Urban Integrity Monitoring

• Urban scenario
  ▪ Limited satellite visibility
  ▪ Multiple faulty measurements

• Methods incorporating temporal information\(^1\)[2] or auxiliary sources\(^3\)[4]

• Challenges
  ▪ Dependency on prior information
  ▪ Extending to a probabilistic likelihood model instead of a single point estimate
  ▪ Scalability to multiple faults

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Particle RAIM Concept

RAIM\(^{[5][6]}\) – non faulty measurement subset using single point.

Particle RAIM\(^{[7]}\) – distinct measurement subset for each particle.

## Particle RAIM Concept

<table>
<thead>
<tr>
<th>Traditional RAIM(^{[5][6]})</th>
<th>Particle RAIM(^{[7]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumes correctness of a single state estimate</td>
<td>Assumes likelihood over the state estimate in the form of particles</td>
</tr>
<tr>
<td>Has disconnected state estimation and fault exclusion</td>
<td>Has joint state estimation and fault exclusion</td>
</tr>
<tr>
<td>Retains a single measurement-fault hypothesis</td>
<td>Retains a particle distribution over different measurement-fault hypotheses</td>
</tr>
</tbody>
</table>

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[7] Gupta et al., ION GNSS+, 2019
Key Contributions

• Developed a particle filter based framework for joint state estimation and fault-exclusion
  ▪ Robustly tracks multiple fault-aware hypotheses
  ▪ Makes no assumptions on availability of additional information sources

• Derived an upper bound for the integrity risk associated with state estimation
  ▪ Is based on robustness of state estimation under perturbed input
  ▪ Evaluates Integrity risk for an arbitrary distribution instead of a single point
Outline

• Proposed Framework
  ▪ Overall Architecture
  ▪ Particle RAIM Filter
  ▪ Integrity Risk Bound

• Experimental Validation
  ▪ Investigation Scenarios
  ▪ State Estimation Comparison
  ▪ Risk Bound Evaluation

• Conclusion
Overall Architecture

GNSS measurements

Particle distribution

Particle RAIM filter

Risk bounding

Integrity risk bound

Particle distribution

Perturbation to particle distribution

Motion sample

Overall Architecture

Particle distribution

Particle distribution

Particle RAIM filter

Risk bounding

Integrity risk bound

Particle distribution

Motion sample
Particle RAIM Filter

A fault-tolerant distribution is computed for each motion sample
RAIM Voting

1. Each particle casts a squared-normal local vote per measurement using its normalized residual.

\[ v_n^k = P_{N^2(0,1)}(r_n^k) \]

2. Overall confidence for each measurement by pooling local votes by all particles.

\[ \pi_k = \frac{\sum_{n=1}^{N} v_n^k}{\sum_{k=1}^{K} \sum_{n=1}^{N} v_n^k} \]

A particle’s vote is its measure of confidence in a measurement.
Mixture Weighting

• Gaussian Mixture Model (GMM) for overall measurement likelihood
  ▪ Multi-modalities by different measurement subsets

• Individual measurement likelihood
  ▪ Gaussian distribution
  ▪ Associated with confidence parameter

\[
p(m^K | x, S^K) = \sum_{k=1}^{K} \pi_k p(m_k | x, S_k)\]
Overall Architecture

- GNSS measurements

Particle distribution at time \( t \) and \( t-1 \)

- Particle RAIM filter

- Perturbation to particle distribution at time \( t \)

- Motion sample

- Particle distribution

Risk bounding

Integrity risk bound at time \( t \)
Probability of Hazardously Misleading Information (pHMI)

Initial particle distribution

Final particle distribution

pHMI computed as the weight allocated to particles outside Alert Limit
Probability of Hazardously Misleading Information (pHMI)

Initial particle distribution

Final particle distribution

Particle weights

Weight computation

pHMI is dependant on the perturbation to initial particles
Probability of Hazardously Misleading Information (pHMI)

Particle weights

Weight computation

Initial particle distribution

Final particle distribution

pHMI is dependant on the reference position
Computing Integrity Risk Bound

In the diagram, the following components are shown:

- **Empirical risk**
- **Divergence risk**
- **Mean-perturbation particle distribution**
- **Inverse Bernoulli Divergence**
- **Gap term**
- **Risk bounding**
- **PAC-Bayesian generalization error bound**

Mathematically, it is expressed as:

\[ R(\pi) \leq \hat{R}_N(\pi) + D_{\text{Ber}}^{-1}(\hat{R}_N(\pi), \epsilon) \]

Where:
- \( R(\pi) \) is the risk
- \( \hat{R}_N(\pi) \) is the empirical risk
- \( D_{\text{Ber}} \) is the inverse Bernoulli divergence
- \( \epsilon \) is a small positive number

The diagram also includes references to McAllester's work in Machine Learning, 1999 and 2003.
Bound Components: Empirical Risk

Empirical risk measures the average pHMI from perturbations

\[ \hat{R}_N(\pi) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{E}_{x \sim \pi} \left[ P(\|x - x_{un}\| \geq r) \right] \]

Mean-perturbation particle distribution

Expectation over reference positions

Average over initial perturbations
Bound Components: Divergence Risk

\[ D_{Ber}^{-1}(\widehat{R}_N(\pi), \epsilon) \]

**KL divergence from prior**

\[ \epsilon = \frac{1}{N} \left( KL(\pi || \pi_{prev}) + \log \frac{N + 1}{\delta} \right) \]

**Gap term**

**Provided error allowance**

Divergence risk estimates the uncertainty due to unseen perturbations
Outline

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  ▪ Integrity Risk Bound

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  ▪ Investigation Scenarios
  ▪ State Estimation Comparison
  ▪ Risk Bound Evaluation

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Investigation Scenarios

• Verification requirements
  ▪ Multiple runs with different trajectories
  ▪ Access to position ground truth of navigation agent
  ▪ GNSS pseudorange measurements and motion model availability
  ▪ Varied noise profiles under controlled settings

• Scenario 1
  ▪ 50 random simulated trajectories
  ▪ Simulated pseudorange and odometry measurements under varied noise settings similar to urban environments

• Scenario 2
  ▪ Highway driving with GNSS pseudorange and Doppler measurements
  ▪ Induced bias errors for imitating urban environments
  ▪ Odometry using Doppler measurements
Evaluation on Scenario 1

- Multiple runs with 50 different trajectories
- Access to true position of navigation agent
- Varied noise profiles under controlled settings
- Particle filter performs 2D state estimation with GNSS ranging measurements and odometry based motion model

Examples from a set of 50 random trajectories of length 4000m

- Start
- End
## Evaluation on Scenario 1

<table>
<thead>
<tr>
<th>Simulation Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td># of satellites</td>
<td>5</td>
</tr>
<tr>
<td>GNSS ranging variance</td>
<td>$15m^2$</td>
</tr>
<tr>
<td>GNSS ranging random bias</td>
<td>$100m$</td>
</tr>
<tr>
<td>Faulty range measurements</td>
<td>1-3</td>
</tr>
<tr>
<td>Bias switching probability</td>
<td>0.2</td>
</tr>
<tr>
<td>Motion variance</td>
<td>$1m^2$</td>
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<th>Filter Parameter</th>
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<tr>
<td># of particles</td>
<td>200</td>
</tr>
<tr>
<td>Measurement variance</td>
<td>$15m^2$</td>
</tr>
<tr>
<td>Propagation variance</td>
<td>$10m^2$</td>
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</tbody>
</table>

- GNSS ranging measurements have non-zero mean gaussian noise
State Estimation Comparison

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Particle RAIM tracks the state more accurately than snapshot RAIM.
State Estimation Comparison

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Particle RAIM tracks the state more accurately than Filter bank RAIM
## Experimental Setup

### Simulation Parameters

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<tr>
<td>Motion variance</td>
<td>$1m^2$</td>
</tr>
<tr>
<td>Number of runs</td>
<td>50</td>
</tr>
</tbody>
</table>

### Filter Parameters

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### Integrity Parameters

<table>
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<tbody>
<tr>
<td>Alert Limit</td>
<td>20m</td>
</tr>
<tr>
<td>Bound error allowance</td>
<td>0.1</td>
</tr>
<tr>
<td>Perturbation variance</td>
<td>$5m^2$</td>
</tr>
</tbody>
</table>

Integrity parameters chosen according to filter performance.
Risk Bound Evaluation

Risk bound successfully upper bounds the reference pHMI

# of perturbation samples : 30

Event Risk Bound Evaluation

Integrity risk bound

Reference pHMI

epoch

<p>| | |</p>
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</table>
Bound gap variation with perturbation samples

Perturbations: 5

Perturbations: 10

Perturbations: 30

Alert Limit | 20m
Bound error allowance | 0.1
Perturbation variance | $5m^2$

Bound improves with more perturbation samples
Empirical-Divergence Risk Tradeoff

Empirical risk contribution increases with more perturbation samples.

- Perturbations: 5
- Perturbations: 10
- Perturbations: 30

Alert Limit: 20m
Bound error allowance: 0.1
Perturbation variance: 5m²
### Performance Metrics

#### Noise parameters

<table>
<thead>
<tr>
<th>bias (m)</th>
<th># of faults</th>
<th>Failure rate (%)</th>
<th>Mean error (m)</th>
<th>Failure error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0</td>
<td>0</td>
<td>7.92</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.05</td>
<td>8.15</td>
<td>14.46</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.77</td>
<td>10.59</td>
<td>27.95</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.82</td>
<td>13.55</td>
<td>25.36</td>
</tr>
<tr>
<td>500</td>
<td>1</td>
<td>0.05</td>
<td>9.04</td>
<td>15.21</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.90</td>
<td>10.61</td>
<td>26.94</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.79</td>
<td>13.96</td>
<td>24.58</td>
</tr>
</tbody>
</table>

# of perturbation samples: 10
# of runs: 50
Alert limit: 20m

- **Failure rate**: % of cases when bounding fails
- **Mean error**: mean filter error across all runs
- **Failure error**: mean filter error in failure cases

Bound performance is robust to large bias
Evaluation on Scenario 2

- Comma2k19 dataset\textsuperscript{[10]}
  - Driving on California’s 280 highway: almost open sky environment
  - 1 minute segment from 2019 segments
  - Limited GNSS measurements (upto 5) with random bias added to a few measurements (upto 2) to imitate urban environment
  - Motion model based on odometry from Doppler measurements
  - “Ground truth” using all the GNSS measurements along with GNSS-INS-Visual Odometry fusion and post-processing (Estimated RMSE <2m)\textsuperscript{[10]}

\textsuperscript{[10]} Schafer et. al, \textit{Arxiv}, 2018
Evaluation on Scenario 2

Particle RAIM Filter (MSE 8.3m)

Integrity risk bound

Successful state estimation and risk bounding on real-world data

# of perturbation samples : 20
Alert limit: 20m
Conclusion

• Devised a framework to compute fault-aware position distribution and associated integrity risk bounds

• Integrated traditional RAIM algorithm with particle filter framework using Gaussian mixtures for measurement model

• Leveraged PAC-Bayesian generalisation error bounds from Statistical Learning Theory for upper bounding the integrity risk

• Experimentation using multiple trajectories (simulated and real) validates that the
  ▪ Algorithm is able to accurately perform localization in scenarios with various faults
  ▪ Computed integrity risk bound is robust on biases in measurements
  ▪ Integrity risk bound provides a low failure rate upper-bound on probability of hazardously misleading information
Acknowledgement

This material is based upon work supported by the National Science Foundation under award number 1750864.
Extra Slides
Risk Bound with Open Sky Measurements

Particle RAIM Filter (MSE 5.8m)

Integrity risk bound

Good state estimation yields lower risk bounds

# of perturbation samples: 20
Alert limit: 20m
Risk Bound with Multiple Sensors

Particle RAIM Filter (MSE 1.4m)

Integrity risk bound

Incorporating additional sensors allows bounds with lower Alert Limit

- Particle distribution
- Ground truth
- Empirical risk
- Divergence risk
- Integrity risk bound
- Reference pHMI

# of perturbation samples: 20
Alert limit: 5m