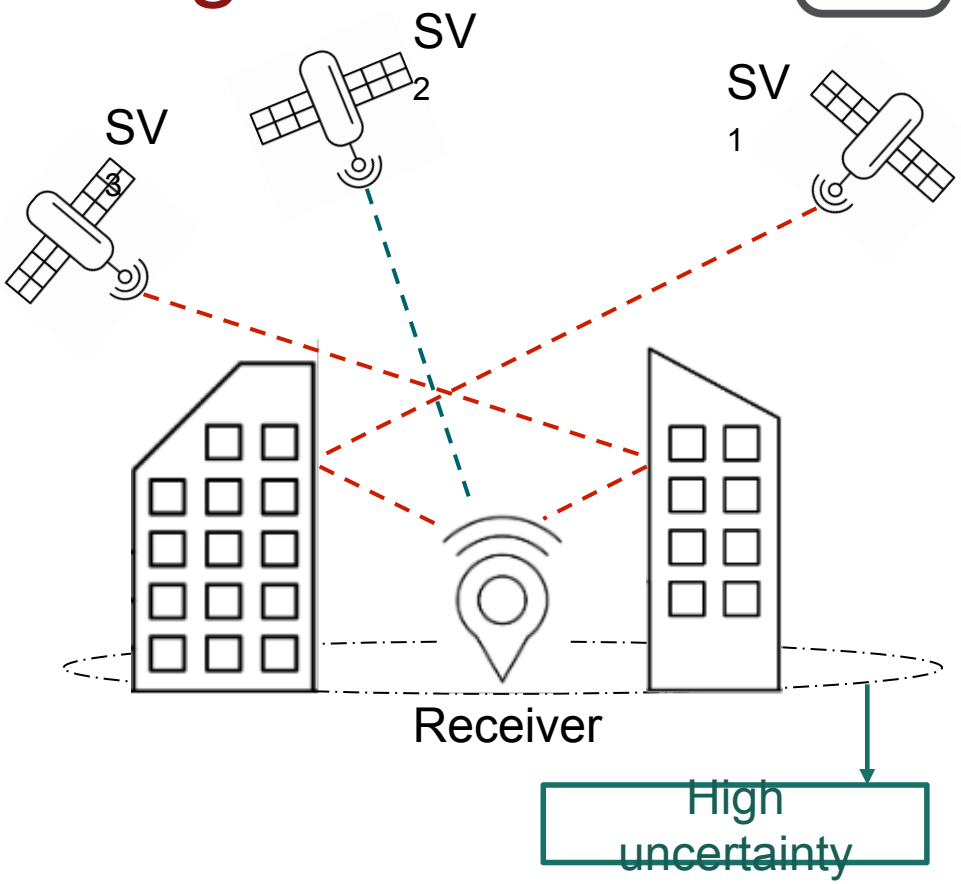
Urban Integrity Monitoring

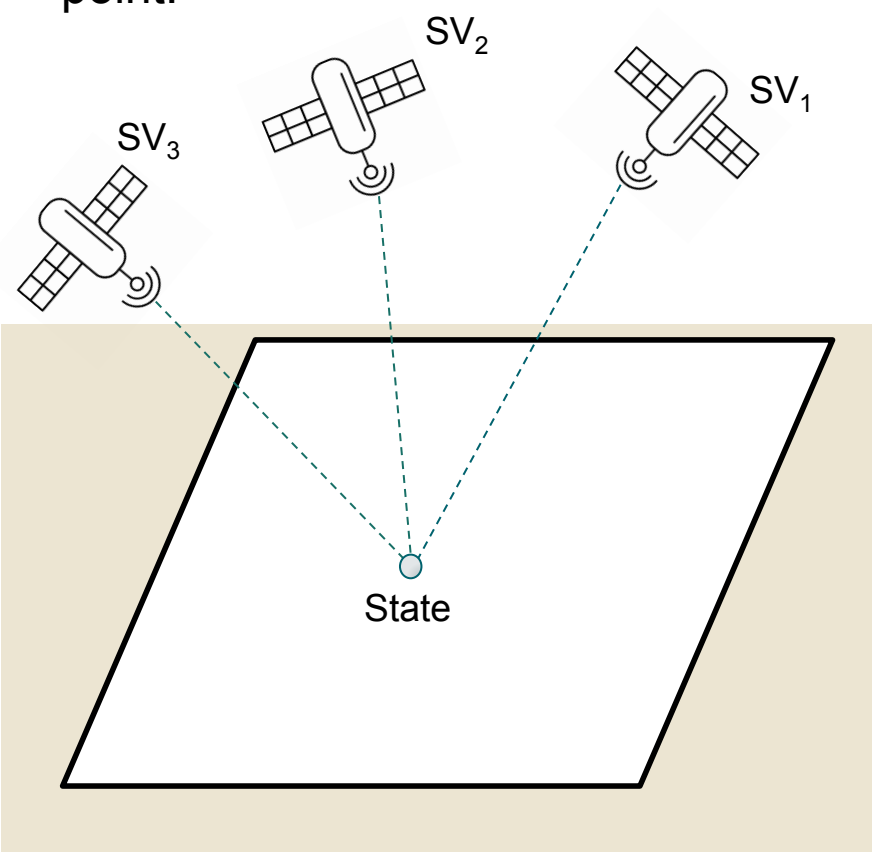
- Urban scenario
  - Limited satellite visibility
  - Multiple faulty measurements
- Methods incorporating temporal information[1][2] or auxiliary sources[3][4]
- Challenges
  - Dependency on prior information
  - Extending to a probabilistic likelihood model instead of a single point estimate
  - Scalability to multiple faults



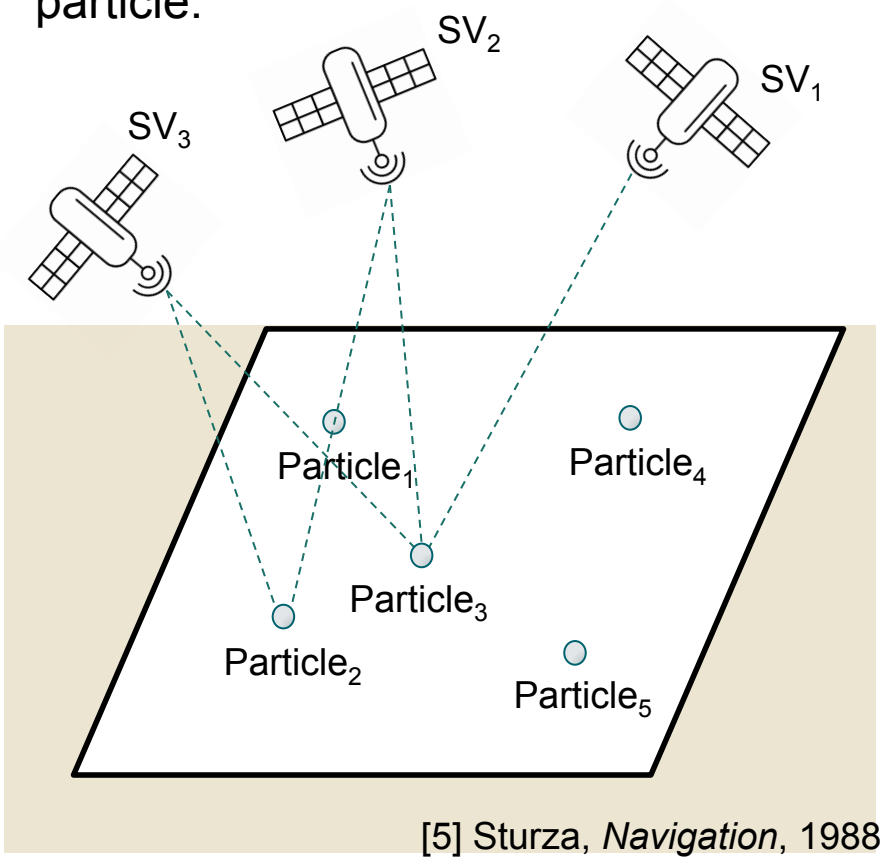
[1] Joerger et. al., *AIAA J.*, 2013
[2] Brenner, *ION GPS*, 1995
[3] Groves et. al., *Navigation*, 2013
[4] Betaille et. al., *Navigation*, 2016

Particle RAIM Concept

RAIM[5][6] – non faulty measurement subset using single point.



Particle RAIM[7] – distinct measurement subset for each particle.



[5] Sturza, *Navigation*, 1988
[6] Parkinson, *Navigation*, 1988
[7] Gupta et. al. *ION GNSS+*, 2019

Particle RAIM Concept

Traditional RAIM[5][6]	Particle RAIM[7]
Assumes correctness of a single state estimate	Assumes likelihood over the state estimate in the form of particles
Has disconnected state estimation and fault exclusion	Has joint state estimation and fault exclusion
Retains a single measurement-fault hypothesis	Retains a particle distribution over different measurement-fault hypotheses

[5] Sturza, *Navigation*, 1988
[6] Parkinson, *Navigation*, 1988
[7] Gupta et. al. *ION GNSS+*, 2019

Key Contributions

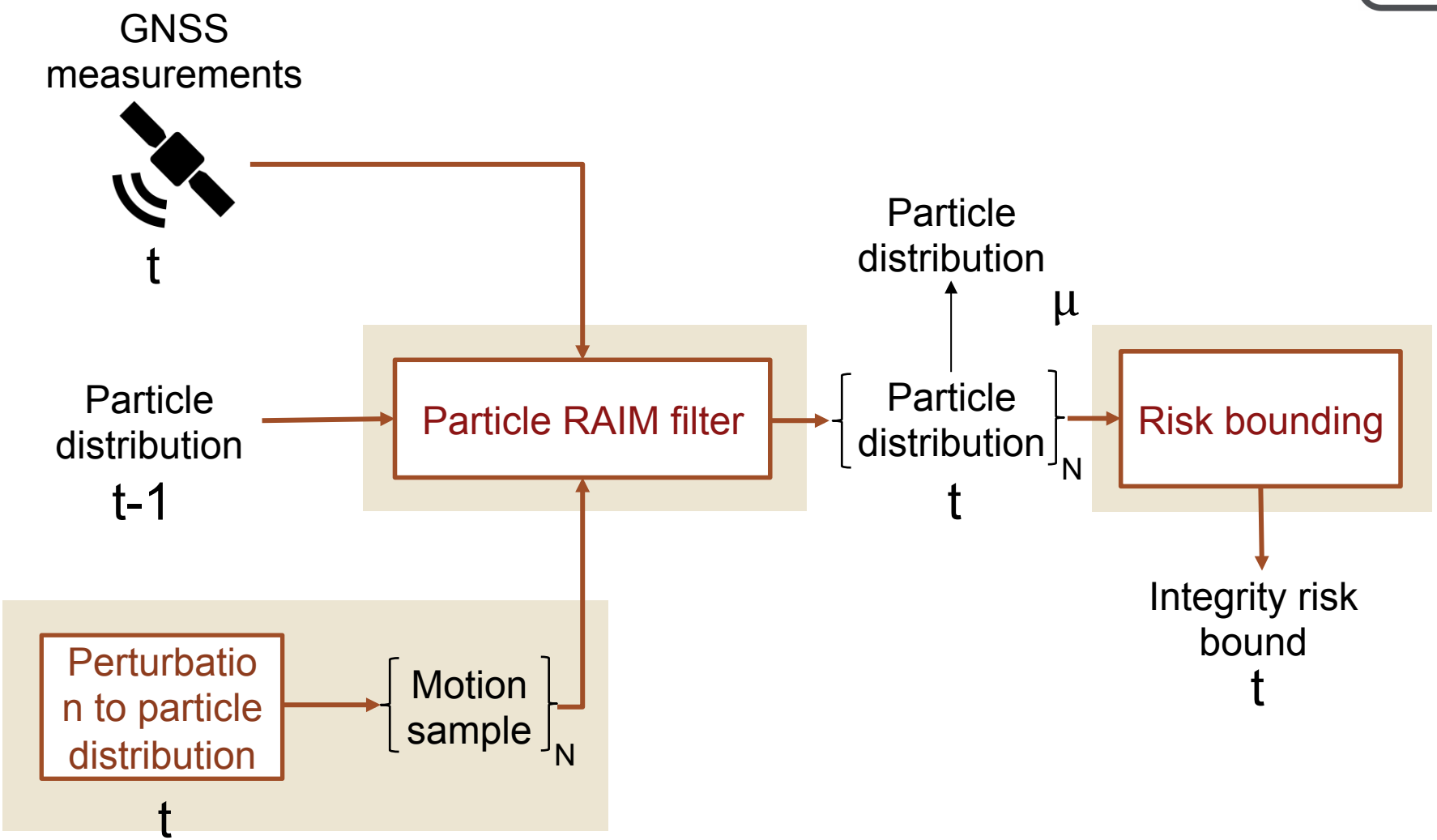
- Developed a particle filter based framework for joint state estimation and fault-exclusion
  - Robustly tracks multiple fault-aware hypotheses
  - Makes no assumptions on availability of additional information sources

- Derived an upper bound for the integrity risk associated with state estimation
  - Is based on robustness of state estimation under perturbed input
  - Evaluates Integrity risk for an arbitrary distribution instead of a single point

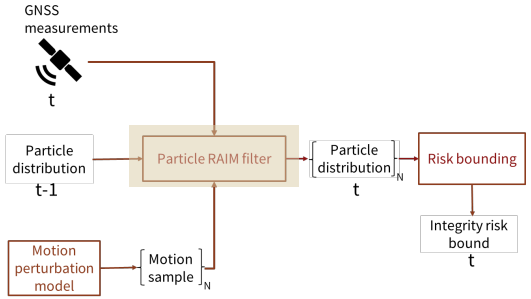
Outline

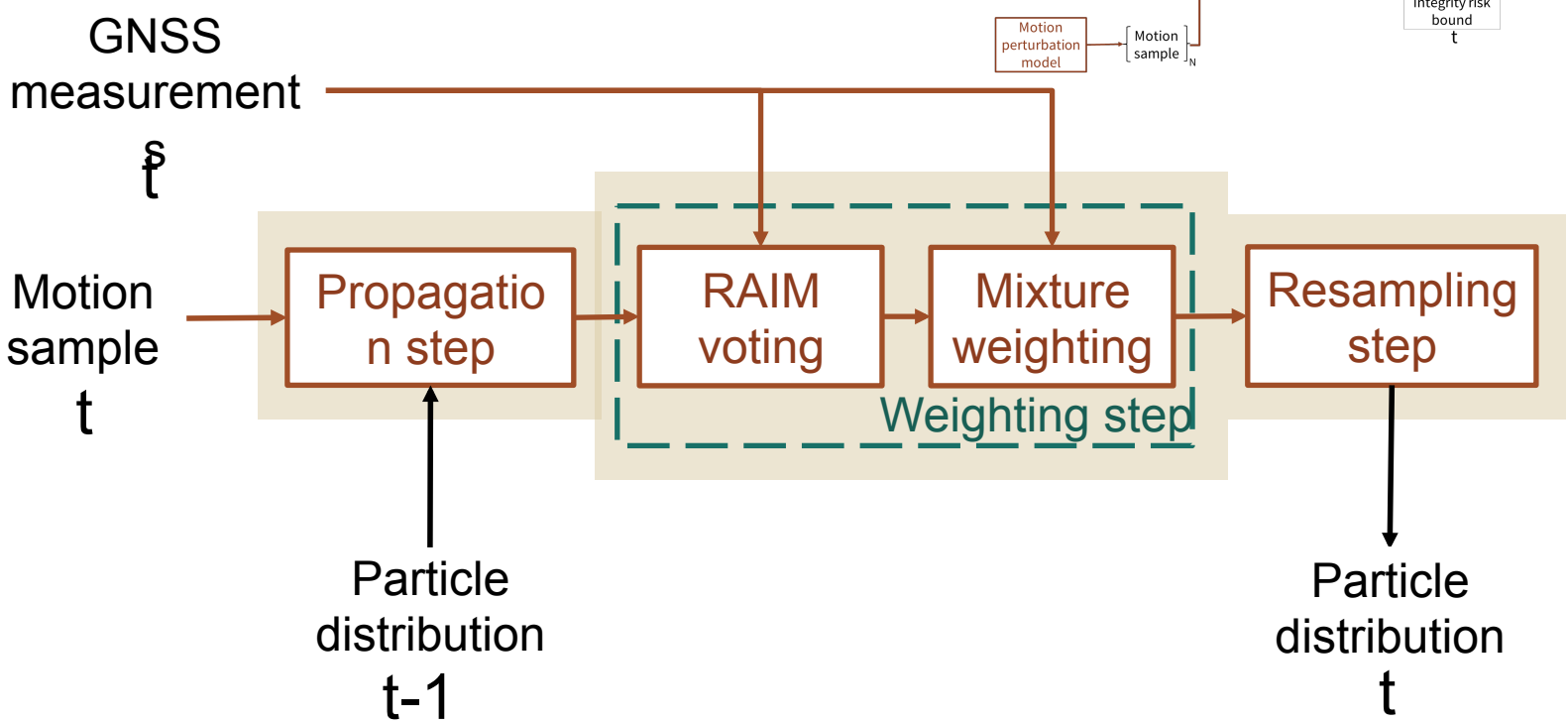
- Proposed Framework
  - Overall Architecture
  - Particle RAIM Filter
  - Integrity Risk Bound
- Experimental Validation
  - Investigation Scenarios
  - State Estimation Comparison
  - Risk Bound Evaluation
- Conclusion

Overall Architecture



Particle RAIM Filter





A fault-tolerant distribution is computed for each motion sample

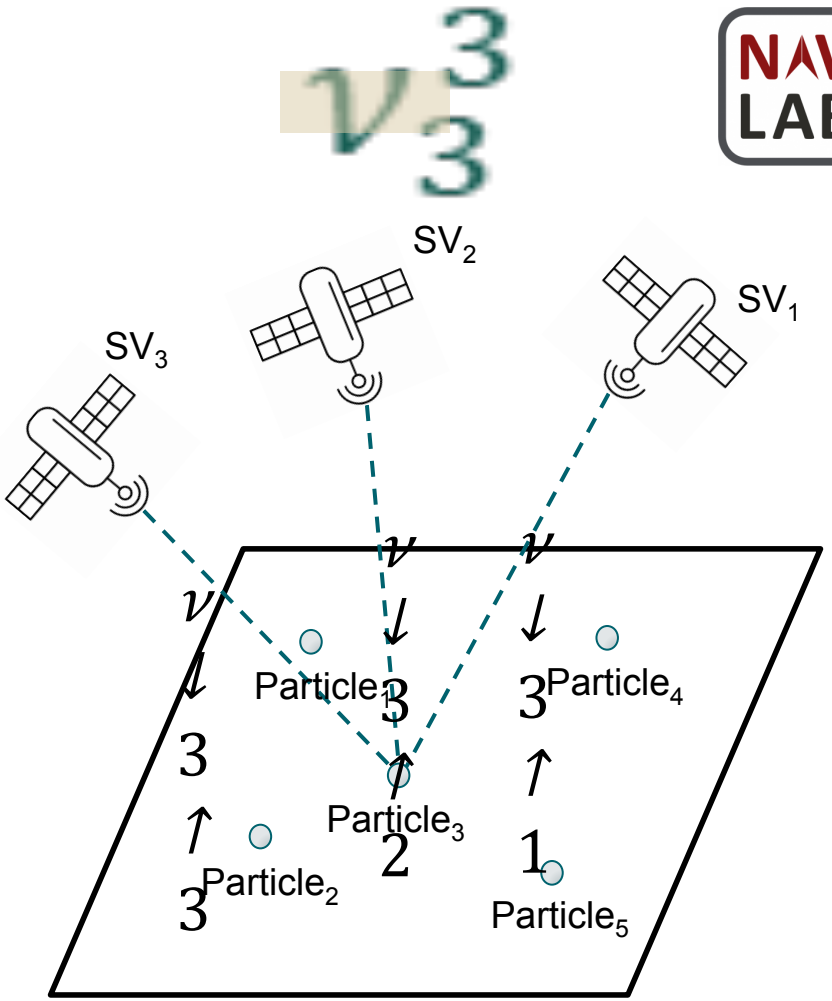
RAIM Voting

1. Each particle casts a squared-normal local vote per measurement using its normalized residual.

$$v_n^k = \mathbf{P}_{\mathcal{N}^2(0,1)}(r_n^k)$$

2. Overall confidence for each measurement by pooling local votes by all particles.

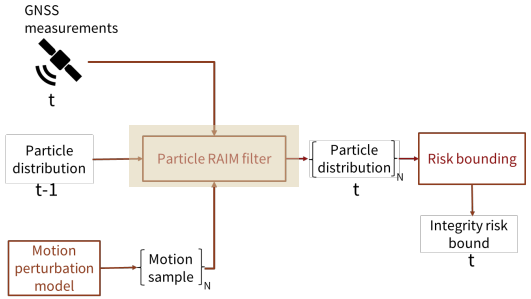
$$\pi_k = \frac{\sum_{n=1}^{N} v_n^k}{\sum_{k=1}^{K} \sum_{n=1}^{N} v_n^k}$$



A particle's vote is its measure of confidence in a measurement

Mixture Weighting

- Gaussian Mixture Model (GMM) for overall measurement likelihood
  - Multi-modalities by different measurement subsets
- Individual measurement likelihood
  - Gaussian distribution
  - Associated with confidence parameter



Set of ranging measurements

# of measurements

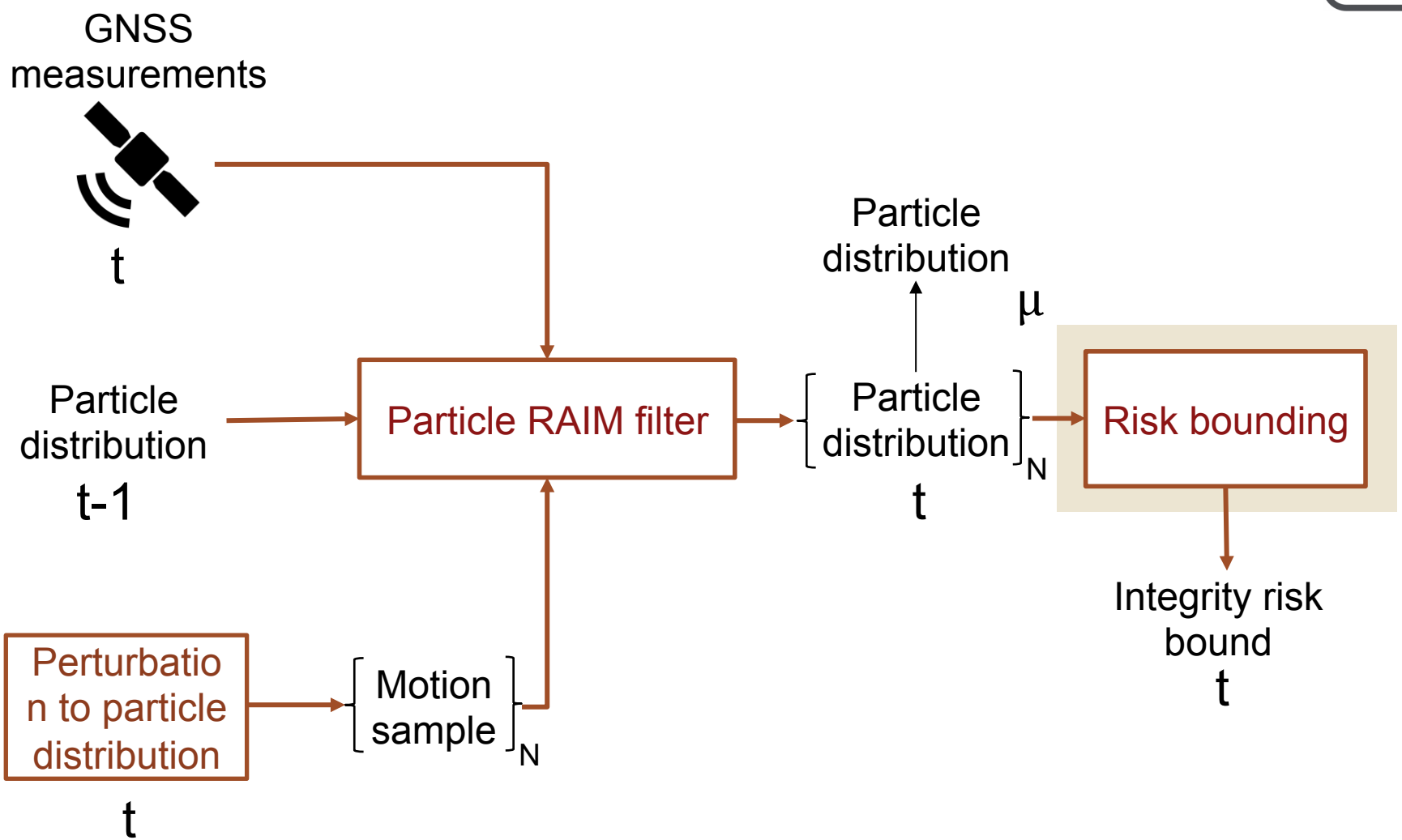
$$p(m^K|x, S^K) = \sum_{k=1}^{K} \pi_k \underbrace{p(m_k|x, S_k)}$$

Gaussian likelihood

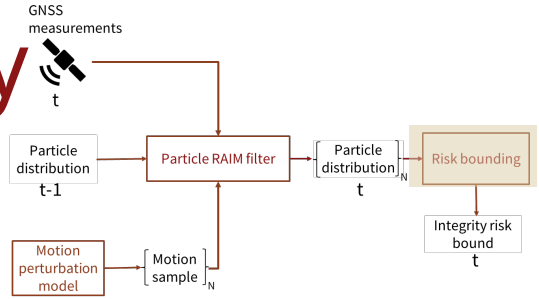
Set of satellite positions

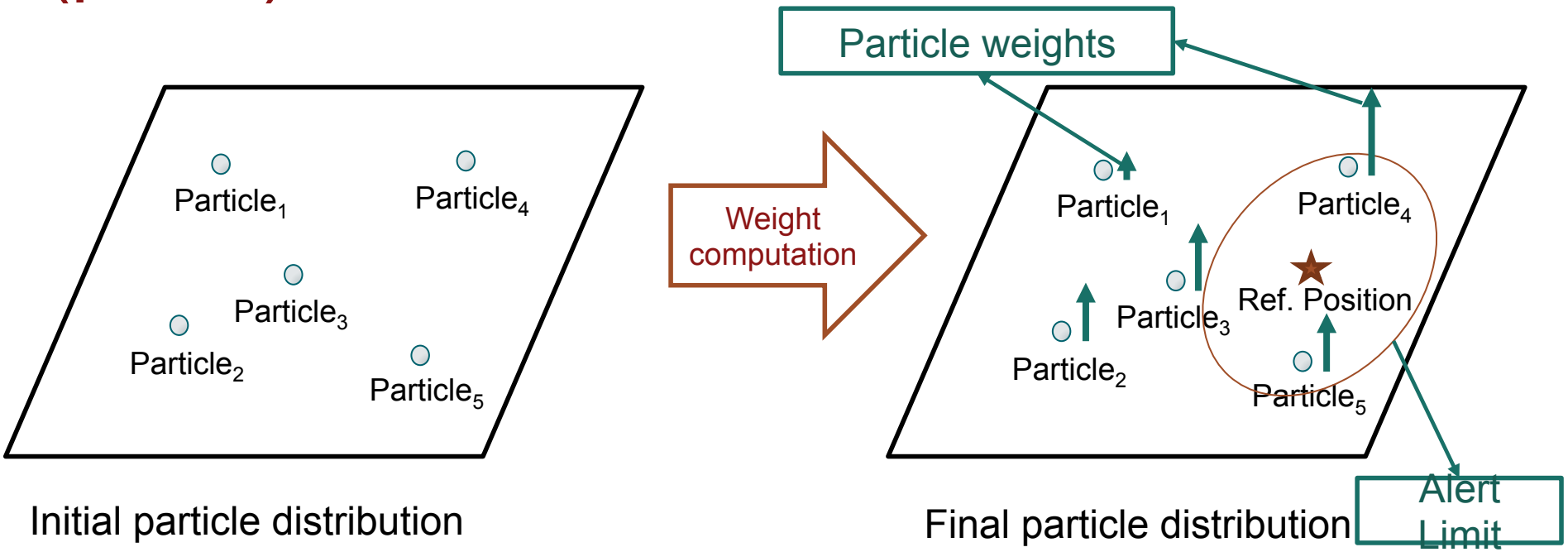
Confidence parameter

Overall Architecture



Probability of Hazardously Misleading Information (pHMI)



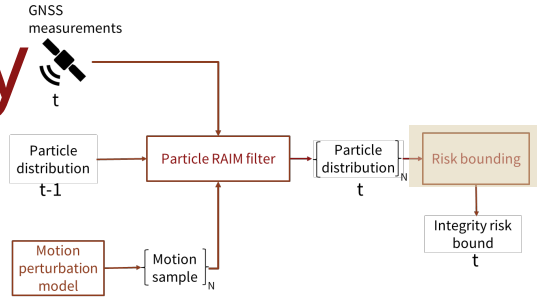


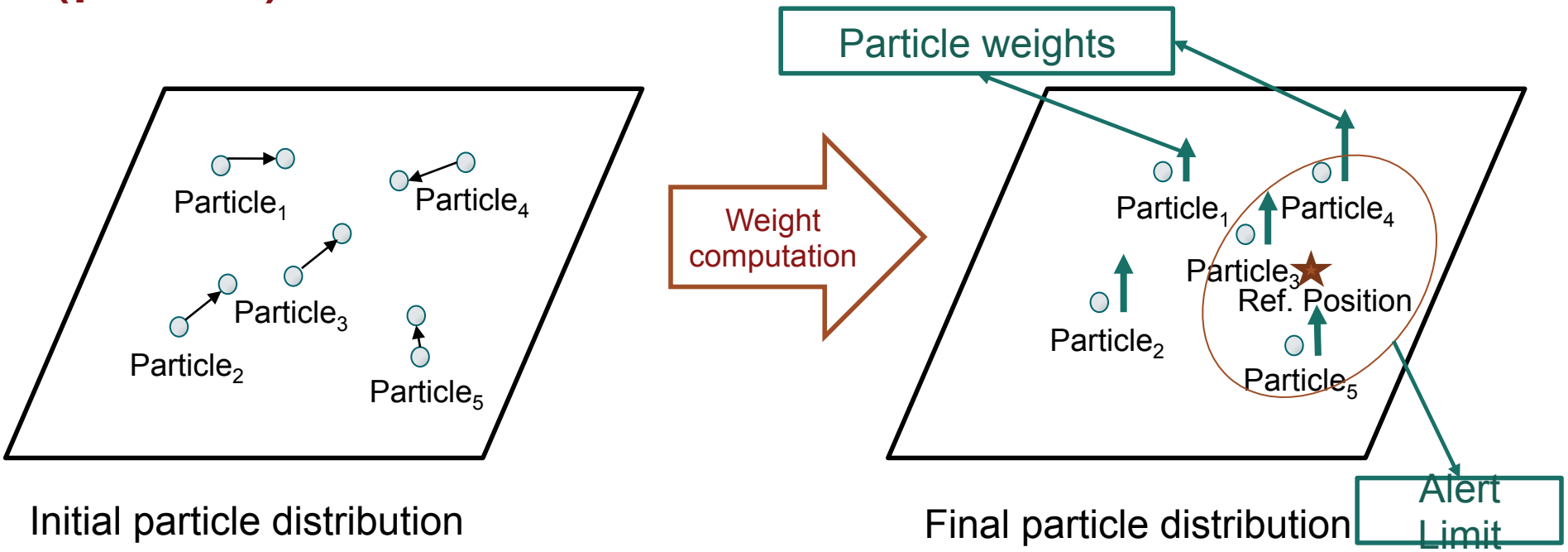
Initial particle distribution

Final particle distribution

pHMI computed as the weight allocated to particles outside Alert Limit

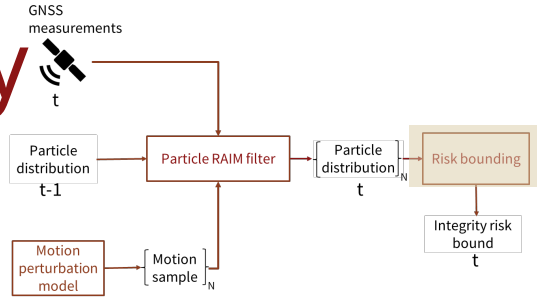
Probability of Hazardously Misleading Information (pHMI)

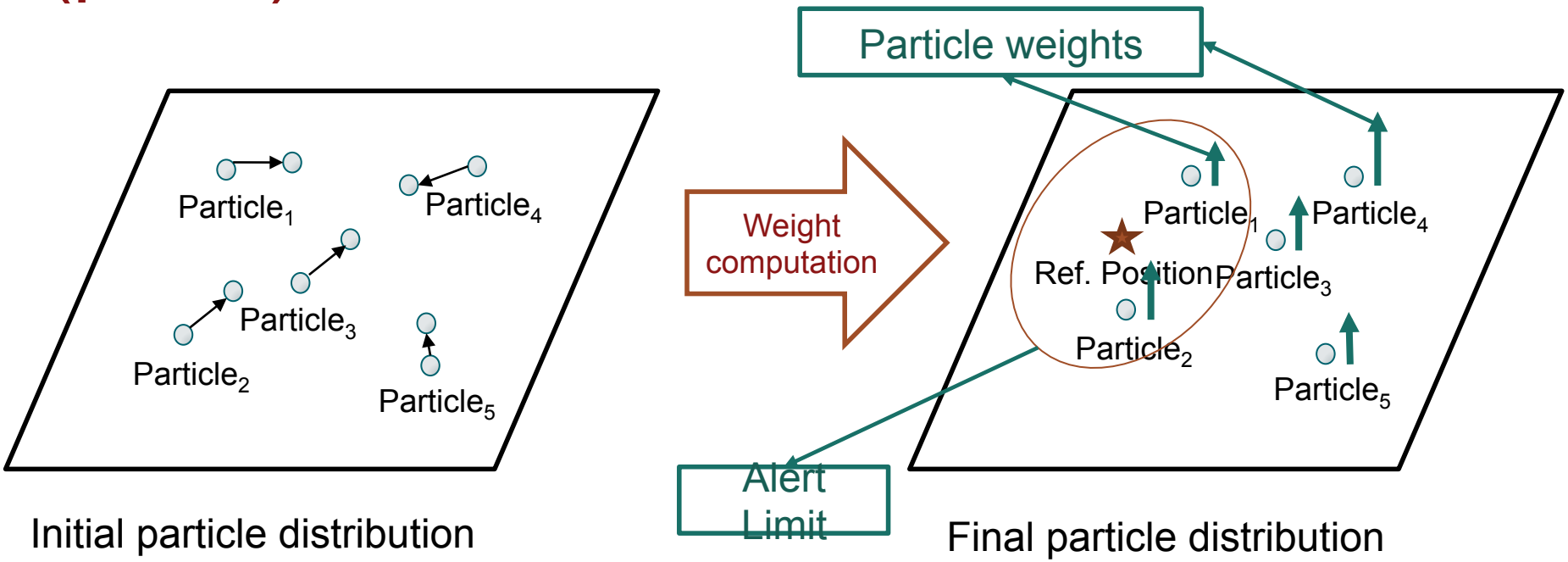




pHMI is dependant on the perturbation to initial particles

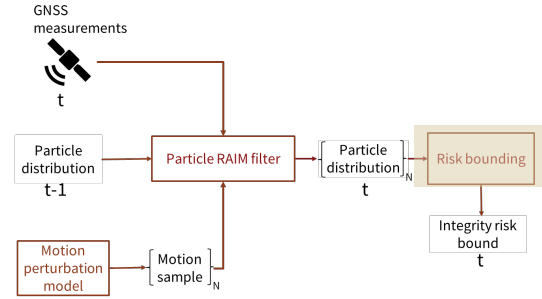
Probability of Hazardously Misleading Information (pHMI)





pHMI is dependant on the reference position

Computing Integrity Risk Bound



Inverse Bernoulli Divergence[8][9]

Empirical risk

Gap term

$$R(\pi) \leq \hat{R}_N(\pi) + D^{-1}_{Ber}(\hat{R}_N(\pi), \epsilon)$$

Divergence risk

Mean-perturbation particle distribution

PAC-Bayesian generalization error bound[8][9]

Particle distribution $\}_N$

Risk bounding

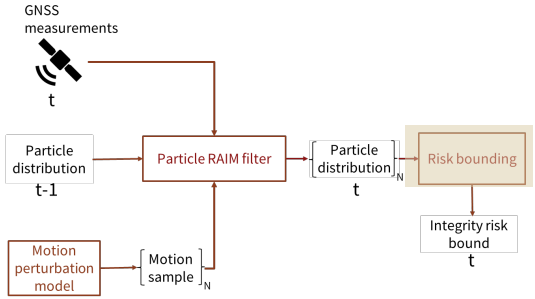
Divergence risk

Empirical risk

Integrity risk bound

[8] McAllester, *Machine Learning*, 1999
[9] McAllester, *Machine Learning*, 2003

Bound Components: Empirical Risk



Mean-perturbation particle distribution

Expectation over reference positions

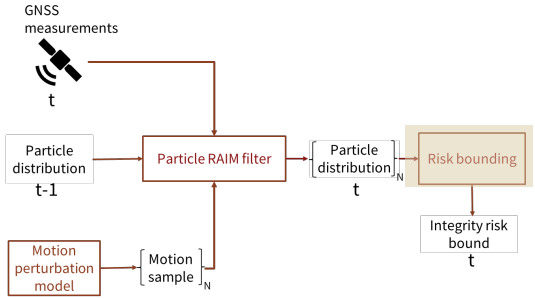
pHM

$$\hat{R}_N(\pi) = \frac{1}{N}\sum_{n=1}^{N} \mathbb{E}_{x\sim\pi}[P(\|x - x_{u_n}\| \geq r)]$$

Average over initial perturbations

Empirical risk measures the average pHMI from perturbations

Bound Components: Divergence Risk



$$D_{Ber}^{-1}(\hat{R}_N(\pi), \boxed{\epsilon})$$

KL divergence from prior

$$\epsilon = \frac{1}{N}(KL(\pi \| \pi_{prev}) + \log \frac{N+1}{\delta})$$

Gap term

Provided error allowance

Divergence risk estimates the uncertainty due to unseen perturbations

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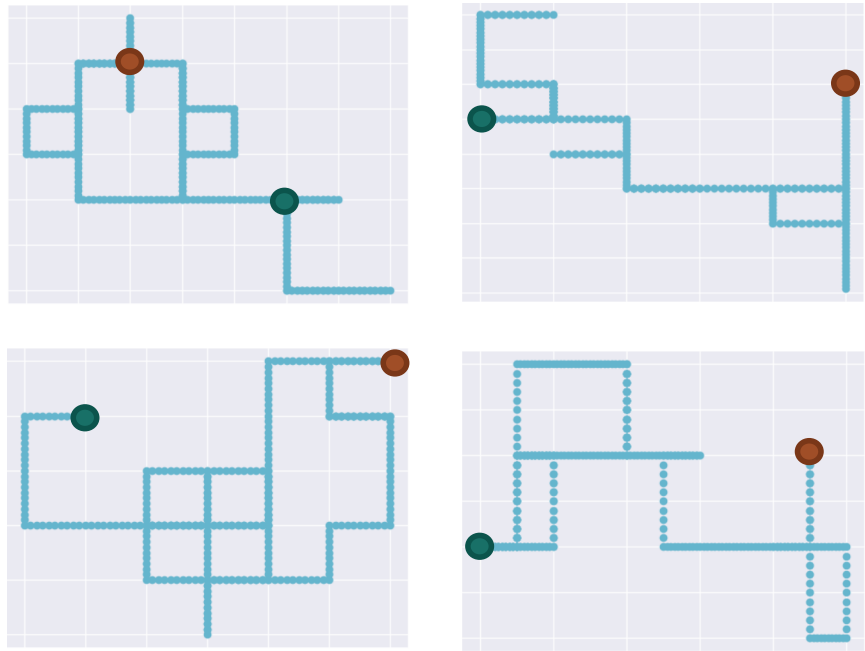
Investigation Scenarios

- Verification requirements
  - Multiple runs with different trajectories
  - Access to position ground truth of navigation agent
  - GNSS pseudorange measurements and motion model availability
  - Varied noise profiles under controlled settings

- Scenario 1
  - 50 random simulated trajectories
  - Simulated pseudorange and odometry measurements under varied noise settings similar to urban environments
- Scenario 2
  - Highway driving with GNSS pseudorange and Doppler measurements
  - Induced bias errors for imitating urban environments
  - Odometry using Doppler measurements

Evaluation on Scenario 1

- Multiple runs with **50 different trajectories**
- Access to **true position** of navigation agent
- **Varied noise profiles** under controlled settings
- Particle filter performs **2D state estimation** with GNSS ranging measurements and **odometry based motion model**



Examples from a set of 50 random trajectories of length 4000m

Start
End

Evaluation on Scenario 1

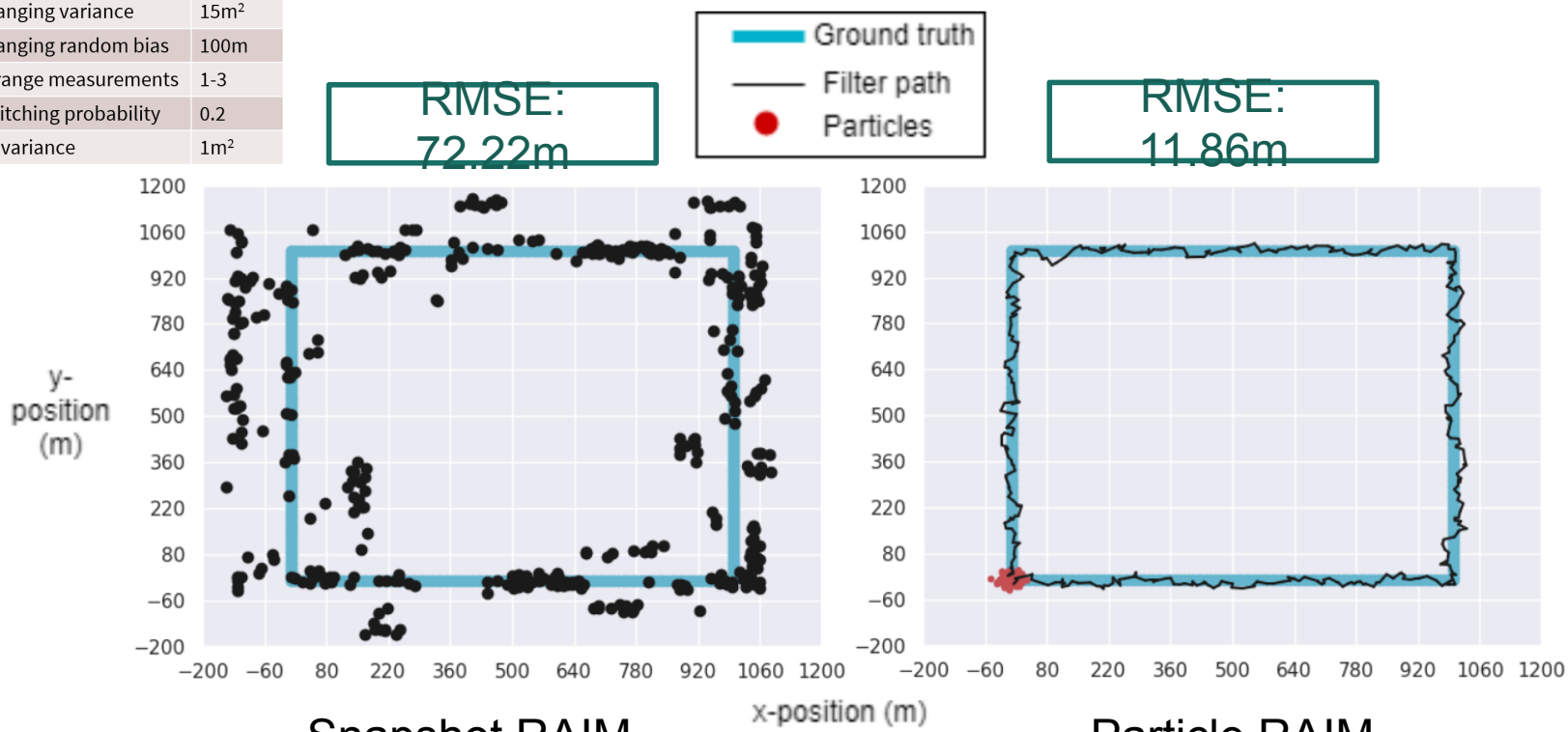
Simulation Parameter	Value
# of satellites	5
GNSS ranging variance	$15m^2$
GNSS ranging random bias	100m
Faulty range measurements	1-3
Bias switching probability	0.2
Motion variance	$1m^2$

Filter Parameter	Value
# of particles	200
Measurement variance	$15m^2$
Propagation variance	$10m^2$

GNSS ranging measurements have non-zero mean gaussian noise

State Estimation Comparison

# of satellites	5
GNSS ranging variance	$15m^2$
GNSS ranging random bias	100m
Faulty range measurements	1-3
Bias switching probability	0.2
Motion variance	$1m^2$



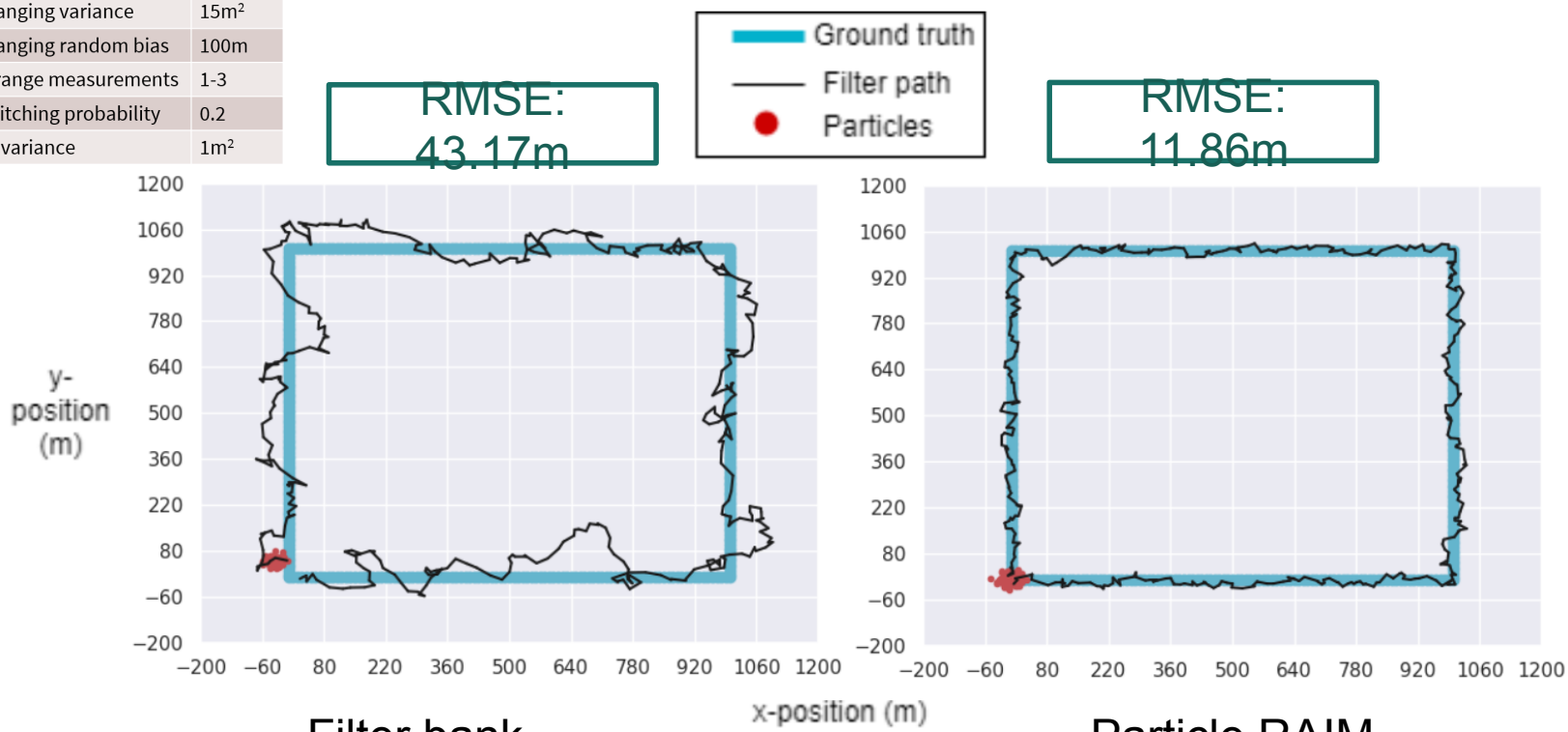
Snapshot RAIM

Particle RAIM

Particle RAIM tracks the state more accurately than snapshot RAIM

State Estimation Comparison

# of satellites	5
GNSS ranging variance	$15m^2$
GNSS ranging random bias	100m
Faulty range measurements	1-3
Bias switching probability	0.2
Motion variance	$1m^2$



Filter bank RAIM

Particle RAIM

Particle RAIM tracks the state more accurately than Filter bank RAIM

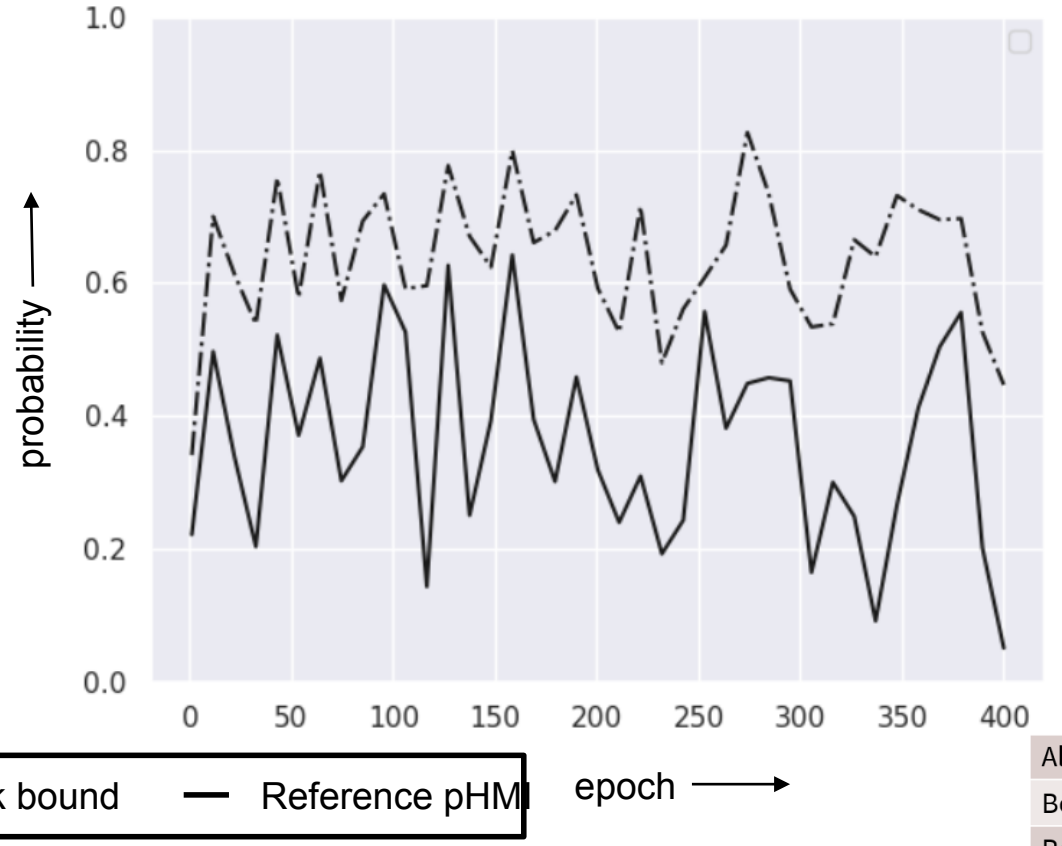
Experimental Setup

Simulation Parameter	Value
# of satellites	5
GNSS ranging variance	$10m^2$
GNSS ranging random bias	100m
Faulty range measurements	1-3
Bias switching probability	0.2
Motion variance	$1m^2$
Number of runs	50

Filter Parameter	Value
# of particles	200
Measurement variance	$15m^2$
Propagation variance	$10m^2$
Integrity Parameter	**Value**
Alert Limit	20m
Bound error allowance	0.1
Perturbation variance	$5m^2$

Integrity parameters chosen according to filter performance

Risk Bound Evaluation

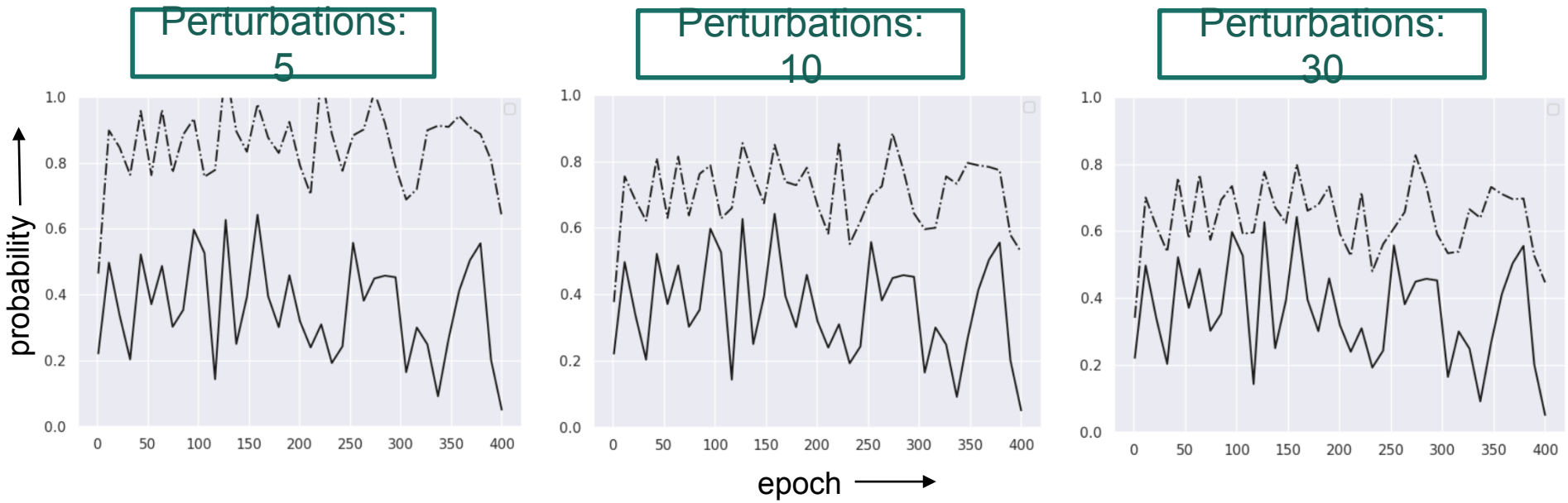


# of perturbation samples : 30

Alert Limit	20m
Bound error allowance	0.1
Perturbation variance	$5m^2$

Risk bound successfully upper bounds the reference pHMI

Bound gap variation with perturbation samples

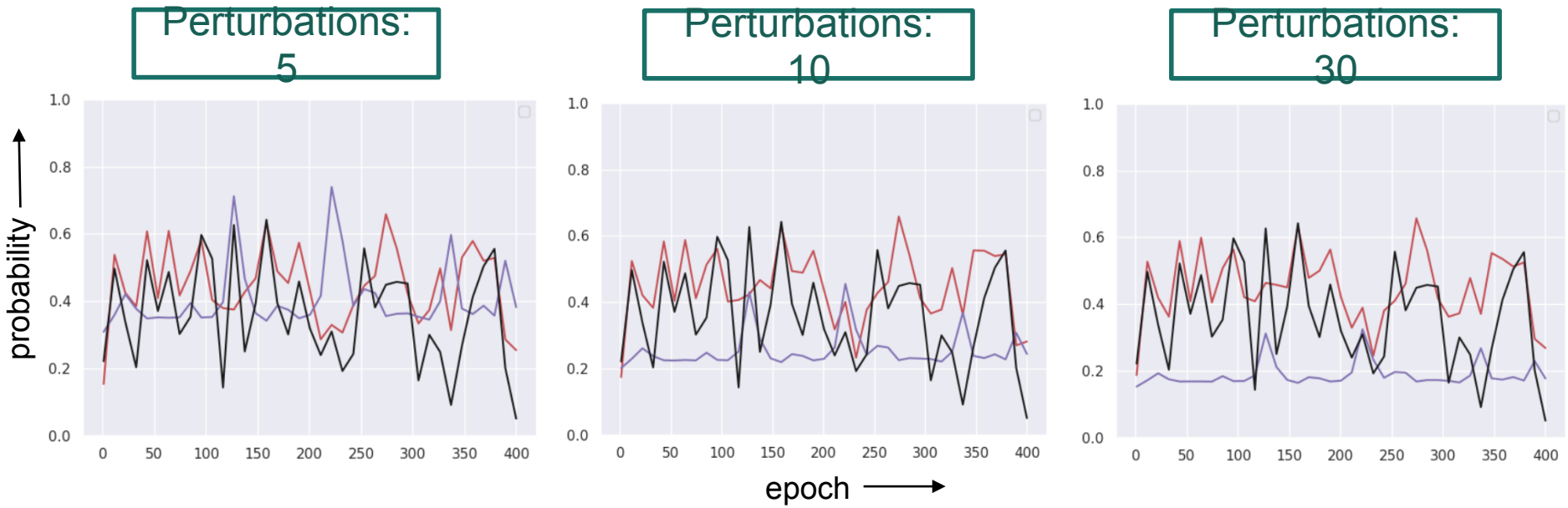


Integrity risk bound — Reference pHMI

Alert Limit	20m
Bound error allowance	0.1
Perturbation variance	$5m^2$

Bound improves with more perturbation samples

Empirical-Divergence Risk Tradeoff



Empirical risk — Divergence risk — Reference pHMI

Alert Limit	20m
Bound error allowance	0.1
Perturbation variance	$5m^2$

Empirical risk contribution increases with more perturbation samples

Performance Metrics

Noise parameters bias (m)	# of faults	Failure rate (%)	Mean error (m)	Failure error (m)
100	0	0	7.92	-
	1	0.05	8.15	14.46
	2	0.77	10.59	27.95
	3	1.82	13.55	25.36
500	1	0.05	9.04	15.21
	2	0.90	10.61	26.94
	3	1.79	13.96	24.58

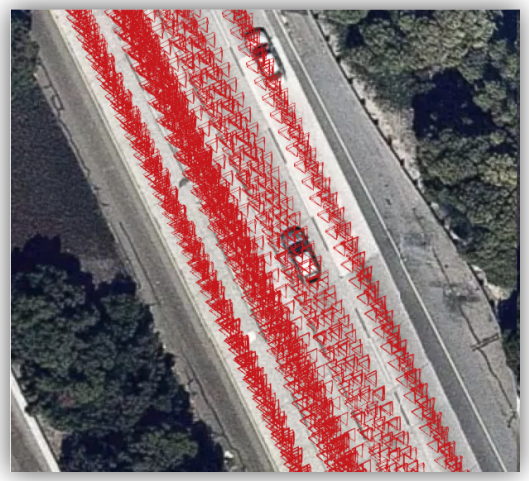
# of perturbation samples : 10
# of runs : 50
Alert limit: 20m

Failure rate : % of cases when bounding fails
Mean error : mean filter error across all runs
Failure error : mean filter error in failure cases

Bound performance is robust to large bias

Evaluation on Scenario 2

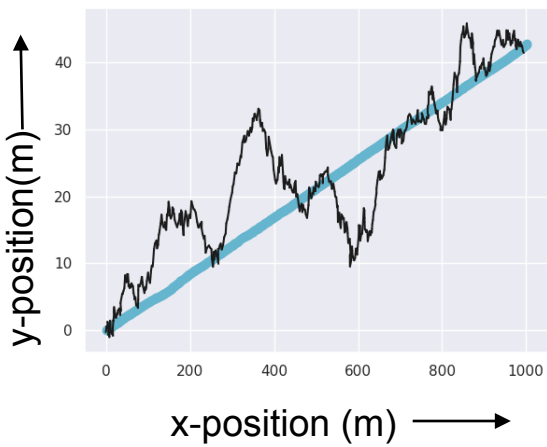
- Comma2k19 dataset[10]
  - Driving on California's 280 highway: almost open sky environment
  - 1 minute segment from 2019 segments
  - **Limited GNSS measurements** (upto 5) with **random bias** added to a few measurements (upto 2) to imitate urban environment
  - Motion model based on odometry from **Doppler measurements**
  - "Ground truth" using **all** the GNSS measurements along with **GNSS-INS-Visual Odometry fusion** and **post-processing** (Estimated



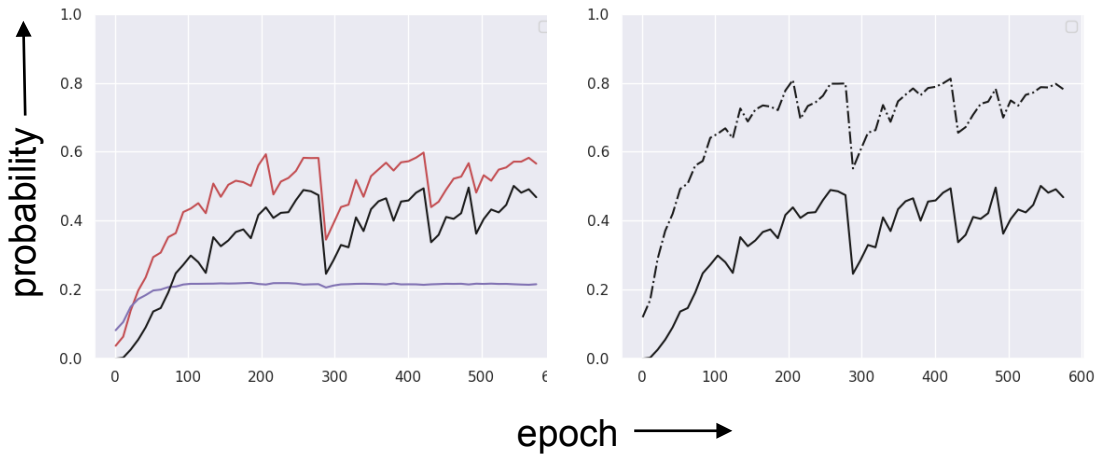
[10] Schafer et. al, *Arxiv*, 2018

Evaluation on Scenario 2

Particle RAIM Filter (MSE 8.3m)



Integrity risk bound



Mean filter estimate | Ground truth | Empirical risk | Divergence risk | Integrity risk bound | Reference pHMI

# of perturbation samples : 20
Alert limit: 20m

Successful state estimation and risk bounding on real-world data

Conclusion

- Devised a framework to compute fault-aware position distribution and associated integrity risk bounds
- Integrated traditional RAIM algorithm with particle filter framework using Gaussian mixtures for measurement model
- Leveraged PAC-Bayesian generalisation error bounds from Statistical Learning Theory for upper bounding the integrity risk
- Experimentation using multiple trajectories (simulated and real) validates that the
  - Algorithm is able to accurately perform localization in scenarios with various faults
  - Computed integrity risk bound is robust on biases in measurements
  - Integrity risk bound provides a low failure rate upper-bound on probability of hazardously misleading information

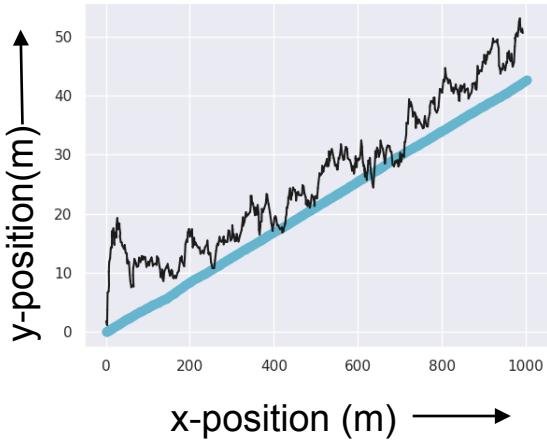
Acknowledgement

This material is based upon work supported by the National Science Foundation under award number 1750864.

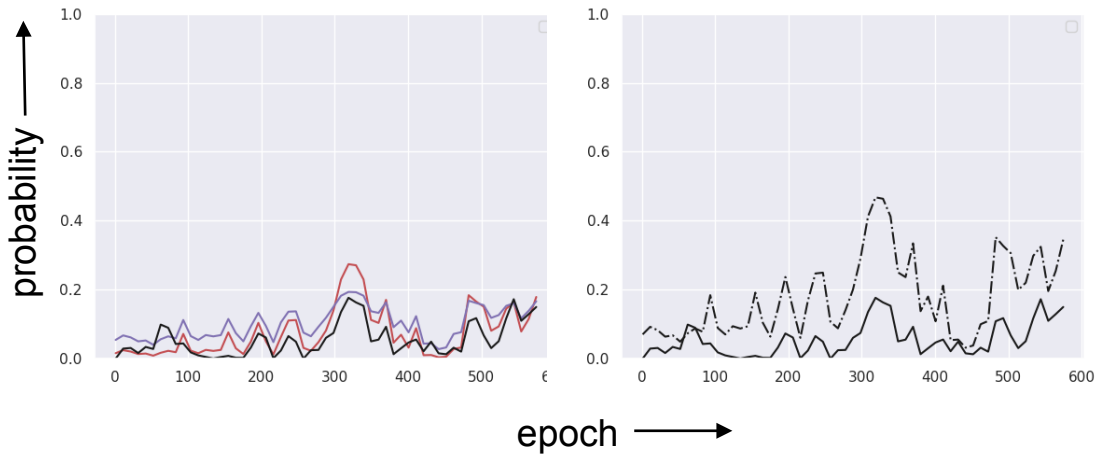
Extra Slides

Risk Bound with Open Sky Measurements

Particle RAIM Filter (MSE 5.8m)



Integrity risk bound



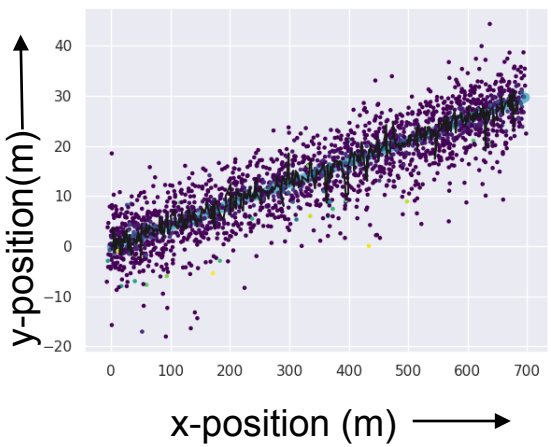
Mean filter estimate | Ground truth | Empirical risk | Divergence risk | Integrity risk bound | Reference pHMI

# of perturbation samples : 20
Alert limit: 20m

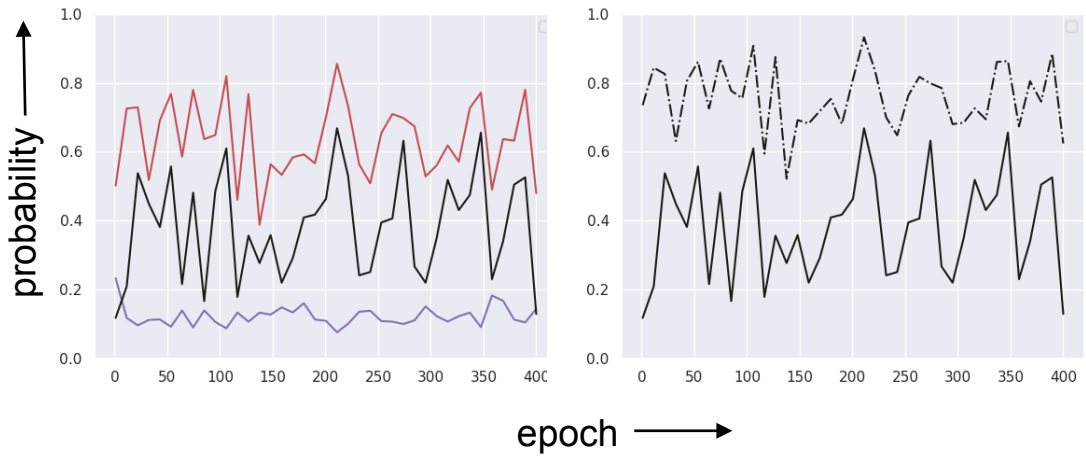
Good state estimation yields lower risk bounds

Risk Bound with Multiple Sensors

Particle RAIM Filter (MSE 1.4m)



Integrity risk bound



Particle distribution | Ground truth | Empirical risk | Divergence risk | Integrity risk bound | Reference pHMI

# of perturbation samples : 20
Alert limit: **5m**

Incorporating additional sensors allows bounds with lower Alert Limit