The SUMO Speaker Series for Undergraduates
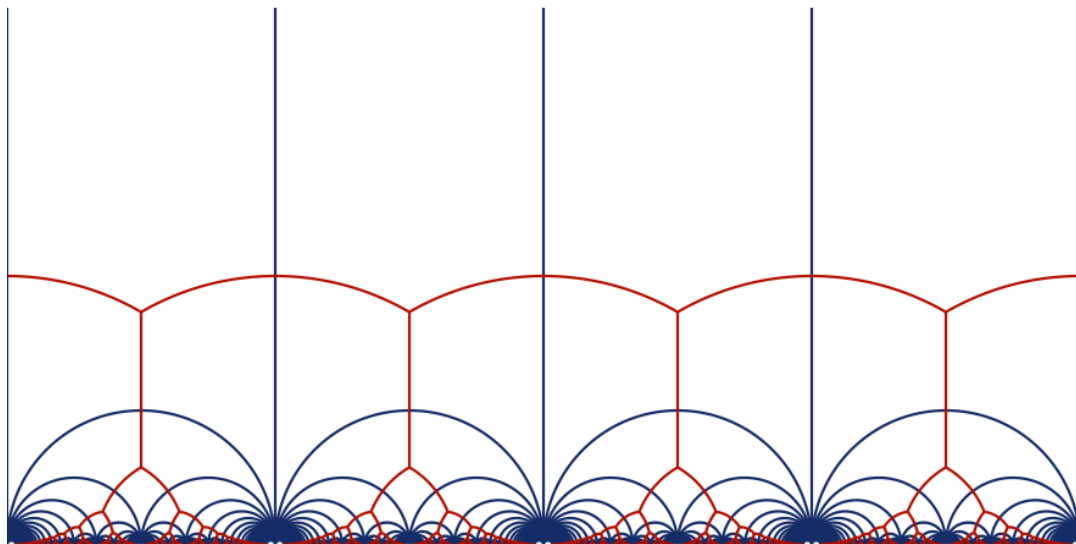## Thursday, November 21st
4:15-5:05, room 380C
(Food Provided)

Positive-definite integral quadratic forms and the 15 theorem
Professor Sheel Ganatra

Abstract:

Are all natural numbers expressible as a sum of two squares (of integers)? What about four squares? The answer to the first question, famously studied by Fermat, is no, whereas the answer to the second, due to Lagrange, is yes. More broadly, the study of integer solutions of integer polynomial equations has a long history, initiated perhaps by Diophantine and continued by luminaries such as Fermat, Lagrange, Gauss, Euler, and Ramanujan.
In this talk, we examine the case of positive-definite integral quadratic forms, a class of quadratics including sums of squares and expressions such as $x^2 + 2xy + 3y^2 + 4z^2 + 6w^2 + 4wx + 20t^2$. Are all natural numbers expressible in such a given fixed form, using integer inputs? A recent theorem of Conway-Schneeberger gives a startling answer: if 1 through 15 are represented, then so is every other number! We will give some ideas behind a simplified proof due to Bhargava, taking a detour through lattices and linear algebra.

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