



# Life and Growth

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## R. Posner (2004) *Catastrophe: Risk and Response*

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*“Certain events quite within the realm of possibility, such as a major asteroid collision, global bioterrorism, abrupt global warming — even certain lab accidents— could have unimaginably terrible consequences up to and including the extinction of the human race... I am not a Green, an alarmist, an apocalyptic visionary, a catastrophist, a Chicken Little, a Luddite, an anticapitalist, or even a pessimist. But... I have come to believe that what I shall be calling the ‘catastrophic risks’ are real and growing...”*

# Should we switch on the Large Hadron Collider?

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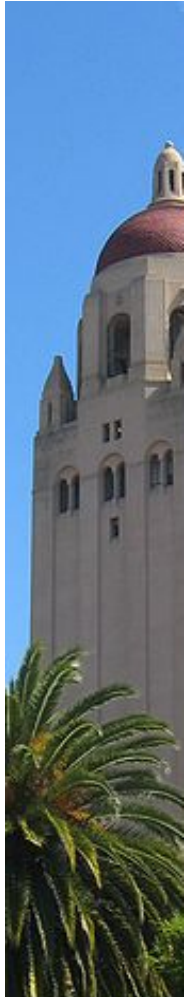
- Physicists have considered the possibility that colliding particles together at energies not seen since the Big Bang could cause a major disaster (mini black hole, strangelets).
- Conclude that the probability is tiny.
- But how large does it have to be before we would not take the risk?
- As economic growth makes us richer, should our decision change?

# Growth involves costs as well as benefits

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- Benefits | Costs
  - Nuclear power | Nuclear holocaust
  - Biotechnology | Bioterror
  - Nanotechnology | Nano-weapons
  - Coal power | Global warming
  - Internal combustion engine | Pollution
  - Radium, thalidomide, lead paint, asbestos
- Technologies (new pharmaceuticals, medical equipment, airbags, pollution scrubbers) can also save lives

**How do considerations of life and death affect the theory of economic growth and technological change?**



# The “Russian Roulette” Model

New ideas raise consumption,  
but a tiny probability of a disaster...

## Simple Model

- Single agent born at the start of each period
- Endowed with stock of ideas  $\Rightarrow$  consumption  $c$ , utility  $u(c)$
- Only decision: to research or not to research
  - **Research:**

With (high) probability  $1 - \pi$ , get a new idea that raises consumption by growth rate  $\bar{g}$ .

But, with small probability  $\pi$ , disaster kills the agent.
  - **Stop:** Consumption stays at  $c$ , no disaster.

- Expected utility for the two options:

$$\begin{aligned} U^{\text{Research}} &= (1 - \pi)u(c_1) + \pi \cdot 0 = (1 - \pi)u(c_1), \quad c_1 = c(1 + \bar{g}) \\ U^{\text{Stop}} &= u(c) \end{aligned}$$

- Taking a first-order Taylor expansion around  $u(c)$ , agent undertakes research if

$$(1 - \pi)u'(c)\bar{g}c > \pi u(c)$$

- Rearranging:

$$\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}$$

## Three Cases

- Consider CRRA utility:

$$u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma}$$

$\bar{u}$  is a key parameter

- Three cases:
  - $0 < \gamma < 1$
  - $\gamma > 1$
  - log utility ( $\gamma = 1$ )



## Case 1: $0 < \gamma < 1$

$$\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}$$

- The value of life relative to consumption:

$$\frac{u(c)}{u'(c)c} = \bar{u}c^{\gamma-1} + \frac{1}{1 - \gamma}.$$

- $\bar{u}$  not important, so set  $\bar{u} = 0$

$$\Rightarrow \frac{u(c)}{u'(c)c} = 1/(1 - \gamma)$$

**Exponential growth, with rare disasters.**

## Case 2: $\gamma > 1$

- Notice that we've implicitly normalized the utility from death to be zero (in writing the lifetime expected utility function)
  - So flow utility must be positive for consumer to prefer life
- But  $\gamma > 1$  implies  $u(c)$  negative if  $\bar{u} = 0$ :

$$u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma}$$

Example:  $\gamma = 2$  implies  $u(c) = -1/c$ .

- Therefore  $\bar{u} > 0$  is required in this case.

## Case 2: $\gamma > 1$ (continued)

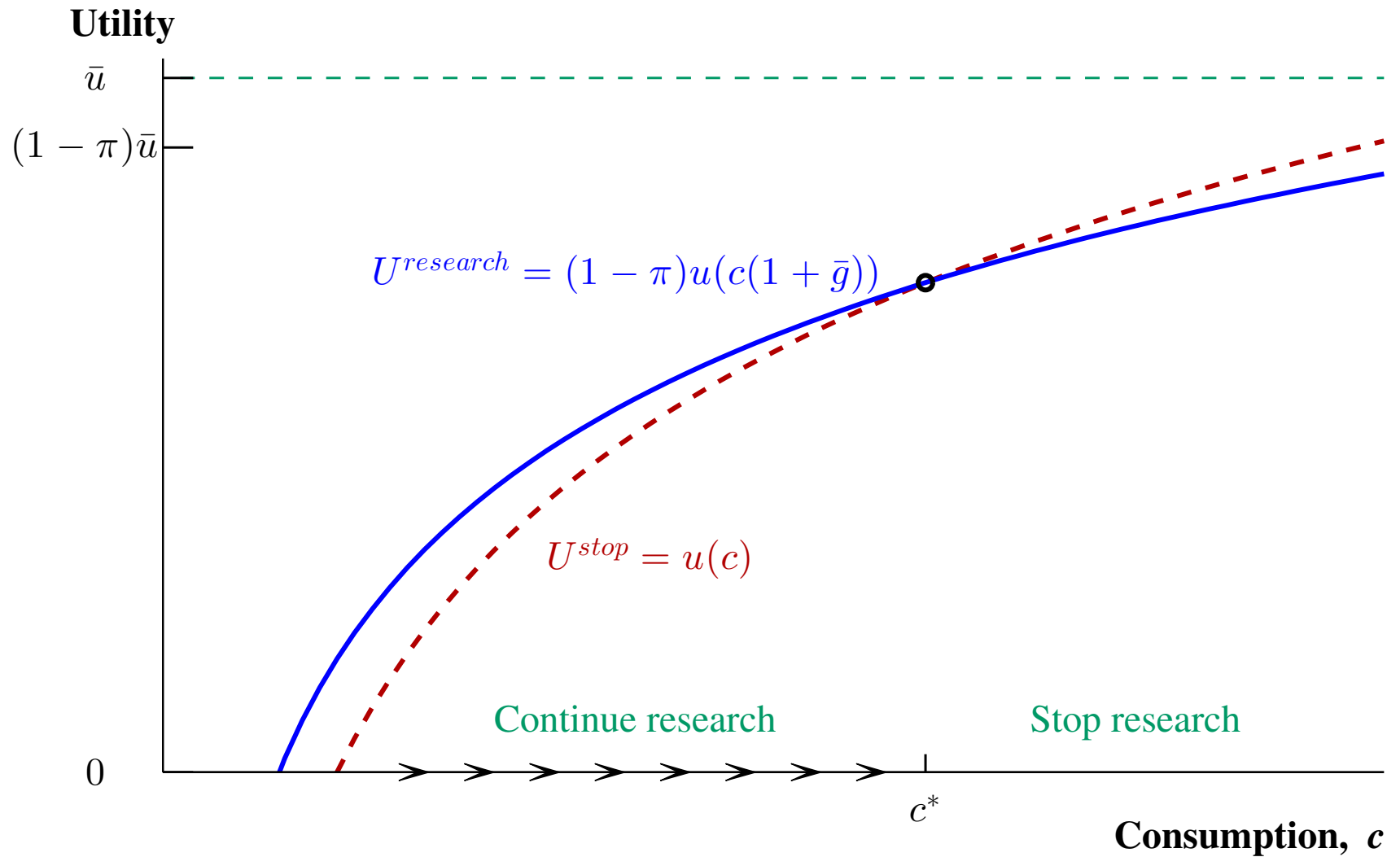
$$\bar{g} > \frac{\pi}{1 - \pi} \cdot \frac{u(c)}{u'(c)c}$$

- With  $\gamma > 1$ , the value of life rises relative to consumption!

$$\frac{u(c)}{u'(c)c} = \bar{u}c^{\gamma-1} + \frac{1}{1 - \gamma}.$$

Eventually, people are rich enough that the risk to life of Russian Roulette is too great and **growth ceases**.

# The Research Decision when $\gamma > 1$



### Case 3: log utility ( $\gamma = 1$ )

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- Flow utility is unbounded in this case
- But the value of life relative to consumption still rises

$$u(c)/u'(c)c = \bar{u} + \log c$$

- So growth eventually ceases in this case as well.



# Microfoundations in a Growth Model

Hall and Jones (2007) meet  
Acemoglu (Direction TechChg)

Production  $C_t = \left(\int_0^{A_t} x_{it}^{1/\alpha} di\right)^\alpha, \quad H_t = \left(\int_0^{B_t} z_{it}^{1/\alpha} di\right)^\alpha$

Ideas  $\dot{A}_t = \bar{a}S_{at}^\lambda A_t^\phi, \quad \dot{B}_t = \bar{b}S_{bt}^\lambda B_t^\phi$

RC (labor)  $L_{ct} + L_{ht} \leq L_t, \quad L_{ct} \equiv \int_0^{A_t} x_{it} di, \quad L_{ht} \equiv \int_0^{B_t} z_{it} di$

RC (scientists, pop)  $S_{at} + S_{bt} \leq S_t, \quad S_t + L_t \leq N_t$

Mortality  $\delta_t = h_t^{-\beta}, \quad h_t \equiv H_t/N_t$

Utility  $U = \int_0^\infty e^{-\rho t} u(c_t) M_t dt, \quad \dot{M}_t = -\delta_t M_t$

Flow util.  $u(c_t) = \bar{u} + \frac{c_t^{1-\gamma}}{1-\gamma}, \quad c_t \equiv C_t/N_t$

Pop growth  $\dot{N}_t = \bar{n}N_t$

## Allocating Resources

- 14 unknowns, 11 equations (not counting utility)
  - $C_t, H_t, c_t, h_t, A_t, B_t, x_{it}, z_{it}, S_{at}, S_{bt}, S_t, L_t, N_t, \delta_t$
- Three allocative decisions to be made
  - ( $s_t$ ) Scientists:  $S_{at} = s_t S_t$
  - ( $l_t$ ) Workers:  $L_{ct} = l_t L_t$
  - ( $\sigma_t$ ) People:  $S_t = \sigma_t N_t$
- Rule of Thumb allocation:  $s_t = \bar{s}$ ,  $l_t = \bar{l}$ , and  $\sigma_t = \bar{\sigma}$



## BGP under the Rule of Thumb

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PROPOSITION 1: As  $t \rightarrow \infty$ , there exists an asymptotic balanced growth path such that growth is given by

$$g_A^* = g_B^* = \frac{\lambda \bar{n}}{1 - \phi}$$

$$\delta^* = 0$$

$$g_c^* = g_h^* = \alpha g_A^* = \alpha g_B^* = \bar{g} \equiv \frac{\alpha \lambda \bar{n}}{1 - \phi}.$$

# The Optimal Allocation

$$\max_{\{s_t, \ell_t, \sigma_t\}} U = \int_0^{\infty} M_t u(c_t) e^{-\rho t} dt \quad s.t.$$

$$c_t = A_t^\alpha \ell_t (1 - \sigma_t)$$

$$h_t = B_t^\alpha (1 - \ell_t) (1 - \sigma_t)$$

$$\dot{A}_t = \bar{a} s_t^\lambda \sigma_t^\lambda N_t^\lambda A_t^\phi$$

$$\dot{B}_t = \bar{b} (1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_t^\phi$$

$$\dot{M}_t = -\delta_t M_t, \quad \delta_t = h_t^{-\beta}$$

# Hamiltonian

- In solving, useful to define

$$\mathcal{H} = M_t u(c_t) + p_{at} \bar{a} s_t^\lambda \sigma_t^\lambda N_t^\lambda A_t^\phi + p_{bt} \bar{b} (1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_t^\phi - v_t \delta_t M_t$$

- Co-state variables:
  - $p_{at}$ : shadow value of a consumption idea
  - $p_{bt}$ : shadow value of a life-saving idea
  - $v_t$ : shadow value of an extra person

## Optimal Growth with $\gamma > 1 + \beta$

**PROPOSITION 2:** Assume  $\gamma > 1 + \beta$ . There is an asymptotic balanced growth path such that  $\ell_t$  and  $s_t$  both fall to zero at constant exponential rates, and

$$g_s^* = g_\ell^* = \frac{-\bar{g}(\gamma - 1 - \beta)}{1 + (\gamma - 1)(1 + \frac{\alpha\lambda}{1-\phi})} < 0$$

$$g_A^* = \frac{\lambda(\bar{n} + g_s^*)}{1 - \phi}, \quad g_B^* = \frac{\lambda\bar{n}}{1 - \phi} > g_A^*$$

$$g_\delta^* = -\beta\bar{g}, \quad g_h^* = \bar{g}$$

$$g_c^* = \alpha g_A^* + g_\ell^* = \bar{g} \cdot \frac{1 + \beta(1 + \frac{\alpha\lambda}{1-\phi})}{1 + (\gamma - 1)(1 + \frac{\alpha\lambda}{1-\phi})}$$

## Intuition

$$\dot{A}_t = \bar{a}s_t^\lambda \sigma_t^\lambda N_t^\lambda A_t^\phi \text{ and } \dot{B}_t = \bar{b}(1 - s_t)^\lambda \sigma_t^\lambda N_t^\lambda B_t^\phi.$$

- $1 - s_t \rightarrow 1$ , but  $s_t$  falls exponentially, slowing growth in  $A_t$ .
- Why? The FOC for allocating  $l_t$  is

$$\frac{1 - l_t}{l_t} = \beta \frac{\delta_t v_t}{u'(c_t)c_t} = \delta_t \tilde{v}_t$$

where  $v_t$  is the shadow value of a life, from the Hamiltonian.

- Numerator is extra lives that can be saved, denominator is extra consumption that can be produced
- Race!

## Optimal Growth with $\gamma < 1 + \beta$

**PROPOSITION 3:** Assume  $1 < \gamma < 1 + \beta$ . There is an asymptotic balanced growth path such that  $\tilde{\ell}_t \equiv 1 - \ell_t$  and  $\tilde{s}_t \equiv 1 - s_t$  both fall to zero at constant exponential rates, and

$$g_A^* = \frac{\lambda \bar{n}}{1 - \phi}, \quad g_B^* = \frac{\lambda(\bar{n} + g_{\tilde{s}}^*)}{1 - \phi} < g_A^*$$

$$g_c^* = \bar{g}, \quad g_{\tilde{\delta}}^* = -\beta g_h^*.$$

$$g_{\tilde{s}}^* = g_{\tilde{\ell}}^* = \frac{-\bar{g}(\beta + 1 - \gamma)}{1 + \beta(1 + \frac{\alpha\lambda}{1-\phi})} < 0$$

$$g_h^* = g_c^* \cdot \left( \frac{1 + (\gamma - 1)(1 + \frac{\alpha\lambda}{1-\phi})}{1 + \beta(1 + \frac{\alpha\lambda}{1-\phi})} \right) < g_c^*.$$

## Optimal Growth with $\gamma = 1 + \beta$

PROPOSITION 4: Assume  $1 < \gamma = 1 + \beta$ . There is an asymptotic balanced growth path such that  $\ell_t$  and  $s_t$  settle down to constants strictly between 0 and 1, and

$$g_A^* = g_B^* = \frac{\lambda \bar{n}}{1 - \phi}$$

$$g_c^* = g_h^* = \bar{g}, \quad g_\delta^* = -\beta \bar{g}.$$



# Empirical Evidence



# Empirical Evidence

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- $\gamma - 1$  versus  $\beta$ 
  - $\gamma > 1$  is the “normal” case
  - Evidence from health “production functions” suggests  $\beta$  is relatively small
- Trends in R&D: Toward health
- Quantifying the “growth slowdown”

## Evidence on $\beta$ (Recall: $\delta_t = h_t^{-\beta}$ )

- Plausible upper bound compares trends in mortality to trends in health spending
  - Attributes all decline in mortality to real health spending
  - Minimal quality adjustment reinforces “upper bound” view for  $\beta$
- Numbers for 1960 – 2007
  - Age-adjusted mortality rates fell at 1.2% per year
  - CPI-deflated health spending grew at 4.1% per year
    - $\Rightarrow \beta^{\text{upper bound}} \approx \frac{1.2}{4.1} \approx .3$
- Hall and Jones (2007) more careful analysis along these lines finds age-specific estimates of between .10 and .25.

## Evidence on $\gamma$

- Risk aversion evidence suggests  $\gamma > 1$ 
  - Asset pricing (Lucas 1994), Labor supply (Chetty 2006)
- Intertemporal substitution elasticity  $1/\gamma$ 
  - Traditional view is  $EIS < 1 \Rightarrow \gamma > 1$  (Hall 1988)
  - Careful micro work supporting this view: Attanasio and Weber (1995), Barsky et al (1997), Guvenen (2006), Hall (2009)
  - Other recent work finds some evidence for  $EIS > 1 \Rightarrow \gamma < 1$  (Vissing-Jorgensen and Attanasio 2003, Gruber 2006)
- Mixed evidence.

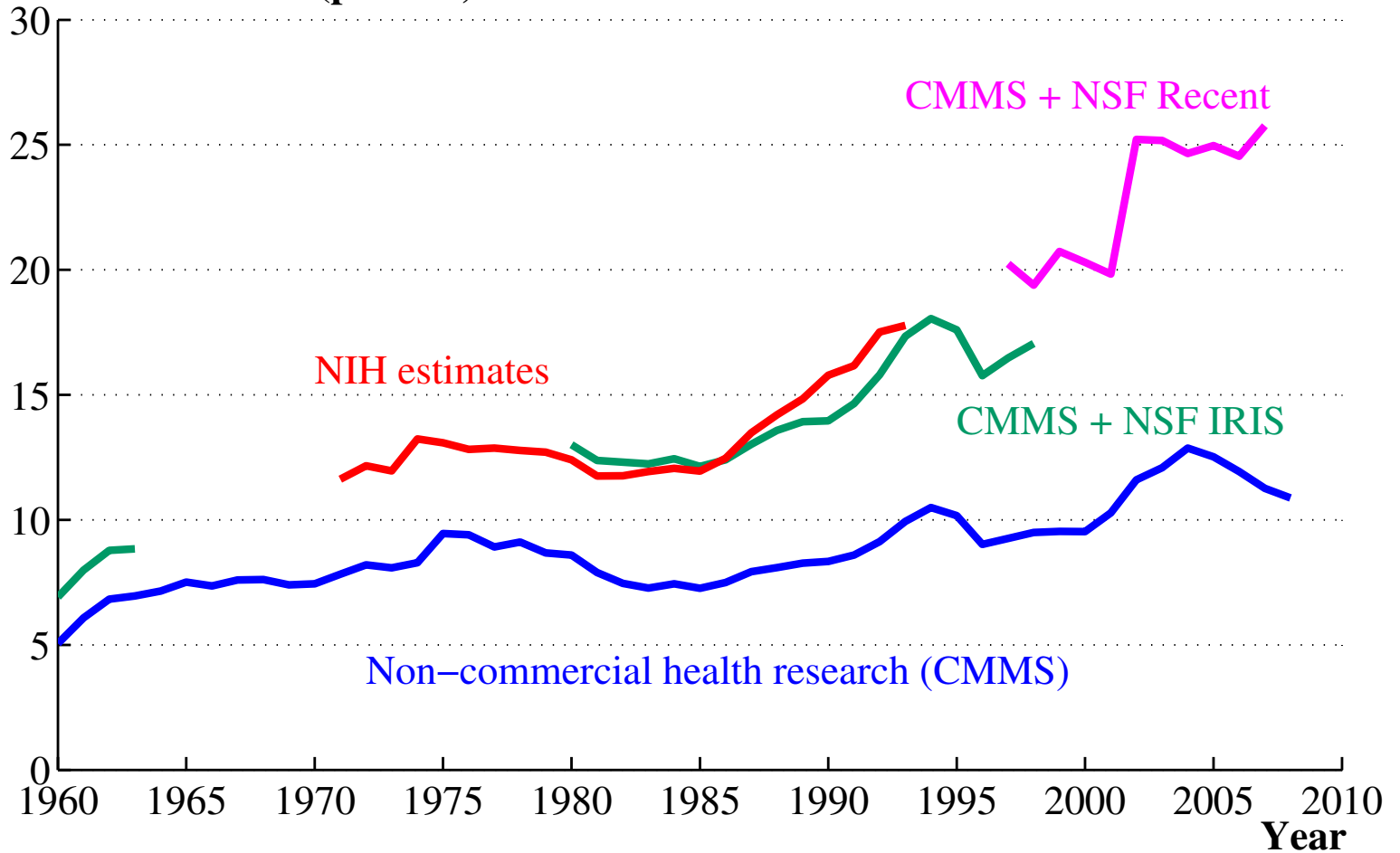
## Evidence on the Value of Life

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- Same force as in Hall and Jones (2007) on health spending
  - Consumption runs into sharply diminishing returns:  $u'(c)$
  - While life becomes increasingly valuable:  $u(c)$
- Evidence on value of life?
  - Nearly all is cross sectional
  - Costa and Kahn (2004), Hammitt, Liu, and Liu (2000)
- Other evidence? Safety standards?

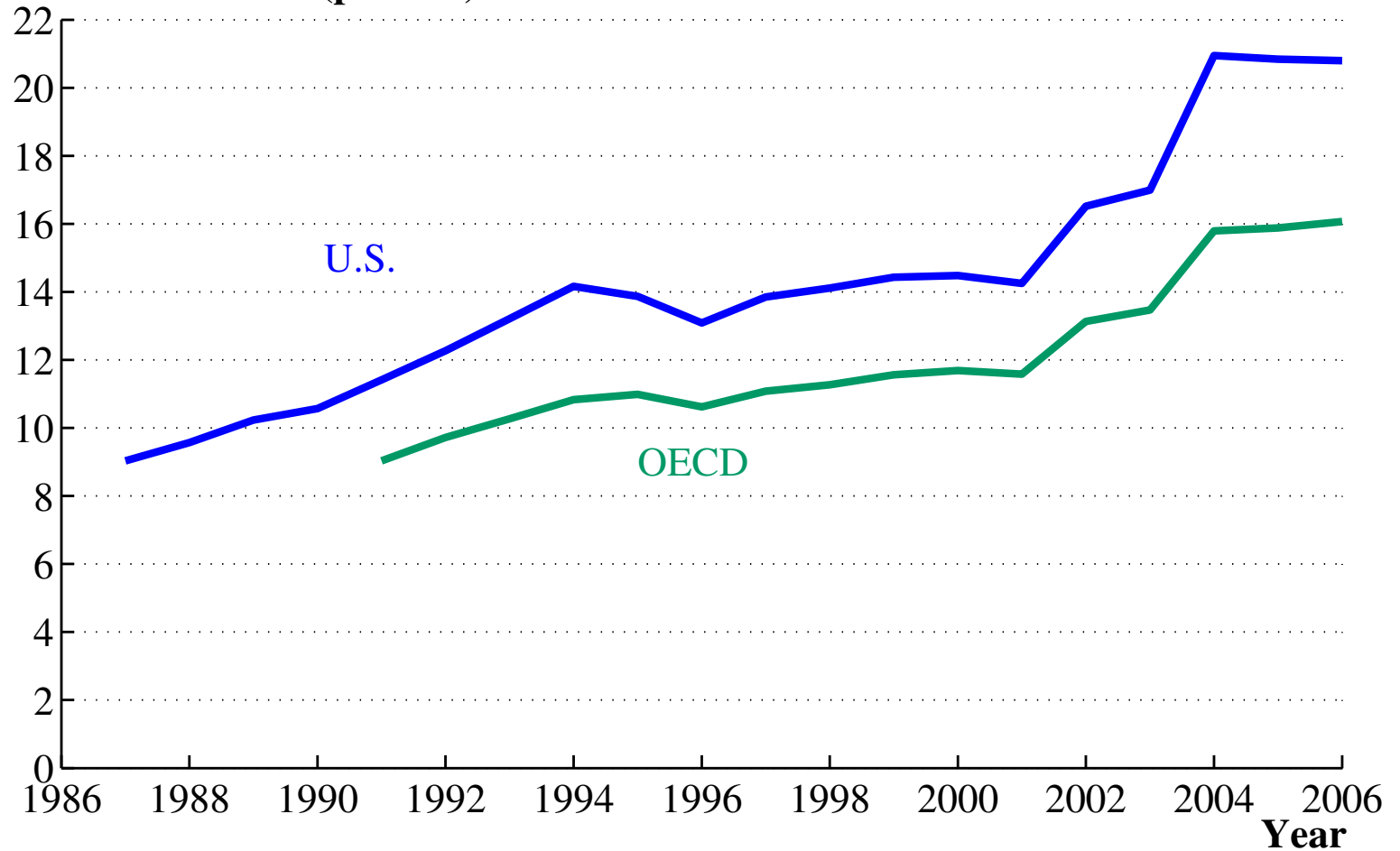
# The Changing Composition of U.S. R&D Spending

Health Share of R&D (percent)

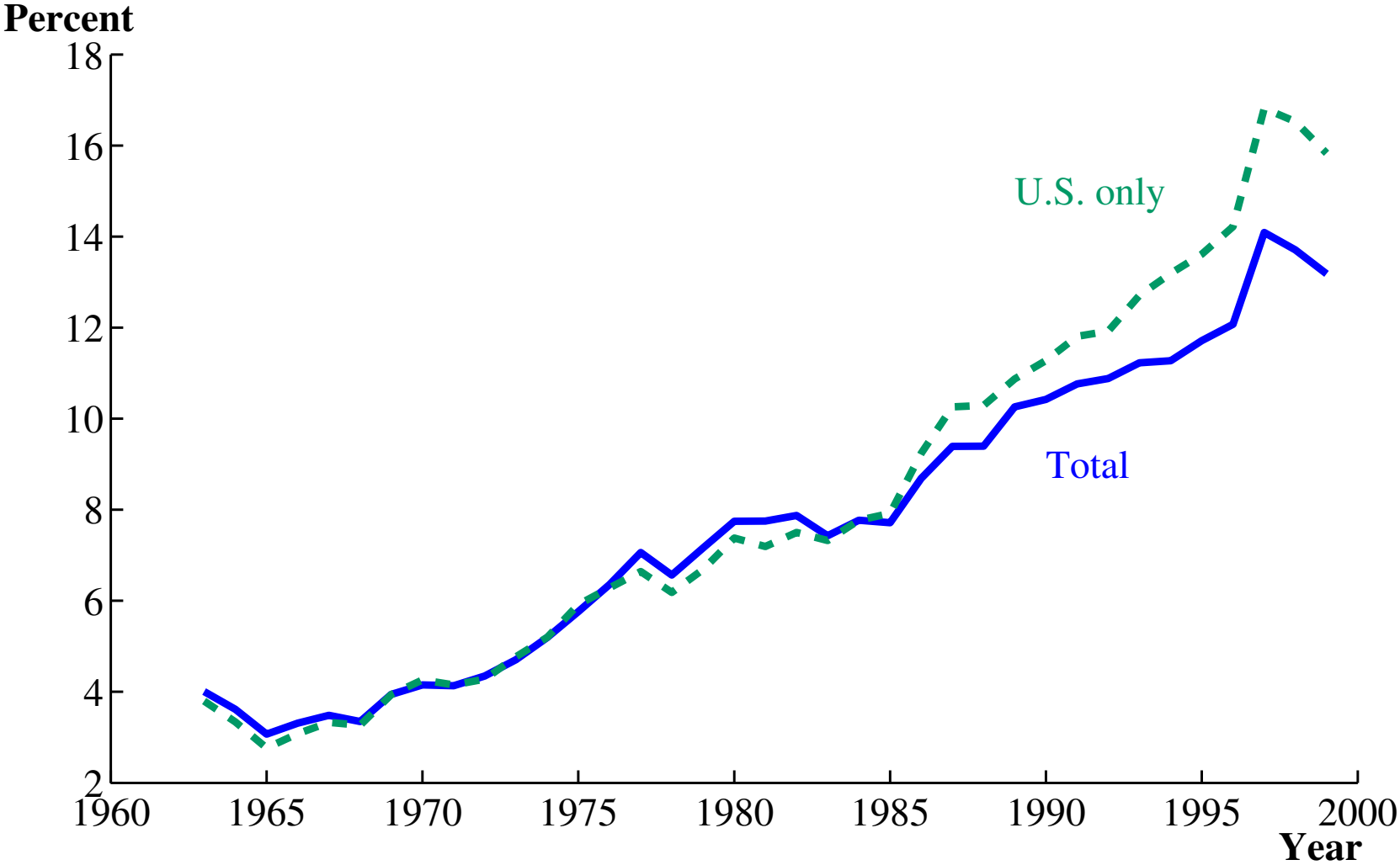


# The Changing Composition of OECD R&D Spending

Health Share of R&D (percent)



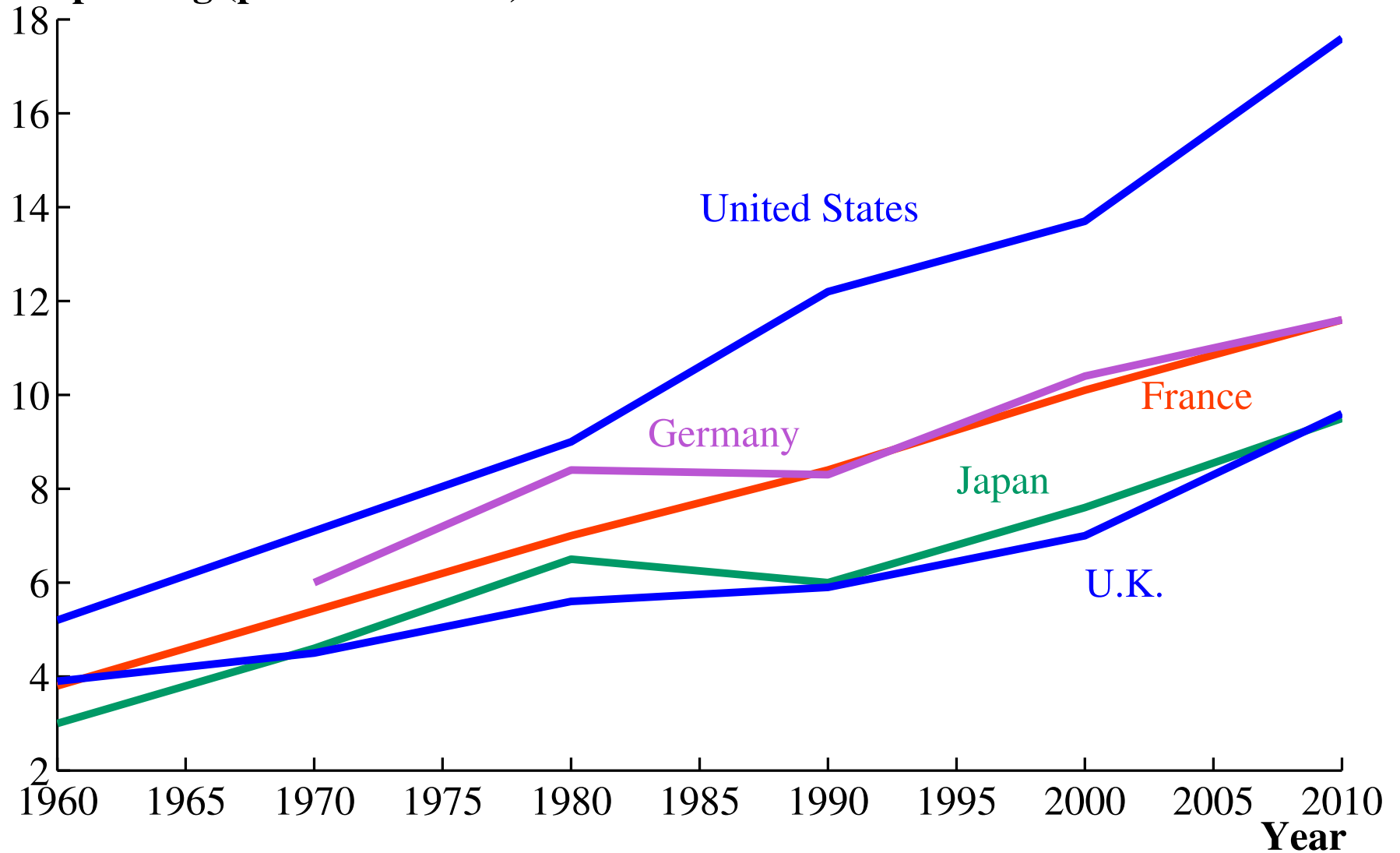
# Fraction of Patents for Medical Eq. or Pharma



Source: Jeffrey Clemens

# An Income Effect in Health Spending

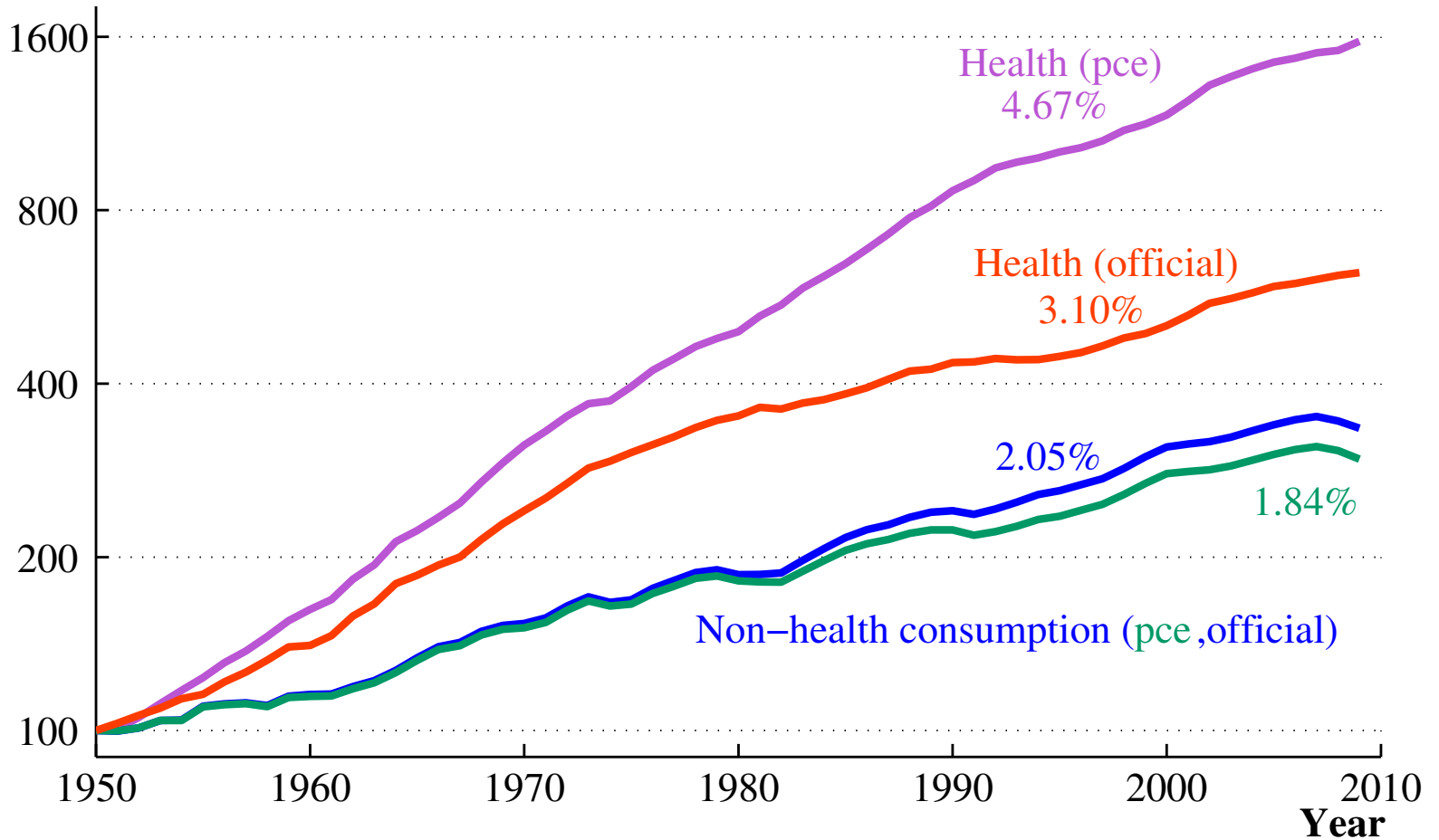
Health spending (percent of GDP)





# Health and Consumption

Real quantity per person (1950=100)



## The Growth Drag: Ratio of $g_c$ to $g_h$

$\frac{\alpha\lambda}{1-\phi}$	$\beta = .25$		$\beta = .10$	
	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 1.5$	$\gamma = 2$
0.50	0.79	0.55	0.66	0.46
1.00	0.75	0.50	0.60	0.40
2.00	0.70	0.44	0.52	0.33

## A Future Slowdown?

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- Calibration to past growth suggests  $\frac{\alpha\lambda}{1-\phi} < 2$ , so that  $\bar{g} < 2\%$ .
  - Therefore growth in  $h$  must slowdown significantly from its  $4 + \%$  rate.
  - And  $g_c \approx \frac{1}{2}g_h < 1\%$  suggests a slowdown of consumption growth as well.
- Intuition:  $h$  has been growing much faster than its steady state rate because of the rising share of research devoted to life-saving technologies.

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## Conclusions

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- Including “life and death” considerations in growth models can have first order consequences.
- For a large class of preferences, safety is a luxury good.
  - Diminishing returns to consumption on any given day means that additional days of life become increasingly valuable.
  - R&D may tilt toward life-saving technologies and away from standard consumption goods.
  - Consumption growth may be substantially slower than what is feasible, possibly even slowing all the way to zero.