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(but not all) Patents are Essential**

By
Daniel Quint
Stanford University
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Stanford Institute for Economic Policy Research
Stanford University
Stanford, CA 94305
(650) 725-1874

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ECONOMICS OF PATENT POOLS WHEN SOME (BUT NOT ALL) PATENTS ARE ESSENTIAL

DANIEL QUINT¹

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ABSTRACT. Patent pools are agreements by multiple patent owners to license certain patents to third parties as a package, and often form in conjunction with the development of a technological standard. A key distinction made by regulators – between patents which are *essential* to a standard and patents for which suitable substitutes exist – has not been captured in existing economic models. I present a model of competition among differentiated technologies, in which some patents are essential and some are not. I show that pools of essential patents are Pareto-improving whenever they occur, while pools of nonessential patents can be welfare-negative, even when the included patents are all complements. I discuss conditions under which certain pools are likely to form, the “outsider problem” which makes some pools inefficiently small, and the effects of compulsory individual licensing.

1 Introduction

When firms with market power sell complementary goods, their combined prices will typically be higher than under a single monopolist. This effect, first understood by Cournot (1838) and later termed “double marginalization,” can be particularly severe in the context of intellectual property. In high-tech fields where innovation is rapid and cumulative, a large number of patents may block the same new technology; double marginalization can make the technology expensive to commercialize, harming downstream producers and consumers as well as the innovators the patent system was designed to reward.

For over 150 years, one tool used to combat this problem has been a *patent pool* – an agreement by multiple patentholders to share a group of patents among themselves or to license them as a package to third parties. Patent pools sometimes form in

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conjunction with the development of a technological standard, as with the MPEG and DVD-video standards in the late 1990s. When patents in a pool are complements, the pool can lower their combined price and increase licensing revenues, as well as reduce transaction costs (by reducing the number of individual licensing agreements required to make use of the technology) and the risk of holdup by the final patentholders.

However, patent pools have also been used to eliminate competition between rival technologies, and even to administer cartels. The Hartford-Empire pool of glassware manufacturing technologies, broken up in 1942, used restrictive licensing terms to effectively maintain production quotas and discourage entry into the market. More recently, in 1998, the FTC challenged a pool containing patents related to two different types of lasers used for laser-eye surgery, claiming that the pool was anticompetitive.

When considering patent pools related to technological standards, a key distinction has been made by U.S. regulators between patents which are *essential* to comply with the standard and patents for which suitable substitutes exist. The inclusion of only essential patents in a pool is seen as a “competitive safeguard” to ensure that the pool does not have anticompetitive effects; both the DVD and MPEG pools are limited to essential patents, and provide for independent experts to determine which patents are essential.²

While patent pools in the United States have a history going back over a century, attempts to rigorously model their economic effects have begun only in the last several years. As I discuss below, the models in the existing literature treat patents as identical. Thus, they fail to distinguish between essential and nonessential patents, treating only the cases where users need licenses to all of the patents (and therefore all are essential), or where any subset of the patents of a particular size is sufficient (and therefore no patent is essential). There are no models of patent pools in an environment in which essential and non-essential patents co-exist. As the title suggests, this is the aim of this paper: to understand the economic effects of a patent pool in an environment where some, but not all, patents are essential.

To do this, I introduce a model of price competition among differentiated technologies. I assume the existence of some essential patents, which block all of the technologies; and some nonessential patents, each of which block only one technology. Given the licensing fees demanded by each patentholder, potential producers choose whether to license the patents required to commercialize one of the technologies, and earn profits accordingly. The results I find are as follows:

- A patent pool containing **only essential patents** reduces the prices of all technologies, increases producer surplus, and increases the profits of every patentholder outside the pool. Such a pool therefore represents a Pareto-improvement

²In business letters issued by the Department of Justice in 1998 and 1999 in regard to the proposed MPEG and DVD pools, Joel Klein writes, “One way to ensure that the proposed pool will integrate only complementary patent rights is to limit the pool to patents that are essential to compliance with the Standard Specifications. Essential patents by definition have no substitutes; one needs licenses to each of them in order to comply with the standard.”

whenever it is profitable for its participants. It is likely to form when there are sufficiently many essential patents, but may be inefficiently small when each patentholder can opt out without disrupting pool formation.

- A patent pool containing **complementary nonessential patents**, or the inclusion of complementary nonessential patents in a pool of essential patents, reduces the price of one technology, but may increase the prices of others. It tends to increase total producer surplus, but may hurt some individual producers; it increases the profits of some outside patentholders, and decreases the profits of others. The overall welfare effect is likely to be positive, but may be negative if the patents being pooled are already inexpensive relative to their substitutes.
- A pool containing **substitute patents** decreases producer surplus and is likely to decrease total welfare.
- Unlike in previous models, robustness to **compulsory individual licensing** is not a sufficient screen for efficiency: pools of complementary nonessential patents are always robust to individual licensing, and are sometimes welfare-negative.

The rest of this paper proceeds as follows. Section 2 discusses existing literature on patent pools. Section 3 introduces a model of competition among patentholders in a differentiated-products setting, and characterizes the equilibrium of the model. Section 4 analyzes the effects (on equilibrium prices and welfare) of certain types of patent pools, and the conditions which favor their formation. Section 5 discusses limitations of the current model and possible extensions. Section 6 concludes. All claims in the text (including those stated informally) are proved in the appendix.³

2 Related Literature

Gilbert (2004) gives a detailed history of antitrust treatment for patent pools, going back to the beginning of the 20th century. He emphasizes the importance of patents in a pool being complements rather than substitutes, and models a few simple cases. He also discusses examples of patent pooling arrangements with restrictive terms being used to enforce cartels, or to prevent the challenging of weak patents (patents unlikely to be upheld in court).

Shapiro (2001) emphasizes the double-marginalization problem of complementary patents. He uses a simple model of Cournot competition to show that a patent pool will lower prices and increase welfare when patents are perfect complements,

³Some lengthy but mechanical calculations can be found in a separate technical appendix, available at my website, <http://www.stanford.edu/~dqint>, under the “papers” link.

and raise prices and destroy welfare when patents are perfect substitutes. He also discusses cross-licensing arrangements and standard-setting boards.

Lerner and Tirole (2004) present a model in which patents need not be either perfect substitutes nor perfect complements. They model a world with n identical (separately-owned) patents and a downward-sloping demand curve provided by a continuum of potential users who derive value based on the number of patents they license. They show that a pool containing all n patents is more likely to be welfare-enhancing when patents are more complementary; and that forcing pool participants to also offer their patents individually (Compulsory Individual Licensing) destabilizes “bad” pools without affecting “good” ones.

Brenner (2005) extends the Lerner and Tirole framework to consider smaller (“incomplete”) pools containing only some of the patents. He explicitly models the fact that some patentholders may do better by remaining outside of the pool, and examines which pools will form under different formation procedures. Brenner compares the welfare achieved under a particular formation protocol to the outcome without a pool, and shows that compulsory individual licensing is a good screen for efficiency under this formation rule.

Aoki and Nagaoka (2005) similarly show that even when a patent pool is welfare-enhancing for the buyers and members, it is often better to remain outside the pool. They employ a sequential coalition-formation model (based on Maskin’s (2003) framework of coalition games with externalities) to show that the grand coalition will not always form. They also consider firms which hold patents and compete in the downstream market, and discuss three particular examples of standards board-based patent pools.

The main limitation of the existing models is that they model the patents in question as interchangeable. That is, in all of these models, users derive value based on the number of patents they license, not which ones. This means that either all or none of the patents are essential. In addition, users are only minimally heterogeneous – every user who enters the market will choose to license the same set of patents. By presenting a model of competing technologies blocked by some essential and some nonessential patents, I try to address this limitation.

3 Model

3.1 Players, Strategies, Payoffs

My model is a static model of price competition among patentholders, who license their patents to producers. Like the prior literature, I take the set of patents, and the technologies they block, as given. I also do not explicitly model the formation of patent pools; my model instead measures the effect of a given pool on equilibrium prices and welfare, taking its existence as exogenous. (I do discuss what conditions make a particular pool profitable, and therefore more likely to form. These limitations

are discussed in Section 5.)

Formally, the players are the set $T = \{1, 2, \dots, T\}$ of patentholders. Strategy sets are the licensing fee each patentholder charges, $p_i \in A_i = \mathfrak{R}^+$, and payoffs are licensing revenues, $u_i = p_i q_i(p_i, p_{-i})$.⁴ I assume that patentholders name prices simultaneously, and that each patent is individually owned (or multiple patents held by the same owner are licensed together as a portfolio).

Next, I assume a set $\mathcal{K} = \{1, 2, \dots, K\}$ of distinct *technologies* which are blocked by one or more of these patents. The technologies are substitutes for each other, and do not have any other close substitutes outside of the set \mathcal{K} . In the laser-eye example, the two technologies were distinct types of lasers; more generally, they could be different techniques to manufacture the same good, or different components to include in a product.

Finally, I introduce a measure 1 of *producers* $l \in L$ who can profit from access to these technologies. The producers are heterogeneous; if producer $l \in L$ gains access to technology $k \in \mathcal{K}$, his profit is

$$v_k + \epsilon_k^l - P_k$$

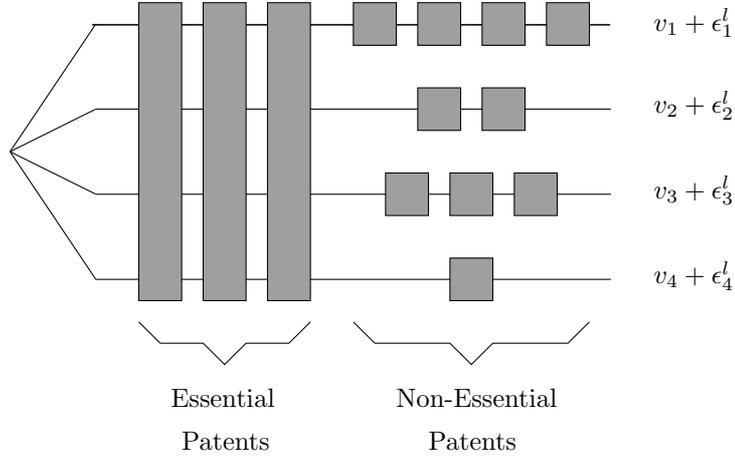
where v_k is a common term reflecting the value of the technology, ϵ_k^l is an idiosyncratic term specific to the producer/technology pair, and P_k is the total cost to license the required intellectual property. Producers do not gain from accessing more than one technology; their payoff from not accessing any of the technologies is ϵ_0^l . I assume that the ϵ terms are independent and identically distributed; I will make a further assumption later about their probability distribution.

I assume that the patents fall into two categories: fundamental patents, which block *all* of the technologies, and more narrow patents, which block *only one* of the technologies.⁵ Since producers must license each of the former or exit the market entirely, I refer to these patents as *essential*; since producers can find substitutes for the latter (by switching to a different technology), I refer to these as *non-essential*. We can visualize the problem facing a given producer as follows, with each horizontal line representing a technology (a path to a given payoff) and the rectangles blocking each path the patents which must be licensed to use the technology:

⁴For simplicity, I assume that patentholders' only revenue is from licensing, that is, patentholders do not compete in the downstream market.

⁵I discuss later a special case in which the model can be extended to allow for patents which block arbitrary subsets of the technologies.

An Example: 4 Technologies and 13 Patents



Let T^E denote the set of essential patents, and T_k^N the set of non-essential patents which block a particular technology k . The demand for a given technology k , and all the patents which block it, is the measure of producers for whom

$$v_k + \epsilon_k^l - \sum_{i \in T^E \cup T_k^N} p_i = \max \left\{ \epsilon_0^l, \max_{k' \in \mathcal{K}} \left\{ v_{k'} + \epsilon_{k'}^l - \sum_{i \in T^E \cup T_{k'}^N} p_i \right\} \right\}$$

Note that I assume there is no bargaining between patentholders and producers, and that essential patentholders do not charge different prices to producers using different technologies. Our framework is similar to the differentiated-products framework explored in Caplin and Nalebuff (1991) and elsewhere. The results there do not extend directly to our setting, since the “goods” in our model are aggregate goods, that is, the price of a technology is the sum of prices set by multiple players.

3.2 Interpretation of the Model

There are two natural interpretations of the model. The first is as it is written – technologies are different manufacturing techniques, and producers are differentiated by their aptitude at each technique. Patent licensing fees are paid as lump sums, and producers are local: they do not compete with each other (since one producer’s decision does not affect another’s profits). Producers are seen as the “end users” of each technology, since consumers are not modeled. However, if consumers are homogeneous within each producer’s locale, or if producers are able to price-discriminate, then consumer surplus is zero and is not missing from the model.

A second (perhaps more conventional) interpretation is that it is consumers, not producers, who have the stochastic utility functions. Between patentholders and consumers is a layer of perfectly-competitive producers with no fixed costs and constant,

identical marginal costs for products made with each technology. Patentholder prices are per-unit licensing fees, producers earn zero profits, producer surplus as described in the paper actually accrues to consumers, and the analysis is otherwise unchanged, except that the mean value of each technology is net of the producers' marginal cost. (This interpretation also answers the question of why patentholders do not compete with producers in the downstream market; they can, but earn no additional profits, so the analysis does not change.) Under this interpretation, it seems more natural to see the technologies as different products, or as different bundles of components. Thus, the model could also be used to consider the bundling of consumer goods, or the pricing of aggregate products made up of components supplied by different firms.

3.3 Equilibrium

Within this general setup, a particular game is defined by the following primitives:

- The number of technologies $K = |\mathcal{K}|$
- The distribution F from which the idiosyncratic terms ϵ_0^l and ϵ_k^l are drawn
- The mean value of each technology, (v_1, v_2, \dots, v_K) , which I will abbreviate \mathbf{v}
- The number of essential patents, $n_E \equiv |T^E|$, and the number of nonessential patents blocking each technology k , $n_k \equiv |T_k^N|$, which together I will abbreviate $\mathbf{n} \equiv (n_E, n_1, n_2, \dots, n_K)$

I will refer to “aggregate prices” as the sums of prices demanded by each set of similar patentholders:

- $P_k^N \equiv \sum_{i \in T_k^N} p_i$ is the combined price of all the nonessential patents blocking technology k
- $P^E \equiv \sum_{i \in T^E} p_i$ is the combined price of all the nonessential patents
- $P_k \equiv P^E + P_k^N$ is the total price to access technology k

I make the following assumption about the distribution of idiosyncratic terms ϵ_k^l :

Assumption 1. ϵ_0^l and ϵ_k^l are *i.i.d* across producers and technologies. The distribution F from which they are drawn is strictly increasing on $(-\infty, \infty)$, and F and $1 - F$ are log-concave.⁶

This condition is sufficient to begin to understand the equilibrium prices demanded by patentholders.

Lemma 1. Fix a game $G = (|\mathcal{K}|, F, \mathbf{v}, \mathbf{n})$.

⁶This is equivalent to the assumption that both ϵ and $-\epsilon$ have increasing hazard rates.

1. An equilibrium exists and is unique
2. The equilibrium value of P^E is increasing in n_E and decreasing in (n_1, n_2, \dots, n_K)
3. The equilibrium value of P_k^N is decreasing in n_E and increasing in (n_1, n_2, \dots, n_K)
4. The total price P_k of technology k is increasing in n_E and in n_k (but $P_{k'}$ ($k' \neq k$) may be increasing or decreasing in n_k)

I show in the appendix that Lemma 1 follows from a set of properties of the demand system – log-demand for each technology is concave in its own price, has increasing differences in its own price and the price of another technology, and satisfies a dominant diagonal condition as in Milgrom and Roberts (1990), and similar conditions hold for the combined demand for all the technologies. These properties occur naturally in our setting under Assumption 1. The pricing game among patentholders is not a supermodular game, due to strategic substitutability between players in the same grouping (T^E or T_k^N); but equilibrium can be shown to be symmetric among players within each grouping, and each set of players can therefore be replaced by an “aggregate” player who mimics their combined actions. The resulting $K + 1$ -player game is a supermodular game when log-payoffs are considered and the sign of the “essential” player’s price is reversed, and is indexed by $(-n_E, n_1, n_2, \dots, n_K)$; the results follow.

To make sharp welfare predictions, we will require one additional regularity condition on the demand for each technology. Since the “aggregate players” do not maximize profits, it is possible for a “positive” change – an increase in the price of a rival technology – to lead to a sufficiently strong overreaction in the price of another technology that patentholders blocking that technology are left worse off. We impose a condition which will rule out this sort of perverse result.

Assumption 2. *The log of the inverse demand function⁷ $P_k(q, \cdot)$ has increasing differences in q and $P_{k'}^N$ ($k' \neq k$), and in q and $-P^E$; and $\log P^E(q, \cdot)$ has increasing differences in q and $-P_k^N$.*

Assumption 1 implies increasing differences in the log-demand functions – an increase in one price raises the demand for a competing technology, but also lowers the price-elasticity of demand for that technology. This implies that an oligopolist pricing a single technology would respond to an increase in a rival technology’s price by raising his own price. Assumption 2 implies that the increase would be small enough to maintain a higher market share than before. The condition holds for logit

⁷Formally, the inverse demand functions $P_k^N(q, \cdot)$ and $P^E(q, \cdot)$ are defined implicitly by

$$\begin{aligned} Q_k(P_1^N + P^E, \dots, P_k^N(q, \cdot) + P^E, \dots, P_K^N + P^E) &= q \\ \sum_{k=1}^K Q_k(P_1^N + P^E(q, \cdot), \dots, P_K^N + P^E(q, \cdot)) &= q \end{aligned}$$

where $Q_k(\cdot)$ is the demand for technology k at a given set of prices.

demand; I expect it to hold more generally, but I have not yet found general conditions on F which guarantee it.

Under Assumptions 1 and 2, we can make precise predictions about the impact of \mathbf{n} on equilibrium payoffs. Let u_k denote the equilibrium profit of each patentholder in T_k^N , and u_E the equilibrium profit of each patentholder in T^E :

Theorem 1. *Under Assumptions 1 and 2,*

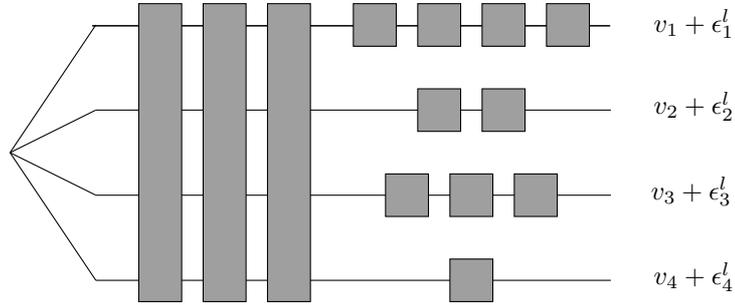
1. $(u_E, u_1, u_2, \dots, u_K)$ are all decreasing in n_E
2. u_E and u_k are decreasing in n_k ; for $k' \neq k$, $u_{k'}$ is increasing in n_k

I show in the next section that many patent pools can be evaluated as changes to the parameters n_E and n_k .

4 The Effects of Patent Pools

4.1 Modeling Pools as Changes in \mathbf{n}

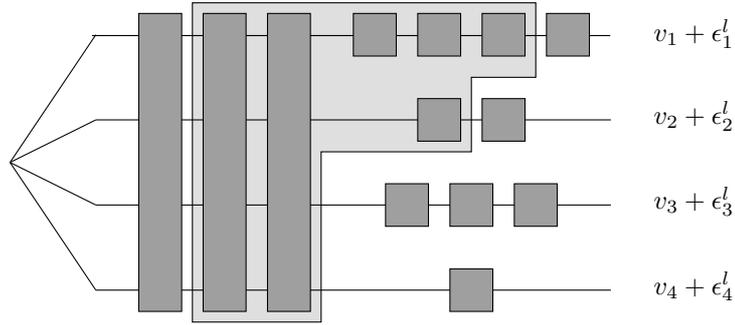
To illustrate my general treatment of patent pools, I begin with the technology diagram from earlier:



Recall that a game is characterized by the primitives $|\mathcal{K}|$, F , $\mathbf{v} = (v_1, v_2, \dots, v_K)$, and $\mathbf{n} = (n_E, n_1, n_2, \dots, n_K)$; this particular game is

$$G_1 = (4, F, (v_1, v_2, v_3, v_4), (3, 4, 2, 3, 1))$$

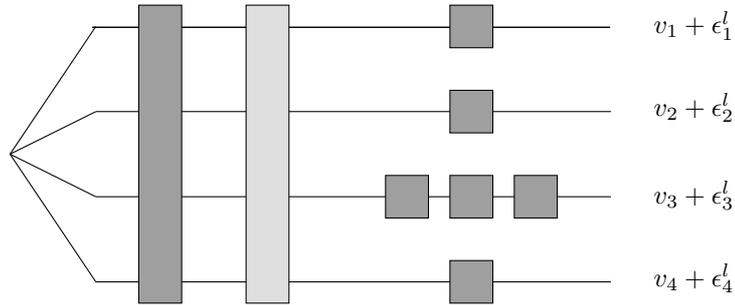
Suppose that a patent pool were to form, consisting of two of the essential patents, three of the nonessential patents blocking technology 1, and one of the nonessential patents blocking technology 2:



Once the pool had formed, the optimization problem it faced would be the same as the one facing an essential patentholder: naming a price, which would be paid by every producer using any of the technologies, to maximize revenue.⁸ Thus, the pool behaves as if it were a single essential patent in a different game with fewer competitors. Equilibrium prices of the game G_1 following the formation of this pool would be equivalent to equilibrium prices in the game

$$G_2 = (4, F, (v_1, v_2, v_3, v_4), (2, 1, 1, 3, 1))$$

pictured below:



Thus, we can compare equilibrium prices before and after the formation of this pool by comparing equilibrium prices in the game G_1 with those in the game G_2 . Since aggregate payoff measures can be expressed as functions of prices, these can be compared as well. And we can begin to answer the question of whether this pool might be likely to form by comparing the joint profits of these six patentholders in the game G_1 to those of a single essential patentholder in the game G_2 .

More generally, a pool containing m_E essential patents and (m_1, m_2, \dots, m_K) nonessential patents blocking the various technologies would transform the game $G = (|\mathcal{K}|, F, \mathbf{v}, (n_E, n_1, \dots, n_K))$ into the game

$$G' = (|\mathcal{K}|, F, \mathbf{v}, (n_E + 1 - m_E, n_1 - m_1, n_2 - m_2, \dots, n_K - m_K))$$

⁸For now, I assume that patents in the pool are only available through the pool. I consider the effect of independent licensing later on.

provided either $m_E > 0$ or $\min_{k \in \mathcal{K}} m_k > 0$, that is, provided the pool contained a patent blocking every technology. A pool of m_k nonessential patents blocking a single technology k would have the same effect as a reduction of n_k to $n_k + 1 - m_k$. (Pools containing no essential patents, and nonessential patents blocking more than one but not all of the technologies, cannot be analyzed in this way in general; as I discuss below, in the special case of logit demand, these can be considered as well.)

Below, I use this approach to examine the effects various patent pools would have on equilibrium prices and payoffs. I begin by considering pools containing only essential patents; from a welfare perspective, the results are favorable. Next, I consider pools containing some nonessential patents, but containing only patents which are pairwise complements; the results are more ambiguous. Finally, I consider two examples of pools containing rival, or substitute, patents; the results are mostly negative.

4.2 The Effect of Prices on Total Welfare

Total welfare is simply the sum of all patentholders' and producers' payoffs. Transfers between players (payments from producers to patentholders) are welfare-neutral; the only source of "value" in the model is the gross profits of each producer,

$$v_k + \epsilon_k^l$$

Thus, there are two sources of inefficiency: producers who could potentially earn positive profits using some technology, but are priced out of the market by licensing fees; and producers who are priced into the "wrong" technology. A decrease in P^E lowers the prices of all technologies by the same amount; it does not change the technology choice of any active producers, but allows more producers into the market, creating value. Thus, total welfare is always decreasing in P^E . A decrease in P_k^N , on the other hand, lowers only the price of technology k . This allows new producers into the market for technology k , creating value; and it induces some producers to switch to technology k , away from other technologies. If technology k was relatively cheap, however, the producers who switch reduce total welfare; and if prices are sufficiently unequal, this effect can dominate.

These effects are easiest to see in the case of logit demand, discussed below. In the case of logit demand, the welfare effects of price changes are

$$\frac{\partial \text{Welfare}}{\partial P_k^N} = Q_k (\bar{P} - P_k) \quad \text{and} \quad \frac{\partial \text{Welfare}}{\partial P^E} = - \left(1 - \sum_{k' \in \mathcal{K}} Q_{k'} \right) \bar{P}$$

where Q_k is the demand for a given technology k and $\bar{P} = \sum_{k' \in \mathcal{K}} Q_{k'} P_{k'}$ is the average price paid by all the producers (including those who license no patents).

As it happens, this same intuition extends to the formation of certain patent pools. From a welfare perspective, the dominant effect of a pool of essential patents is to lower the prices of *all* technologies, increasing welfare. The dominant effect of a

pool of non-essential patents blocking a single technology is to lower the price of that particular technology, which can have a positive or negative welfare effect, depending on whether that technology is relatively expensive or cheap.

4.3 General Patent Pool Results

Previous work has suggested that pools are generally welfare-positive when the patents being pooled are perfect complements. However, I offer contrasting results on pools of complementary patents: pools of essential patents are Pareto-improving when they occur, but pools of complementary nonessential patents can be welfare-destroying.

Theorem 2. *A pool containing only essential patents will:*

- *Lower the price of each technology*
- *Increase the surplus of each individual producer*
- *Increase the profits of every patentholder outside of the pool*

Thus, when such a pool is profitable for its members, it represents a Pareto-improvement.

Pools of essential patents correspond to a decrease in n_E ; the results therefore follow from Lemma 1 and Theorem 1. This is the natural analog to the perfect-complements case in previous models.

What ensures that these pools are Pareto-improving is not that the patents in the pool have no substitutes within the pool, but that they have no substitutes *outside* the pool either. In contrast, consider pools containing some nonessential patents, but only ones blocking the same technology, so that the patents in the pool are all complements:

Theorem 3. *Consider a pool of nonessential patents which block a single technology k , or the addition of these patents to an existing pool of essential patents. The effects will be:*

- *A decrease in the price P_k of technology k*
- *An increase in the profits of the essential patentholders, and in the profits of nonessential patentholders who block technology k but remain outside the pool*
- *A decrease in the profits of nonessential patentholders blocking the other technologies*

The total prices of the other technologies $P_{k'}$ may increase or decrease, and the net effect on welfare may be positive or negative.

This type of pool corresponds to a decrease in n_k ; the results again follow from Lemma 1 and Theorem 1. In the next section, I offer an example where the sign of the welfare effect can be calculated, and where the effect of this pool on total producer surplus is always positive.

Pools containing some essential patents and some nonessential patents blocking a single technology k will correspond to decreases in both n_k and n_E . They will therefore lower the price of technology k , increase the profits of nonessential patentholders who block technology k and remain outside the pool, and increase the profits of essential patentholders who remain outside the pool; the other effects are ambiguous.

4.4 Sharper Results under Logit Demand

In this section, I illustrate and sharpen these results, and show some new ones, under the special case of logit demand. I assume that F (the distribution of the ϵ_k^l terms) is the standard double-exponential distribution,⁹ leading to market shares

$$Q_k = \frac{e^{v_k - P_k}}{1 + \sum_{k'=1}^K e^{v_{k'} - P_{k'}}$$

for each technology k .

Logit demand is appealing for a number of reasons. First, under logit demand, existence and uniqueness of equilibrium extend to a more general model in which each patent is allowed to block an arbitrary subset of the technologies, rather than all or only one. In this more general model, it can be shown that for any two patentholders i and j ,

$$\frac{\partial^2 \log u_i}{\partial p_i \partial p_j} = \frac{\partial^2 \log u_j}{\partial p_i \partial p_j}$$

This ensures existence of a *potential function* for the game, that is, a continuous and differentiable function $P : A_1 \times A_2 \times \dots \times A_{|T|} \rightarrow \Re$ with

$$\frac{\partial P}{\partial p_i} = \frac{\partial \log u_i}{\partial p_i}$$

(See Monderer and Shapley (1996) for more on potential games.) The potential function P can be shown to be strictly concave, with the equilibrium of the game corresponding to the unique maximum of P , ensuring uniqueness and making it straightforward to calculate equilibrium prices numerically.

Second, in our original game where each nonessential patent blocks only a single technology, equilibrium prices under logit demand can be characterized as the unique

⁹ $F(x) = \exp(-\exp(-(x + \gamma)))$, where $\gamma \approx 0.5772$ is Euler's constant. See McFadden (1974) or Anderson, de Palma and Thisse (1992) for more on logit demand.

solution to the system of equations

$$\begin{aligned}
P_1^N(1 - Q_1) &= n_1 \\
P_2^N(1 - Q_2) &= n_2 \\
&\vdots \\
P_K^N(1 - Q_K) &= n_K \\
P^E(1 - Q_1 - Q_2 - \dots - Q_K) &= n_E
\end{aligned}$$

The parameters n_E and n_k were defined as numbers of patentholders, but there is nothing lost by treating them here as continuous variables. Price elasticity terms have a closed form, so these equations can be differentiated with respect to a given parameter n_i , and the effect of a change in n_i on equilibrium prices (and therefore payoffs and welfare) can be explicitly calculated, leading to a clearer understanding of the equilibrium effects of particular pools.

We return to the pool of complementary nonessential patents considered in Theorem 3, with the additional assumption of logit demand:

Result 1. *Suppose a pool formed consisting of m out of the n nonessential patents which blocked technology k . Under logit demand, the pool is certain to increase total producer surplus, although some individual producers may be hurt. The total price of technology $k' \neq k$ will increase if*

$$P^E \geq P_{k'}^N \frac{Q_{k'}}{1 - Q_{k'}}$$

before the pool formed. The net effect of the pool on total welfare can be expressed as

$$\int_{n-m+1}^n \left[\sum_{k' \in \mathcal{K}} w_{k'} (P_k - P_{k'}) + w_0 P_k \right] dn_k$$

where $w_{k'}$ and w_0 are positive weights which vary with n_k .

It is clear that when P_k is high relative to the prices of the other technologies, the integrand in the welfare expression is positive, and the pool, though not Pareto-improving, is overall welfare-positive. However, when P_k is relatively low, the integrand may be negative, so the pool is potentially welfare-destroying.

Example. *Consider a model with three technologies, logit demand, $\mathbf{v} = (10, 10, 5)$, $n_E = 1$, and $n_1 = n_2 = n_3 = 3$. A pool consisting of all three nonessential patents blocking technology 3 nearly doubles the profits of its members, and increases total producer surplus; but it raises the total prices of the other two technologies and is overall welfare-negative.¹⁰*

¹⁰A pool of the three nonessential patents blocking technology 2, on the other hand, would be welfare-enhancing; but is unlikely to occur, as it would decrease the revenue of its participants. Equilibrium prices and payoffs for this (and later) numerical examples are given in the appendix.

In the case of logit demand with only two competing technologies, however, this particular combination – a profitable but welfare-destroying pool of nonessential complements – cannot occur:

Result 2. *With logit demand, when $K = 2$, a pool of nonessential patents blocking the same technology is welfare-positive if it is profitable for its members.*

4.5 Two Examples of Pools Containing Substitutes

Earlier papers have shown that when pooled patents are not available to be licensed separately, pools of substitute patents lead to higher prices and lower welfare. I give two examples along the same lines in the current setting.

Result 3. *Under logit demand, a pool consisting of two substitute patents (nonessential patents blocking different technologies) is certain to decrease total producer surplus.*

Result 4. *Under logit demand, a pool consisting of one nonessential patent blocking each technology is certain to decrease total welfare.*

Of course, a pool containing several nonessential patents blocking each technology might well be welfare-enhancing overall. However, we could decompose the formation of that pool into the formation of K smaller pools, each containing several nonessential patents blocking a single technology; and then the merging of all these pools into a single one. Result 4 states that while the net effect may be welfare-positive, this last step – combining competing pools into a single one – would be inefficient, leaving total welfare lower than if separate pools were maintained.

4.6 Conditions Favoring Pool Formation

I can begin to answer the question of when a particular pool is likely to form by considering the conditions under which it would increase the profits of its participants. All the results in this section are for a model with logit demand. (Results stated informally here are stated more formally, and proved, in the appendix.)

Not surprisingly, when the number of essential patentholders is sufficiently large, the double-marginalization problem is severe, and a pool containing all, or most, of the essential patents will always be profitable.¹¹ However, in Theorem 2, we saw that a pool of essential patents has a positive externality on the patentholders outside the pool. Essential patentholders are able to capture much of the benefit of pool formation without joining the pool. As in many coalitional games with externalities, this leads to an “outsider problem”: if 19 patentholders have already joined a pool,

¹¹Formally, fix $m \geq 0$ and the primitives of a game G except for n_E ; if n_E is sufficiently large, a pool of all but m of the essential patents increases the profits of its participants.

the twentieth may prefer what he can get on his own to 1/20 of the pool’s revenue after he joins, even if the combined profits are higher with him in the pool.

This outsider problem is considered in Aoki and Nagaoka and in Brenner. Aoki and Nagaoka base their model on Maskin’s (2003) axiomatic approach to coalition formation in games with externalities. When all patents are essential, they show that the grand coalition will only occur when the number of patentholders is small; for large numbers of patentholders, “the emergence of [an] outsider is inevitable, so that. . . voluntary negotiation cannot secure the socially efficient outcome.” Brenner proposes an “exclusionary” formation rule, under which a patent pool proposed by one player must be unanimously accepted by its members or fail to form; this allows the grand coalition to be achieved when it is efficient but could not be reached by sequential negotiations.

In both Brenner’s and Aoki and Nagaoka’s models, the grand coalition maximizes the total profits of the patentholders, but may not be achieved because a single patentholder would prefer to remain outside the pool. In our setting, the problem is even more severe. While a pool of all the essential patents is likely socially optimal (relative to a smaller pool), it may *not* maximize the joint profits of the essential patentholders due to strategic effects, since nonessential patentholders raise their prices as more essential patents join the pool.

Example. *Return to our previous example, with three technologies, $\mathbf{v} = (10, 10, 5)$, and $n_1 = n_2 = n_3 = 3$, but this time with $n_E = 6$. A pool of all six essential patents increases their profits. However, once five of them have formed a pool, the addition of the final essential patentholder **reduces** their combined profits.*¹²

As in previous models, while patentholders might prefer joining a large pool to the pool failing to form, if individual patentholders can opt out without destroying the pool, “good” pools may not reach their efficient size. Such “outsider” problems do occur in reality. In 1995, the DVD standard was agreed to by ten firms; per Aoki and Nagaoka, “While they originally aimed at providing a one-stop shopping facility for licensing the standard information, essential patents, and logo, Thomson decided to license its patents independently.” The remaining nine firms split into two patent pools, one containing the essential patents of six firms and one containing the essential patents of the other three. Thus, even when a pool is desirable, there remains the question of how it can be achieved.¹³

¹²We can show generally that with logit demand, while n_E remains sufficiently large, each incremental entrant into the pool increases the combined profits of the pool and the new entrant. This is shown in the appendix, along with a sufficient condition for this not to hold when n_E is small.

¹³When a pool forms in conjunction with the development of a technological standard, the set of patents which are essential is endogenous, as it depends on the standard selected. Holdout in a standard-setting context also occurs. In August 2006, the Federal Trade Commission ruled against chipmaker Rambus for “taking part in JEDEC’s DRAM standards-setting activities. . . .while secretly developing patents involving specific memory technologies ultimately adopted in JEDEC’s standards” (see Chin (2006)). Similar accusations against Dell during the setting of the VL-bus

As with essential patents, a large number of nonessential patents blocking the same technology will favor pool formation. We can also consider the incentive to allow nonessential patents into an existing pool of essential patents. By Theorem 3, this would impose a positive externality on the essential patentholders outside of the pool, and the other nonessential patentholders blocking the same technology; when the pool is able to capture enough of this externality, then adding nonessential patents to the pool is profitable. Thus, when a pool already contains all of the essential patents, and is able to add all the nonessential patents blocking a given technology, this will always increase the combined profits of the pool and the patents being added. However, when the pool does not contain every essential patent or only a single patent is added, there are conditions under which the combined profit of the pool and the newly added patentholder go down.

Example. *Continuing to rely on the same example as before: if a pool has already formed containing all the essential patents, then adding one (of the three) nonessential patents which block technology 2 would decrease the combined profit of the pool and the newcomer; adding two or three, however, would increase combined profits.*

4.7 Compulsory Individual Licensing

Lerner and Tirole propose compulsory individual licensing – mandating that participants in a patent pool also offer their patents individually – as a solution to welfare-negative pools. Define a pool to be *stable* to individual licensing if equilibrium prices are the same with and without individual licensing, and *weakly unstable* to individual licensing if in at least some equilibria, after the pool names its profit-maximizing price, individual licensing causes prices to revert to their pre-pool levels. Lerner and Tirole show that in their setting, welfare-positive pools are stable to compulsory individual licensing, while welfare-negative pools are at least weakly unstable. Thus, they claim that compulsory individual licensing functions as a screen for efficient pools.

In Brenner’s setting (an extension of Lerner and Tirole’s model where pools smaller than the grand coalition are considered), all welfare-positive pools are stable to individual licensing, but some welfare-negative pools may be as well. However, pools which form in equilibrium under an “exclusive” formation rule – where participants must agree unanimously to a proposed pool or it does not form, rather than patentholders being able to opt in or out individually – are stable to individual licensing if and only if they are welfare-enhancing.

In our setting, compulsory individual licensing does not always distinguish welfare-positive from welfare-negative pools. In particular:

Theorem 4. *Pools containing only essential patents are stable to individual licensing.*

Pools containing only complementary nonessential patents are also stable to individual licensing.

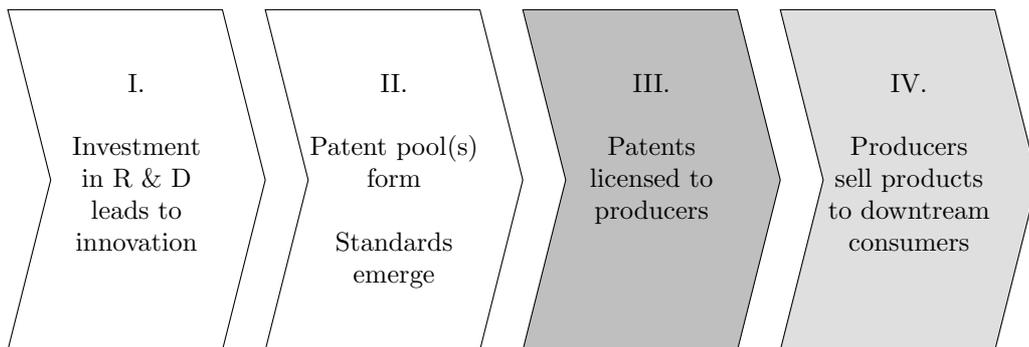
standard led to a 1995 consent decree; see Shapiro for details.

Both these types of pools are unaffected by compulsory individual licensing; but we saw in Theorem 3 that pools of complementary nonessential patents can be welfare-negative. Thus, compulsory individual licensing is not a sufficient screen against socially inefficient pools.

Pools containing substitute patents, however, would be unstable to individual licensing. Consider a pool containing one nonessential patent blocking each of several technologies. Each patentholder in the pool has an incentive to slightly undercut the pool's price, to capture the entire revenue from producers selecting the technology they block rather than sharing it with the rest of the pool; prices would collapse to the prices set in the absence of the pool. Thus, compulsory individual licensing in our setting is disruptive to some welfare-negative pools, but is not a perfect screen for efficiency.

5 Scope of This Model

The following flowchart shows the stages from innovation to commercialization of a new technology:



My model only explicitly captures box 3, the licensing of patents to producers.

Producers who license these patents do so with the intention of selling products based on these technologies to end users. I do not make a general attempt to model this downstream competition; however, under two particular sets of assumptions discussed earlier, the model captures both boxes 3 and 4. (The two sets of assumptions: either producers are local and fully extract consumer surplus, or it is consumers with differentiated tastes, and producers are perfectly competitive and simply pass along patentholders' prices, plus their own marginal costs, to consumers.) A more general model of downstream competition might allow for patentholder participation in the downstream market as well.

Box 2 covers the formation of a patent pool, which may be intertwined with the selection or emergence of a technological standard. I do not explicitly model this stage; welfare analysis was performed as if pools formed exogenously. Pool formation (and standard setting) is a difficult coalition-formation problem, and would likely be sensitive to the exact procedure by which patents were selected for the pool.

In box 1, firms make decisions on how much to invest in research and in what areas to focus those efforts; these research decisions then lead to innovations. Research decisions are of course influenced by expectations about what will occur in the later stages. My long-term goal is to present a model with endogenous innovation, standard selection, *and* pool formation, to understand the effect of various regulatory regimes on the speed and direction of innovation.

6 Conclusion

I have introduced a differentiated-products framework to model price competition for intellectual property when some patents are essential and some are not, and used the model to examine the equilibrium effects of various potential patent pools. In particular, I distinguish between pools consisting only of essential patents, which have only positive externalities and are Pareto-improving when they occur; and pools consisting of complementary but non-essential patents, which decrease the profits of competing patentholders and can be welfare-positive or welfare-negative. In contrast to previous models, complementarity between patents in a pool is not a sufficient condition for the pool to be welfare-positive, and compulsory individual licensing does not screen perfectly for efficiency.

I have focused on the application of the model to patent pools, but the same framework could also be applied to questions of consumer product bundling or the aggregation of products made from individually-priced components.

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7 Appendix – Proof of Lemma 1

First, I show that the conclusions of Lemma 1 follow from a set of conditions on the system of demand for the different technologies; then I show that under Assumption 1, the demand system described in the paper satisfies these conditions.

7.1 Lemma 2

Lemma 2. *Define a demand system for K technologies as a function of $K + 1$ prices as*

$$Q_k(P_1^N + P^E, P_2^N + P^E, \dots, P_K^N + P^E)$$

and let $Q_A(\cdot) \equiv \sum_{k=1}^K Q_k(\cdot)$. Suppose that Q_k is everywhere strictly positive and differentiable, and for any technologies k and $k' \neq k$,

1. Q_k is decreasing in P_k^N , increasing in $P_{k'}^N$, and decreasing in P^E
2. $\frac{\partial}{\partial P_k^N} \log Q_k$ is decreasing in P_k^N , increasing in $P_{k'}^N$, and decreasing in P^E
3. Q_A and $\frac{\partial}{\partial P^E} \log Q_A$ are decreasing in P_k^N (and therefore in P^E as well)
4. $\arg \max_{P_k^N} P_k^N Q_k(P_k^N, \cdot)$ is bounded above at $P^E = 0$ as all prices $P_{k'}^N$ ($k' \neq k$) go to $+\infty$.

Then the conclusions of Lemma 1 hold: equilibrium exists and is unique, with $(-P^E, P_1^N, P_2^N, \dots, P_K^N)$ increasing in $(-n_E, n_1, n_2, \dots, n_K)$, and $P_k^N + P^E$ increasing in n_E and n_k .

Maximizing $p_i q_i$ is the same as maximizing $\log p_i + \log q_i$, so each player $i \in T_k^N$ maximizes $\log p_i + \log Q_k$, with

$$\frac{\partial \log u_i}{\partial p_i} = \frac{1}{p_i} + \frac{\partial}{\partial P_k^N} \log Q_k$$

Since the latter term is negative and decreasing, $\log u_i$ is strictly concave, and the unique best-response is equivalent to a solution to the first-order condition. For $i, j \in T_k^N$, equilibrium prices are

$$\frac{1}{p_i} = -\frac{\partial}{\partial P_k^N} \log Q_k = \frac{1}{p_j}$$

and summing up,

$$\frac{n_k}{P_k^N} = -\frac{\partial}{\partial P_k^N} \log Q_k$$

Similarly, each $i \in T^E$ best-responds by playing the unique solution to

$$\frac{\partial \log u_i}{\partial p_i} = \frac{1}{p_i} + \frac{\partial}{\partial P^E} \log Q_A = 0$$

Summing up, equilibrium of the pricing game is equivalent to a solution to the $K + 1$ simultaneous equations

$$\frac{n_k}{P_k^N} = -\frac{\partial}{\partial P_k^N} \log Q_k, \quad \frac{n_E}{P^E} = -\frac{\partial}{\partial P^E} \log Q_A$$

By the same logic, then, it is equivalent to the equilibrium of a different game in which $K + 1$ players maximize $n_k \log P_k^N + \log Q_k(\cdot)$ and $n_E \log P^E + \log Q_A(\cdot)$, respectively. Conditions 2 and 3 establish that this is a supermodular game in $(-P^E, P_1^N, \dots, P_K^N)$, and the first-order conditions establish it is indexed by $(-n_E, n_1, n_2, \dots, n_K)$.

Let \bar{p}_k be any number strictly above the upper bound on $\arg \max_{P_k^N} P_k^N Q_k$ at $P^E = 0$ as all $P_{k'}^N$ ($k' \neq k$) go to ∞ . Supermodularity and concavity ensure that strategies $P_k^N \geq n_k \bar{p}_k$ are strictly dominated, as are strategies for player E above his best-response to $(0, 0, \dots, 0)$. After these strategies are eliminated, one more iteration removes strategies for any player which are arbitrarily close to 0. We now have a continuous game defined over a compact strategy space, so the usual existence arguments hold (see, for example, Glicksberg (1952)).

To show uniqueness, suppose there were two equilibrium profiles $P = (P_1^N, \dots, P_K^N, P^E)$ and $\tilde{P} = (\tilde{P}_1^N, \dots, \tilde{P}_K^N, \tilde{P}^E)$. Let $\Delta_k = \tilde{P}_k^N - P_k^N$ and $\Delta_E = \tilde{P}^E - P^E$. Suppose $\Delta_k \neq 0$ and assume without loss that it is positive. I claim that

$$\Delta_k < \max\{\Delta_1, \dots, \Delta_{k-1}, \Delta_{k+1}, \dots, \Delta_K, -\Delta_E\}$$

If not, then $\Delta_k + \Delta_E \geq 0$, so the total price of technology k is higher at \tilde{P} ; and $\Delta_k \geq \Delta_{k'}$, so no other technology's price increased more. Then we could see the move from P to \tilde{P} in two stages: in the first stage, the total price of each technology increased by the amount $\Delta_k + \Delta_E \geq 0$; and in the second stage, the price of each technology $k' \neq k$ decreases by an amount $\Delta_k - \Delta_{k'} \geq 0$. Condition 2 implies that both these changes decrease $\frac{\partial}{\partial P_k^N} \log Q_k$; since $\tilde{P}_k^N > P_k^N$, these combined mean that $\frac{\partial \log u_k}{\partial P_k^N}$ is strictly lower at \tilde{P} than at P , contradicting \tilde{P}_k^N and P_k^N both being best-responses. The same argument shows that if $\Delta_E > 0$, then $\Delta_E < \max\{-\Delta_1, \dots, -\Delta_K\}$. But then if the two profiles are different, pick any player with $|\Delta_i|$ maximal and this condition is violated for them.

Once we have uniqueness, $(-P^E, P_1^N, \dots, P_K^N)$ increasing in $(-n_E, n_1, \dots, n_K)$ follows by the properties of supermodular games (see, for example, the corollary to Theorem 6 in Milgrom and Roberts (1990)). A decrease in n_E leads to a decrease in P^E and an increase in P_k^N for every k . Letting $\hat{k} = \arg \max_k \Delta_k$,

$$\Delta_{\hat{k}} < \max\{\Delta_1, \dots, \Delta_{\hat{k}-1}, \Delta_{\hat{k}+1}, \dots, \Delta_K, -\Delta_E\}$$

and so $\Delta_k \leq \Delta_{\hat{k}} < -\Delta_E$, so $P_k^N + P^E$ falls in response to a decrease in n_E . Similarly, a drop in n_k leads to a drop in P_k^N and a rise in P^E , with

$$\Delta_E < \max\{-\Delta_1, \dots, -\Delta_K\}$$

If $k \neq \arg \min \Delta_i$, then the condition for the change in some other nonessential player's best-response is violated; so $\Delta_E < -\Delta_k$, or $P_k^N + P^E$ falls in response to a drop in n_k .

7.2 Proof the Conditions for Lemma 2 Hold under Assumption 1

I prove two more lemmas before arguing that the conditions for Lemma 2 are met.

Lemma 3. *Let X_1 and X_2 be independent random variables with distributions F_1 and F_2 , and let F_3 be the distribution of $X_1 + X_2$. Let F_1 , F_2 , $1 - F_1$, and $1 - F_2$ be log-concave. Let the distribution of X_2 be related to a parameter a , and X_1 be independent of a .*

- If f_2/F_2 is increasing in a then f_3/F_3 is increasing in a
- If $f_2/(1 - F_2)$ is decreasing in a then $f_3/(1 - F_3)$ is decreasing in a

We mimic the proof in Barlow and Proschan (1975, p. 100), who use techniques from Karlin (1968) to show that $1 - F_3$ is log-concave. Note that

$$F_3(t, a) = \int_{-\infty}^{\infty} F_1(t - s)f_2(s, a)ds$$

We want to show that F_3 is log-supermodular in t and a , that is, for $t_1 > t_2$ and $a_1 > a_2$,

$$F_3(t_1, a_1)F_3(t_2, a_2) > F_3(t_1, a_2)F_3(t_2, a_1)$$

which is the same as showing the positivity of the determinant

$$D = \begin{vmatrix} F_3(t_1, a_1) & F_3(t_1, a_2) \\ F_3(t_2, a_1) & F_3(t_2, a_2) \end{vmatrix} = \begin{vmatrix} \int_{-\infty}^{\infty} F_1(t_1 - s)f_2(s, a_1)ds & \int_{-\infty}^{\infty} F_1(t_1 - s)f_2(s, a_2)ds \\ \int_{-\infty}^{\infty} F_1(t_2 - s)f_2(s, a_1)ds & \int_{-\infty}^{\infty} F_1(t_2 - s)f_2(s, a_2)ds \end{vmatrix}$$

By the Basic Composition Formula (see Karlin (1968)), this is equivalent to

$$\iint_{s_1 < s_2} \begin{vmatrix} F_1(t_1 - s_1) & F_1(t_1 - s_2) \\ F_1(t_2 - s_1) & F_1(t_2 - s_2) \end{vmatrix} \begin{vmatrix} f_2(s_1, a_1) & f_2(s_1, a_2) \\ f_2(s_2, a_1) & f_2(s_2, a_2) \end{vmatrix} ds_2 ds_1$$

or, integrating by parts,

$$\iint_{s_1 < s_2} \begin{vmatrix} F_1(t_1 - s_1) & f_1(t_1 - s_2) \\ F_1(t_2 - s_1) & f_1(t_2 - s_2) \end{vmatrix} \begin{vmatrix} f_2(s_1, a_1) & f_2(s_1, a_2) \\ F_2(s_2, a_1) & F_2(s_2, a_2) \end{vmatrix} ds_2 ds_1$$

The sign of the first integral is the same as the sign of

$$\frac{f_1(t_2 - s_2) F_1(t_2 - s_2)}{F_1(t_2 - s_2) F_1(t_2 - s_1)} - \frac{f_1(t_1 - s_2) F_1(t_1 - s_2)}{F_1(t_1 - s_2) F_1(t_1 - s_1)}$$

Now,

$$\frac{f_1(t_2 - s_2)}{F_1(t_2 - s_2)} > \frac{f_1(t_1 - s_2)}{F_1(t_1 - s_2)}$$

because $t_1 > t_2$ and f_1/F_1 is decreasing, and

$$\frac{F_1(t_2 - s_2)}{F_1(t_2 - s_1)} > \frac{F_1(t_1 - s_2)}{F_1(t_1 - s_1)}$$

because $s_1 < s_2$ and F_1 is log-concave; so the first determinant is strictly positive. The second determinant has the same sign as

$$\frac{f_2(s_1, a_1)}{F_2(s_1, a_1)} \frac{F_2(s_1, a_1)}{F_2(s_2, a_1)} - \frac{f_2(s_1, a_2)}{F_2(s_1, a_2)} \frac{F_2(s_1, a_2)}{F_2(s_2, a_2)}$$

Again,

$$\frac{f_2(s_1, a_1)}{F_2(s_1, a_1)} > \frac{f_2(s_1, a_2)}{F_2(s_1, a_2)}$$

because $a_1 > a_2$ and f_2/F_2 is increasing in a_1 , and

$$\frac{F_2(s_1, a_1)}{F_2(s_2, a_1)} > \frac{F_2(s_1, a_2)}{F_2(s_2, a_2)}$$

is the log-supermodularity of F_2 (the same condition). So the second determinant is strictly positive as well, so $D > 0$ and we're done.

To show the second dot point, let $X'_1 = -X_1$ and $X'_2 = -X_2$, with distributions $F_{1'}$ and $F_{2'}$; and let $\beta = -\alpha$. If $f_{2'}(t)/(1 - F_{2'}(t))$ is decreasing in β , then $f_2(-t)/F_2(-t)$ is increasing in α , so f_3/F_3 is increasing in α ; if $F_{3'}$ is the distribution of $X'_1 + X'_2$, this is equivalent to $f_{3'}/(1 - F_{3'})$ decreasing in β .

Lemma 4. *Let $\epsilon_1, \epsilon_2, \dots, \epsilon_M$ be i.i.d. random variables drawn from a distribution F with density f , where f has full support on $(-\infty, \infty)$ and F and $1 - F$ are strictly log-concave. Let a_2, a_3, \dots, a_M be arbitrary constants. Define another random variable*

$$y = \max_{i>1} \{a_i + \epsilon_i\} - \epsilon_1$$

and let F_y and f_y be its distribution function and density. Then

1. $\frac{f_y(s)}{F_y(s)}$ is strictly decreasing in s
2. $\frac{f_y(s)}{1 - F_y(s)}$ is strictly increasing in s
3. $\frac{f_y(s)}{F_y(s)}$ is strictly increasing in a_i ($i > 1$)
4. $\frac{f_y(s)}{1 - F_y(s)}$ is strictly decreasing in a_i

Note that $1-F$ and F log-concave imply that $\frac{f}{1-F}$ is increasing and $\frac{f}{F}$ is decreasing, respectively. To begin, define $z = \max_{i>1} \{a_i + \epsilon_i\}$ and let F_z be its distribution. Note that

$$F_z(s) = \prod_{i>1} F(s - a_i)$$

and so

$$f_z(s) = \sum_{i>1} \left(f(s - a_i) \prod_{j>1, j \neq i} F(s - a_j) \right)$$

Then

$$\frac{f_z(s)}{F_z(s)} = \sum_{i>1} \frac{f(s - a_i)}{F(s - a_i)}$$

By assumption, $f(s - a_i)/F(s - a_i)$ is decreasing in s , so f_z/F_z is decreasing.

Next, define

$$A_i(s) \equiv \frac{f(s - a_i) \prod_{j>1, j \neq i} F(s - a_j)}{1 - \prod_{j>1} F(s - a_j)}$$

and note that

$$\frac{f_z(s)}{1 - F_z(s)} = \sum_{i>1} A_i(s)$$

I will show that each A_i is increasing in s , and so $f_z/(1 - F_z)$ is increasing:

$$\begin{aligned} A_i &= \frac{f(s - a_i)}{1 - F(s - a_i)} \frac{[1 - F(s - a_i)] \prod_{j>1, j \neq i} F(s - a_j)}{1 - \prod_{j>1} F(s - a_j)} \\ &= \frac{f(s - a_i)}{1 - F(s - a_i)} \left(\frac{\prod_{j>1, j \neq i} F(s - a_j) - 1}{1 - \prod_{j>1} F(s - a_j)} + \frac{1 - \prod_{j>1} F(s - a_j)}{1 - \prod_{j>1} F(s - a_j)} \right) \\ &= \frac{f(s - a_i)}{1 - F(s - a_i)} \left(\frac{\prod_{j>1, j \neq i} F(s - a_j) - 1}{1 - \prod_{j>1} F(s - a_j)} + 1 \right) \end{aligned}$$

By assumption, $\frac{f(s - a_i)}{1 - F(s - a_i)}$ is increasing in s ; since F is increasing, the term in parentheses is increasing in s as well, so A_i is increasing in s , and therefore $f_z/(1 - F_z)$ is increasing.

Barlow and Proschan (Theorem 4.2, p. 100) state that if X_1 and X_2 are independent random variables whose distributions have increasing hazard rates, then the random variable $X_1 + X_2$ has an increasing hazard rate as well. In our case, z and $-\epsilon_1$ are independent and have increasing hazard rates, so $y = z - \epsilon_1$ has an increasing hazard rate; and $-z$ and ϵ_1 are independent with increasing hazard rates, so $-y = -z + \epsilon_1$ does as well. (The proof in Barlow and Proschan extends to a strict inequality in our case, since the first inequality on page 101 holds strictly.) This proves that $\frac{f_y}{1 - F_y}$ is increasing and $\frac{f_y}{F_y}$ is decreasing.

To prove parts 3 and 4, we will first show that f_z/F_z is increasing in a_i and $f_z/(1 - F_z)$ decreasing in a_i ; we will then extend these properties to y . Recall that

$$\frac{f_z(s)}{F_z(s)} = \sum_{i>1} \frac{f(s - a_i)}{F(s - a_i)}$$

Since only the i term depends on a_i , and f/F is decreasing, f_z/F_z is increasing in a_i . Similarly,

$$\frac{f_z(s)}{1 - F_z(s)} = \sum_{i>1} A_i$$

where

$$A_i = \frac{f(s - a_i)}{1 - F(s - a_i)} \left(\frac{\prod_{j>1, j \neq i} F(s - a_j) - 1}{1 - \prod_{j>1} F(s - a_j)} + 1 \right)$$

If $j \neq i$, A_j is decreasing in a_i , as it reduces the term $F(s - a_j)$ in the numerator and the negative one in the denominator. If $j = i$, then the initial $f/(1 - F)$ term of A_i decreases in a_i , and the negative $F(s - a_i)$ term in the denominator decreases, reducing A_i . So $f_z/(1 - F_z)$ is decreasing in a_i . Setting $X_1 = -\epsilon_1$ and $X_2 = z$ and applying Lemma 3 completes the proof.

Finally, on to our demand system. Consider a measure 1 of producers, each with utility $v_k + \epsilon_k^l - p_k$ for each technology k and ϵ_0^l for no technology. Normalize $v_0 = 0$, and assign a negative “price” p_0 to the outside option (corresponding to a price of $-p_0 \geq 0$ added to all technologies). Then a share

$$\begin{aligned} Q_k &= \Pr \left(v_k + \epsilon_k^l - p_k \geq \max_{i=0,1,\dots,K} \{v_i + \epsilon_i^l - p_i\} \right) \\ &= \Pr \left(-p_k \geq y_k = \max_{i \neq k} \{v_i + \epsilon_i^l - p_i - v_k\} - \epsilon_k^l \right) \end{aligned}$$

of producers will buy technology k , which is $F_{y_k}(-p_k)$ with $a_i = v_i - p_i - v_k$.

The conditions we need to show:

1. Q_k is decreasing in p_k , increasing in p_i ($i \neq k$), and falls when all prices rise together, $\sum_{i=1}^K \frac{\partial}{\partial p_i} \log Q_k < 0$

By construction, y is stochastically increasing in a_i , which is decreasing in p_i , so F_{y_k} is increasing in p_i and p_0 (corresponding to a simultaneous decrease in all prices); F_{y_k} is a distribution function, so $F_{y_k}(-p_k)$ is decreasing in p_k .

2. $\frac{\partial}{\partial p_k} \log Q_k$ is decreasing in p_k , increasing in p_i ($i \neq k$), and falls when all prices rise together, $\sum_{i=1}^K \frac{\partial}{\partial p_i} \frac{\partial \log Q_k}{\partial p_k} < 0$.

$\frac{\partial}{\partial p_k} \log Q_k = \frac{\partial}{\partial p_k} \log F_{y_k}(-p_k) = -f_{y_k}(-p_k)/F_{y_k}(-p_k)$. By lemma 4, this is decreasing in p_k , and decreasing in a_i and therefore increasing in p_i and p_0 .

3. Q_A is decreasing in p_i (and therefore in $\sum_i p_i$), and $\sum_{i=1}^K \frac{\partial \log Q_A}{\partial p_i}$ is decreasing in p_i (and therefore in $\sum_i p_i$).

$Q_A = 1 - F_{y_0}(-p_0)$ is increasing in a_i and therefore decreasing in p_i . Increases in all p_i correspond to a decrease in p_0 ; $\partial \log(1 - F_{y_0}(-p_0))/\partial(-p_0) = -f_{y_0}(-p_0)/(1 - F_{y_0}(-p_0))$, which is increasing in a_i and therefore decreasing in p_i , and decreasing in its argument (therefore increasing in p_0 and falling when all prices rise).

4. For any $t > 0$, there exists \bar{p}_k sufficiently large that for any $p_k > \bar{p}_k$, $-p_k \frac{\partial \log Q_k}{\partial p_k} < t$ as opponent prices go to ∞ .

All we need is that player $k \neq 0$'s best-response is bounded as his opponents' prices go to ∞ . Consider the case where all the competing technologies are prohibitively expensive and can be ignored completely, and each producer chooses between technology k and the outside option. All the same properties still hold for F_{y_k} , and so a finite best-response will exist. Since $\log p_k + \log Q_k$ is concave and has increasing differences in p_k and p_i , any price higher than that is strictly dominated.

8 Proof of Theorem 1

From Lemma 1, we know an increase in n_E leads to decreases in each P_k^N , and increases in P^E and $P^E + P_k^N$.

We claim that P^E/n_E , the price demanded by each individual patentholder, falls when n_E rises. To see this, note that an essential patentholder i 's equilibrium price satisfies the first-order condition

$$\frac{\partial \log u_i}{\partial p_i} = \frac{1}{p_i} + \frac{\partial \log Q_A}{\partial p_A} = 0$$

We showed above that $\partial \log Q_A/\partial p^E$ is decreasing in each price, and therefore decreases when n_E rises; if p_i increased, $\partial \log u_i/\partial p_i$ would decrease, and the first-order condition would not be satisfied. Since every price increased, the fraction of producers licensing any technology, $\sum_k Q_k$, declines; so each essential patentholder charges a lower price and sells to fewer producers, for lower profits.

When n_E increases, P_k^N falls and n_k is unchanged, so the price each nonessential patentholder charges falls. To see that they also get lower profits, consider the shift from old equilibrium prices to new ones in two stages. In the first, every price but P_k^N adjusts to its new level; in the second, the "aggregate player" for technology k best-responds. In the first stage, P^E rises and each competing $P_{k'}^N$ falls. Under Assumption 2, this leads the aggregate player to select a lower quantity in the second stage. So P_k^N/n_k and Q_k are both decreasing in n_E , and therefore u_k is decreasing in n_E .

When n_k increases, P_k^N and $P_{k'}^N$ rise and P^E falls. By the same logic, under Assumption 2, $Q_{k'}^N$ rises for every $k' \neq k$, so $u_{k'}$ increases; but $\sum_{k'=1}^K Q_k$ falls, resulting in lower u_E . Q_k must fall for both of the above to hold. Since (by the logic in the uniqueness proof) P_k^N increases by more than $P_{k'}^N$ when n_k increases, and by more than P^E decreases, we can see the overall change in total prices as an increase in all prices by some amount, followed by drops in prices of the competing technologies $k' \neq k$, both of which lower the value of $\partial \log Q_k / \partial p_k$; for each individual patentholder $i \in T_k^N$ to be best-responding at the new equilibrium prices, $1/p_i$ must go up, meaning p_i falls. With lower price and lower marketshare, u_k is lower.

9 Proof of Theorems 2 and 3

A pool of essential patents has the same effects as a decrease in n_E , and a pool of nonessential patents blocking technology k has the same effects as a decrease in n_k . Both these theorems then follow from Lemma 1 and Theorem 1.

10 Logit Demand

10.1 Potential function and strict concavity.

One of the nice properties of logit demand is that

$$\frac{\partial Q_k}{\partial P_{k'}} = \begin{cases} Q_k Q_{k'} & \text{if } k' \neq k \\ -Q_k(1 - Q_k) & \text{if } k' = k \end{cases}$$

Consider the more general model where each patent i blocks an arbitrary subset of the technologies. If we let S_i be the market share of player i (that is, the sum of Q_k over the technologies blocked by patent i), we can calculate that

$$\frac{\partial S_i}{\partial p_j} = S_i S_j - S_{i,j}$$

where $S_{i,j}$ is the fraction of producers who license both patents i and j . Then

$$\frac{\partial}{\partial p_i} \log u_i = \frac{\partial}{\partial p_i} \log p_i + \frac{\partial}{\partial p_i} \log S_i = \frac{1}{p_i} + \frac{-S_i(1 - S_i)}{S_i} = \frac{1}{p_i} - 1 + S_i$$

and so

$$\frac{\partial^2}{\partial p_i \partial p_j} \log u_i = S_i S_j - S_{i,j} = \frac{\partial^2}{\partial p_i \partial p_j} \log u_j$$

Per Monderer and Shapley (1996), this is necessary and sufficient for the existence of a potential function P with

$$\frac{\partial P}{\partial p_i} = \frac{\partial \log u_i}{\partial p_i}$$

Since $\log u_i$ is strictly concave and best-responses are all interior, best-responses are characterized by the first-order condition $\frac{\partial u_i}{\partial p_i} = 0$; so equilibrium is equivalent to solutions to $\nabla P = 0$. We can calculate that

$$D^2P = \begin{bmatrix} -p_1^{-2} & & & & \\ & -p_2^{-2} & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & -p_N^{-2} \end{bmatrix} - \begin{bmatrix} S_1(1-S_1) & S_{1,2} - S_1S_2 & \dots & S_{1,N} - S_1S_N \\ S_{1,2} - S_1S_2 & S_2(1-S_2) & \dots & S_{2,N} - S_2S_N \\ \vdots & \vdots & \ddots & \vdots \\ S_{1,N} - S_1S_N & S_{2,N} - S_2S_N & \dots & S_N(1-S_N) \end{bmatrix}$$

The matrix on the right is a variance-covariance matrix, and is therefore positive semidefinite, so D^2P is negative definite; so P is strictly concave, leading to $\nabla P = 0$ only at the unique global maximum. As before, we can bound best-responses above to make the strategy space compact, ensuring existence of the unique equilibrium.

10.2 Characterization of equilibrium as simultaneous equations.

Returning to the model in which nonessential patents block only one technology, recall that best-responses are solutions to

$$\frac{1}{p_i} = -\frac{\partial}{\partial p_i} \log S_i = 1 - S_i$$

For $i \in T_k^N$, the right-hand side is $1 - Q_k$; summing over the patents in T_k^N gives

$$n_k = P_k^N (1 - Q_k)$$

For $i \in T^E$, the right-hand side is $1 - \sum Q_k$; summing over the essential patentholders gives

$$n_E = P^E \left(1 - \sum_{k \in \mathcal{K}} Q_k \right)$$

Thus, equilibrium prices correspond to solutions to these $K + 1$ simultaneous equations.

10.3 Effect of prices on welfare.

Another property of logit demand is that

$$PS = \mathbb{E}_{\epsilon_0^l, \epsilon_1^l, \epsilon_2^l, \dots, \epsilon_K^l} \max \left\{ \epsilon_0^l, \max_{k \in \mathcal{K}} \{v_k - P_k + \epsilon_k^l\} \right\} = \log \left(1 + \sum_{k \in \mathcal{K}} e^{v_k - P_k} \right)$$

Combined profits of the patentholders in T_k^N are $P_k^N Q_k$, and total profits of the essential patentholders are $P^E \sum_{k \in \mathcal{K}} Q_k$, so

$$\begin{aligned} W &= \log \left(1 + \sum_{k \in \mathcal{K}} e^{v_k - P^E - P_k^N} \right) + \sum_{k \in \mathcal{K}} P_k^N Q_k + P^E \sum_{k \in \mathcal{K}} Q_k \\ &= \log \left(1 + \sum_{k \in \mathcal{K}} e^{v_k - P_k} \right) + \sum_{k \in \mathcal{K}} P_k Q_k \end{aligned}$$

Differentiating,

$$\begin{aligned} \frac{\partial W}{\partial P_i} &= \frac{-e^{v_i - P_i}}{1 + \sum_{k \in \mathcal{K}} e^{v_k - P_k}} + Q_i - P_i Q_i (1 - Q_i) + \sum_{k \in \mathcal{K} - \{i\}} P_k Q_i Q_k \\ &= -Q_i + Q_i - P_i Q_i + Q_i \sum_{k \in \mathcal{K}} P_k Q_k = Q_i \left(\sum_{k \in \mathcal{K}} P_k Q_k - P_i \right) = Q_i (\bar{P} - P_i) \end{aligned}$$

where $\bar{P} = \sum_{k \in \mathcal{K}} Q_k P_k$. We can see a change in P^E as simultaneous changes to all the prices P_k , so

$$\frac{\partial W}{\partial P^E} = \sum_{k \in \mathcal{K}} \frac{\partial W}{\partial P_k} = \sum_{k \in \mathcal{K}} Q_k (\bar{P} - P_k) = \sum_{k \in \mathcal{K}} Q_k \bar{P} - \sum_{k \in \mathcal{K}} Q_k P_k = - \left(1 - \sum_{k \in \mathcal{K}} Q_k \right) \bar{P}$$

10.4 Result 1 – Price and Welfare Effects of n_k

The impact of changes in n_E and n_k on equilibrium prices, market shares, and payoffs is calculated explicitly in a separate technical appendix, available online at <http://www.stanford.edu/~dqunt> under the “papers” link. Total producer surplus is shown to be decreasing in n_k . The effect of n_k on the price of a competing technology is

$$\frac{\partial}{\partial n_k} (P_{k'}^N + P^E) = \left(\frac{P_{k'}^N Q_{k'}}{1 - Q_{k'} + P_{k'}^N Q_{k'}} - \frac{P^E}{1 + P^E} \right) \Lambda$$

where Λ is a positive term. Multiplying by both denominators and canceling repeated terms, this has the same sign as $P_{k'}^N Q_{k'} - P^E (1 - Q_{k'})$. Since $P_{k'}$ and $Q_{k'}$ are increasing in n_k and P^E decreasing in n_k , this term decreases as n_k decreases, so if it's negative at the original level of n_k , it remains negative as n_k decreases to its post-pool level, implying that $P_{k'}$ increases in response to the pool.

The effect of n_k on welfare is

$$\frac{\partial}{\partial n_k} W = \left(\sum_{k' \in \mathcal{K}} \frac{Q_{k'} (1 - Q_{k'})}{1 - Q_{k'} + P_{k'}^N Q_{k'}} (P_{k'} - P_k) - \frac{1 - \sum_{k' \in \mathcal{K}} Q_{k'}}{1 + P^E} P_k \right) \Lambda$$

The change in welfare due to the pool can be written as

$$- \int_{n_{+1-m}}^n \frac{\partial W}{\partial n_k} dn_k$$

and the result follows.

10.5 Result 2 – $K = 2$

We consider the effect of a change in n_1 on everyone other than the essential patentholders blocking technology 1. We show in the technical appendix that

$$\frac{\partial}{\partial n_1}(W - P_1^N Q_1) = \left(\sum_{k \in \mathcal{K} - \{1\}} \frac{P_k^N Q_k}{1 - Q_k + P_k^N Q_k} - 1 \right) \Lambda$$

When $K = 2$, the term inside parentheses is

$$\frac{P_2^N Q_2}{1 - Q_2 + P_2^N Q_2} - 1 < 0$$

so the net effect of a pool of patents in T_1^N on everyone outside of T_1^N is positive. Since the effect on patentholders in T_1^N who are excluded from the pool is also positive, if the pool is good for its participants, it is welfare-positive overall. (When $K > 2$, the term in parentheses cannot generally be signed.)

10.6 Result 3 – Pool of Two Substitutes

Suppose the patents block technologies 1 and 2. The pool sets price p^* to maximize $p(Q_1 + Q_2)$, leading to first-order condition $p^*(1 - Q_1 - Q_2) = 1$, which is equivalent to

$$p^*(1 - Q_1) = \frac{1 - Q_1}{1 - Q_1 - Q_2} = 1 + \frac{Q_2}{1 - Q_1 - Q_2}$$

and

$$p^*(1 - Q_2) = \frac{1 - Q_2}{1 - Q_1 - Q_2} = 1 + \frac{Q_1}{1 - Q_1 - Q_2}$$

Along with the best-response functions for the other patentholders in T_1^N and T_2^N , this leads to

$$P_1^N(1 - Q_1) = n_1 + \frac{Q_2}{1 - Q_1 - Q_2} \quad \text{and} \quad P_2^N(1 - Q_2) = n_2 + \frac{Q_1}{1 - Q_1 - Q_2}$$

in equilibrium, along with the usual conditions

$$P_k^N(1 - Q_k) = n_k \quad \text{and} \quad P^E \left(1 - \sum_{k \in \mathcal{K}} Q_k \right) = n_E$$

for the other aggregate prices. Thus, the equilibrium effect of the pool is equivalent to increases in n_1 and n_2 , each of which reduces producer surplus.

10.7 Result 4 – Pool of One Patent for Each Technology

This pool corresponds to an increase in n_E by 1, and decreases in each n_k by 1. The welfare effects of each of these are

$$\frac{\partial W}{\partial n_E} = -\frac{1}{1+P^E} \sum_{k \in \mathcal{K}} \frac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k} (P_k^N + P^E) \delta$$

and

$$\frac{\partial W}{\partial n_i} = \frac{Q_i}{1-Q_i+P_i^N Q_i} \left(\sum_{k \in \mathcal{K}-\{i\}} \frac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k} (P_k^N - P_i^N) - \frac{1-\sum_{k \in \mathcal{K}} Q_k}{1+P^E} (P_i^N + P^E) \right) \delta$$

with $\delta > 0$. Straight calculation (in the technical appendix) shows that

$$\frac{\partial W}{\partial n_E} - \sum_{i \in \mathcal{K}} \frac{\partial W}{\partial n_i} < 0$$

proving that the pool is welfare-destroying.

11 Claims on Pool Formation under Logit Demand

11.1 Large Pools of Essential Patents

Claim 1. *Fix $m \geq 0$. If n_E is sufficiently large, a pool containing all but m of the essential patents increases the profits of its participants.*

Let π be the equilibrium profits of each essential patentholder in the game with the same primitives as G except that there are only $m+1$ essential patents. Note that $\pi > 0$. Should a pool form containing all but m of the n_E essential patents, π would be the pool's total revenue; divided up equally, this would give each participant a payoff of

$$u_{n_E}^{pool} = \frac{\pi}{n_E - m}$$

If no pool forms, each essential patentholder's profits are

$$u_{n_E}^{nopool} = \frac{1}{n_E} P^E \sum_{k \in \mathcal{K}} Q_k$$

We know that

$$\sum_{k \in \mathcal{K}} Q_k = \frac{\sum_{k \in \mathcal{K}} e^{v_k - P_k^N - P^E}}{1 + \sum_{k \in \mathcal{K}} e^{v_k - P_k^N - P^E}} < \sum_{k \in \mathcal{K}} e^{v_k - P_k^N - P^E} < e^{-P^E} \sum_{k \in \mathcal{K}} e^{v_k}$$

and so

$$u_{n_E}^{nopool} < \frac{1}{n_E} P^E e^{-P^E} V$$

where $V \equiv \sum_{k \in \mathcal{K}} e^{v_k}$. To prove the result, we need to show that for n_E sufficiently large,

$$\frac{\pi}{n_E - m} > \frac{1}{n_E} P^E e^{-P^E} V$$

Since $n_E > n_E - m$, a sufficient condition is

$$\frac{\pi}{V} > P^E e^{-P^E}$$

π/V is a constant, and $x e^{-x}$ decreases to 0 as x increases; since $P^E > n_E$ increases without bound as $n_E \rightarrow \infty$, the result follows.

11.2 Incremental Pooling of Essential Patents

Claim 2. *Fix $k \geq 1$. If n_E is sufficiently large, the formation of a pool of $k + 1$ essential patents increases the joint profits of its participants.*

(Note that this is the same as saying, the addition of k essential patents to an already-essential pool increases the joint profits of the new entrants and the pool.)

Joint profits before pool formation are

$$\frac{k+1}{\tilde{n}_E} \pi^E(\tilde{n}_E)$$

Joint profits after are

$$\frac{1}{\tilde{n}_E - k} \pi^E(\tilde{n}_E - k)$$

We can express these both as

$$(k+1)^r \frac{\pi^E(\tilde{n}_E - (1-r)k)}{\tilde{n}_E - (1-r)k}$$

where $r = 0$ corresponds to profits after the pool forms, and $r = 1$ corresponds to profits before the pool. Thus, we can write the change in joint profits as

$$d\pi = - \int_0^1 \frac{d}{dr} \left((k+1)^r \frac{\pi^E(\tilde{n}_E - (1-r)k)}{\tilde{n}_E - (1-r)k} \right) dr$$

Fix $r \in [0, 1]$, and let $n_E = \tilde{n}_E - (1-r)k$. I show in the technical appendix that the integrand has the same sign as

$$\frac{1}{k} \log(k+1) Q_0^2 \frac{wtd\ avg\ P_k^N Q_k}{wtd\ avg\ (1-Q_k)} + \frac{1}{k} \log(k+1) P^E (1-Q_0) Q_0 - \left(1 - \frac{1}{k} \log(k+1) Q_0 \right)$$

where $Q_0 = 1 - \sum_{k \in \mathcal{K}} Q_k$ and

$$wtd\ avg\ f(k) \equiv \left(\sum_k \frac{Q_k}{1 - Q_k + P_k^N Q_k} f(k) \right) / \left(\sum_k \frac{Q_k}{1 - Q_k + P_k^N Q_k} \right)$$

Now, since

$$Q_k = \frac{e^{v_k - P_k}}{1 + \sum_{k' \in \mathcal{K}} e^{v_{k'} - P_{k'}}} < e^{v_k} e^{-P^E}$$

and similarly

$$P^E(1 - Q_0) < \left(\sum_{k \in \mathcal{K}} e^{v_k} \right) P^E e^{-P^E}$$

and since $P^E > n_E$ and P^E therefore grows without bounds as n_E increases, and since P_k^N decreases as n_E increases, we can drive all the $P_k^N Q_k$ terms, as well as $P^E(1 - Q_0)$, to 0 by increasing n_E . Since the $(1 - Q_k)$ terms increase, this means that the first two terms of the expression above vanish, leaving (in the limit, as n_E grows arbitrarily large)

$$\approx - \left(1 - \frac{1}{k} \log(k+1) Q_0 \right)$$

which is negative and bounded away from 0. We can choose \tilde{n}_E sufficiently large so that the whole expression is negative on the range $n_E \in [\tilde{n}_E - k, \tilde{n}_E]$, and then $d\pi$ is the negative of the integral of a negative expression, making it positive.

Claim 3. *But on the other hand, when $(m-1)(1-Q_0) > 0.442$, adding an incremental patent the pool decreases combined profits.*

By the same logic, letting $k = 1$ above, when

$$(\log 2)P^E(1 - Q_0)Q_0 + (\log 2)Q_0 - 1 > 0$$

at every point while $n_E \in (m, m+1)$, then the integrand will be everywhere positive and adding a single essential patent to the pool will decrease joint profits. Since $P^E Q_0 = n_E$, this requires

$$n_E(1 - Q_0) + Q_0 > \frac{1}{\log 2} \approx 1.442$$

or $(n_E - 1)(1 - Q_0) > 0.442$. $(1 - Q_0)$ is decreasing in n_E , so it is smallest at $n_E = m + 1$; since we're working in the range $n_E \in [m, m+1]$, $n_E - 1$ is at least $m - 1$, so the expression will be positive everywhere in the range if

$$(m - 1)(1 - Q_0) > 0.442$$

where Q_0 is evaluated at $n_E = m + 1$, that is, before the additional patent is added.

11.3 Pools of Nonessential Complements

The proofs that when n_k is sufficiently large, both large and incremental pools of nonessential patents blocking technology k are profitable are nearly identical to the corresponding proofs for essential patents, and can be found in the technical appendix.

11.4 Adding Nonessentials to an Essential Pool

To begin, we show that under logit demand, an essential patentholder cannot gain by charging different prices to producers using different technologies. If he could price-discriminate on this basis, he would choose prices $p_i^1, p_i^2, \dots, p_i^K$ for users of each technology to maximize

$$u_i = \sum_{k \in \mathcal{K}} p_i^k Q_k$$

Differentiating with respect to a particular price p_i^k gives

$$\begin{aligned} \frac{\partial u_i}{\partial p_i^k} &= Q_k - p_i^k Q_k (1 - Q_k) + \sum_{k' \neq k} p_{k'} Q_k Q_{k'} \\ &= Q_k - p_i^k Q_k + Q_k \sum_{k' \in \mathcal{K}} p_{k'} Q_{k'} \end{aligned}$$

The first-order condition, then, is

$$\begin{aligned} Q_k(p_i^k - 1) &= Q_k \sum_{k' \in \mathcal{K}} p_{k'} Q_{k'} \\ p_i^k &= 1 + \sum_{k' \in \mathcal{K}} p_{k'} Q_{k'} \end{aligned}$$

The right-hand side does not depend on k ; so even when unconstrained, the patentholder maximizes profits by demanding the same price from every “type” of producer.

We claim that when a pool already exists containing all the essential patents, combined profits of the pool and the newcomers increases when all the nonessential patents blocking technology k are added to the pool. Consider the effects of the addition of these patents to the pool as occurring in four stages:

1. The profits of these nonessential patents begin to accrue to the pool, without any changes in any prices
2. The pool reduces the prices of these patents to 0 and adjusts the price of the pool, as a static best-response to the prices of the nonessential patents blocking other technologies
3. The nonessential patentholders blocking other technologies change their prices to the new equilibrium levels
4. The pool changes its prices to its new equilibrium level

Stage 1 does not change the joint profits of the pool and the patents being added, it just allows us to focus on the profits going to essential patentholders in the last three stages.

We showed above that an essential patentholder cannot gain by charging different prices to producers accessing different technologies. Thus, stage 2 increases essential patentholder (pool) profits, increasing joint profits.

Since the new equilibrium corresponds to a decrease in n_k , the prices of nonessential competing patents $P_{k'}^N$ decrease in equilibrium; this increases essential patentholder profits. So stage 3 increases the joint profits we're looking at.

Finally, stage 4 increases joint profits, since the pool will only change prices to increase profits.

Thus, each stage increases the joint profits, so the net effect is to increase joint profits.

On the other hand, when $n_E > 1$ (there is at least one outsider to the pool of essential patents), $n_k \geq 2$, and $Q_k \geq \frac{1}{n_E}$, the addition of a single nonessential patent blocking k to the pool decreases combined profits. The proof is similar to the incremental pooling results above, and can be found in the technical appendix.

12 Proof of Theorem 4 – Compulsory Individual Licensing

We assume, as in Lerner and Tirole and Brenner, that in the presence of a patent pool with compulsory individual licensing, pricing occurs in two stages:

1. The patent pool, and patentholders outside the pool, name prices
2. The members of the patent pool name prices for their individual patents

Producers then choose a technology, licensing pooled patents as needed, either individually or through the pool, whichever is cheaper.

Claim 4. *Suppose that each patent in the pool is worthless without every patent in the pool. (That is, all patents in the pool are essential, or all patents in the pool are nonessential and block the same technology.) Then if the pool's price in Stage 1 is a static best-response to the prices of the patents outside the pool, then Stage 2 changes nothing.*

The logic is as follows. In stage 2, each member of the pool, by naming a sufficiently high price, can “sabotage” individual licensing, that is, ensure that nobody will license *any* of the pooled patents individually. Thus, if pool members name prices which sum to less than the price of the pool, it must be that each of them is earning weakly higher profits than they would without individual licensing. Since the price of the pool was the unique maximizer of the pooled patentholders’ combined profits,

this is only possible if the sum of the individual prices of the pooled patents is the same as the price of the pool; so total prices, and each patentholder's revenue, is the same as without individual licensing. Anticipating this in stage 1, the pool has no reason to play anything other than a static best-response, and the same prices emerge as in equilibrium without individual licensing of pooled patents.

13 The Numerical Example

We make three separate claims with regard to a particular numerical example: three technologies, logit demand, and $\mathbf{v} = (10, 10, 5)$. In each case below, one can verify that these are the unique equilibrium prices by checking the four necessary and sufficient conditions

$$\begin{aligned} P^E(1 - Q_1 - Q_2 - Q_3) &= n_E \\ P_1^N(1 - Q_1) &= n_1 \\ P_2^N(1 - Q_2) &= n_2 \\ P_3^N(1 - Q_3) &= n_3 \end{aligned}$$

The first claim is that when $\mathbf{n} = (1, 3, 3, 3)$, a pool of the three nonessential patents blocking technology 3 would be profitable but welfare-decreasing, while a pool of the three nonessential patents blocking technology 2 would be welfare-increasing but unprofitable. Consider the resulting equilibrium prices and payoffs in each case. (π_E and π_k refer to the combined profits of *all* the nonessential patentholders and *all* the nonessential patentholders blocking technology k ; PS and W refer to total producer surplus and welfare, respectively.)

\mathbf{n}	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	π_E	π_1	π_2	π_3	PS	W
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	3.578	1.861	<u>1.861</u>	<u>0.048</u>	1.521	<u>8.869</u>
(1,3,3,1)	4.818	4.647	4.647	1.091	.354	.354	.084	3.818	1.647	1.647	<u>0.091</u>	1.572	<u>8.775</u>
(1,3,1,3)	6.011	3.777	2.666	3.009	.206	.625	.003	5.011	0.777	<u>1.666</u>	0.009	1.794	<u>9.256</u>

A decrease in n_3 from 3 to 1 increases the profits of the nonessential patentholders blocking technology 3 from 0.048 to 0.091, but decreases total welfare from 8.869 to 8.775. A decrease in n_2 from 3 to 1 increases welfare from 8.869 to 9.256, but decreases the profits of the nonessential patentholders blocking technology 2.

The second claim is that when $\mathbf{n} = (6, 3, 3, 3)$, a pool of all six essential patents would be profitable, but the addition of the last essential patent to a pool of the first five would decrease their combined profits. The equilibrium outcomes in each case:

\mathbf{n}	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	π_E	π_1	π_2	π_3	PS	W
(6,3,3,3)	8.224	3.466	3.466	3.004	.135	.135	.001	<u>2.224</u>	0.466	0.466	0.004	0.315	3.477
(2,3,3,3)	5.685	4.410	4.410	3.026	.320	.320	.009	<u>3.685</u>	1.410	1.410	0.026	1.045	7.577
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	<u>3.578</u>	1.861	1.861	0.048	1.521	8.869

Finally, we claim that when $\mathbf{n} = (1, 3, 3, 3)$, corresponding to a pool of all the essential patents already having formed, that adding one nonessential patent blocking

technology 2 would decrease their combined profits, but that adding two or three would increase combined profits. The equilibrium outcomes:

\mathbf{n}	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	π_E	π_1	π_2	π_3	PS	W
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	<u>3.578</u>	1.861	<u>1.861</u>	0.048	1.521	8.869
(1,3,2,3)	5.171	4.354	3.904	3.024	.311	.488	.008	<u>4.171</u>	1.354	1.904	0.024	1.643	9.096
(1,3,1,3)	6.011	3.777	2.666	3.009	.206	.625	.003	<u>5.011</u>	0.777	1.666	0.009	1.794	9.256

Combined profits of the pool and a single patent in T_2^N are $3.578 + \frac{1}{3}(1.861) = 4.198$ to begin, and 4.171 after the patent is added to the pool. On the other hand, combined profits of the pool and two such patents are $3.578 + \frac{2}{3}(1.861) = 4.819$ to begin, and 5.011 after both are added to the pool. That adding all three is profitable was proven in a previous appendix.