Online Appendix for
Population and Welfare:
The Greatest Good for the Greatest Number

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A. Roles of Birth and Death Rates

This section of the appendix provides details on the exercise discussed in Section 4.4 of the paper. The goal is to quantify the contributions of fertility (birth rates) and longevity (death rates) to population growth.

Notation. In a given year t, total population $N(t)$ is the sum of population of different ages:

$$N(t) = \sum_{a=0}^{A} N_a(t).$$

The law of motion for $N_a(t)$ is given by:

$$N_a(t) = \begin{cases} N_{a-1}(t-1) + M_a(t) - D_a(t) & \text{if } a > 0 \\ B(t) + M_a(t) - D_a(t) & \text{if } a = 0, \end{cases}$$

where $M_a(t)$ is the net inflow of migrants of age a in year t, $D_a(t)$ is the total number of deaths at age a in year t, and $B(t)$ is the total number of births. It is useful to rewrite the law of motion in terms of death rate:

$$d_a(t) := \frac{D_a(t)}{N_a(t)} \implies N_a(t) = \begin{cases} \frac{N_{a-1}(t-1)+M_a(t)}{1+d_a(t)} & \text{if } a > 0 \\ \frac{B(t)+M_a(t)}{1+d_a(t)} & \text{if } a = 0, \end{cases}$$

Methodology. To isolate the contribution of longevity, we consider a counterfactual where we fix the death rates by age. Specifically, we start with the total population and age distribution as of 1960, and simulate the evolution of population assuming the death rates by age remained constant at their 1960 levels, but births and migration by age evolved as in the data:

$$N^{\text{sim}}(t) = \sum_{a=0}^{A} N_a^{\text{sym}}(t) \quad \text{where} \quad N_a^{\text{sim}}(t) = \begin{cases} N_a(0) & \text{if } t = 0 \\ \frac{B(t)+M_a(t)}{1+d_a(0)} & \text{if } t > 0 \text{ and } a = 0 \\ \frac{N_{a-1}^{\text{synth}}(t-1)+M_a(t)}{1+d_a(0)} & \text{if } t > 0 \text{ and } a > 0. \end{cases}$$

We refer to the growth rate of population in this simulation as the counterfactual population growth rate - the one that would have prevailed had death rates by age remained constant at 1960 levels. The gap between this counterfactual growth rate and the actual reflects the contribution of longevity (falling death rate by age) to population growth.

Data. We implement the exercise using annual data on $N_a(t)$, $D_a(t)$ and $B(t)$ from the Human Mortality Database for 24 countries: Australia, Austria, Belgium, Canada, Czechia, Denmark, Finland, France, Luxembourg, Norway, Spain, UK, Italy, Japan, Netherlands, Sweden, Switzerland, Iceland, USA, Portugal, Israel, Hong Kong, Croatia, and South Korea. For all except the last four of these countries, the data start in 1960. Table 1 shows the results of this exercise, contrasting actual and average population growth for each of the countries.

Table 1: Population Growth Holding Longevity Constant

Country	Start	End	g_N	g_N^{sim}
Australia	1960	2019	1.5%	1.4%
Austria	1960	2019	0.4%	0.2%
Belgium	1960	2019	0.4%	0.2%
Canada	1960	2019	1.3%	1.1%
Switzerland	1960	2019	0.8%	0.6%
Czechia	1960	2019	0.2%	0.0%
Denmark	1960	2019	0.4%	0.3%
Spain	1960	2019	0.7%	0.5%
Finland	1960	2019	0.4%	0.2%
France	1960	2019	0.6%	0.4%
UK	1960	2019	0.4%	0.2%
Hong Kong	1986	2019	0.9%	0.8%
Croatia	2001	2019	-0.3%	-0.4%
Iceland	1960	2019	1.2%	1.1%
Israel	1983	2016	2.3%	2.1%
Italy	1960	2019	0.3%	0.1%
Japan	1960	2019	0.5%	0.1%
Korea	2003	2019	0.4%	0.2%
Luxembourg	1960	2019	1.1%	1.0%
Netherlands	1960	2019	0.7%	0.6%
Norway	1960	2019	0.7%	0.6%
Portugal	1960	2019	0.3%	0.0%
Sweden	1960	2019	0.5%	0.4%
USA	1960	2019	1.0%	0.9%
All countries - pop weighted			0.72%	0.53%

B. "Beyond Consumption" Calculations

This section provides details for the derivation and implementation of the framework in Section 5 of the paper. Recall that the welfare function this framework starts with is:

$$W(N_t^p,\, N_t^k,\, c_t^p,\, l_t,\, c_t^k,\, h_t^k,\, b_t) \;=\; N_t^p \cdot u(c_t^p,\, l_t,\, c_t^k,\, h_t^k,\, b_t) + N_t^k \cdot \widetilde{u}(c_t^k).$$

To avoid cumbersome notation, we will use the shorthand:

$$U_t = u(c_t^p, l_t, c_t^k, h_t^k, b_t).$$

B.1 Derivation of CEW growth

Let ω_t^p and ω_t^k be the total welfare shares of parents and kids in year t:

$$\omega_t^p := \frac{N_t^p \cdot U_t}{N_t^P \cdot U_t \;+\; N_t^K \cdot \widetilde{u}(c_t^k)} \quad ; \quad \omega_t^k := \frac{N_t^K \cdot \widetilde{u}(c_t^k)}{N_t^P \cdot U_t \;+\; N_t^K \cdot \widetilde{u}(c_t^k)}.$$

Define adjusted social welfare as in the main text:

$$W(\lambda_t) = \; N_t^p \cdot u\left(\lambda_t c_t^p, l_t, \lambda_t c_t^k, h_t^k, b_t\right) + N_t^k \cdot \widetilde{u}\left(\lambda_t c_t^k\right).$$

Totally differentiating $W(\lambda_t)$ yields:

$$\begin{aligned}\frac{dW_t}{W_t} = \omega_t^p \cdot \Bigg[& \frac{dN_t^P}{N_t^P} \;+\; \frac{u_{c_t^p} \cdot c_t^p \cdot \lambda_t}{U_t} \cdot \left(\frac{d\lambda_t}{\lambda_t} \;+\; \frac{dc_t^p}{c_t^p}\right) \;+\; \frac{u_{lt} \cdot l_t}{U_t} \cdot \frac{dl_t}{l_t} \\ & + \frac{u_{c^k t} \cdot c_t^k \lambda_t}{U_t} \cdot \left(\frac{d\lambda_t}{\lambda_t} \;+\; \frac{dc_t^k}{c_t^k}\right) \;+\; \frac{u_{h^k t} \cdot h_t^k}{U_t} \cdot \frac{dh_t^k}{h_t^k} \;+\; \frac{u_{bt} b_t}{U_t} \cdot \frac{db_t}{b_t}\Bigg] \\ + \omega_t^k \cdot \Bigg[& \frac{dN_t^K}{N_t^K} \;+\; \frac{\tilde{u}'(\lambda_t \cdot c_t^k) \cdot \lambda_t \cdot c_t^k}{\tilde{u}(\lambda_t \cdot c_t^k)} \cdot \left(\frac{d\lambda_t}{\lambda_t} \;+\; \frac{dc_t^k}{c_t^k}\right)\Bigg].\end{aligned}$$

To obtain CEW growth, we set $\frac{dW_t}{W_t} = 0$ and solve for $g_\lambda = -\frac{d\lambda_t}{\lambda_t}$ around $\lambda_t = 1$:

$$g_\lambda = \kappa_t \cdot \Bigg[\omega_t^p \cdot \Bigg(\frac{dN_t^P}{N_t^P} + \frac{u_{c_t^p} \cdot c_t^p}{U_t} \cdot \frac{dc_t^p}{c_t^p} + \frac{u_{lt} \cdot l_t}{U_t} \cdot \frac{dl_t}{l_t} + \frac{u_{c^k t} \cdot c_t^k}{U_t} \cdot \frac{dc_t^k}{c_t^k} + \frac{u_{h^k t} \cdot h_t^k}{U_t} \cdot \frac{dh_t^k}{h_t^k} + \frac{u_{bt} b_t}{U_t} \cdot \frac{db_t}{b_t} \Bigg) + \omega_t^k \cdot \Bigg(\frac{dN_t^K}{N_t^K} + \frac{\tilde{u}'(c_t^k) \cdot c_t^k}{\tilde{u}(c_t^k)} \cdot \frac{dc_t^k}{c_t^k} \Bigg) \Bigg],$$

$$\text{where} \quad \kappa_t := \left[\omega_t^p \cdot \left(\frac{u_{c_t^p} \cdot c_t^p}{U_t} + \frac{u_{c^k t} \cdot c_t^k}{U_t} \right) + \omega_t^k \cdot \frac{\tilde{u}'(c_t^k) \cdot c_t^k}{\tilde{u}(c_t^k)} \right]^{-1}.$$

The optimality conditions of the parent's utility maximization problem will help us map some of these weights to observables. These optimality conditions are:

$$\frac{u_l}{u_{c^p}} = wh \quad ; \quad \frac{u_{c^k}}{u_{c^p}} = b \quad ; \quad \frac{u_b}{u_{c^p}} = whe + c^k \quad ; \quad \frac{u_{h^k}}{u_{c^p}} = \frac{whbe}{\eta h^k}.$$

The log specification in Assumptions 1 and 2 from the main text respectively yield:

$$v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = u\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \quad \text{and} \quad \tilde{v}\left(c_t^k\right) = \tilde{u}\left(c_t^k\right).$$

With these functional forms, and using parental FOCs, one can show that:

$$\kappa_t = \frac{N_t^P \cdot v(c_t^p, c_t^k, \vec{x}_t) + N_t^K \tilde{v}(c_t^k)}{(1 + \alpha \cdot b_t^\theta) \cdot N_t^P + N_t^K}.$$

Plugging back in the expression for g_λ yields:

$$\begin{aligned} g_{\lambda_t} &= \pi_t^p \cdot v\left(c_t^p, c_t^k, \vec{x}_t\right) \cdot \frac{dN_t^p}{N_t^p} + \pi_t^k \cdot \tilde{v}(c_t^k) \cdot \frac{dN_t^k}{N_t^k} \\ &+ \pi_t^p \cdot \frac{dc_t^p}{c_t^p} + (1 - \pi_t^p) \cdot \frac{dc_t^k}{c_t^k} \\ &+ \pi_t^p \cdot \left(\frac{u_{lt}\, l_t}{u_{c^p t}\, c_t^p} \cdot \frac{dl_t}{l_t} + \frac{u_{bt}\, b_t}{u_{c^p t}\, c_t^p} \cdot \frac{db_t}{b_t} + \frac{u_{h^k t}\, h_t^k}{u_{c^p t}\, c_t^p} \cdot \frac{dh_t^k}{h_t^k} \right), \end{aligned}$$

where

$$\pi_t^p = \frac{N_t^p}{(1+\alpha b_t^\theta)N_t^p + N_t^k} \quad ; \quad \pi_t^k = \frac{N_t^k}{(1+\alpha b_t^\theta)N_t^p + N_t^k} \quad ;$$

$$v\left(c_t^p, c_t^k, \vec{x}_t\right) = v\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) = \frac{u\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right)}{u_{c^p}\left(c_t^p, l_t, c_t^k, h_t^k, b_t\right) \cdot c_t^p} \quad ; \quad \tilde{v}(c_t^k) = \frac{\tilde{u}(c_t^k)}{\tilde{u}'(c_t^k) \cdot c_t^k}.$$

Using the optimality conditions and budget constraint from the parent's utility maximization problem, we can map the weights to observables:

$$\begin{aligned} g_{\lambda_t} &= \pi_t^p \cdot v\left(c_t^p, c_t^k, \vec{x}_t\right) \cdot \frac{dN_t^p}{N_t^p} + \pi_t^k \cdot \tilde{v}(c_t^k) \cdot \frac{dN_t^k}{N_t^k} \\ &+ \pi_t^p \cdot \frac{dc_t^p}{c_t^p} + (1-\pi_t^p) \cdot \frac{dc_t^k}{c_t^k} \\ &+ \pi_t^p \cdot (1+\alpha b_t^\theta) \cdot \frac{l_t}{l_{ct}} \cdot \frac{dl_t}{l_t} \\ &+ \pi_t^p \cdot \left(\alpha b_t^\theta + (1 + \alpha b_t^\theta) \cdot \frac{b_t \cdot e_t}{l_{ct}}\right) \cdot \frac{db_t}{b_t} \\ &+ \pi_t^p \cdot (1+\alpha b_t^\theta) \cdot \frac{b_t \cdot e_t}{l_{ct}} \cdot \frac{1}{\eta} \cdot \frac{dh_t^k}{h_t^k}. \end{aligned}$$

B.2 Implementation

In addition to parameters values for α, θ, and η, to implement these "micro" calculations, we need time series for N_t^p (# of adults), N_t^k (# of kids), $b_t = \frac{N_t^k}{N_t^p}$ (# of kids per adult), c_t^p (adult's consumption), c_t^k (kid's consumption), l_t (adult's leisure time), l_{c_t} (adult's work time), and $b_t e_t$ (adult's childcare time).

- The data inputs are:
  - Total population (N_t), consumption (c_t), average hours worked, and number of employed (emp) from the Penn World Tables;[1]
  - Population 0-19 year old (N_t^k) from the World Bank;
  - Population 20-65 from the World Bank (used to calculate hours worked per adult l_{ct} as described below);

[1] The specific data series we use from PWT are **pop** for N_t, **avh** for average hours worked, and **emp** for number of employed. For consumption per capita, we use the same definition from our baseline calculation.

– Total childcare $b_t \cdot e_t$ from time-use surveys. For each country, we keep all respondents who are 20 years or older. Whenever the time-survey is not available at annual frequency, we annualize total childcare assuming a constant annual growth rate between two consecutive time use surveys.

- We calculate the remaining variables from these data inputs using the following relationships:

$$
\begin{aligned}
N_t^p &= N_t - N_t^k \\
b_t &= \frac{N_t^k}{N_t^p} \\
l_{c_t} &= \frac{\text{average hours worked} \times \text{number of employed}}{\text{Population 20-65 years old}} \\
l_t &= 16 \text{ hours/day} - l_{c_t} - b_t e_t \\
c_t^p &= \frac{1 + b_t}{1 + \alpha b_t^\theta} \cdot c_t \\
c_t^k &= \alpha b_t^{\theta - 1} \cdot c_t^p \, .
\end{aligned}
$$

  The last two expressions combine accounting:

$$
c_t := \frac{N_t^p}{N_t^p + N_t^k} \cdot c_t^p \; + \; \frac{N_t^k}{N_t^p + N_t^k} \cdot c_t^k \, ,
$$

  with the parent's optimality conditions.

We calibrate the growth rate in kid's human capital, $\frac{dh_t^k}{h_t^k}$, as follows. We first assume that growth rate is constant within a country in our sample. That implies:

$$
\frac{dh_t^k}{h_t^k} = \frac{dh^k}{h^k} = \frac{dh}{h} := \text{ average of } \frac{dh_t}{h_t} \, .
$$

That is, the average growth rate in kid's human capital can be captured by its growth rate for the adult population.

To measure $\frac{dh}{h}$, we assume that growth in labor productivity reflects equal contributions from growth in human capital and growth in productivity per unit of human capital (growth in w_t). The latter growth would reflect growth in TFP and physical capital. Note labor productivity can expressed in term of consumption per working hour as:

$$w_t h_t = \frac{(1+\alpha b_t^{\theta}) c_t^p}{l_{c_t}} .$$

So attributing half of productivity growth to human capital growth implies:

$$\frac{dh_t}{h_t} = \frac{1}{2} \frac{d\left(\frac{(1+\alpha b_{t+1}^{\theta}) c_{t+1}^p}{l_{ct+1}}\right)}{\frac{(1+\alpha b_{t+1}^{\theta}) c_{t+1}^p}{l_{ct+1}}} .$$

Finally, to implement these calculations, we need the value of a year of life relative to consumption for both kids and parents for each country-year. Given our assumed utility function for kid's utility:

$$\tilde{v}(c_t^k) = \bar{u}_k + \log(c_t^k) ,$$

a parent's first-order condition ties c_t^k to parent's consumption, and so to average consumption. But we still need to calibrate $\bar{u}_k$ to get $\tilde{v}(c_t^k)$ in all country-years. Our calibration remains hinged to the U.S. in 2006. Furthermore, we still target an adult's value of life relative to consumption of 4.87 for the U.S. in 2006. This remains the same as in the baseline calculation, reflecting that individuals sampled in the VSL studies we cite are adults.

Assume a specific ratio, call it μ, for the flow utility of a kid to that of an adult for the U.S. in 2006. Given the log utility specifications, this directly implies that the ratio of $\tilde{v}(c^k_{\text{US, 2006}})$ to $v^p_{\text{US, 2006}}$ is also μ. Thus, conditional on a value for μ, we arrive at $\tilde{v}(c^k_{\text{US, 2006}})$ and thereby $\bar{u}_k$. This yields:

$$\bar{u}_k = \mu \cdot 4.8705 - \log\left(\alpha b_{\text{US, 2006}}^{\theta-1} \cdot \frac{1 + b_{\text{US, 2006}}}{1 + \alpha b_{\text{US, 2006}}^{\theta}}\right) .$$

For the parents, since we do not fully parameterize the utility function, we proceed as follows:

- conditional on establishing $v(c_t^p, c_t^k, \vec{x}_t)$ in a specific country for a base year, we chain weight to obtain its values in other years. Specifically, because of the log assumption on utility from consumption, we have:

$$v\left(c_t^p, c_t^k, \overrightarrow{x_t}\right) = \frac{u\left(c_t^p, c_t^k, \overrightarrow{x_t}\right)}{u_{c_t^p}\left(c_t^p, c_t^k, \overrightarrow{x_t}\right)\cdot c_t^p} = u\left(c_t^p, c_t^k, \overrightarrow{x_t}\right) \;;\; \text{so } \boxed{v_t = v_{t-1} + dU_t},$$

where:

$$dU_t = u_{c_t^p}\cdot c_t^p \cdot \frac{dc_t^p}{c_t^p} + u_{c_t^k}\cdot c_t^k \cdot \frac{dc_t^k}{c_t^k} + u_{l_t}\cdot l_t \cdot \frac{dl_t}{l_t} + u_{h_t^k}\cdot h_t^k \cdot \frac{dh_t^k}{h_t^k} + u_{b_t}\cdot b_t \cdot \frac{db_t}{b_t}\,.$$

That is, compared to our baseline treatment, the mapping of $v\left(c_t^p, c_t^k, \overrightarrow{x_t}\right)$ through time and across countries reflects, not only parent's consumption growth, but also growth in kid's consumption, leisure, and the number and quality of kids.

Using parent's optimality conditions and budget constraint:

$$\begin{aligned} dU_t = &\frac{dc_t^p}{c_t^p} + \alpha b_t^\theta \cdot \frac{dc_t^k}{c_t^k} + \left(1+\alpha b_t^\theta\right)\frac{l_t}{l_{c_t}}\cdot\frac{dl_t}{l_t} \\ &+ \left(1+\alpha b_t^\theta\right)\frac{b_t e_t}{l_{c_t}}\cdot\frac{1}{\eta}\frac{dh_t^k}{h_t^k} + \left[\left(1+\alpha b_t^\theta\right)\frac{b_t\left(\phi+e_t\right)}{l_{c_t}} + \alpha b_t^\theta\right]\cdot\frac{db_t}{b_t}\,. \end{aligned} \tag{1}$$

- For all countries, the base year is 2006

- For the US, v^p in the base year is pinned down by calibration target:

$$v^p_{\text{US, 2006}} = 4.87$$

- Using chain-weighting across countries in 2006, we get v^p_{2006} for each other country in base year 2006, with the U.S. acting as the "base country". To do this chain weighting, we first rank the six countries based on their per capita consumptions in 2006. We then use equation 1 to calculate the change (percent differential) in v^p across any two "consecutive" countries. In calculating that percent differential:

- We use arc growth rates:

$$\frac{dx}{x} = \frac{x_i - x_{i-1}}{1/2 \cdot x_{i-1} + 1/2 \cdot x_i} \ .$$

- We employ Tornqvist weights — that is, weights in equation 1 are the average of the corresponding values in the 2 consecutive countries;
- For the g_h^k terms: We assume $g_h^k = g_h$; we then back out g_h from the budget constraint (income accounting), assuming one-half of labor productivity differences across the two consecutive countries in 2006 reflect human capital differences.