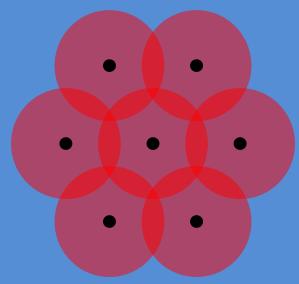
II. Real-space multiple-scattering theory of EXAFS and XANES & FEFF

J. J. Rehr, J. J. Kas and F. D. Vila





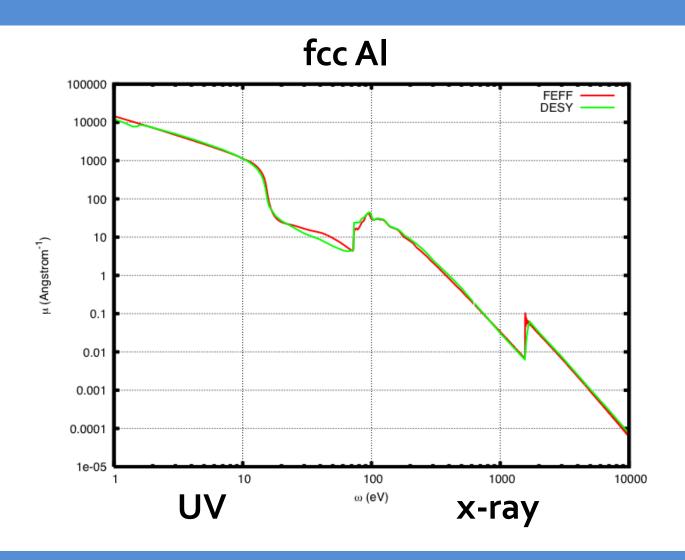


Outline

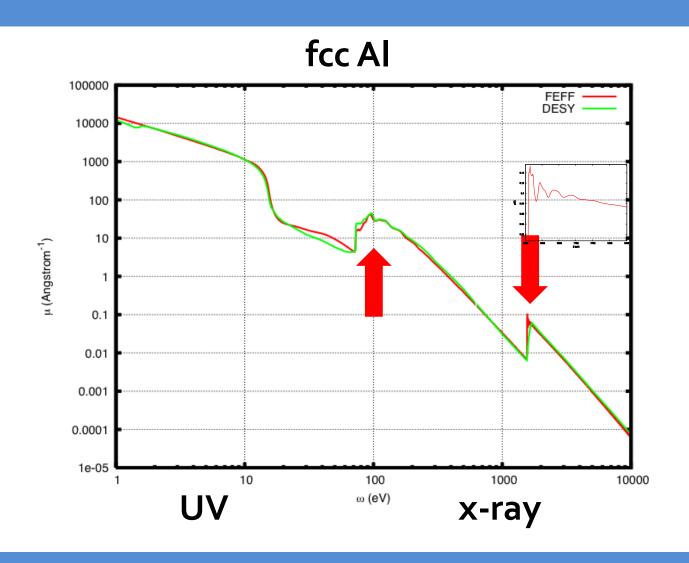
Talk:

- -Real-space multiple-scattering (RSMS) Theory akα Real-space Green's function (RSGF) theory -Implementation of RSMS in FEFF
- Key approximations and limitations Effects of structure and disorder Some advanced developments

Full spectrum XAS: Expt. Vs Theory

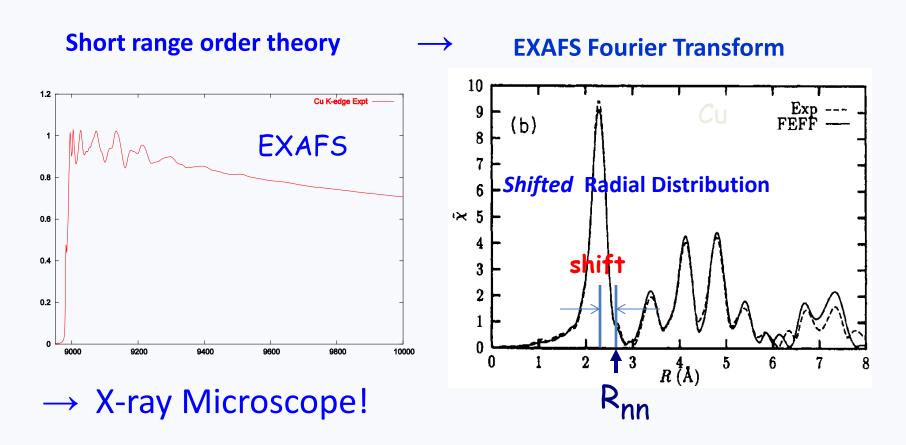


The devil is in the details: edges, fine-structure ...



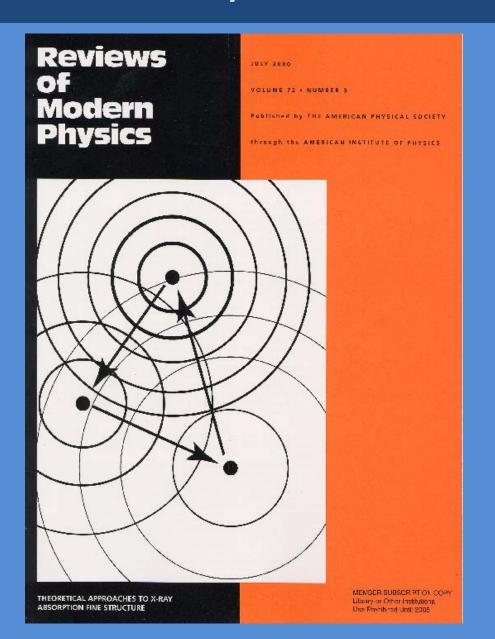
Historical interpretation of EXAFS*

*Stern Sayers Lytle, UW 1971



BUT needed to calibrate experiment with "Standard"

EXAFS Theory

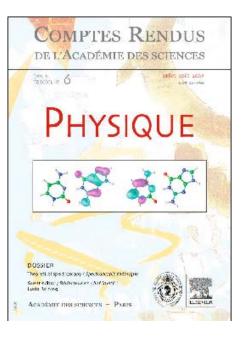


Quantitative theory of EXAFS:

Theory behind FEFF6

J. J. Rehr & R.C. Albers Rev. Mod. Phys. 72, 621 (2000)

Advances in Theory – FEFF9





Available online at www.sciencedirect.com



C. R. Physique 10 (2009) 548–559 http://france.elsevier.com/direct/COMREN/

PHYSIQUE

Theoretical spectroscopy / Spectroscopie théorique

Ab initio theory and calculations of X-ray spectra

John J. Rehr*, Joshua J. Kas, Micah P. Prange, Adam P. Sorini, Yoshinari Takimoto, Fernando Vila

Department of Physics, University of Washington, Seattle, WA 98195-1560, USA

Available online 5 December 2008

Update of Rehr & Albers: Advanced techniques and ab initio treatment of many-body effects

Other references:

PHYSICAL REVIEW B

VOLUME 58, NUMBER 12

15 SEPTEMBER 1998-II

Real-space multiple-scattering calculation and interpretation of x-ray-absorption near-edge structure

A. L. Ankudinov

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PERSPECTIVE

www.rsc.org/pccp | Physical Chemistry Chemical Physics

Parameter-free calculations of X-ray spectra with FEFF9

John J. Rehr,*^a Joshua J. Kas,^a Fernando D. Vila,^a Micah P. Prange^{bc} and Kevin Jorissen^a

Received 15th December 2009, Accepted 27th April 2010 First published as an Advance Article on the web 6th May 2010

DOI: 10.1039/b926434e

RSGF in the hierarchy of spectroscopy methods

mostly intractable except in small molecules

```
Atomic models:
     e.g. de Groot. Atomic cross-sections, multiplet s
     theory with fitted parameters, crystal field model Hamiltonians
DFT (Density Functional Theory):
     WIEN2k, ABINIT, VASP, CASTEP, StoBe, Orca...,
     Accurate for ground-state properties, not reliable for excited states,
     Delta-SCF "Final State Rule" with core-hole
Quasi-particle Green's Function Theory: FEFF9
     Appropriate for excited states, NOT full potential
BSE (Bethe-Salpeter Equation):
     Exc!ting, OCEAN, Al2NBSE. Accurate but demanding.
     Less user friendly. Misses excitations & satellites
QC methods:
     MRCI, MRCC, CASPT2, QMC, etc, highly accurate but
```

FEFF development philosophy

"Pretty good" spectra

Advantages:

Real-space

Fully relativistic, all-electron

Semi-automated, user-friendly, easy to use

Built for EXAFS and related x-ray spectroscopies

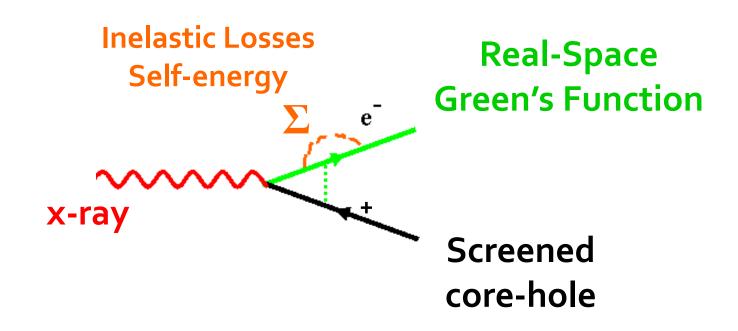
Applicable to materials throughout the periodic table

Disadvantages: Not always the best tool:

Spherical potentials – can lose accuracy near edges

Quasi-particle theory only – ignores multiplets, satellites

FEFF quantitative XANES theory in one Feynman diagram



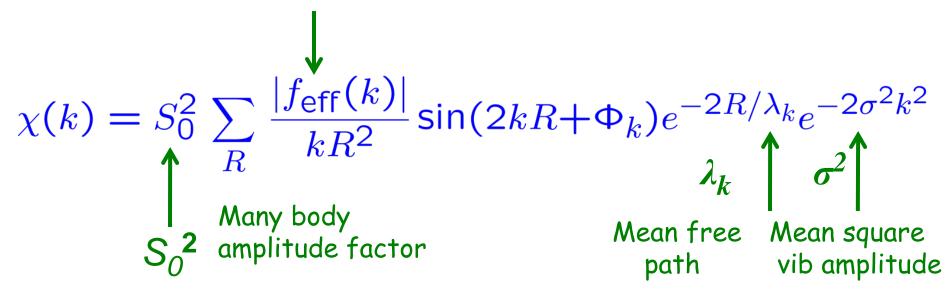
"Can you write an equation

for the theory?"

P.A.M. Dirac

Answer: Exact EXAFS Equation*

Effective Scattering Amplitude $f_{
m eff}$



EXAFS measures local structure & disorder

Distance R Coordination N Disorder σ^2

*JJR, RC Albers, CR Natoli, EA Stern, Phys Rev B34, 4350 (1986)

BUT: need many parameters!

Question: Can the EXAFS parameters

$$k f_{\text{eff}} \Phi_k \sigma^2 \lambda_k S_0^2$$

be calculated theoretically?

FEFF: Many-body → effective single particle

XAS absorption coefficient

Many-body Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{F} |\langle I|\Delta|F\rangle|^2 \delta(E_F - E_I - \omega)$$

Effective Single particle Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{i,f} |\langle i|d|f\rangle|^2 \delta\left(E_f - E_i - \omega\right) S_0^2$$

$$H = -(1/2)\nabla^2 + V$$

$$H' = H + V_{ch} + \Sigma(E)$$

Effective Single particle Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{ij} \langle i | d^{\dagger} | f \rangle \langle f | d | i \rangle \delta(E_f - E_i - \omega) S_0^2$$

Effective Single particle Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{ij} \langle i | d^{\dagger} | f \rangle \langle f | d | i \rangle \delta(E_f - E_i - \omega) S_0^2$$

$$\rho(r,r',E) = \sum_{f} |f\rangle\langle f| \,\delta\left(E_f - E\right)$$

Density Matrix

Effective Single particle Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{ij} \langle i | d^{\dagger} | f \rangle \langle f | d | i \rangle \delta(E_f - E_i - \omega) S_0^2$$

$$ho(r, r', E) = \sum_{f} |f\rangle\langle f| \; \delta\left(E_f - E\right)$$

$$\operatorname{Im}\left[G(r, r', E)\right] = -\frac{1}{\pi}\rho(r, r', E)$$

$$G = \left[E - H + i\Gamma\right]^{-1}$$

Density matrix from Green's function

Effective Single particle Fermi's Golden Rule

$$\mu(\omega) \propto \sum_{ij} \langle i | d^{\dagger} | f \rangle \langle f | d | i \rangle \delta(E_f - E_i - \omega) S_0^2$$

$$\rho(r, r', E) = \sum_{f} |f\rangle\langle f| \,\delta\left(E_f - E\right)$$

$$\operatorname{Im}\left[G(r, r', E)\right] = -\frac{1}{\pi}\rho(r, r', E)$$

$$G = \left[E - H + i\Gamma\right]^{-1}$$

Substitute sum over final states with Green's function

$$\mu(\omega) \propto \operatorname{Im} \sum_{i} \langle i | d^{\dagger} G(\omega + E_{i}) d | i \rangle \theta_{\Gamma}(\omega + E_{i} - E_{Fermi}) S_{0}^{2}$$

What's a Green's function?

Wave function in QM

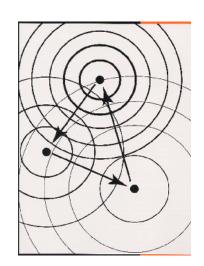
$$H \Psi = E \Psi$$

 $\Psi(r)$ = Amplitude to find particle at r

Green's function
$$(H - E) G = -\delta(r-r')$$

G(r,r',E) = aka Propagator

= Amplitude to go from r to r'



FEFF: Local basis and matrix elements

$$\mu(\omega) \propto \operatorname{Im} \sum_{i} \langle i | d^{\dagger} G(\omega + E_{i}) d | i \rangle \theta_{\Gamma}(\omega + E_{i} - E_{Fermi}) \quad S_{0}^{2}$$

Insert complete set of states

$$1 = \sum_{L} |i, L\rangle\langle i, L|$$

FEFF: Local basis and matrix elements

$$\mu(\omega) \propto \operatorname{Im} \sum_{i} \langle i | d^{\dagger} G(\omega + E_{i}) d | i \rangle \theta_{\Gamma}(\omega + E_{i} - E_{Fermi}) S_{0}^{2}$$

Insert complete set of states $1 = \sum_{i} |i,L\rangle\langle i,L|$

$$1 = \sum_{L} |i, L\rangle\langle i, L|$$

$$\mu(\omega) \propto \operatorname{Im} \sum_{iLL'} \langle i | d^{\dagger} | i, L \rangle G_{LL'}(\omega + E_i) \langle i, L' | d | i \rangle \theta_{\Gamma}(\omega + E_i + E_{Fermi}) S_0^2$$

FEFF: Local basis and matrix elements

$$\mu(\omega) \propto \operatorname{Im} \sum_{i} \langle i | d^{\dagger} G(\omega + E_{i}) d | i \rangle \theta_{\Gamma}(\omega + E_{i} - E_{Fermi}) S_{0}^{2}$$

Insert complete set of states $1 = \sum_{i} |i,L\rangle\langle i,L|$

$$1 = \sum_{L} |i, L\rangle\langle i, L|$$

$$\mu(\omega) \propto \text{Im} \sum_{iLL'} \left\langle i \left| d^{\dagger} \right| i, L \right\rangle G_{LL'}(\omega + E_i) \left\langle i, L' \left| d \right| i \right\rangle \theta_{\Gamma}(\omega + E_i + E_{Fermi}) \, S_0^2$$
 Green's Function

matrix

Getting G: Multiple Scattering Theory

Dyson's equation: $G = G^0 + G^0 VG$

Iterating: $G = G^0 + G^0VG^0 + G^0VG^0VG^0 + \cdots$

Dyson's equation:

$$G = G^0 + G^0 V G$$

Iterating:

$$G = G^{0} + G^{0}VG^{0} + G^{0}VG^{0}VG^{0} + \cdots$$



$$G(r,r') = G_0(r,r') + \int d^3r'' G_0(r,r'') V(r'') G_0(r'',r') + \cdots$$

Dyson's equation:

$$G = G^0 + G^0 V G$$

Iterating:

$$G = G^{0} + G^{0}VG^{0} + G^{0}VG^{0}VG^{0} + \cdots$$

$$V = \sum_{i} v_{i}$$

Atomic pot.
$$V = \sum_i v_i$$
 $G = G^0 + \sum_i G^0 v_i G^0 + \sum_{ij} G^0 v_i G^0 v_j G^0 + ...$

Dyson's equation:

$$G = G^0 + G^0 V G$$

Iterating:

$$G = G^{0} + G^{0}VG^{0} + G^{0}VG^{0}VG^{0} + \cdots$$

Atomic pot. partition

$$V = \sum_{i} v_{i}$$

$$V = \sum_{i} v_{i}$$
 $G = G^{0} + \sum_{i} G^{0} v_{i} G^{0} + \sum_{ij} G^{0} v_{i} G^{0} v_{j} G^{0} + ...$

Site scatt. matrix

$$t_i = v_i + v_i G_0 t_i$$

$$t_i = v_i + v_i G_0 t_i$$
 $G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i \neq j} G^0 t_i G^0 t_j G^0 + ...$

Dyson's equation:

$$G = G^0 + G^0 V G$$

Iterating:

$$G = G^{0} + G^{0}VG^{0} + G^{0}VG^{0}VG^{0} + \cdots$$

Atomic pot. partition

$$V = \sum_{i} v_{i}$$

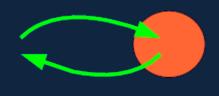
$$V = \sum_i v_i \qquad G = G^0 + \sum_i G^0 v_i G^0 + \sum_{ij} G^0 v_i G^0 v_j G^0 + ...$$

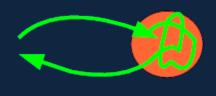


Site scatt. matrix

$$t_i = v_i + v_i G_0 t_i$$

$$t_i = v_i + v_i G_0 t_i$$
 $G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i \neq i} G^0 t_i G^0 t_j G^0 + ...$





$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i \neq j} G^0 t_i G^0 t_j G^0 + ...$$

$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i
eq j} G^0 t_i G^0 t_j G^0 + ...$$

Central atom contrib.

$$G_c = G_0 + G_0 t_c G_0$$



EXAFS

$$G = G_c + G_c \sum_{i \neq c} t_i G_c + G_c \sum_{i \neq c} \sum_{j \neq c} t_i G_0 t_j G_c + \cdots$$

$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i \neq j} G^0 t_i G^0 t_j G^0 + ...$$

Central atom contrib.

$$G_c = G_0 + G_0 t_c G_0$$



EXAFS

$$G = G_c + G_c \sum_{i \neq c} t_i G_c + G_c \sum_{i \neq c} \sum_{j \neq c} t_i G_0 t_j G_c + \cdots$$

Graphically: Path expansion

$$G = G_c + \overline{G_c}TG_c + G_cTG^0TG_c + G_cTG^0TG^0TG_c + \dots$$

$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i \neq j} G^0 t_i G^0 t_j G^0 + \dots$$

$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i
eq j} G^0 t_i G^0 t_j G^0 + ...$$

Total scatt. matrix

$$T_{LL'}^{ij} = t_{LL'}^i \delta_{ij}$$



$$G = G^0 + G^0 T G^0 + G^0 T G^0 T G^0 + \dots$$

$$G = G^0 + \sum_i G^0 t_i G^0 + \sum_{i
eq j} G^0 t_i G^0 t_j G^0 + ...$$

Total scatt. matrix

$$T_{LL'}^{ij} = t_{LL'}^i \delta_{ij}$$



$$G = G^0 + G^0 T G^0 + G^0 T G^0 T G^0 + \dots$$

Sum and invert



XANES

$$G = \left[1 - G^0 T\right]^{-1} G^0$$

Implementation: FEFF Code

PHYSICAL REVIEW B

VOLUME 58, NUMBER 12

15 SEPTEMBER 1998-II

Real-space multiple-scattering calculation and interpretation of x-ray-absorption near-edge structure

A. L. Ankudinov

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B. Ravel

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J. J. Rehr

Department of Physics, University of Washington, Seattle, Washington 98195-1560

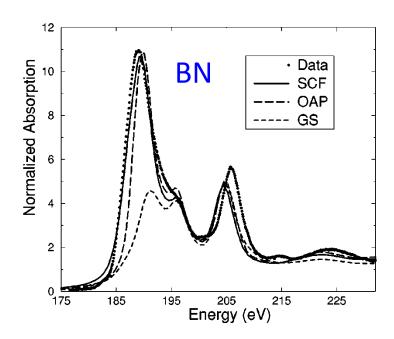
89 atom cluster

S. D. Conradson

MST-11, Los Alamos National Laboratory, Los Alamos, New Mexic

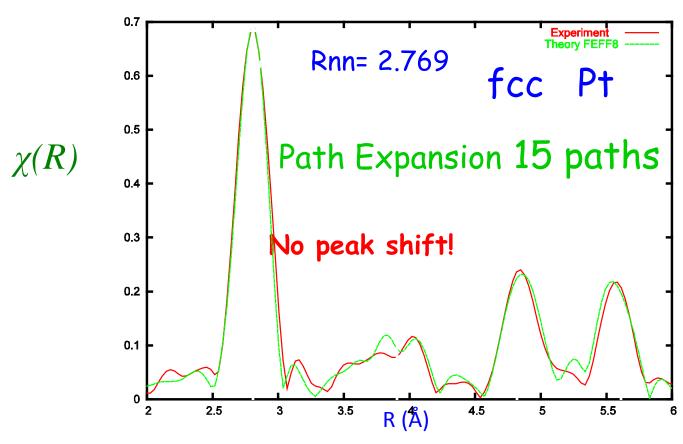
Core-hole, SCF potentials

Essential!



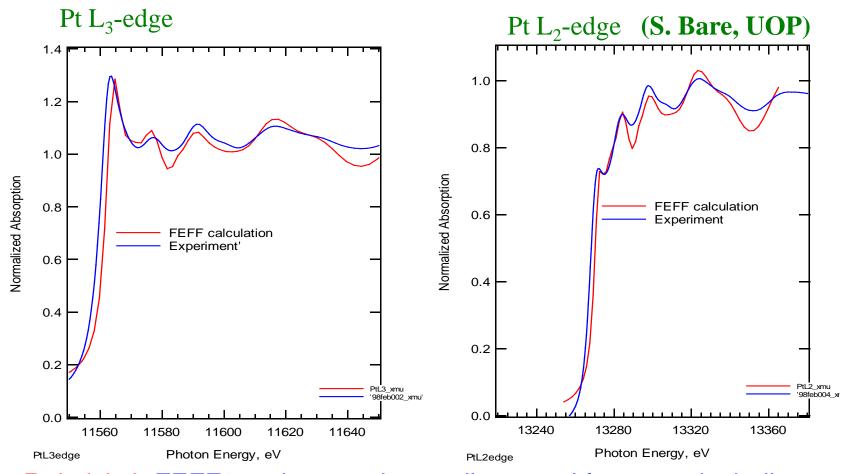
Example: Pt EXAFS – path expansion





*Theoretical phases accurate distances to < 0.01 Å

Example: Pt XANES full multiple-scattering



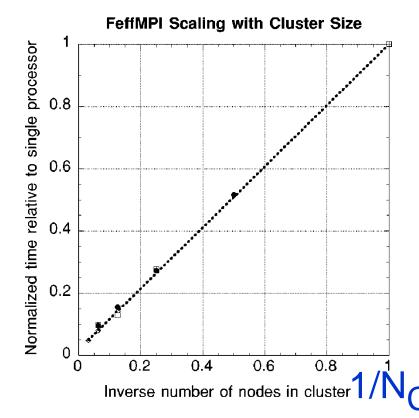
- Relativistic FEFF8 code reproduces all spectral features, including absence of white line at L₂-edge.
- Self-consistency essential: position of Fermi level strongly affects white line intensity.

Green's Functions and Parallel Computation

PHYSICAL REVIEW B, VOLUME 65, 104107

Parallel calculation of electron multiple scattering using Lanczos algorithms

A. L. Ankudinov, ¹ C. E. Bouldin, ² J. J. Rehr, ¹ J. Sims, ² and H. Hung ² ¹Department of Physics, University of Washington, Seattle, Washington 98195 ²National Institute of Standards and Technology, Gaithersburg, Maryland 20899



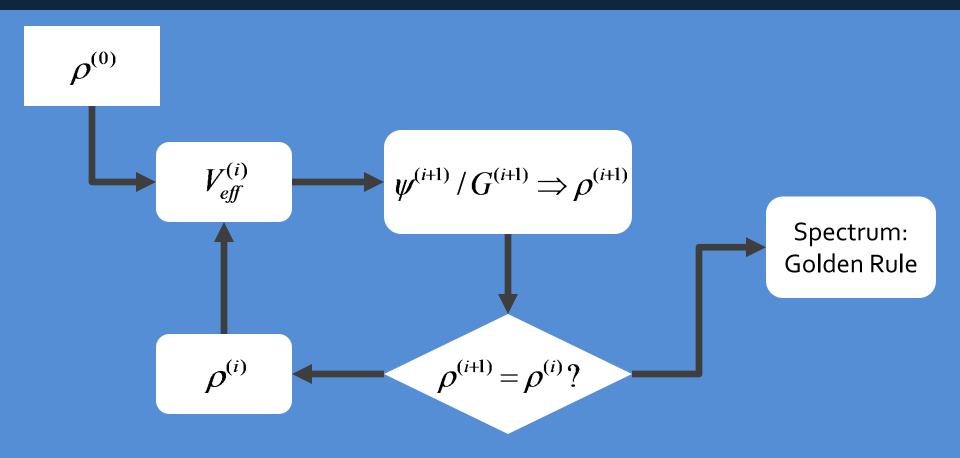
$$\mu(E) \sim -\frac{1}{\pi} \mathrm{Im} \left\langle \mathrm{i} \right| \hat{\epsilon} \cdot \mathbf{r}' \, \mathrm{G}(\mathbf{r}',\mathbf{r},\mathrm{E}) \, \hat{\epsilon} \cdot \mathbf{r} \, \left| \mathrm{i} \right\rangle$$
 Energy E is just a parameter !

"Natural parallelization"

Each CPU does one energy

Self-consistent Densities and Potentials

$$\rho(r) = -\frac{1}{\pi} \int_{-\infty}^{E_{Fermi}} \text{Im}[G(r, r; E)] dE$$



Key approximations in FEFF

Dirac-Fock relativistic atomic states; semi-relativistic scattering states

Spherical overlapped muffin-tin potentials: Huge simplification of the problem

Quasi-particle approximation:

Electron propagates in lossy medium described by

Approximate self-energy

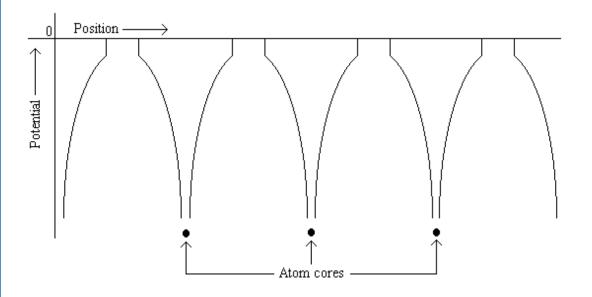
Core-hole treatment:

RPA or DFT-Screened core-hole

The muffin-tin potential

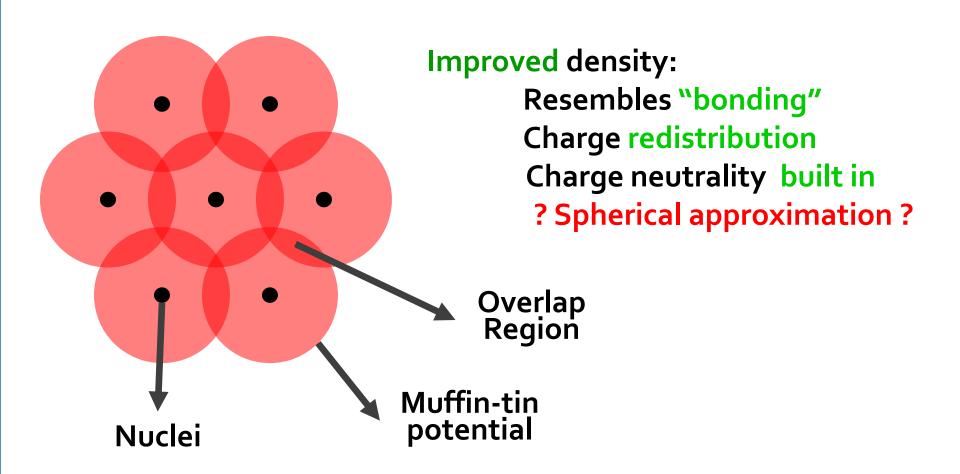
Scattering potential partition into muffin-tins

$$V = \sum_{i} v_{i}$$

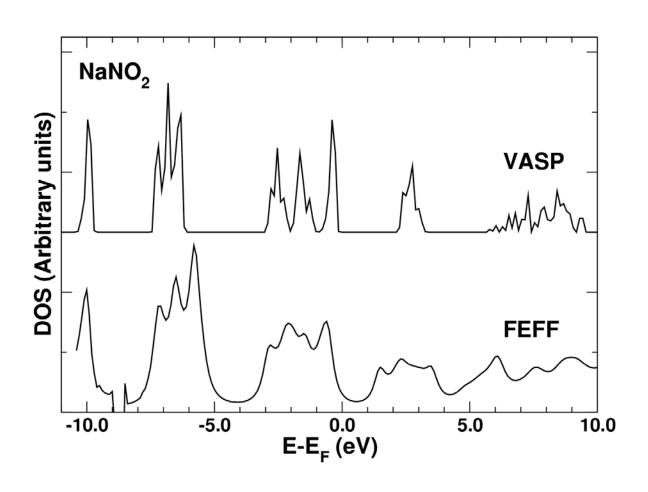




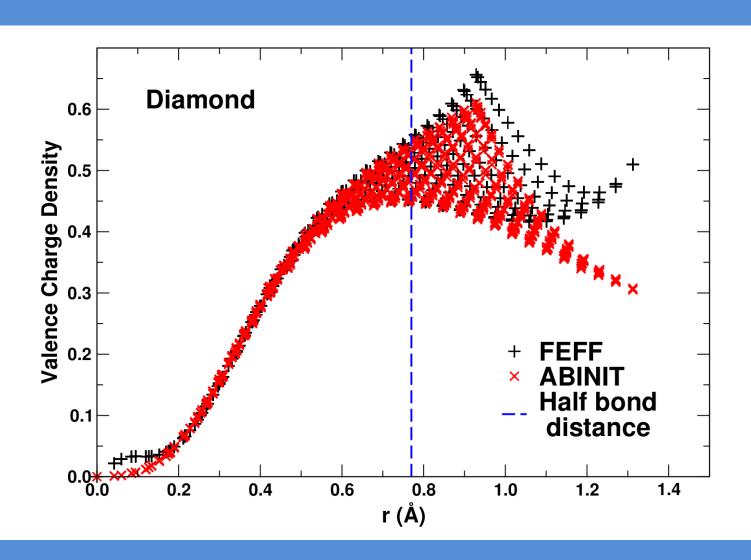
The overlapped muffin-tin potential (~ 10%)



FEFF Density of States vs full potential codes



FEFF electron density in real space vs full potential codes



Disorder and Debye-Waller factors

DW factors:

Crucial for EXAFS

Very little effect in XANES region

Can be included anyway in single-scattering approx.

Both ab initio and model forms

Static Disorder:

May be important in XANES

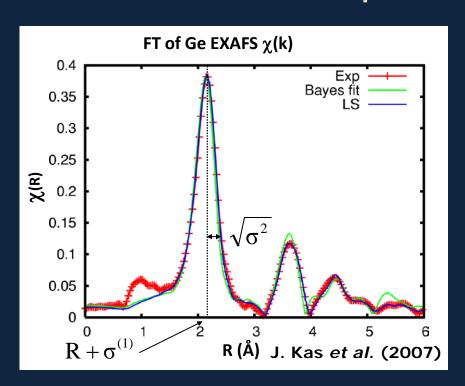
Need external input for FEFF simulations

Methods MD trajectories, MC sampling ...

Quick intro to DW factors

Multiple Scattering Path XAFS DW Factor

Average commonly expressed in terms of the cumulant expansion



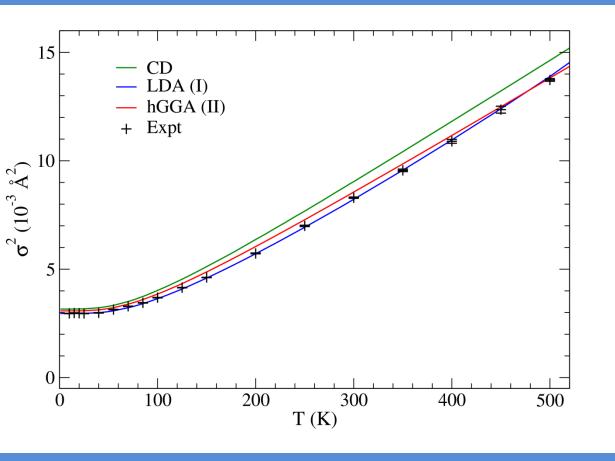
$$\left\langle e^{i2kr} \right\rangle \equiv e^{2ikR_0}e^{-W(T)}$$

$$W(T) = -\sum_{n=1}^{\infty} \frac{(2ik)^n}{n!} \sigma^{(n)}(T)$$

Leading cumulants

$$\begin{cases} \boldsymbol{\sigma}^{(1)} = \langle r - R_0 \rangle \\ \boldsymbol{\sigma}^{(2)} = \langle (r - \bar{r})^2 \rangle \equiv \boldsymbol{\sigma}^2(T) \\ \boldsymbol{\sigma}^{(3)} = \langle (r - \bar{r})^3 \rangle \end{cases}$$

EXAFS near-neighbor DW Factor of Cu

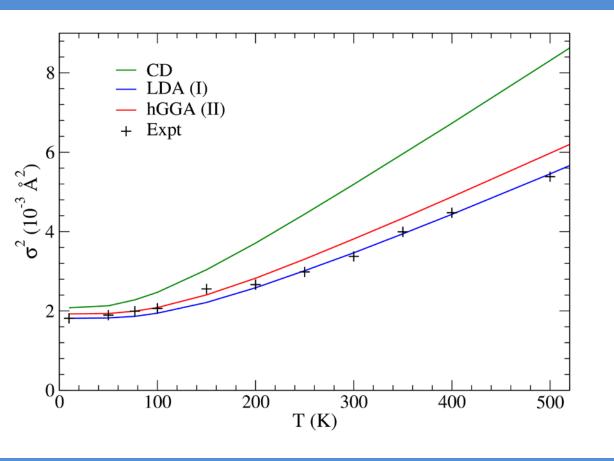


CD (Correlated Debye): Standard FEFF

LDA, hGGA:
Ab initio DW

Isotropic bonding:
Good CD results

EXAFS near-neighbor DW Factor of Ge



CD (Correlated Debye): Default in FEFF

LDA, hGGA:
Ab initio DW

Directional bonding: Needs AIDW

Ab Initio DW factors: Lanczos algorithm

XAFS DW Factor for path R:

$$\sigma_R^2(T) = \frac{\hbar}{2\mu_R} \int_0^\infty \frac{1}{\omega} \coth\left(\frac{\beta\hbar\omega}{2}\right) \rho_R(\omega) d\omega$$

VDOS expressed as imaginary part of the phonon propagator

$$\rho_R(\boldsymbol{\omega}) = -\frac{2\boldsymbol{\omega}}{\pi} \operatorname{Im} \left\langle 0 \left| \frac{1}{\boldsymbol{\omega}^2 - \mathbf{D} + i\boldsymbol{\varepsilon}} \right| 0 \right\rangle$$

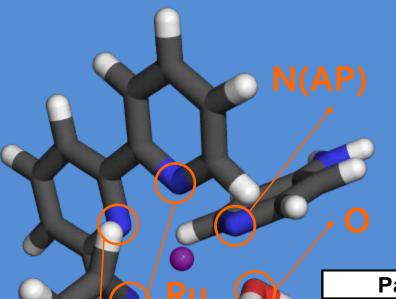
Seed state: Displacement

along path

Dynamical Matrix: Calculated using ab initio methods (Abinit, Gaussian, VASP, etc)

Ab Initio DW Factors in Metal-Ligand Complexes

$Ru(bpy)^2(AP)(H2O)^{++}$



Good agreement for tight ligands (bpy)

Useful agreement for weak ligands (AP and H2O)
Still within error margin

	R _{M-L} (In A)		σ^2 (in 10 ⁻³ A ²)	
Path	Theory	Exp	Theory	Exp.
Ru-N(bpy)	2.08	2.05±0.01	2.49	2.6±0.9
	2.04		2.32	
	2.10		2.60	
	2.09		2.50	
Ru-N(AP)	2.14	2.10±0.03	2.61	4±3
Ru-O	2.22	2.06±0.05	4.93	9±7

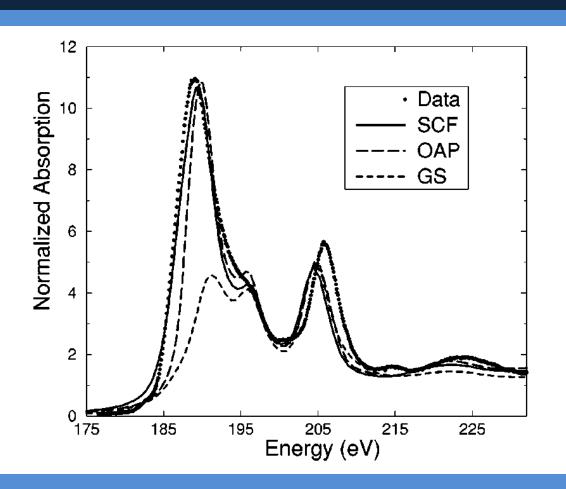
N(bpy)

Expt: Salassa et al., J. of Physics: Conference Series 190, 012141 (2009)

Beyond DFT: Quasi-particle Self-Energy Effects

Quasi-particle (QP) effects:

$$G = [E - H - \Sigma]^{-1}$$



BN 89 atom cluster

Ground state potential:

Usually insufficient

Need QP effects

and SCF potentials

Improvements to the theory: key many body effects Lecture III. Inelastic losses and Manybody effects

Treatment of the core hole:

Screening DFT or RPA Chemical shifts

Self-energy approximations:

Need more than single-pole self-energy

Many-body effects:

Multi-electron excitations S_{θ}^{2}

Charge transfer excitations:

Other FEFF capabilities

Quick overview of other FEFF capabilities:

XES

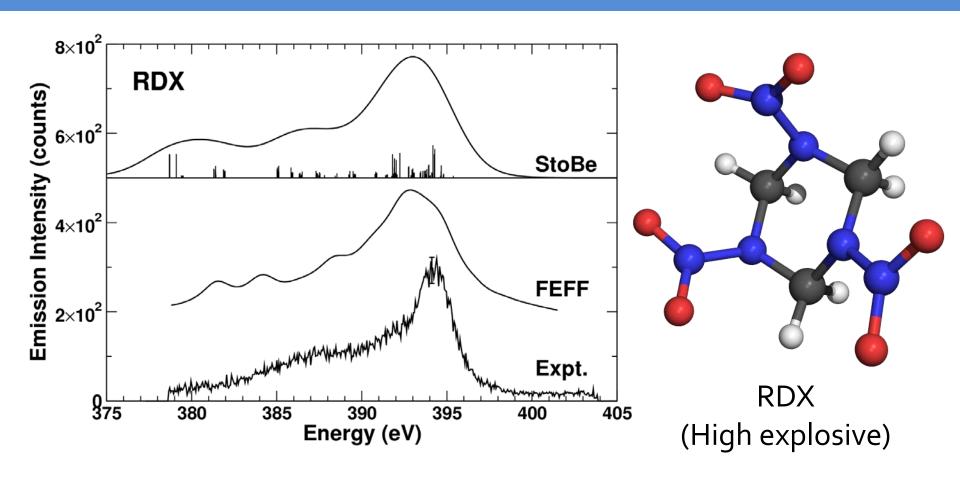
RIXS

Compton Profiles

Reciprocal space: EELS and impurity GF

Hubbard U method

Non-resonant X-ray Emission (XES)



Resonant Inelastic X-ray Scattering (RIXS)

PHYSICAL REVIEW B 83, 235114 (2011)

Real-space Green's function approach to resonant inelastic x-ray scattering

J. J. Kas,¹ J. J. Rehr,^{1,*} J. A. Soininen,² and P. Glatzel³
¹Department of Physics, Box 351560, University of Washington, Seattle, Washington 98195-1560, USA
²Department of Physics, P.O. Box 64, University of Helsinki, FI-00014 Helsinki, Finland
³European Synchrotron Radiation Facility, B.P. 220, F-38043 Grenoble, France

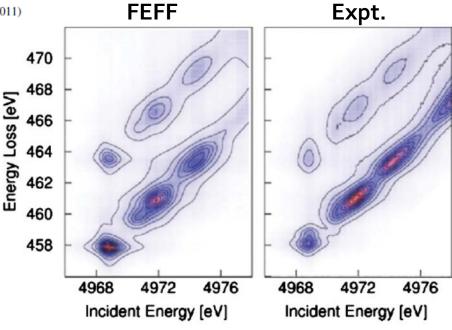
(Received 21 January 2011; revised manuscript received 7 April 2011; published 8 June 2011)

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{\omega}{\Omega} \sum_{F} \left| \frac{\sum_{M} \langle F | \Delta_2^{\dagger} | M \rangle \langle M | \Delta_1 | \Psi_0 \rangle}{E_M - \Omega - E_0 + i \Gamma_M} \right|^2 \times \delta(\Omega - \omega + E_0 - E_F)$$



$$\frac{d^2\sigma}{d\Omega d\omega} \propto \frac{\omega}{\Omega} \int d\omega_1 \frac{\mu_e(\omega_1)\mu(\Omega-\omega-\omega_1+E_b)}{|\omega-\omega_1-i\Gamma_b|^2}$$

 TiO_2 (Ti K α)



Compton Profiles

PHYSICAL REVIEW B 85, 115135 (2012)

Real-space Green's function calculations of Compton profiles

Brian A. Mattern, Gerald T. Seidler, Joshua J. Kas, Joseph I. Pacold, and John J. Rehr Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA (Received 2 February 2012; revised manuscript received 16 March 2012; published 29 March 2012)

PHYSICAL REVIEW B 94, 214201 (2016)

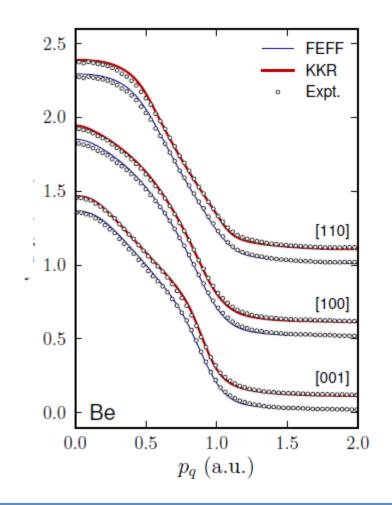
Finite-temperature calculations of the Compton profile of Be, Li, and Si

E. Klevak, F. D. Vila, J. J. Kas, J. J. Rehr, and G. T. Seidler Department of Physics, University of Washington, Seattle, Washington 98195, USA (Received 3 August 2016; published 2 December 2016)

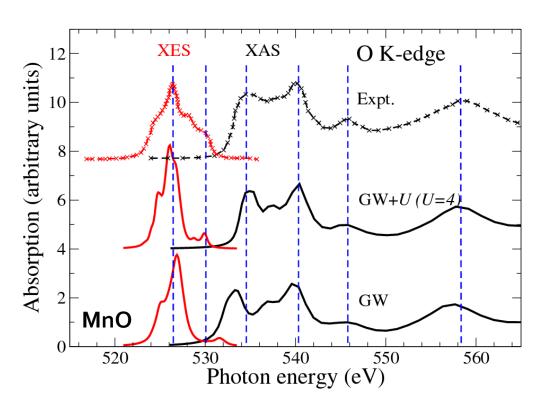
$$S(\mathbf{q},\omega) = \sum_{F} \left| \langle F | \sum_{j} \exp(i\mathbf{q} \cdot \mathbf{r}_{j}) | I \rangle \right|^{2} \delta(E_{F} - E_{I} - \hbar \omega)$$

$$S(\mathbf{q},\omega) = (m/\hbar q) J(p_{q})$$

$$J(p_{q}) \equiv \int d^{3} p \rho(\mathbf{p}) \, \delta(p_{q} - (\omega m/q - \hbar q/2))$$

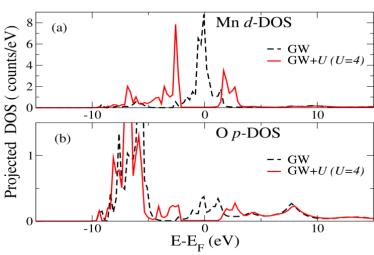


Strongly correlated systems: Hubbard GW+U



U calculated using constrained RPA within RSMS

(Nearly) parameter free



Further information

The FEFF Project website:

URL: feffproject.org

The FEFF Users Guide:

URL: feffproject.org/feffproject-feff-documentation.html

Developers contact:

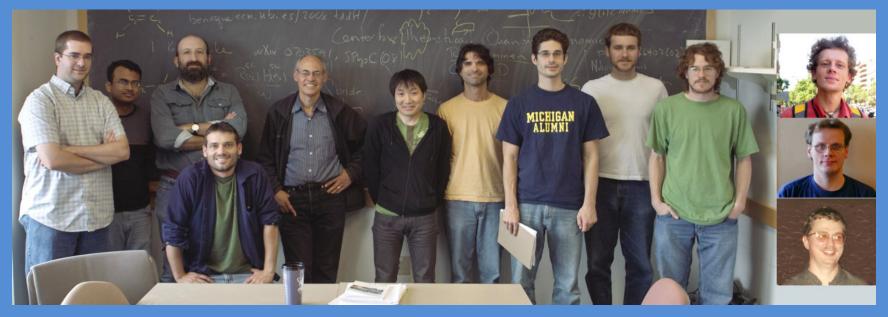
URL: feffproject.org/feffproject-contact.html

Summary

Take away messages

Know the basics of RSMS/RSGF theory
Understand the key approximations in FEFF
Know some of FEFF's advanced capabilities

Acknowledgments: the FEFF group



Ken Nagle Towfiq Ahmed Fernando Vila Micah Prange John Rehr Yoshi Takimoto Hadley Lawler Adam Sorini John Vinson Josh Kas Kevin Jorissen Aleksi Soininen Alex Ankudinov Shauna Story Egor Clevac

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