. Therefore, a frequency band in which the GPS signal broadcasts should be protected. The L1 broadcasts in one of the Aeronautical Radionavigation Services (ARNS) frequency bands, which ranges from 1559 MHz to 1610 MHz. It is assigned for an exclusive use of radionavigation satellites, such as the GPS. The L2 does not broadcast

<table>
<thead>
<tr>
<th>Link</th>
<th>P (5 Degree Elev.)</th>
<th>C/A (5 Degree Elev.)</th>
<th>P (Zenith)</th>
<th>C/A (Zenith)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>-163 dBW</td>
<td>-160 dBW</td>
<td>-155 dBW</td>
<td>-153 dBW</td>
</tr>
<tr>
<td>L2</td>
<td>-166 dBW</td>
<td>-166 dBW</td>
<td>-158 dBW</td>
<td>-158 dBW</td>
</tr>
</tbody>
</table>

Table 4-1. Received GPS Signal Power Level at 5 Degree Elevation Angle and at Zenith
in an ARNS frequency band. It broadcasts in a frequency band from 1215 MHz to 1240 MHz, a region shared by Radionavigation Satellites and Radio Location Devices, such as aviation and weather radar. Therefore, the L2 signal is not as well protected as the L1 signal, and is considered a non-safety-of-life signal, according to the U.S. Federal Radionavigation Plan [75]. Like the L1, the selected third frequency, Lc, is located in one of the

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Systems using the band</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>108 - 117.975 MHz</td>
<td>Localizer, VOR, SCAT-1</td>
<td>Worldwide ARNS exclusive</td>
</tr>
<tr>
<td>328.6 - 335.4 MHz</td>
<td>Glide slope</td>
<td>Worldwide ARNS exclusive</td>
</tr>
<tr>
<td>960 - 1215 MHz</td>
<td>Tacan, DME, ATCRB, Mode S, JTIDS (U.S. only)</td>
<td>Worldwide ARNS except in U.S.</td>
</tr>
<tr>
<td>1300 - 1350 MHz</td>
<td>Radio location</td>
<td>Shared with radio location devices</td>
</tr>
<tr>
<td>1559 - 1610 MHz</td>
<td>GPS</td>
<td>Exclusive for GPS</td>
</tr>
<tr>
<td>2700 - 2900 MHz</td>
<td>Radio location</td>
<td>Shared with radio location devices</td>
</tr>
<tr>
<td>4200 - 4400 MHz</td>
<td>Radio altimeter</td>
<td>Worldwide ARNS exclusive</td>
</tr>
<tr>
<td>5000 - 5150 MHz</td>
<td>MLS</td>
<td>Shared with MSS feeder link (5091 - 5150 MHz)</td>
</tr>
</tbody>
</table>

Table 4-2. ARNS Bands from 100 MHz to 5150 MHz

ARNS frequency bands, ranging from 960 MHz to 1215 MHz. Even though use of this band is not exclusive to the GPS, interference from other systems in the band is considered minimal as it is under the control of the aviation community. For this reason, the Lc is considered to meet the needs of safety-of-life applications [75]. For the analysis on the different third frequency effect of a on the CIR, only candidate third frequencies in the ARNS bands are considered. Figure 4-1 shows the wavelength of beat frequencies when the L1 and L2 frequencies are combined with a varying third frequency, from 900 MHz to 2500 MHz. It includes three ARNS bands, 960-1215 MHz, 1300-1350 MHz and 1559-1610 MHz, as described in Table 4-2. The figure is divided into four frequency regions, A1, A2,
B1 and B2. In the A1 region, a wavelength of the L2-Lc beat frequency is longer than that of the L1-L2 combination (.86 cm), and a wavelength of the L1-Lc is shorter than the L1-L2 wavelength. In the B1 region, both the L2-Lc and L1-Lc wavelengths are longer than the L1-L2 wavelength. The A2 and B2 regions are mirror images of the A1 and B1 regions, except that the relative magnitude between the L2-Lc and L1-Lc wavelength is reversed. Wavelengths of both the L2-Lc and L1-Lc in frequency ranges below the lower limit of the A1 region or above the upper limit of the A2 region are shorter than that of the L1-L2. These outer regions are excluded from the analysis.
4.2 Performance of the CIR with a Varying Third Frequency

4.2.1 Performance of the CIR with a Short Baseline Distance

The conditional probability analysis used an integrity parameter $\lambda/\sigma$ from each integer ambiguity resolution step in the CIR process. This term is plotted over the ARNS band in the A1 region in Figure 4-2, and the B1 region in Figure 4-3. A short baseline distance

![Graph showing the $\lambda/\sigma$ ratio for each integer ambiguity resolution step in the CIR process in Region A1.](image)

**Figure 4-2.** The $\lambda/\sigma$ Ratio for each Integer Ambiguity Resolution Step of the CIR Process in Region A1

between the reference receiver and user receivers is assumed for the analysis, and residual differential ionospheric error is ignored. Standard deviation of the double difference carrier phase multipath and receiver error is 2% of its wavelength, as discussed in Chapter 2. In the A1 region, as the candidate third frequency increases toward the L2, a wavelength of the L2-Lc beat frequency becomes longer whereas a standard deviation of the Lc code measurement error remains the same. Therefore, the integrity parameter, $\lambda/\sigma$, for resolving
The $\lambda/\sigma$ Ratio for each Integer Ambiguity Resolution Step of the CIR Process in Region B1

the L2-Lc beat frequency integer ambiguity by using the Lc code measurement increases as the candidate third frequency becomes higher. However, standard deviation of multipath and receiver error in the L2-Lc beat frequency measurement grows as its wavelength increases over the region. Therefore, the integrity parameter for resolving the L1-L2 beat frequency integer ambiguity by using the L2-Lc beat frequency measurement decreases as the candidate third frequency becomes higher. Behavior of the integrity parameter curves for resolving the L1-Lc integer ambiguity using the L1-L2 measurement, and the Lc integer ambiguity using the L1-Lc measurement display similar trends, respectively.

The maximum level of integrity over the range of different third frequencies is found by looking at the lowest intersection of the four integrity parameter curves. Although one can resolve integer ambiguities of all steps in the CIR down to the Lc integer, the maximum
level of integrity is set by the minimum $\lambda/\sigma$ ratio due to the CIR's cascading nature. For example, for the selected third frequency at 1176.45 MHz, the maximum level of integrity is defined by the L1-L2 integer resolving step using the L2-Lc measurement at the $\lambda/\sigma$ ratio of 7.4 (ο mark on the Figure 4-2). Using the same analysis, an optimal third frequency for the CIR with a short baseline distance in the A1 region is located at 1109.7 MHz, where the maximum level of integrity is defined by the L1-L2 integer resolving step using the L2-Lc measurement at the $\lambda/\sigma$ ratio of 17 (χ mark on the Figure 4-2).

In the B1 region, the maximum level of integrity is defined by the Lc integer resolving step using the L1-L2 measurement for most of the frequency range (Figure 4-3). Since the wavelength and standard deviation of measurement error of the L1-L2 beat frequency is fixed over the possible third frequency range, the maximum $\lambda/\sigma$ ratio is constant at 14.9 between 1307 MHz and 1350 MHz. For the frequency range from 1300 MHz to 1307 MHz, the maximum $\lambda/\sigma$ ratio is determined by the L1-Lc integer ambiguity resolution step. The $\lambda/\sigma$ value in this region varies from 13 to 14.9. Therefore, in the B1 region, the optimal third frequency for the CIR with a short baseline distance can be placed anywhere in between 1307 MHz to 1350 MHz.

The optimal spectral location of the third frequency for a short baseline distance with multipath and receiver error of 2% of wavelength is at 1109.7 MHz (A1 region). It yields the highest integrity parameter ($\lambda/\sigma = 17$), which is 2.3 times greater than the integrity parameter at the chosen Lc frequency (1176.45 MHz, $\lambda/\sigma = 7.4$).
4.2.2 Effect of Multipath Reduction

If the multipath and receiver error mitigation technique discussed in Section 3.4.5 is applied to reduce the double difference carrier phase measurement multipath and receiver error from 2% of wavelength to 1%, then the value of the optimal frequency also changes.

![Diagram showing the relationship between wavelength error and frequency for different steps of the CIR, with a peak at 1144.2 MHz.]  

**Figure 4-4.** The $\lambda/\sigma$ Ratio for Integer Ambiguity Resolution Steps of the CIR, Region A1, k=1%

The optimal third frequency for the CIR with a short baseline in the A1 region is now at 1144.2 MHz, where the maximum level of integrity is defined by the L1-L2 integer resolving step using the L2-Lc measurement at the $\lambda/\sigma$ ratio of 24 (x mark on the Figure 4-4). In Figure 4-5, the integrity parameter for the steps in the CIR process in the B1 region is plotted. The optimal third frequency for the CIR with a short baseline in this region is now 1301.6 MHz at the $\lambda/\sigma$ ratio of 27 (x mark on Figure 4-5).
Figure 4-5. The $\lambda/\sigma$ Ratio for Integer Ambiguity Resolution Steps for the CIR, Region B1, $k=1\%$

The optimal spectral location of the third frequency for a short baseline distance with multipath and receiver error of 1% of wavelength is at 1301.6 MHz (B1 region). It yields the highest integrity parameter ($\lambda/\sigma = 27$), which is 2.2 times greater than the integrity parameter at the chosen Lc frequency (1176.45 MHz, $\lambda/\sigma = 12.3$). The optimal third frequency for the CIR with a short baseline distance is summarized in Table 4-3.

<table>
<thead>
<tr>
<th>$\sigma_{mpr} = 0.01\lambda$</th>
<th>$\sigma_{mpr} = 0.02\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region B1, 1301.6 MHz</td>
<td>Region A1, 1109.7 MHz</td>
</tr>
</tbody>
</table>

Table 4-3. Optimal Third Frequency for the CIR with Short Baseline Distance

4.2.3 Performance of the CIR over Long Baseline Distance

As baseline distance between the reference and user receivers increases, residual differential ionospheric error grows. Effect of a different third frequency on the CIR with long base-
line distance is examined by calculating the double difference measurement error at each increasing baseline distance. Five different baseline distances, 0 km, 3 km, 5 km, 7 km and 9 km, are selected for this analysis. Since ionospheric delay is an inverse function of frequency squared, the resulting error at each candidate third frequency is also calculated at each selected baseline distance. Linear spatial gradient of 2 ppm is assumed for the standard deviation of residual differential ionospheric error. Also, standard deviation of the double difference carrier phase multipath and receiver error is 2% of its wavelength, as discussed in Chapter 2. The integrity parameter is plotted over the ARNS band in the A1 region in Figure 4-6. The integrity parameters of all steps in the CIR decrease over the range of candidate third frequencies as the baseline distance increases. The highest intersection of the four curves at each baseline distance is used to find the maximum level of integrity of the
Figure 4-7. The Maximum Integrity Parameter Over the Third Frequency, Region A1, Increasing Distance
CIR at a given distance. These are plotted in Figure 4-7. An optimal third frequency for the CIR is found by using the figure, which is bounded by the L1-L2 integer resolving step using the L2-Lc measurement. It does not have a fixed value over distance, however. The optimal third frequency increases as the baseline distance increases. This behavior is expected, since ionospheric error is an inverse function of frequency squared. As a third frequency becomes higher, it is less affected by the ionosphere. However, as the third frequency reaches toward the L2, it also increases the wavelength of the L2-Lc beat frequency. As seen in Figure 4-8, the L2-Lc beat frequency measurement error is dominated by multipath and receiver error as the third frequency increases toward the L2. Therefore, the integrity of the L1-L2 beat frequency integer ambiguity resolution with the L2-Lc measurement decreases as its wavelength becomes longer with the higher third frequency.
Figure 4-8. Multipath and Receiver Error and Residual Differential Ionospheric Error versus Third Frequency

Figure 4-9. The $\lambda/\sigma$ Ratio of Integer Ambiguity Resolution Steps in the CIR, Region B1, Increasing Distance
Figure 4-10. The Maximum Integrity Parameter Over the Third Frequency, Region B1, Increasing Distance

Figures 4-9 and 4-10 describe the effect of baseline distance on CIR with a varying third frequency in the B1 region. In this region, the maximum level of integrity is defined by the Lc integer resolving step using the L1-L2 measurement between 1307 MHz and 1350 MHz at all baseline distances. As mentioned, the maximum $\lambda/\sigma$ ratio is a constant over this frequency range, and decreases over increasing baseline distance as residual differential ionospheric error increases. Therefore, in the B1 region, the optimal third frequency for the CIR with a long baseline distance can be at any frequency between 1307 MHz to 1350 MHz at all tested baseline distances.

The optimal spectral location of the third frequency for a long baseline distance can be at any frequency between 1307 MHz to 1350 MHz (B1 region). It yields the highest integrity parameter ($\lambda/\sigma=8.7$) at 9 km baseline distance, which is 1.2 times greater than the integ-
rity parameter at the chosen Lc frequency (1176.45 MHz, λ/σ = 7.1) at the same baseline distance.
5

Optimization Using Linear Combinations of Carrier Frequencies

5.1 Linear Combinations of Carrier Frequencies: What Can Be Gained?

5.1.1 Use of Linear Combinations of the L1 and L2 Carrier Frequencies

Under the present GPS signal structure, linear combinations of the L1 and L2 carrier frequencies can generate beat frequencies with many different characteristics. In a study by Han and Rizos [25], the linear combinations in Table 5-1 are suggested to be used in carrier phase integer ambiguity resolution techniques. Wavelengths of beat frequencies are generated by using Equation (5-1). $m_1$ and $m_2$ represent the design space for linear combination with the L1 and L2 frequencies. $\alpha_i$ is the ratio value of ionospheric error. $\alpha_\phi$ is the measurement noise in the carrier phase combination with respect to the L1 carrier phase measurement.

$$\lambda = c/(m_1 f_{L1} + m_2 f_{L2}) \text{ (m)} \quad (5-1)$$

There are a number of linear ambiguity resolution techniques that utilize the unity beat frequency ($m_1 = 1$, $m_2 = -1$) between the L1 and L2 frequencies (Widelane). The WL has a wavelength of 86 cm, which is longer than the L1’s 19 cm and the L2’s 24 cm, and the increased wavelength makes resolving its integer ambiguity easier. For example, the same
change in a satellite geometry yields greater observability with the WL measurements than with either the L1 or L2 carrier phase measurements. This translates to quicker resolution of the WL integer ambiguity as compared to either the L1 or L2 resolution.

<table>
<thead>
<tr>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$f$ (MHz)</th>
<th>$\lambda$ (m)</th>
<th>$\alpha_i$</th>
<th>$\alpha_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1575.42</td>
<td>0.1903</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1227.60</td>
<td>0.2442</td>
<td>1.6469</td>
<td>1.0000</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>2455.20</td>
<td>0.1221</td>
<td>1.6469</td>
<td>2.0000</td>
</tr>
<tr>
<td>77</td>
<td>-60</td>
<td>47651.34</td>
<td>0.0063</td>
<td>0.0000</td>
<td>97.6166</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>347.82</td>
<td>0.8619</td>
<td>-1.2833</td>
<td>1.4142</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2803.02</td>
<td>0.1070</td>
<td>1.2833</td>
<td>1.4142</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>879.78</td>
<td>0.3408</td>
<td>2.8054</td>
<td>2.2361</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>695.64</td>
<td>0.4310</td>
<td>-1.2833</td>
<td>2.8284</td>
</tr>
<tr>
<td>-3</td>
<td>4</td>
<td>184.14</td>
<td>1.6281</td>
<td>18.2518</td>
<td>5.0000</td>
</tr>
<tr>
<td>-7</td>
<td>9</td>
<td>20.46</td>
<td>14.6526</td>
<td>350.3500</td>
<td>11.4018</td>
</tr>
</tbody>
</table>

Table 5-1. Characteristics of Certain Dual-Frequency Combinations (L1 and L2)

One of the advantages of using a different linear combination of the L1 and L2 carrier frequencies in the integer ambiguity resolution is that one can generate a beat frequency with even longer wavelength than the unity beat frequency case. For example, using a linear combination of the -7L1+9L2 (Table 5-1, last row) will yield a beat frequency with a wavelength of 14.65 meters. If this beat frequency is used instead of the WL in the dual-frequency CIR, the longer wavelength dramatically lowers the probability of wrong beat frequency integer resolution.

However, the increase in wavelength comes with a certain penalty. Since each carrier frequency of the -7L1+9L2 beat frequency is multiplied by a factor greater than one, its measurement error is greater than the unity beat frequency measurement error. According to Table 5-1, measurement noise ($\alpha_\phi$, last column) of the -7L1+9L2 linear combination beat
frequency is eight times greater than the unity case in cycles. Due to this increase in measurement error, resolution of the L1 or L2 integer ambiguity through use of the \(-7L1+9L2\) pseudorange is much harder than resolution via the WL pseudorange. Therefore, the characteristics of beat frequencies generated by linear combination of carrier frequencies should be carefully examined before they are used in the CIR.

5.1.2 Effect of Linear Combinations of the L1, L2 and Lc Carrier Frequencies on the CIR

The increase in measurement noise due to the multiplication factors of each carrier frequency in the linear combination beat frequency is acceptable if the frequency’s pseudorange accuracy is suitable for a desired application. When used in the CIR, an increase in wavelength of the linear combination beat frequency will decrease probability of incorrect integer ambiguity resolution. Conversely, if there is a beat frequency in the CIR process which yields an exceptionally low probability of incorrect integer resolution, it can be replaced with a linear combination beat frequency with a shorter wavelength. If the replacement beat frequency has the higher pseudorange accuracy, the original beat frequency’s exceptionally low probability of incorrect integer ambiguity resolution is traded with higher pseudorange accuracy of the latter’s linear combination beat frequency. Therefore, when linear combination beat frequencies are used in the CIR, a trade-off between accuracy of the beat frequency pseudorange and integrity of resolving its integer ambiguity should be possible. Probability of incorrect integer ambiguity resolution of each linear combination beat frequency can also be calculated over distance. It can be used to determine the service distance of the CIR, which depends on the desired level of integrity and pseudorange
accuracy. Thus, trade-off between accuracy of the beat frequency pseudorange and integrity of resolving its integer ambiguity also affects the service distance of the CIR.

With the current GPS signal architecture, the CIR can only use two linearly independent beat frequency measurements. Linear independence between the beat frequency measurements is required in each CIR step since linearly dependent measurement does not add new information (Section 3.4.3). Also, the design space for linear combinations is limited in two dimensions, $m_1$ and $m_2$, which are shown in Equation (5-1). However, with the third civil frequency, the CIR can utilize three linearly independent beat frequencies, and the design space for linear combinations is expanded into three dimensions, $m_1$, $m_2$, and $m_c$, as shown in Equation (5-2).

$$\lambda_{GWL} = c/(m_1 f_1 + m_2 f_2 + m_c f_c) \ (m)$$  \hspace{1cm} (5-2)

Figure 5-1 shows a conceptual trade-off between accuracy and integrity when linear combinations among three carrier frequencies are used to generate three linearly independent
beat frequencies. The beat frequencies for each CIR step are designated as Generalized Widelane (GWL) 1, 2, and 3, since their wavelengths are longer than a wavelength of the L1, L2, and Lc carrier frequency. In the figure, the sum of each step represents accuracy of the final CIR step: The higher the sum of the steps, the higher the accuracy of the final CIR step. Height of each step represents the difference between wavelengths of the top and the bottom beat frequency. It also represents the difficulty in resolving the integer ambiguity of the top beat frequency by using measurements in the lower steps. Therefore, the higher the step, the higher the probability of incorrect integer resolution of the top beat frequency measurement. The left side of the figure describes the CIR process with unity beat frequencies, the EWL, WL and ML, where accuracy of the end product is high, at the Lc carrier measurement error level. As discussed in Chapter 3, the Lc integer can be resolved with probability of incorrect integer resolution, or integrity risk, of $10^{-4}$ up to 2.4 km from the reference station. The right side of the figure describes a theoretical trade-off between accuracy and integrity in the CIR with three Generalized Widelanes. The accuracy of each CIR step in this case is lower than the unity beat frequency case. However, getting to the next step, or resolving the integer ambiguity is easier since each beat frequency has longer wavelength, or lower step height, than the unity beat frequency case. Therefore, Figure 5-1 represents a possible trade between the final pseudorange accuracy and level of integrity for each CIR step by using linear combinations of three carrier frequencies.
5.2 The Generalized Widelane Design Space and Selection Criteria

5.2.1 Design Space for the Generalized Widelane

Figure 5-1 describes only parts of theoretical trading possibilities among accuracy, integrity and service distance when linear combinations among three carrier frequencies are used to create Generalized Widelanes. In order to correctly assess the effect of using the GWL in the CIR, increase in measurement noise due to algebraic multiplication of each carrier phase measurement and scaling factor for ionospheric error must be examined in addition to wavelength. Only then, can an optimal set of GWL be selected for the CIR with a desired accuracy and integrity level.

A measurement equation for the GWL, which combines three available carrier measurements, is

\[
\lambda_{GWL} \phi_{GWL} = R + \lambda_{GWL} N_{GWL} - \lambda_{GWL} \left( \frac{m_1}{\lambda_1} + \frac{Y_{12}m_2}{\lambda_2} + \frac{Y_{1c}m_c}{\lambda_c} \right) I_1 + T + \nu_{GWL} \quad (m) (5-3)
\]

\[
\lambda_{GWL} = c/(m_1f_1 + m_2f_2 + m_c f_c) \quad (m)
\]

\[
\phi_{GWL} = m_1\phi_1 + m_2\phi_2 + m_c\phi_c
\]

\[
N_{GWL} = m_1N_1 + m_2N_2 + m_cN_c
\]

\[
\nu_{GWL} = \lambda_{GWL} \left( \frac{m_1}{\lambda_1} + \frac{Y_{12}m_2}{\lambda_2} + \frac{Y_{1c}m_c}{\lambda_c} \right)
\]

Two design constants for the GWL selection, CI and CM, are derived from the GWL measurement equation, Equation (5-3).
\[ CI = \lambda_{GWL} \left( \frac{m_1}{\lambda_1} + \frac{Y_{12}m_2}{\lambda_2} + \frac{Y_{1c}m_c}{\lambda_c} \right) = \frac{I_{GWL}}{I_1} \]  

(5-4)

Where \( I_{GWL} \) is ionospheric error at the GWL frequency.

\[ CM = \lambda_{GWL} \sqrt{m_1^2 + m_2^2 + m_c^2} \]  

(5-5)

CI is an ionospheric error scale factor for the GWL measurement, which is the ratio value with respect to the L1 carrier phase ionospheric error. CM is a noise multiplier for the GWL measurement, which represents an increase in the standard deviation of measurement error due to algebraic multiplication of each carrier phase measurement. Ideally, one could use these two design constants and three design parameters, \( m_1, m_2 \) and \( m_c \), which define the these constants, to find candidate beat frequencies for the CIR, given desired accuracy and level of integrity. Unfortunately, this is a non-linear problem in the sense that selection

Figure 5-2. Available Beat Frequency Combinations among \( m_1, m_2 \) and \( m_c \).
of one beat frequency affects selection of the others, and it cannot be solved in closed form. Instead, a global search algorithm is used to find optimal sets of beat frequencies for different accuracy and integrity requirements.

Figure 5-2 plots possible algebraic combinations among the design parameters, $m_1$, $m_2$ and $m_c$, which yield beat frequencies with wavelengths less than 30 meters. The maximum wavelength is equal to a wavelength of the Lc code (30 m). Range of each design parameter is limited to $\pm 15$, since larger range yields CM and CI that are too large to be useful in the CIR. With the given constraints, there are 3318 candidate beat frequencies.

### 5.2.2 The Global Search Algorithm

The following criteria are used in a global search for an optimal set of beat frequencies for the CIR with a desired level of integrity risk among the candidates (Figure 5-2).

1. Probability of incorrect integer resolution of selected beat frequency integer ambiguity must be less then a desired integrity risk ($K_{MD} \sigma_{N_{OL}} < 1/2$).
2. Selected beat frequency measurements must be linearly independent.
3. To preserve the integer nature of the integer ambiguity of three carrier measurements, the volume of a matrix of algebraic multipliers, $\bar{m}$, as shown in Equation (5-6), should be conserved (Equation (5-7)).

$$\text{Algebraic multiplier matrix } = \bar{m} = \begin{bmatrix} m_{11} & m_{12} & m_{1c} \\ m_{21} & m_{22} & m_{2c} \\ m_{c1} & m_{c2} & m_{cc} \end{bmatrix} \quad (5-6)$$

$$|\text{det}(\bar{m})| = 1 \quad (5-7)$$
\[
\lambda_{GWL1} = \frac{c}{(m_{11}f_1 + m_{12}f_2 + m_{1c}f_c)} \\
\lambda_{GWL2} = \frac{c}{(m_{21}f_1 + m_{22}f_2 + m_{2c}f_c)} \\
\lambda_{GWLC} = \frac{c}{(m_{c1}f_1 + m_{c2}f_2 + m_{cc}f_c)}
\]

When the global search algorithm, shown in Figure 5-3, is implemented, the first criterion is checked by calculating probability of incorrect integer resolution for the selected beat.
frequency. It is done so by using the covariance analysis, which is better suited to calculate probability of incorrect integer ambiguity resolution of each CIR step. The conditional probability analysis requires conversion between the $\lambda / \sigma$ ratio and probability of incorrect integer ambiguity resolution. The covariance matrix of states, $P_u$, is calculated with a short baseline distance measurement error model. Effect of residual differential ionospheric error is examined later by applying the selected set of GWL in a performance analysis of the CIR with increasing baseline distance.

The second criterion is checked by calculating the rank of matrix of algebraic multipliers of each selected beat frequency. The rank must be full if the beat frequency measurements are linearly independent. Linearly dependent measurement does not add new information regarding the integer ambiguity, thus it can not be regarded as a CIR step.

The third criterion is met when the absolute value of the determinant of the algebraic multiplier matrix, $\bar{m}$, is 1. If it is a volume conserving matrix, the integer nature of the integer ambiguity of three carrier phase measurements, used to generate the GWL, is preserved.

**5.3 Optimization Results**

**5.3.1 Performance Trade-off Between Accuracy and Integrity**

Using the global search, a set of Generalized Widelanes (GWLs) yielding the highest level of integrity at a short baseline distance is found. As mentioned above, the probability of incorrect GWL integer ambiguity resolution is calculated in the global search by using the covariance analysis of states assuming a short baseline distance. Finding a set of GWLs with the highest level of integrity is done by repeating the global search while increasing
the desired level of integrity until the search returns a null set of GWLs. Once candidate sets of GWLs are acquired, the covariance analysis developed in Section 3.4.4 is used to find performance of the CIR over increasing distance for each GWL set. The result of the performance analysis is used to determine the service distance for each GWL set with varying levels of integrity requirements.

Using this method, the following set of GWLs (Table 5-2) is selected to demonstrate the possible trade-off between accuracy and integrity by replacing the unity beat frequency, the EWL, WL and Lc, used earlier in the CIR. Probability of incorrect resolution of each GWL integer ambiguity over increasing baseline distance is calculated, and the results are shown in Figure 5-4.

<table>
<thead>
<tr>
<th></th>
<th>m1</th>
<th>m2</th>
<th>mc</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GWL1</td>
<td>0</td>
<td>3</td>
<td>-3</td>
<td>1.9m</td>
</tr>
<tr>
<td>GWL2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0.75m</td>
</tr>
<tr>
<td>GWL3</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0.15m</td>
</tr>
</tbody>
</table>

**Table 5-2. Selected GWL for Trade-off Demonstration**

- **GWL1 Selection**

In Section 3.4.4, it was shown that the first unity beat frequency CIR step, resolving the EWL integer ambiguity, yielded exceptionally low integrity risk (far less then \( 10^{-9} \)). Therefore, the GWL1, which has the longest wavelength among the GWLs, is selected such that it would have a shorter wavelength than the EWL, thus trading its exceptional level of integrity for better accuracy. As shown in the figure, due to GWL1’s shorter wavelength, the probability of wrong integer resolution is increased from less then \( 10^{-9} \) to \( 10^{-6} \).

- **GWL2 Selection**
Figure 5-4. Trade-off Result: Probability of Wrong Integer Estimation versus Baseline Distance

Accuracy of the WL measurement could not be traded for increased integrity of the GWL2 integer resolution since there was no available GWL that was linearly independent from GWL1, had a longer wavelength than WL, and had equal or lower CM than the WL measurement. However, there is a GWL which has about the same probability of incorrect integer resolution as the WL, but a slightly shorter wavelength. Therefore, the GWL2 is selected to increase pseudorange accuracy in exchange for a small loss in integrity level.

- GWL3 Selection

It was shown in Section 3.4.4 that, due to its short wavelength, the probability of incorrect resolution of the Lc integer ambiguity is affected primarily by residual differential iono-
spheric error. Therefore, the GWL3, which has the shortest wavelength among the GWLs, is selected such that it would have a lower ionospheric error scale factor, CI, then the Lc carrier phase measurement. GWL3 is chosen with the lower CI in order to perform better with growing residual differential ionospheric error over increasing baseline distance.

### 5.3.2 Analysis of Trade-off Results

The result of the trade-off between accuracy and integrity performed in Section 5.3.1 is compared with performance of the unity beat frequency case, where the EWL, WL and Lc defined in Section 2.1.6 is used in the CIR. Figure 5-5 shows the performance of the CIR.

![Figure 5-5. Level of Accuracy of the CIR vs. Baseline Distance with GWL](image)

1E-4 Integrity
- Trade-off Analysis
- Unity combination
with probability of incorrect integer resolution equal to $10^{-4}$ over different baseline distances. The solid line represents performance of the CIR with the selected set of GWLs from the trade-off analysis, which the dashed line represents the CIR with the unity beat frequencies.

<table>
<thead>
<tr>
<th></th>
<th>EWL</th>
<th>GWL1</th>
<th>WL</th>
<th>GWL2</th>
<th>Lc</th>
<th>GWL3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>5.9 m</td>
<td>1.9 m</td>
<td>0.86 m</td>
<td>0.75 m</td>
<td>0.25 m</td>
<td>0.15 m</td>
</tr>
<tr>
<td>CM</td>
<td>8.29</td>
<td>8.29</td>
<td>1.22</td>
<td>1.06</td>
<td>0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>CI</td>
<td>1.72</td>
<td>1.72</td>
<td>1.28</td>
<td>1.34</td>
<td>1.79</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 5-3. Wavelength, CM and CI of GWLs and Unity Beat Frequencies

- **The Lc vs. the GWL3**

As expected, the probability of incorrect integer resolution of the selected GWL3 increases more slowly than the Lc over increasing baseline distance, due to its lower CI. However, the GWL3 measurement has a higher noise multiplier, CM, than the Lc measurement. Therefore, at the zero baseline length, the GWL3 has higher probability of incorrect integer resolution than the Lc. The service distance of the CIR is 2.4 km for both the GWL3 and Lc (carrier phase) pseudorange measurements with integrity risk of $10^{-4}$.

- **The WL vs. GWL2**

The GWL2 measurement has a smaller CM than the WL measurement. Therefore, it has a lower probability of incorrect integer resolution than the WL at 2.4 km. However, due to its slightly higher CI, the integrity risk of the GWL2 integer resolution is higher than the WL as the baseline distance becomes longer than 12 km. The service distance of the CIR is 22 km for both the GWL2 and WL pseudorange measurements with integrity risk of $10^{-4}$.
• The EWL vs. GWL3

Since the EWL and GWL3 have the same CI and CM, there is no performance difference when one or the other is used in the CIR.

5.3.3 Summary

This analysis shows that the trade-off between accuracy and integrity level using linear combinations of carrier frequencies did not work as expected. Further examination of the available GWL, CI and CM revealed that the trade-off did not work well due to lack of variety in wavelength of GWL with low CI and CM. Figures 5-6, 5-7, and 5-8 show 2-dimensional projections of the three parameter space, which consists of the wavelength of GWL, CM and CI. In the figures, the EWL, WL and Lc are identified with ‘x’ marks, and candidate GWLs are marked with ‘o.’ From the figures, it seems that there are many candidate

![Figure 5-6. GWL Wavelength vs. CM](image-url)
Figure 5-7. GWL Wavelength vs. CI

Figure 5-8. CM vs. CI

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GWLs with lower CM and CI than the unity beat frequencies. However, in order to work, the CIR requires a set of GWLs with not only low CM and CI but also with well separated wavelengths. The conditional probability analysis developed in Section 3.3.1 can be applied here for easier understanding. First, a GWL with a relatively long wavelength is required in order for the CIR to resolve its integer ambiguity by using the available code measurements. However, its CM cannot be too high, since pseudorange from the first GWL measurement will be used to resolve the integer ambiguity of the second GWL. As seen in Figures 5-6 and 5-7, there are only a handful of “long” wavelength GWLs, most of them with high CI and CM. Once the first GWL is selected, the second GWL can be chosen among the candidates. Its wavelength cannot be much shorter than the first GWL in order to resolve its integer ambiguity with a certain level of integrity, and its CM cannot be too high if the pseudorange from the second GWL measurement is to be used to resolve integer ambiguity of the third GWL. Also, the second GWL must be linearly independent from the first GWL. The third GWL is selected by using the same process.

The global search is designed to select GWLs to fit these requirements. Unfortunately, the selected set of GWLs is not much different than the unity beat frequencies, in terms of their wavelength, CM and CI. Therefore, a trade-off among accuracy, integrity level and service distance by using linear combinations of carrier frequencies did not have much effect on performance of the CIR.

Although the optimization effort using the GWLs did not work as expected, performance of the CIR is successfully increased by estimating the spatial gradient of residual ionospheric error. Details of this work are described in Chapter 6. The linear combinations of
carrier frequencies are also useful when one of the three civil GPS frequencies is accidentally or intentionally made unavailable. With the unity combination, only one beat frequency is generated with two carrier frequencies. Analysis of CIR performance with only two civil frequencies is included in Chapter 7, where the effect of RF interference on various GPS receivers is examined.
6 Estimation of Spatial Gradient of Residual Differential Ionospheric Error in the CIR

6.1 Correction of Measurement Error due to Ionosphere

6.1.1 Ionospheric Range Error Correction for Single Frequency Users

The pseudorange measurement error due to ionospheric effect varies from a few meters to tens of meters at the zenith, if uncorrected [41]. For a single frequency, stand alone user, a simple algorithm developed by Klobuchar [42] is used to correct for approximately 50% of the ionospheric range error. The algorithm corrects for the error by using the user’s approximate geodetic latitude, longitude, elevation angle and azimuth to each GPS satellite, along with eight ionosphere coefficients included in the navigation message modulated on the C/A code [61].

By using DGPS, the range error due to ionosphere is reduced further. If the reference and user receivers are receiving a GPS satellite signal through the same ionospheric conditions, DGPS correction eliminates the range error due to ionosphere. However, the condition of the ionosphere decorrelates as the distance between the reference and user receivers increases. The spatial gradient of standard deviation of residual differential ionospheric error is assumed at 2 ppm in the analysis of the CIR, which uses double difference mea-
measurements. Figure 3-9 shows the increase in residual differential ionospheric error in the double difference measurement over growing baseline distance due to the spatial gradient.

### 6.1.2 Ionospheric Range Error Correction for Multiple Frequency Users

For a user with a multiple frequency receiver, ionospheric range error is directly observed as it is a function of the carrier frequency of the GPS signal. For example, if the code and carrier phase measurements from the L1 and L2 frequencies are available, ionospheric range error is calculated by using the following equations.

\[
\rho_{L1} - \rho_{L2} = (1 - \gamma_{12}) I_{L1} + \mu \quad (\text{m})
\]  
\[\lambda_{L1}(\phi_{L1} - N_{L1}) - \lambda_{L2}(\phi_{L2} - N_{L2}) = (\gamma_{12} - 1) I_{L1} + \nu \quad (\text{m})
\]  

- \(\rho\) is the code measurement
- \(\gamma_{12}\) is an ionospheric scale factor for the L2 frequency
- \(I_{L1}\) is pseudorange error due to ionosphere in the L1 frequency
- \(\mu\) is code measurement error, including multipath and receiver error
- \(\lambda_{L1}, \lambda_{L2}\) are wavelengths of the L1 and L2 frequencies, respectively
- \(\phi\) is carrier phase measurement
- \(N\) is the integer ambiguity, which is assumed to be known
- \(\nu\) is carrier phase measurement error, including multipath and receiver error

Also, a linear combination of measurements from multiple frequencies can be formed in such a way as to eliminate the effect of ionosphere. The ionospheric-free linear combination of the L1 and L2 carrier phase measurement, shown in Equation (6-3), has a significant disadvantage, however. The integer ambiguity of the \(\phi_{ionoFree}\) measurement is not an
integer, since the integer ambiguity of the L2 carrier phase measurement is multiplied by 
$f_{L2}/f_{L1}$, which is a real number (0.7792), and the result is then subtracted from the L1
integer ambiguity.

$$\phi_{ionoFree} = \phi_{L1} - \frac{f_{L2}}{f_{L1}} \phi_{L2}$$  \hspace{1cm} (6-3)

The non-integer ratio between the L1, L2 and Lc frequencies is one of the reasons why opti-
mization of the CIR with linear combinations of carrier frequencies in Chapter 5 was not
successful. If the ratio were an integer, ionospheric error-free beat frequencies with various
wavelengths could be formed, which in turn could be used in the CIR to reduce probability
of incorrect integer resolution at long distances.

6.2 Improving Performance of the CIR by Estimating
Spatial Gradient of Residual Differential Ionospheric Error

6.2.1 Estimating the Spatial Gradient of Residual Differential
Ionospheric Error

In Chapter 3, it was shown that the probability of wrong integer estimation in the CIR is
driven by measurement error. For a short baseline distance between the reference and user
receivers, performance of the CIR is determined by double difference multipath and
receiver error. However, as the baseline distance increases, probability of wrong integer
estimation grows larger due to the increase in residual differential ionospheric error.

The spatial gradient of residual differential ionospheric error can be observed and treated
as a state if measurements from two or more separate locations are used in the CIR. The
measurement vector, $z$, the observation matrix, $H$, and the state vector, $u$, in Equation (3-13) are modified to carry out the covariance analysis with the spatial gradient as an additional state, by using measurements from two locations with a separation distance of $\Delta b$, as shown in Equation (6-5) and Figure 6-1.

$$z = Hu + \delta z$$

(6-4)

$$
\begin{bmatrix}
\rho_{1\alpha} \\
\rho_{2\alpha} \\
\rho_{c\alpha} \\
\phi_{1\alpha} \\
\phi_{2\alpha} \\
\phi_{c\alpha} \\
\rho_{1\beta} \\
\rho_{2\beta} \\
\rho_{c\beta} \\
\phi_{1\beta} \\
\phi_{2\beta} \\
\phi_{c\beta}
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 & b & 0 & 0 & 0 \\
1 & 0 & -\frac{b}{\lambda_1} & 0 & 0 & 0 \\
1 & 0 & -\frac{b}{\lambda_c} & 0 & 0 & 0 \\
\frac{1}{\lambda_1} & 0 & -\frac{1}{\lambda_1} & 1 & 0 & 0 \\
\frac{1}{\lambda_2} & 0 & -\frac{1}{\lambda_2} & 0 & 1 & 0 \\
\frac{1}{\lambda_c} & 0 & -\frac{1}{\lambda_c} & 0 & 0 & 1 \\
0 & 1 & (b + \Delta b) & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_1} & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_2} & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_c} & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_1} & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_2} & 0 & 0 & 0 \\
0 & 1 & -\frac{1}{\lambda_c} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \rho_{1\alpha} \\
\delta \rho_{2\alpha} \\
\delta \rho_{c\alpha} \\
\delta \phi_{1\alpha} \\
\delta \phi_{2\alpha} \\
\delta \phi_{c\alpha} \\
\delta \rho_{1\beta} \\
\delta \rho_{2\beta} \\
\delta \rho_{c\beta} \\
\delta \phi_{1\beta} \\
\delta \phi_{2\beta} \\
\delta \phi_{c\beta}
\end{bmatrix}
\begin{bmatrix}
R_\alpha \\
R_\beta \\
\nabla I_1 \\
N_1 \\
N_2 \\
N_c
\end{bmatrix}
$$

(6-5)

- $z$ is a measurement vector, including six double difference measurements from the location $\alpha$, three code and three carrier phase, and also six measurements from the location $\beta$. 

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• H is an observation matrix. Scale factors for ionospheric effect on frequencies other than the L1 are included.

• u is a state vector, containing pseudorange from the location $\alpha$, $R_\alpha$, and from the location $\beta$, $R_\beta$, the spatial gradient of residual differential ionospheric effect at the L1 frequency, $VI_1$, and three integer ambiguities for the L1, L2 and Lc carrier phase measurements, $N_1$, $N_2$, and $N_c$.

• $\delta z$ is a measurement error vector.

### 6.2.2 Result of the Covariance Analysis

With the updated observation matrix and measurement error vector, the same covariance analysis of the state estimation developed in Chapter 3 is used to calculate the probability of incorrect integer resolution of each step in the CIR.

![Figure 6-2. Probability of Wrong Integer Estimation, $\Delta b =$10 km](image)

Figure 6-2. Probability of Wrong Integer Estimation, $\Delta b =$10 km
Figure 6-2 shows improvements in the probability of wrong WL and Lc integer estimation when the spatial gradient of residual differential ionospheric error is estimated by using measurements from two locations separated by 10 km. The probability of wrong EWL inte-

![Graph showing improved performance with estimation of spatial gradient](image)

**Figure 6-3. Improved Performance of the CIR with Estimation of the Spatial Gradient**

ger estimation by using the Lc code measurements in both cases is lower than $10^{-10}$ and hence is not shown on the plot. In Figure 6-3, a comparison of performance of the CIR over distance for a desired level of integrity with (circle in Figure 6-3) and without estimation (triangle in Figure 6-3) of the spatial gradient of residual differential ionospheric error is shown. When the spatial gradient is estimated, the CIR can be used to resolve the Lc integer ambiguity up to 4 km, instead of up to 2.4 km without the estimation, with probability of incorrect integer resolution of $10^{-4}$. It can also resolve the WL integer ambiguity up to 40 km with the estimation, instead of up to 22 km without the estimation, with the same prob-
ability. Clearly, by estimating the spatial gradient, performance of the CIR is improved. Also, as seen in Figure 6-4, the larger the separation distance between the measurements, $\Delta b$, the greater the improvement due to increased observability on the spatial gradient.

**Figure 6-4.** Probability of Wrong Integer Estimation, $\Delta b = 20$ km

However, since the estimated states now include the effect of change in geometry of the satellite as the user moves from one location to the other for multiple measurements, the CIR with spatial gradient of residual differential ionospheric error is no longer a geometry-free process. Although the effect of change in satellite geometry should further improve performance of the CIR, it is not analyzed in this dissertation. Also, the user receiver is assumed to maintain the lock on the GPS carrier as the user moves between the measure-
ment points. If the lock is reset, additional integer ambiguity resolution is required for each carrier phase measurement at the new measurement location.
7

Effect of Radio Frequency Interference on GPS Receivers

7.1 Introduction

GPS satellite signals are extremely weak. A power spectral density of the C/A code signal in the L1 frequency is actually below the ambient noise power spectral density [62]. To acquire the very weak GPS signal, a replica of the C/A code is generated in a GPS receiver and correlated with the received signal. Once the signal is acquired, it is tracked, and the navigational message is decoded. However, the GPS receiver may fail to acquire or track the GPS signal when Radio Frequency Interference (RFI) is present. RFI emanates from man-made devices, such as TV, FM radio, radar, Personal Communication Systems, etc. [49]. One of the main motivations for three civil frequencies is to provide robust GPS service to users in the presence of RFI by incorporating signal redundancy.

The current civil GPS signal at the L1 frequency broadcasts in a portion of the ARNS/GNSS frequency band, from 1559 MHz to 1610 MHz. The International Civil Aviation Organization (ICAO) Special Communication/Operation Division Meeting in 1995 agreed that this frequency band must be protected for the operation of satellite-based radio-navigation systems [36],[37]. However, there are many potential sources of RFI which could compromise integrity of the GPS signal. One worrisome example is Ultra-wideband
(UWB) transmitters. UWB transmitters have the potential to knock out signals from the L1, L2 and Lc simultaneously.

A preliminary study on understanding how RFI from UWB transmitters affects GPS receivers is carried out, and the results are described in the following section. The UWB interference test is carried out in collaboration with Konstantin Gromov and Interval Corporation. The test results are also presented in the paper by Aiello, et al [1].

7.2 Effect of RFI from an Ultra-wideband Transmitter on GPS Receivers

7.2.1 Ultra-wideband Technology and Its Regulatory Status in the U.S.

Unlike conventional radio frequency communication systems, Ultra-wideband (UWB) technology uses a radio signal whose fractional bandwidth, $\eta$, is larger than 25%, as expressed in Equation (7-1), where $f_H$ and $f_L$ indicate the highest and lowest frequencies of interest [67]. For example, the C/A code in the L1 frequency has fractional bandwidth of 0.13%. Therefore, it is not an UWB signal.

$$\eta = \frac{2(f_H - f_L)}{f_H + f_L} \quad \text{(7-1)}$$

UWB devices transmit very short impulses of radio energy. These impulses are very weak in general, but the spectrum signature extends across a wide range of radio frequencies, and may include the frequency band used by GPS signals. The U.S. Federal Communications Commission (FCC) specifies that if a radio signal falls in a restricted frequency band, such as a GNSS band, is radiated unintentionally, and has signal power of less than -65 to -71 dBW/MHz, it is in compliance with the regulation [76]. Since UWB transmission is inten-
tional, it is not allowed to operate in the U.S. However, in 1999, the Office of Engineering and Technology within the FCC granted waivers for UWB technologies for certain applications in limited quantities [80]. Although this waiver in no way prejudices any action the FCC may take regarding UWB devices, the FCC did announce a Notice of Proposed Rule Making (NPRM) on the authorization of UWB technology in 2000, and is seeking comments on its proposal. Therefore, the need exists to understand how the UWB signal affects the GPS in order to protect GPS signal integrity from potential interference.

7.2.2 UWB Interference Test Setup

The Stanford University GPS laboratory and Interval Corporation conducted a series of collaborative trials on the effect of UWB transmitters on GPS receivers. Since UWB communication is an emerging technology with no set standards, the trials are used to develop test methods and identify test variables. The results of the trials are also used to get a first look at the interference effect of UWB signals on GPS receivers.

To carry out the trials, four different types of GPS receivers are used: a survey grade dual frequency capable receiver, a narrow correlator receiver, an inexpensive hand-held receiver and an experimental receiver.

A calibrated horn antenna collocated with the GPS antennae and a spectrum analyzer are used to measure the level of emission from the UWB transmitter in a portion of the GNSS frequency band, from 1565.42 to 1585.42 MHz. The transmitting antenna is connected with a pattern generator, a pulse generator and a programmable attenuator to test different UWB signal configurations. The transmitting antenna and GPS antennae are 1 meter above ground, and 3 meters apart from each other. Figure 7-1 describes the experiment setup.
**Figure 7-1. UWB Interference Test Setup**

### 7.2.3 Test Assumptions and UWB Signal Generation

The test proceeded with the following assumptions. First, a single UWB transmitter is assumed. Second, a level of UWB interference is determined by measuring the distance between the transmitter and GPS antennae when loss of a GPS satellite in the receivers occurs. Although the distances between the UWB antenna and GPS antennae are fixed at 3 meters, the experimental UWB transmitter power was adjusted to simulate a changing distance between the antennae from 1 to 100 meters. Third, the reference emission power level from the UWB antenna is 500 microvolts per meter at 3 meters from the GPS antennae, which is the current FCC limit on unintentional radiation into the GNSS frequency band.
[76]. Fourth, the UWB signal is assumed to be attenuated by the square of the distance from the antenna.

A waveform used to generate the UWB spectrum for testing is shown in Figure 7-2. The left figure shows a single pulse in time, and the right figure shows a frequency spectrum of the UWB signal broadcast from 1 to 4 GHz using the pulse.

With the specified pulse, different UWB signals are generated by varying the following parameters: Pulse Repetition Frequency (PRF), transmitter duty cycle, and modulation pattern.
- **Pulse Repetition Frequency (PRF)**

The PRF is the rate of transmission of the pulses. It determines the period between the pulses in the UWB signal. For example, the PRF of 20 MHz means the pulses are repeated every 50 nanosecond.

- **Transmitter Duty Cycle**

The transmitter duty cycle is the percentage of the transmitter period during which it is on. The transmitter was turned on for 10 μs bursts. So, if the transmitter duty cycle is 50%, the transmitter is on for 10 μs and off for 10 μs. During the off period, no energy was emitted from the antenna. The upper right figure in Figure 7-3 shows how the pulse burst determines transmitter duty cycle.

![PRF = 20 MHz with No Modulation](image1)

![Duty Cycle (Pulse Bursts) = 60%](image2)

![Frequency Spectrum of Dithering Modulation](image3)

![Frequency Spectrum of Random Modulation](image4)

**Figure 7-3. UWB Signal Parameters**
• **Modulation Pattern**

The modulation pattern determines how each pulse is modified for transmission. The UWB signal can be modulated in an unlimited patterns. For example, dithering, which affects the time between the UWB pulses, can be used as the modulation pattern.

In Figure 7-3, a few sample UWB signals based on the pulse in Figure 7-2 are shown. The upper left figure shows the frequency spectrum of the PRF at 20 MHz without a modulation pattern. The upper right figure shows how the transmitter duty cycle is defined. The lower figures show the effects of two different modulation patterns, dithering and random.

**7.2.4 Test Results**

**7.2.4.1 Effect of Varying UWB Pulse Location Near the L1 Frequency**

A PRF of 20 MHz is selected for the interference test. With the given PRF, periodic waveforms create discrete lines in a frequency spectrum. Since the GPS portion of the GNSS band, from 1565.42 to 1585.42 MHz, is 20 MHz wide, the UWB signal is fine tuned to place only one spectrum line in the band. Furthermore, the single spectrum line is placed near 1575.42 MHz, to maximize sensitivity. This strategy is not optimal as the actual interference effect will depend on whether the UWB spectrum line falls on a GPS spectrum line.

Figure 7-4 shows the difference in sensitivity between a survey grade receiver and an inexpensive hand-held receiver. The y-axis shows the number of satellites each receiver is tracking. The x-axis is the location of the UWB spectral line. As shown in the figure, the survey receiver is affected by the UWB pulse even when it is placed 9 MHz below the L1 frequency. However, the hand-held unit is only affected when the UWB pulse is placed very near the L1 frequency. The difference comes from variations in receiver design. The
survey grade receiver samples a wider region of frequency than the hand-held one. Therefore, it is affected by the UWB pulse in a wider frequency range than the hand-held receiver.

7.2.4.2 Effect of Varying the Transmitter Duty Cycle

Using the 2 nanosecond pulse and the PRF of 20 MHz, the transmitter duty cycle is varied by changing the transmission-off time after the transmitter is on for 10 microseconds. Four different duty cycles, 100, 70, 50, and 40%, are tested by applying four off-periods, 0, 4, 10 and 15 microseconds after each 10 microseconds of transmission period. For each duty cycle, the UWB transmitter was first turned off to let the GPS receivers acquire and track GPS satellites under a normal condition. Then, the UWB transmitter was turned on, and the number of tracked satellites for each receiver was recorded as the UWB signal strength increased.
Figure 7-5. Narrow Correlator Receiver vs. Duty Cycle (PRF = 20 MHz)

Figure 7-6. Hand-held Receiver vs. Duty Cycle (PRF = 20 MHz)
Figures 7-5 and 7-6 show the effect of change in transmitter duty cycle on a narrow correlator and a hand-held receiver, respectively. In both figures, the y-axis represents the number of tracked satellites. The numbers immediately below the x-axis represent the average electric field strength in dB-microvolt per meter, which is measured by the spectrum analyzer and calibrated horn antenna. The second set of numbers below the x-axis represent the extrapolated distance between the UWB signal transmitting antenna and GPS antennae. The reference emission power level from the UWB antenna is 500 microvolts per meter at 3 meters from the GPS antennae, as discussed in the original assumptions.

The figures show that the hand-held receiver tracks more satellites with a stronger electric field strength, or at shorter distance between antennae, as the transmitter duty cycle decreases from 100 to 40%. The result for the narrow correlator receiver is different, as it tracks the most satellites with 100% duty cycle at the same electric field strength.

7.2.4.3 Effect of Different Modulation Pattern

There are an unlimited number of possible modulation patterns for the UWB signal. For the preliminary study, however, the effect of two types of modulation schemes on GPS receivers is examined. The first modulation is a random pattern, offering 50% probability of a pulse being generated at the nominal time. The second modulation is dithering, which affects the time between the pulses. Essentially, it is changing the PRF by $50ns + \Gamma \times 5ns$, where the integer $\Gamma$ varies randomly from -4 to 4 with uniformly distributed probability. The maximum time dithering window is 25 MHz, or 40 ns.
Figure 7-7. Narrow Correlator Receiver vs. Modulation (PRF = 20 MHz)

Figure 7-8. Hand-held Receiver vs. Modulation (PRF = 20 MHz)
Figures 7-7 and 7-8 show the effect of the modulated UWB signal on a narrow correlator and a hand-held receiver, respectively. The x- and y-axes of the figures are the same as in Figures 7-5 and 7-6. A pulse of 2 nanoseconds and PRF of 20 MHz is used to generate the unmodulated UWB signal. Both modulation patterns, random and dithering, are tested and plotted with the unmodulated case.

Due to varying behavior of the receivers in the presence of different modulation patterns, it is difficult to draw any conclusion from this test series. However, in general, the receivers tracked more satellites when the UWB signal was modulated.

7.2.5 Summary and Conclusions

The distances between the UWB signal transmitter and GPS antennae where one satellite is lost from varying PRF (Figure 7-4), varying duty cycle (Figures 7-5 and 7-6), and change in modulation pattern (Figures 7-7 and 7-8) are shown in Table 7-1. The separation distance

<table>
<thead>
<tr>
<th></th>
<th>Narrow Correlator Receiver</th>
<th>Survey Grade Receiver</th>
<th>Inexpensive Hand-held Receiver</th>
<th>Experimental receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Sweep</td>
<td>1-3m</td>
<td>1-3m</td>
<td>5-15m</td>
<td>30-100m</td>
</tr>
<tr>
<td>Duty cycle 100%</td>
<td>3-30m</td>
<td>10-30m</td>
<td>30-100m</td>
<td>3-100m</td>
</tr>
<tr>
<td>Duty cycle 70%</td>
<td>3-10m</td>
<td>10-30m</td>
<td>3-30m</td>
<td>5-15m</td>
</tr>
<tr>
<td>Duty cycle 50%</td>
<td>1-3m</td>
<td>10-30m</td>
<td>3-30m</td>
<td>5-15m</td>
</tr>
<tr>
<td>Duty cycle 40%</td>
<td>1-3m</td>
<td>10-30m</td>
<td>1-3m</td>
<td>3-30m</td>
</tr>
<tr>
<td>Random modulation</td>
<td>3-10m</td>
<td>30-100m</td>
<td>10-100m</td>
<td>10-100m</td>
</tr>
<tr>
<td>Dithering</td>
<td>3-10m</td>
<td>10-30m</td>
<td>10-30m</td>
<td>10-100m</td>
</tr>
</tbody>
</table>

Table 7-1. UWB Interference Range (One Satellite Lost)

is regarded as an important parameter for determining a co-location standard for UWB devices and GPS receivers in the future. Uncertainty in the range measurement in the table
reflects errors in the electric field strength measurement. Factors impacting the uncertainty of this measurement include possible UWB signal multipath, imperfection in the component calibration and change in pulse sequence in later tests.

The test results show that in some configurations, the UWB signal does interfere with the GPS. The factor of greatest concern is distance between UWB transmitters and GPS receivers. In Table 7-1, many of the test results suggest that a GPS receiver will lose lock to a GPS satellite when the two units are 100 meters apart. If proliferation of this UWB technology occurs, it could create environments in which GPS receivers are seriously affected by RFI. Affected applications would include car navigation, survey, construction, and aircraft landing, while examples of UWB presence would include personal communication systems, ground penetrating radar, local area networks, etc. However, determining the exact nature of the interference on GPS due to a UWB signal is difficult to quantify. Since the standard for the UWB signal is not yet defined and these trials only tested portions of various aspects of the UWB technology, the results should be considered as preliminary, and used as a stepping stone for more detailed and controlled studies in the future.

7.3 Cascade Integer Resolution with One Frequency Disabled

7.3.1 Using Linear Combinations of Carrier Frequencies

In the current GPS signal configuration, a civil user cannot use the GPS if the L1 frequency is severely interfered with, either accidently or intentionally. Since the C/A code, on which the navigation message is modulated, only broadcasts in the L1, a dual frequency receiver will also not work if the L1 frequency is not available. However, with three civil signals in
the future GPS configuration, loss of one signal can be tolerated through redundancy. In fact, the CIR can still be applied to resolve integer ambiguity when one of the civil GPS signals is not available by using linear combinations of the remaining two carrier frequencies.

With two civil signals, two linearly independent beat frequencies are formed and used in the CIR. The global search algorithm developed in Chapter 5 is used to find an optimal set of Generalized Widelanes (GWLs) when one of the L1, L2 or Lc signals is not available.

7.3.2 Performance of the CIR with Two Carrier Frequencies

Figures 7-9, 7-10, and 7-11 show probability of wrong integer resolution of each CIR step over distance when the L1, L2 and Lc signals are not available. Table 7-2 shows the set of GWLs used in each case. Standard deviation of the double difference measurement error due to multipath and receiver noise is 2% of carrier wavelength, and the spatial gradient of residual differential ionospheric error is 2 ppm. The probability of wrong integer resolution

![Figure 7-9. Performance of the CIR Without the L1 Signal](image-url)
Figure 7-10. Performance of the CIR Without the L2 Signal

Figure 7-11. Performance of the CIR Without the Lc Signal

of the CIR with a lost signal is higher than when all three GPS signals are available. The best performance (resolving the EWL ambiguity) is available when the L1 signal is lost, since linear combination of the L2 and Lc carrier frequencies yields a GWL with very long wavelength and relatively low CM. When the L2 or Lc signal is lost, linear combinations of the remaining carrier frequencies do not yield a GWL with small CM and long wave-
length. Therefore, probability of wrong integer resolution on the first step (resolution of the
GWL1) in these cases is higher than when the L1 signal is lost.

<table>
<thead>
<tr>
<th></th>
<th>GWL1 Measurement</th>
<th>GWL2 Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lost the L1 signal</td>
<td>$0L_1 + 3L_2 - 3L_c$</td>
<td>$0L_1 - 4L_2 + 5L_c$</td>
</tr>
<tr>
<td>Lost the L2 signal</td>
<td>$1L_1 + 0L_2 - 1L_c$</td>
<td>$0L_1 + 0L_2 + 1L_c$</td>
</tr>
<tr>
<td>Lost the Lc signal</td>
<td>$1L_1 - 1L_2 + 0L_c$</td>
<td>$0L_1 + 1L_2 + 0L_c$</td>
</tr>
</tbody>
</table>

Table 7-2. GWL Combination Used

7.4 Summary

Radio Frequency Interference from various man-made devices has the potential to interfere
with the GPS signal, which is very weak. In the present configuration, civil users cannot
use GPS if the L1 frequency is severely interfered with. The future civil configuration of
three frequencies provides the necessary redundancy in the GPS signal to yield robust GPS
service in the presence of either accidental or intentional interference. Among the many
potential sources of RFI which could compromise integrity of the GPS signal, UWB trans-
mitters stand out due to their capacity to disable all three frequencies with their wideband
broadcasts. A preliminary study shows that certain configurations of UWB signal can cause
a receiver to lose lock on a GPS satellite when placed within 100 meters of the UWB
device. UWB is an emerging technology which is continuously being developed. Because
the potential RFI effect on GPS signals from UWB technology is significant, more in-depth
studies should be carried out in preparation for development of UWB standards and appli-
cations.

When a narrowband RFI disables a single GPS signal, the remaining two signals can be
used in the CIR by forming two linearly independent beat frequencies. The global search
algorithm developed in Chapter 5 is used to find an optimal set of GWLs when one of the
three civil signals is lost. Analysis of the two frequency CIR shows that it can resolve the
GWL integer ambiguity with a greatly reduced level of integrity from that of the three fre-
quency CIR.
8 Conclusions

8.1 Summary of Contributions

8.1.1 Development of the Cascade Integer Resolution (CIR) and Analysis of Its Performance

A very accurate pseudorange can be acquired by using carrier phase measurement of the GPS signal, if its integer ambiguity is resolved. The Cascade Integer Resolution uses the longer wavelength of beat frequencies of multiple GPS carrier frequencies as "stepping stones" to resolve the integer ambiguity. With the present GPS signal structure, only one beat frequency with 86 cm wavelength, commonly called "Widelane" (WL), is available. However, in the future, there will be two additional civil signals, the L2 and Lc, yielding two additional beat frequencies: the "Extra Widelane" (EWL) with 5.9 m wavelength, and the "Medium lane" (ML) with 75 cm wavelength. These multiple beat frequencies are utilized in the CIR to eventually resolve the L1, L2 or Lc integer ambiguity. The name comes from the fact that this process resolves the integer ambiguities from the longest to the shortest wavelength successively using the earlier measurement. The higher chipping rate of the Lc code also enables the CIR with future three civil frequencies to start the cascading process with less code measurement error than that of the current CIR.

Due to its cascading nature, integrity of the integer estimation of each step must meet a desired level before the CIR process progresses. The integrity of each integer resolution
step is investigated by first finding the conditional probability of estimating the right integer with a given measurement distribution. Then covariance analysis is carried out with five states: pseudorange, ionosphere delay, integer ambiguity for the L1, L2 and Lc, and with six measurements: three C/A code and three carrier phase measurements. Covariance analysis is used to calculate the probability of estimating the right EWL, WL and ML integers. Results from these integrity analyses are used to find the service distance or coverage area of the CIR, with a given desired probability of incorrect integer resolution. For an application requiring integrity risk on the order of $10^{-4}$, the CIR can be used to resolve the Lc integer ambiguity up to 2.4 km, the WL integer ambiguity up to 22 km, and the EWL integer ambiguity beyond 22 km. Table 8-1 summarizes the performance of the CIR.

Effect of multipath and receiver error reduction on performance of the CIR is also investigated. Table 8-2 summarizes the performance of the CIR when multipath and receiver error is reduced to 1% of carrier wavelength.

For both analyses, the C/A code and carrier phase measurement distributions are assumed to be bounded by normal distributions. The measurement error model has been verified by analyzing the actual GPS data collection (L1 and L2 only). The spatial gradient of residual differential ionospheric error is assumed to be fixed at 2 parts per million.

<table>
<thead>
<tr>
<th>Desired Level of Integrity</th>
<th>Resolved</th>
<th>Integer</th>
<th>Ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lc</td>
<td>WL</td>
<td>EWL</td>
</tr>
<tr>
<td>$10^{-8}$</td>
<td>cannot resolve</td>
<td>cannot resolve</td>
<td>up to 40+km</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>cannot resolve</td>
<td>cannot resolve</td>
<td>up to 40+km</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>up to 2.4 km</td>
<td>up to 22 km</td>
<td>up to 40+km</td>
</tr>
</tbody>
</table>

Table 8-1. Resolved Integer Ambiguity by the CIR for Different Levels of Integrity, k=2% of Wavelength
### Table 8-2. Resolved Integer Ambiguity by the CIR for Different Levels of Integrity, with Multipath and Receiver Error Reduction, k=1% of Wavelength

<table>
<thead>
<tr>
<th>Desired Level of Integrity</th>
<th>Resolved</th>
<th>Integer</th>
<th>Ambiguity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lc</td>
<td>WL</td>
<td>EWL</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>up to 2 km</td>
<td>up to 6.7 km</td>
<td>up to 40+km</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>up to 2.6 km</td>
<td>up to 40+km</td>
<td>up to 40+km</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>up to 2.6 km</td>
<td>up to 40+ km</td>
<td>up to 40+km</td>
</tr>
</tbody>
</table>

#### 8.1.2 Selection of the Third Civil Frequency and Its Effect on the CIR

Although the third civil frequency is now selected at 1176.45 MHz, performance of the CIR is affected if a different third frequency is used. This frequency generates a set of beat frequencies, each with its own wavelength and measurement error when combined with the L1 and L2. Therefore, effect of a different spectral location of the third frequency on performance of the CIR was examined and the optimal third frequency for the CIR was investigated with an assumption that it must be in the Aeronautical Navigation Services frequency band.

The optimal third frequency is searched for in the frequency band from 900 to 2500 MHz, which includes three ARNS bands, 960-1215 MHz, 1300-1350 MHz and 1559-1610 MHz. The optimal third civil frequency for a short baseline distance, where effect of residual differential ionosphere is ignored, and for a long baseline distance are shown in Table 8-3.

### Table 8-3. Optimal Third Frequencies

<table>
<thead>
<tr>
<th>Short Baseline</th>
<th>Long Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>1109.7 MHz</td>
<td>1307 - 1350 MHz</td>
</tr>
</tbody>
</table>
8.1.3 Optimization of the CIR with Linear Combinations of Carrier Frequencies

In Chapter 4, it was shown that the change in beat frequency due to change in the third civil frequency affects performance of the CIR. The idea behind using linear combinations of carrier frequencies is similar to this, except that a different beat frequency is generated not by selecting different carrier frequencies but by choosing different linear combinations of fixed carrier frequencies. The unity combination beat frequencies, the L1-L2, L1-Lc, L2-Lc have been used in the CIR up until now. The optimal set of linear combinations of beat frequencies, or Generalized Widelanes (GWLs) for a desired level of accuracy and probability of incorrect integer ambiguity resolution is obtained by using the global search algorithm developed in Chapter 5.

![Diagram](image)

Figure 8-1. Using the GWL on the CIR

It was envisioned that a trade off among desired accuracy, probability of incorrect integer ambiguity resolution and service distance of the CIR would be possible by using an appropriate set of GWLs. For example, Figure 8-1 shows a conceptual trade-off between accu-
racy and integrity, where accuracy of the unity beat frequency is traded to get lower probability of incorrect integer estimation. However, the optimization results show that the trade-off did not work well due to lack of variety in wavelength of GWL with low measurement error. As a result, the optimal set of GWLs performance closely resembles that of the unity combination beat frequencies.

8.1.4 Estimation of Spatial Gradient of Residual Differential Ionospheric Error

It is shown in the performance analysis of the CIR that measurement error is the limiting factor. For a short baseline distance between the reference and user receivers, performance of the CIR is determined by double difference multipath and receiver error. Such errors can be reduced and baseline distance can be extended when multipath mitigation techniques are applied. However, as the baseline distance increases, performance of the CIR worsens due to increased residual differential ionospheric error.

The spatial gradient of standard deviation of residual differential ionospheric error is assumed to be 2 ppm. It is used as a priori information in the covariance analysis developed in Chapter 3. However, the spatial gradient is observable if measurements from two or more separate locations are used in the CIR. When measurements from locations 10 km apart are used to estimate the spatial gradient, the performance of the CIR improved. For a desired probability of incorrect integer ambiguity resolution of $10^{-4}$, the Lc integer is resolved up to 4 km instead of up to 2.4 km without estimation of the spatial gradient. Also, the WL integer is resolved up to 40 km with the estimation, instead of 22 km without, for the same probability.
8.1.5 Effect of RFI on GPS Receivers

In the present configuration, civil users cannot use GPS if the L1 frequency is disabled. The two additional civil frequencies provide the necessary redundancy in the GPS signal to allow graceful degradation of GPS service in the presence of interference.

Among the many potential sources of RFI which could compromise integrity of the GPS signal, UWB technology stands out due to its capacity to disable all three civil frequencies simultaneously. A preliminary study shows that certain configurations of UWB signal can cause a receiver 100 meters from the UWB device to lose lock on a GPS satellite. Since the standard for UWB signals is not yet defined and possible RFI on GPS signals from UWB technology is significant, determining the exact nature of interference on GPS from this source requires additional studies.

8.2 Application of the CIR and Future Work

The CIR is a geometry free, instantaneous integer resolution technique which utilizes multiple civil GPS signals and beat frequencies of their carrier frequencies. It can be applied to CDGPS applications, such as precision farming, mining operation, harbor navigation, etc. The CIR is also well suited for precision landing of aircraft, since accuracy and integrity of the CIR increases as the user (aircraft) approaches the reference station (airport).

For future work, the measurement error model for the second and third civil GPS signals should be verified as the new signals become available. Also, use of multipath mitigation techniques should be investigated further to enhance performance of the CIR. Using a mea-
measurement error model that is a function of satellite elevation angle is another possible refinement of the CIR.

Also, although the CIR in the present form works in the pseudorange domain, where it resolves the integer ambiguity of a GPS satellite, it can be transformed to work in the position domain. In this domain, redundancy of measurements from multiple GPS satellites can be used to enhance integrity of the integer ambiguity estimation and spatial gradient of residual differential ionospheric error. Also, the effect of change in satellite geometry can be incorporated into the CIR for further performance enhancement.

8.3 Closing

The Cascade Integer Resolution is developed to utilize multiple civil GPS signals to guide users to their destinations accurately and reliably. By instantaneously estimating the integer ambiguity with relatively low probability of incorrect resolution, the CIR is well suited for use in CDGPS with the three civil GPS signals available in the near future.
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