Stanford University Mathematical Organization (SUMO) Speaker Series

Diophantine Approximations

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3–4 PM @ 380-382T

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Abstract

Cantor (and in a different sense Lebesgue) proved in the 19th century that, in a well-defined sense, almost every real number is not the root of any integer polynomial. However, it is surprisingly difficult to give, with proof, an example of such a real number. This difficulty is so profound that mathematicians gave to such numbers the term "transcendental" because they transcend attempts at (algebraic) definition. In this lecture, instead of attempting to prove that familiar numbers such as pi or e are transcendental (which is very hard), I will construct an (uncountably) infinite number of concrete transcendental examples. To prove that these examples are indeed transcendental, we will develop the theory of diophantine approximation, construct a measure of "how irrational a number is", show that "infinitely irrational" numbers are actually transcendental, and then construct explicit examples of "infinitely irrational" numbers using infinite sums.