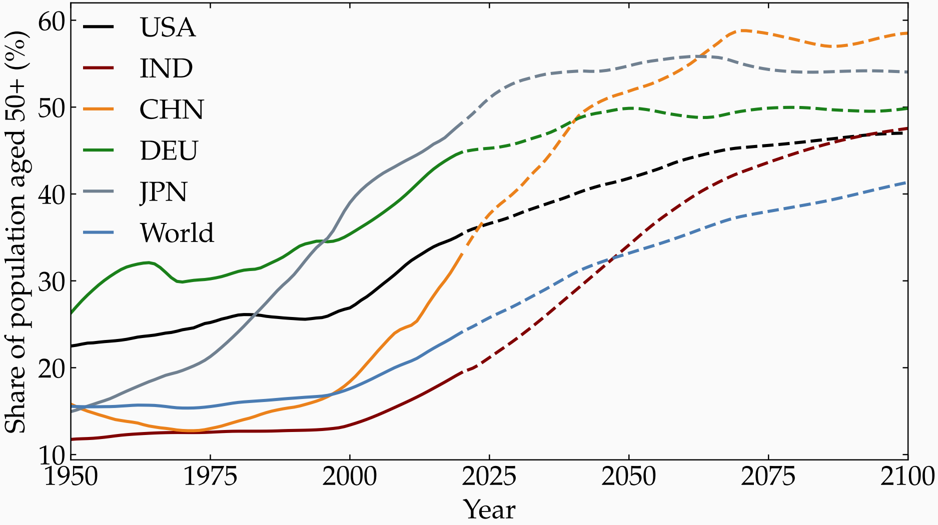
Demographics, Wealth, and Global Imbalances in the Twenty-First Century

Adrien Auclert, Hannes Malmberg, Frédéric Martenet and Matthew Rognlie

Harvard, December 2023

The world population is aging...

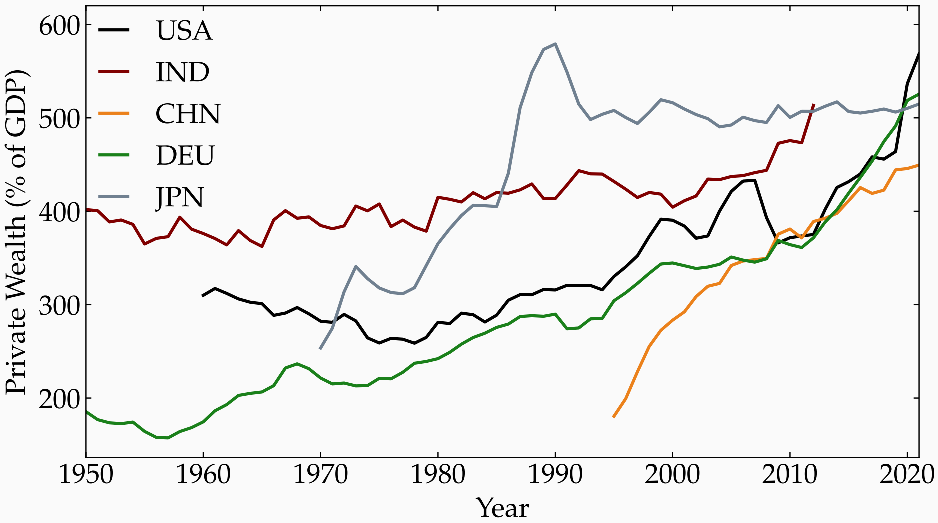
▸ 65+



Source: 2019 United Nations World Population Prospects

...wealth-to-GDP ratios are increasing...

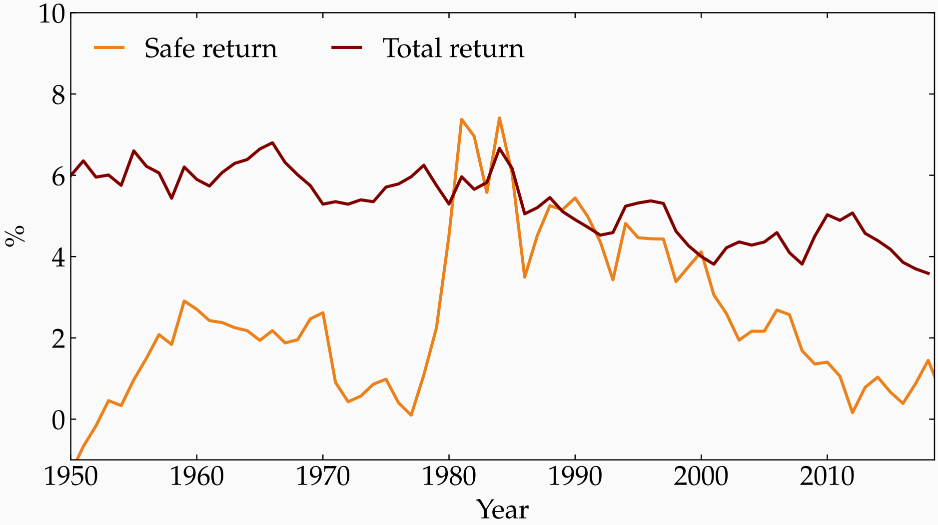
▸ SCF vs WID



*IND: National rather than Private Wealth. Source: World Inequality Database (WID)

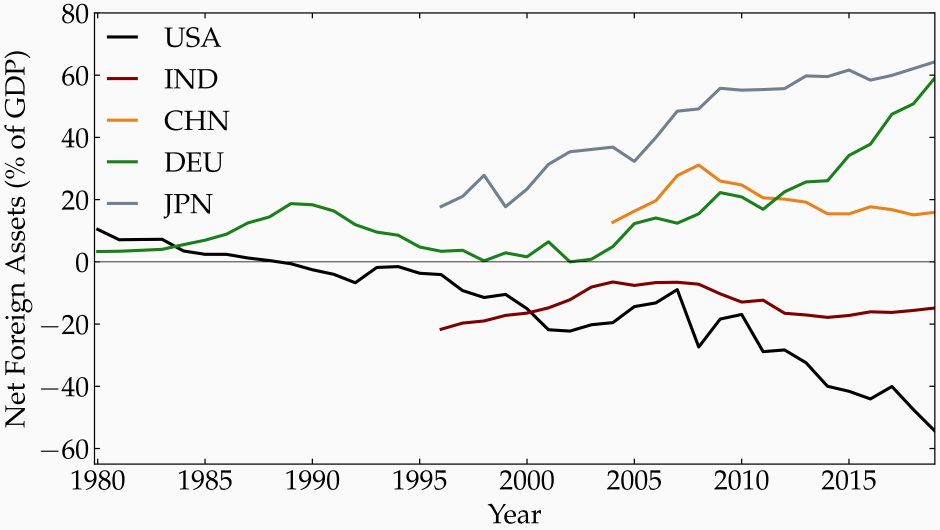
...rates of return on wealth are falling...

▸ Calculation



Source: National Accounts, Flow of Funds, WID.

…and “global imbalances” are rising



Source: International Monetary Fund (IMF), Penn World Table (PWT) 9.1

How will demographics shape these trends in the 21st century?

- Broad agreement that population ageing has contributed to historical trends in W/Y, real returns (r), and NFA imbalances
  - Why? An aging population saves more, and aging is uneven across countries

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  "**great demographic reversal**" hypothesis [Goodhart-Pradhan 2020]

This paper: a sufficient statistic approach to this question

In a baseline multi-country GE overlapping generations (OLG) model,
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1. Age profiles of wealth, labor income, and consumption
2. Demographic projections
3. The elasticity of intertemporal substitution σ
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Conclusions are robust to quantitative simulations of richer model

A bridge between reduced-form and structural approaches

- Existing literature follows two broad approaches:

1. **Reduced-form**, based on shift-share exercises
   - Projected asset demand [Poterba 2001, Mankiw-Weil 1989], projected savings rates [Summers-Carroll 1987, Auerbach-Kotlikoff 1990, Mian-Straub-Sufi 2021...]
   - Projected labor supply [Cutler et al 1990], demographic dividend lit. [Bloom-Canning-Sevilla 2003...]
2. **Structural**, based on fully specified GE OLG models
   - Demographics and **wealth** + social security [Auerbach Kotlikoff 1987, İmrohoroğlu-İmrohoroğlu-Joines 1995, De Nardi-İmrohoroğlu-Sargent 2001, Abel 2003, Geanakoplos-Magill-Quinzii 2004, Kitao 2014...]
   - Demographics and **interest rates** [Carvalho-Ferrero-Necchio 2016, Gagnon-Johannsen-Lopez Salido 2016, Eggertsson-Mehrotra-Robbins 2019, Lisack-Sajedi-Thwaites 2017, Jones 2018, Papetti 2019, Rachel-Summers 2019...]
   - Demographics and **capital flows** [Henriksen 2002, Domeij-Flodén 2006, Börsch-Supan-Ludwig-Winter 2006, Krueger-Ludwig 2007, Backus-Cooley -Henriksen 2014, Bárány-Coeurdacier-Guibaud 2019, Sposi 2021...]

- **Sufficient statistic approach** bridges the gap between both

Baseline model

Environment: demographics, production, and government

OLG model, demographic change + multiple countries facing $\{r_t\}$

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Demographics [drop country subscripts]

- Exogenous, time-varying sequence of births N_{0t}
- Exogenous, constant sequence of mortality rates ϕ_j ▸ Mortality contribution
- No migration

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Production

- Aggregate production fn with capital and effective labor, elasticity of substitution η
- Constant growth rate of labor-augmenting technology γ
- Perfect competition, free capital adjustment

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Government

- Flow budget constraint

$$G_t + w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} \ell_j + B_{t+1},$$

- Balance budget by changing G_t, not τ_t or tr_{jt}, to keep $B_t / Y_t \equiv \text{cst}$

Environment: heterogeneous agents

Problem for **heterogeneous agents** of cohort k (age $j \equiv t - k$):

$$\max \quad \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right]$$

$$\text{s.t.} \quad c_{jt} + \phi_j a_{j+1,t+1} \leq w_t((1-\tau)\ell(z_{jt}) + tr(z^{jt})) + (1 + r_t) a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}\,(1+\gamma)^t$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j: age-specific discount rate
- Φ_j: survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_{jt})$: risky labor supply driven by arbitrary stochastic process z_t
- $\tau, tr(z^{jt})$: taxes and (state-contingent) government transfers
- a_{jt}: annuity holdings

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Equilibrium

Given demographics and policy, in an **integrated world equilibrium**:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c \underbrace{N_t^c \mathbb{E} a_{jt}^c}_{W_t^c} = \sum_c (K_t^c + B_t^c) \quad \forall t$$

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Next: consider small country aging alone, with rest of world at steady state

$\rightarrow$ r constant (will adjust later)

Compositional effects as sufficient statistics

Proposition

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

where $a_{j0} \equiv \mathbb{E} a_{j,0}$ and $h_{j0} = \mathbb{E} w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j.

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Proposition

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}$$

where $a_{jo} \equiv \mathbb{E} a_{j,o}$ and $h_{jo} = \mathbb{E} w_o \ell_{j,o}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j.

$\Rightarrow$ change in log wealth to GDP ratio:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_o}{Y_o}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}\right) - \log\left(\frac{\sum_j \pi_{jo} a_{jo}}{\sum_j \pi_{jo} h_{jo}}\right) \equiv \Delta_t^{comp}$$

measurable from demographic projections and household surveys

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Why? Demographics do not affect individual decisions, just their aggregation

Measuring compositional effects

Measuring Δ^{comp}

- Calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{comp} \equiv \log\left(\frac{\sum \pi_{jt} a_{jo}}{\sum \pi_{jt} h_{jo}}\right) - \log\left(\frac{\sum \pi_{jo} a_{jo}}{\sum \pi_{jo} h_{jo}}\right)$$

- Data:
  - π_{jt}: projections of age distributions over individuals

    2019 UN World Population Prospects
  - a_{jo}, h_{jo} age-wealth and labor income profiles in base year

    For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)

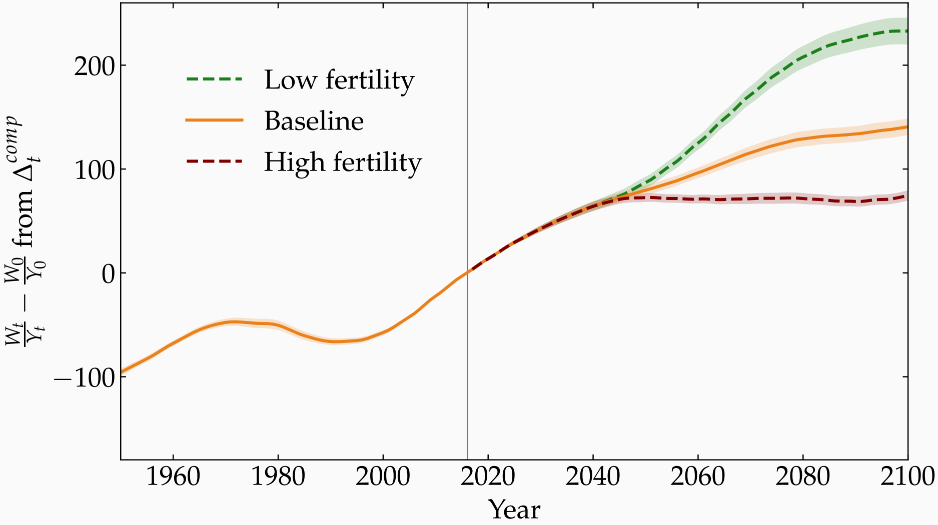
    a_{jo} includes funded part of DB pensions

    Household $\rightarrow$ individual (j) by splitting wealth among adults

- Report implied level change $\frac{W_t}{Y_t} - \frac{W_o}{Y_o} = \frac{W_o}{Y_o}\left(\exp\left\{\Delta_t^{comp}\right\} - 1\right)$

Δ^{comp} in the United States: 1950-2100 (base year: 2016)

Alternative base years



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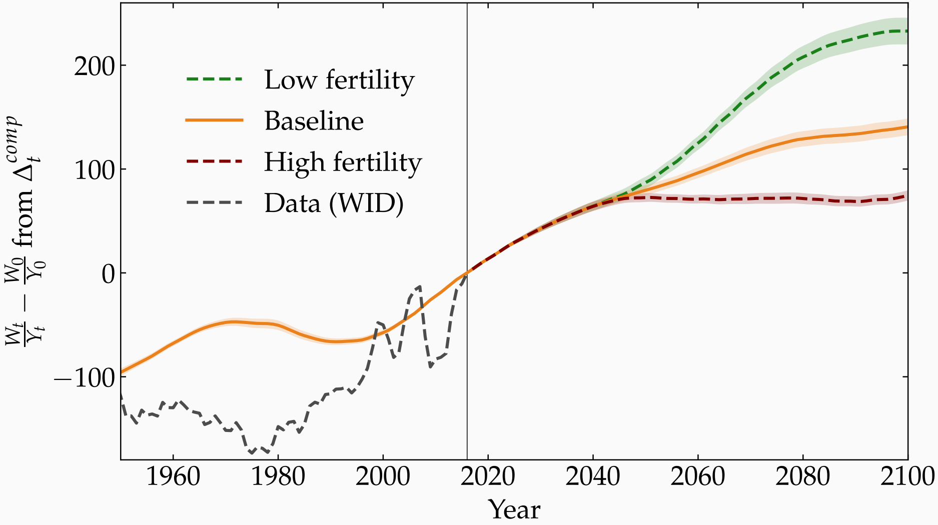
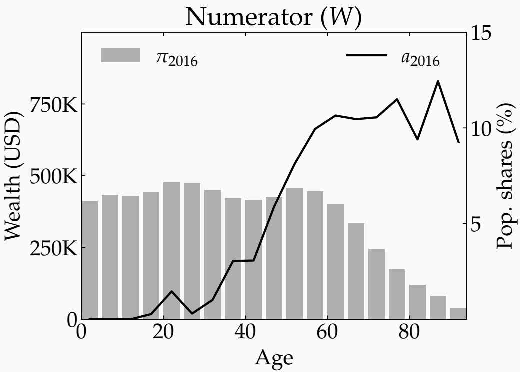


Where do these large effects come from?

▸ Alt. profiles



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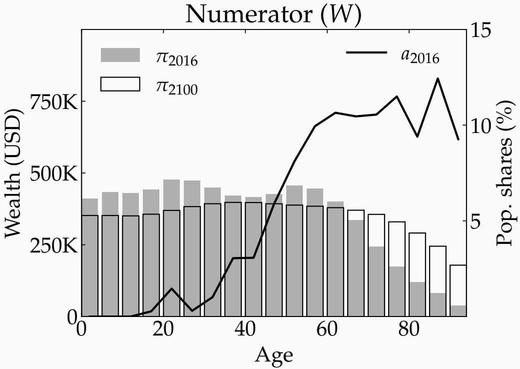

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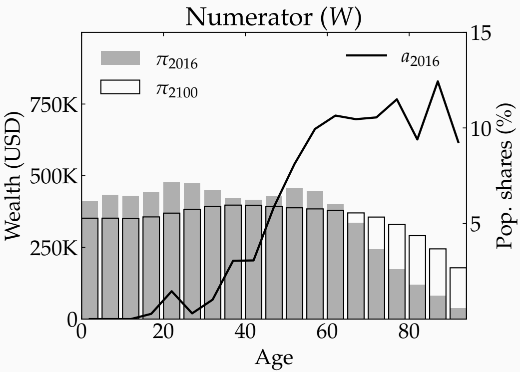

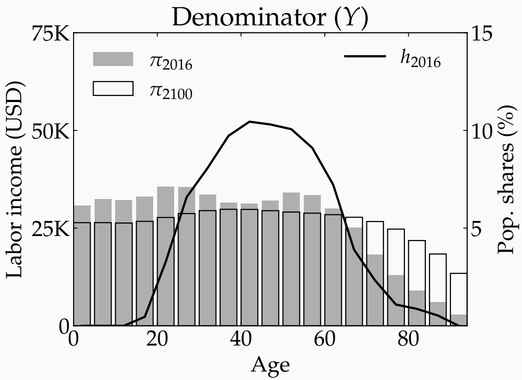

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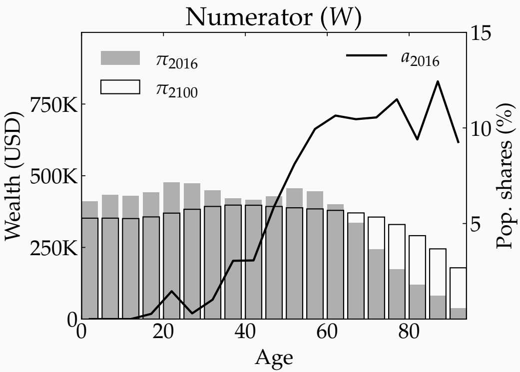
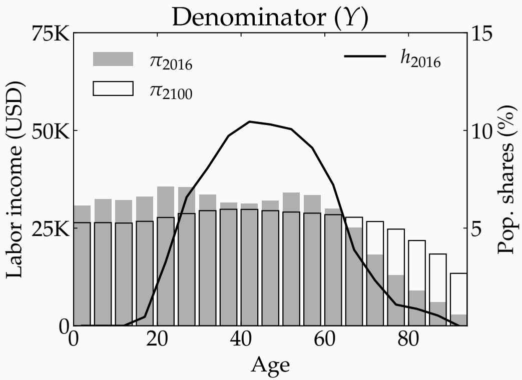
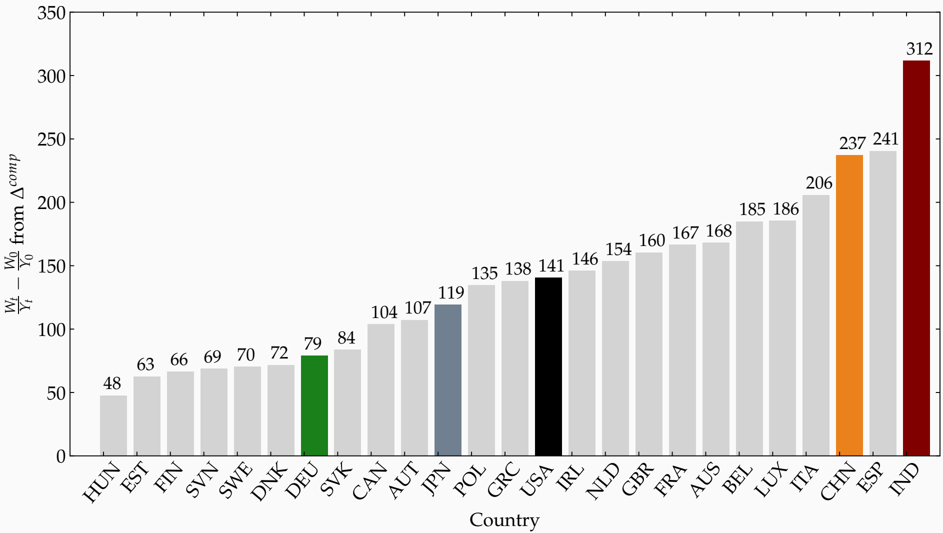




- In paper: separate contribution of numerator and denominator
- Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
  - Historically demographic dividend pushed Y up, reversed in 2010

Δ^{comp} large and heterogeneous by 2100

▸ Aging vs. profiles



General equilibrium implications

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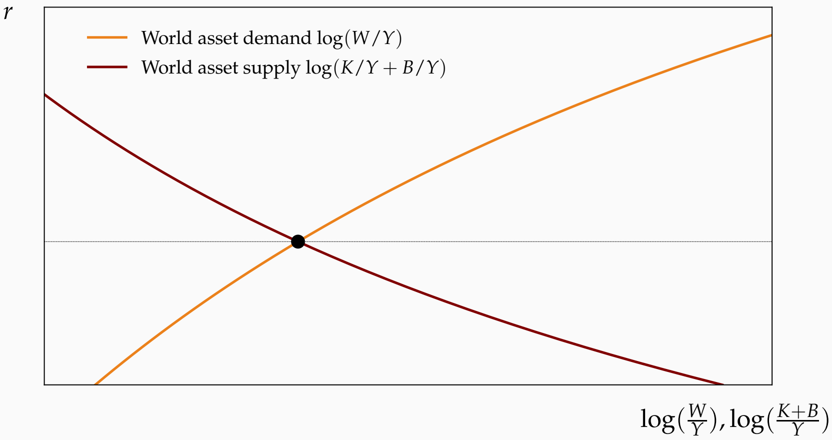
Semielasticity formulas



Semielasticity of asset demand $\bar{\epsilon}_d$: depends on σ, η and observables

Semielasticity of asset supply $\bar{\epsilon}_s$: depends on η and observables

General equilibrium implications

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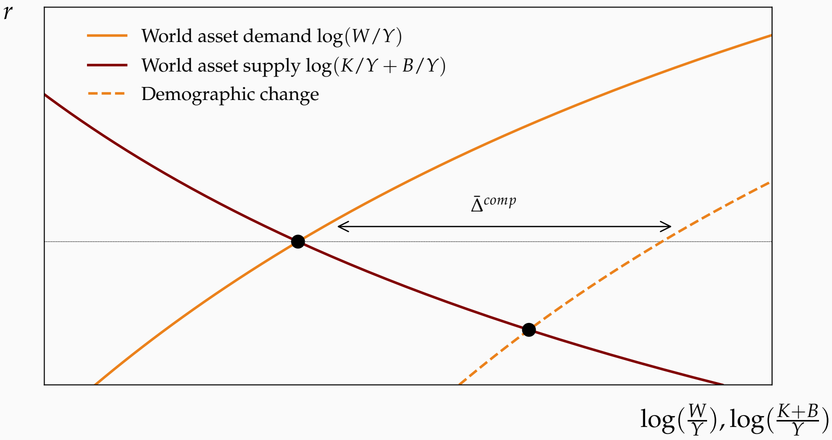

Asset demand shift of $\bar{\Delta}^{comp}$: wealth-weighted average of $\Delta^{comp,c}$

Large and positive in the data.

General equilibrium implications

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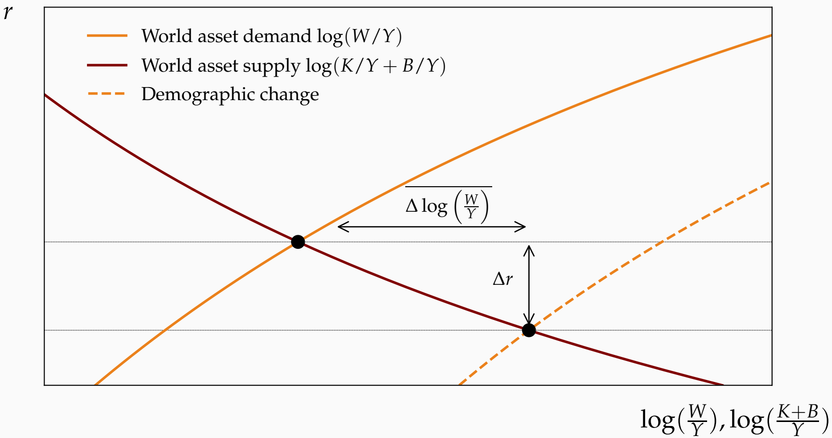


$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}_s + \bar{\epsilon}_d} < 0, \quad \overline{\Delta \log\left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}_s}{\bar{\epsilon}_s + \bar{\epsilon}_d}\bar{\Delta}^{comp} > 0$$

Changes in r and W/Y: 2016 to 2100

▸ Multiple assets

$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$$

$$\overline{\Delta \log \left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s} \bar{\Delta}^{comp}$$

A. Change in world r

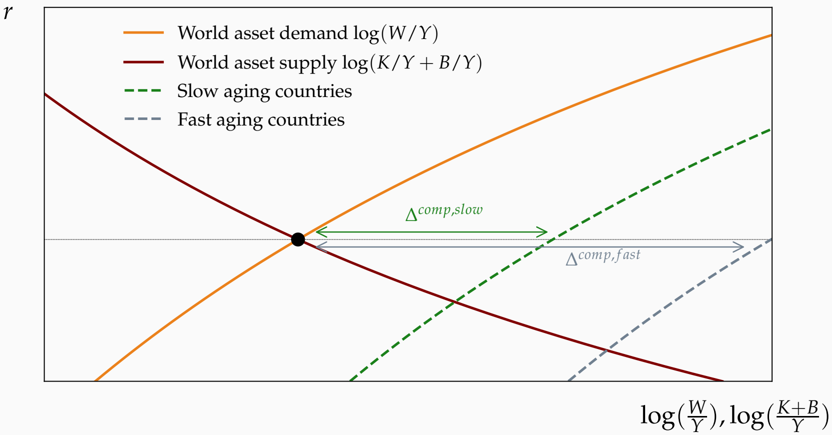
	σ		
η	0.25	0.50	1.00
0.60	-3.03	-1.56	-0.79
1.00	-2.00	**-1.23**	-0.70
1.25	-1.65	-1.09	-0.65

B. Change in avg. $\log W/Y$

	σ		
η	0.25	0.50	1.00
0.60	14.6	7.5	3.8
1.00	16.0	**9.9**	5.6
1.25	16.5	10.9	6.5

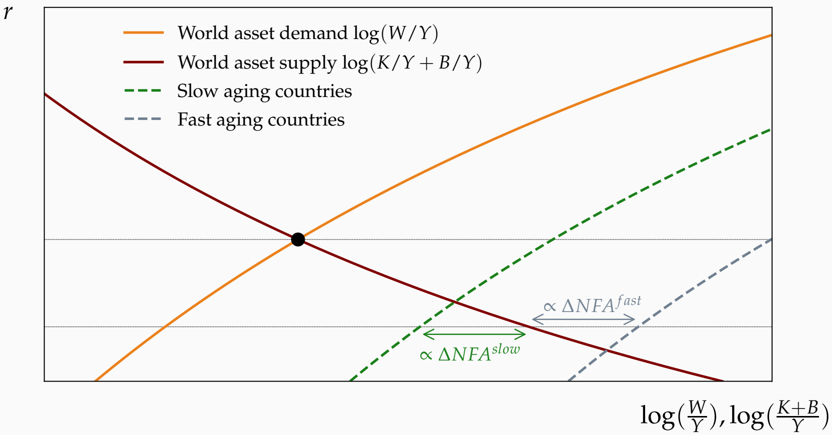
- We'll tend to obtain very similar outcomes for same σ, η in general model

General equilibrium implications, part 2



Country-specific shifts Δ^{comp} large and heterogeneous in data

General equilibrium implications, part 2



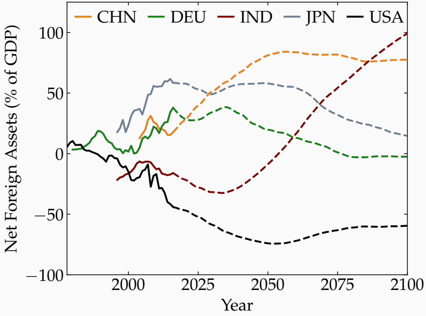
$$\Delta\left(\frac{NFA}{Y}\right) \approx \frac{W_0}{Y_0}\left(\Delta^{comp} - \bar{\Delta}^{comp}\right)$$

Demeaned compositional effect and NFAs

▸ More validation

$$\Delta\left(\frac{NFA^c}{Y^c}\right) \simeq \frac{W_0^c}{Y_0^c}\left(\Delta_t^c - \bar{\Delta}_{comp}\right)$$

A. NFA projection



→ Data suggests large global imbalances for the 21st century

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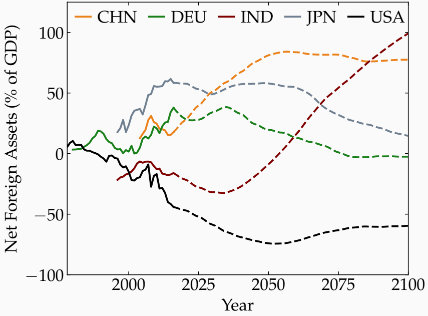
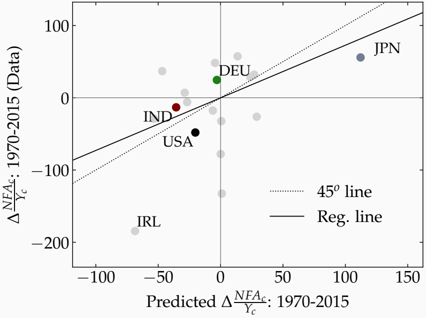


B. Historical performance



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Quantitative model

Updated environment

Calibration

Household problem becomes (with $\nu \geq \frac{1}{\sigma}$):

$$\max \mathbb{E}_k \sum_j \beta_j \Phi_{jk} \left[\frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \Upsilon Z_t^{\nu-\frac{1}{\sigma}} \left(1-\phi_{jt}\right) \frac{\left(a_{jt}\right)^{1-\nu}}{1-\nu} \right]$$

$$\text{s.t.} \quad c_{jt} + a_{j+1,t+1} \leq w_t \left((1-\tau_t)\ell_{jt}(z_j)(1-\rho_{jt}) + tr_{jt}(z_j)\right) + (1+r_t)a_{jt} + b_{jt}^r(z_j)$$

$$a_{j+1,t+1} \geq -\bar{a} Z_t$$

- Introducing bequests rather than annuities:
  - assets become bequests at death, distributed as $b_{jt}^r(z_j)$
- Time-variation in mortality Φ_{jk}, labor supply ℓ_{jt}, retirement age ρ_{jt}
- Fiscal rule with adjustments in taxes and transfers
- Income process with intergenerational persistence
- Migration

Robustness of conclusions: steady-state

▸ Literature

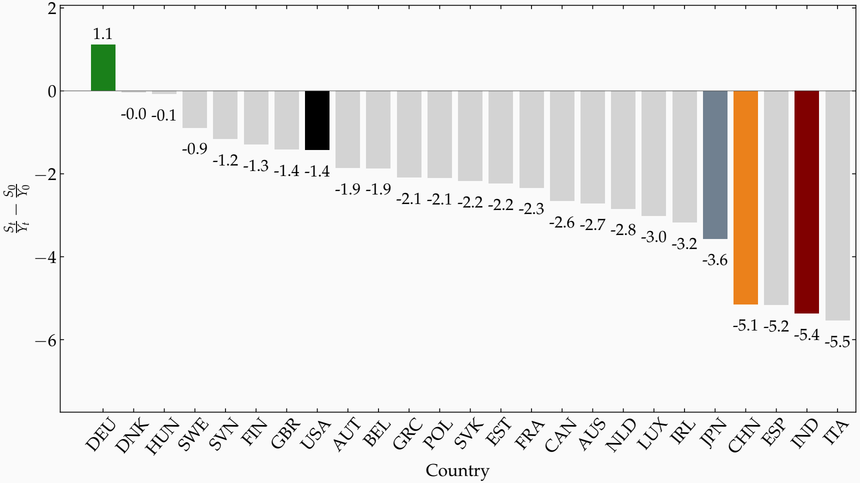
- Assume $\sigma = 0.5$, $\eta = 1$. Let $\overline{\Delta}^{soe} \equiv$ response of W/Y to demographics at fixed r.

	Δr	$\overline{\Delta \log \frac{W}{Y}}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^{d}$	$\bar{\epsilon}^{s}$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
Alternative model specifications						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
Alternative fiscal rules						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

A great demographic reversal?

Worldwide: decreasing S_t/Y_t everywhere

- Perform same exercise, but projecting S/Y from composition



Declining r despite falling savings?

- Will dissaving of the old reverse the effects of demographics?
  [Lane 2020, Goodhart-Pradhan 2020, Mian-Straub-Sufi 2021, Summers 2023]
- Measured S_t/Y_t from composition does decline
- **But**: r does not increase

Declining r despite falling savings?

- Will dissaving of the old reverse the effects of demographics?

  [Lane 2020, Goodhart-Pradhan 2020, Mian-Straub-Sufi 2021, Summers 2023]

- Measured S_t/Y_t from composition does decline
- **But**: r does not increase
- Why? Savings is misleading with declining pop. growth. In steady state

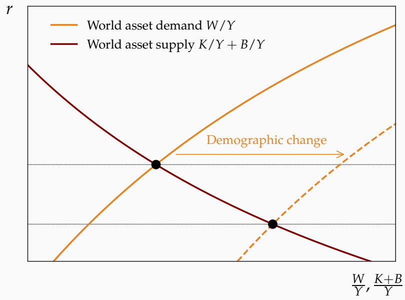
$$\frac{W}{Y} = \frac{S/Y}{g}$$

  where g is GDP growth

- With demographic change, S/Y falls, but g falls by more!

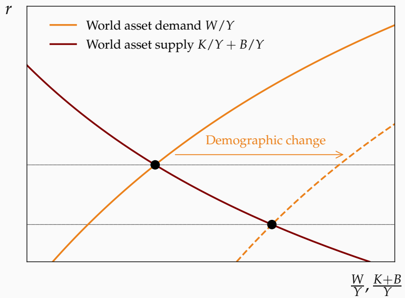
Flows can give the wrong sign for the change in r!

A. Asset demand vs supply

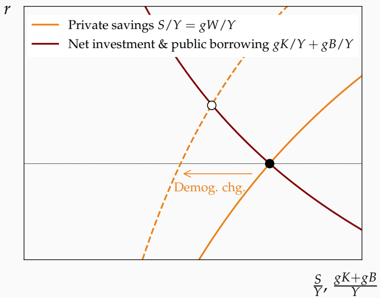


Flows can give the wrong sign for the change in r!

A. Asset demand vs supply

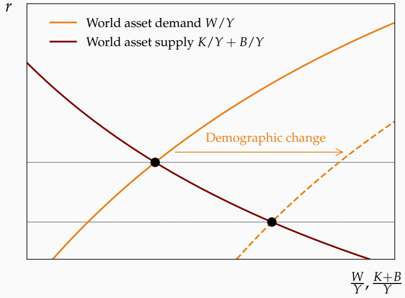


B. Net savings vs investment

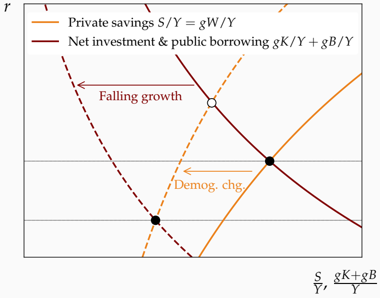


Flows can give the wrong sign for the change in *r*!

A. Asset demand vs supply



B. Net savings vs investment



Conclusion

- How do demographics affect wealth-output ratios, real interest rates, capital flows?

  $\rightarrow$ what matters most is the compositional effect Δ^{comp}

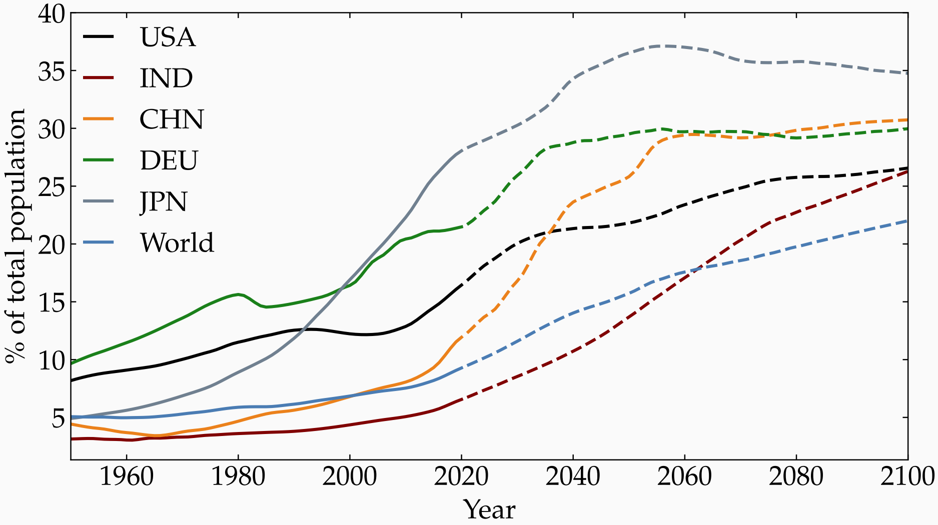
  large and **heterogeneous** in the data

- For the 21st century, our approach:
  - Refutes great demographic reversal hypothesis: r definitively falls
  - Suggests the “global savings glut” has just begun

Thank you!

Additional slides

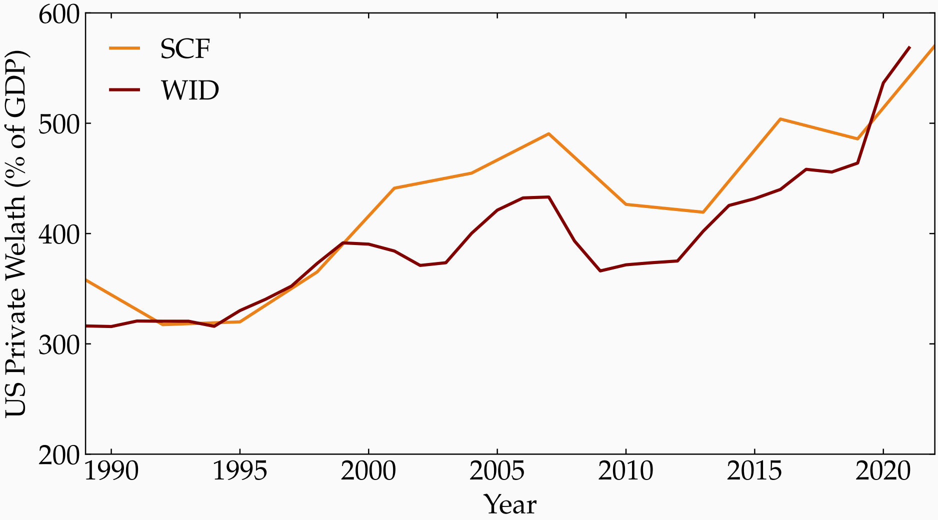
Share of the population aged 65+



Source: 2019 United Nations World Population Prospects

US Wealth-to-GDP from SCF vs World Inequality Database

Back



Source: World Inequality Database (WID), Survey of Consumer Finances (SCF)

Back

- Baseline safe return r_t^{safe} is 10 year constant maturity interest rate minus HP-filtered PCE deflator
- Baseline total return is

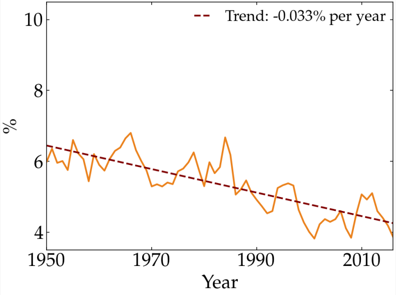
$$r_t = \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t}$$

where $(s_K Y - \delta K)_t$ is net capital income

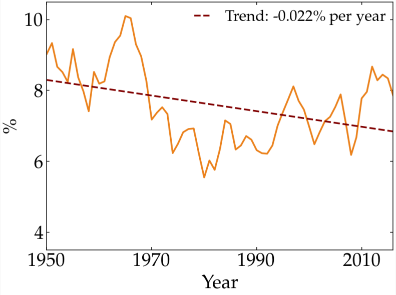
Calculating return: wealth or capital

Back

A. *W* in denominator (baseline)

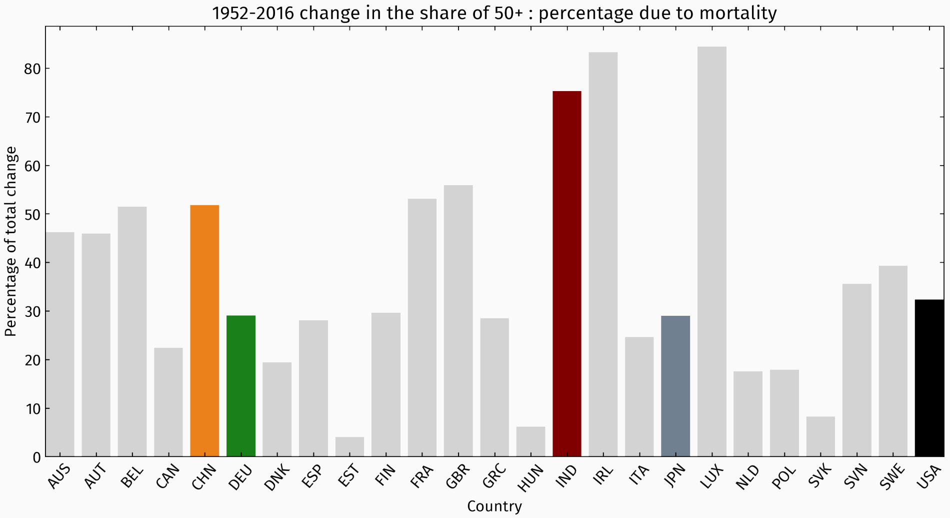


B. *K* in denominator



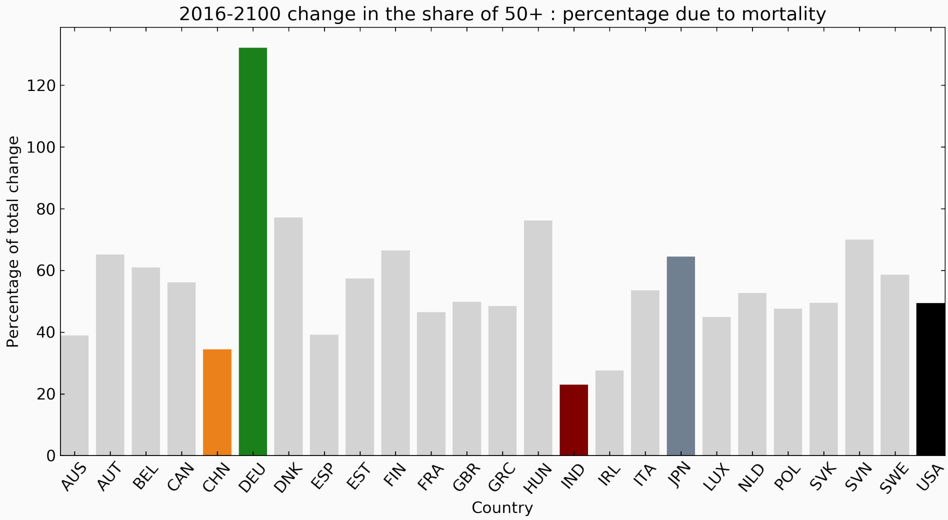
Contribution of mortality to aging since 1950s

◂ Back



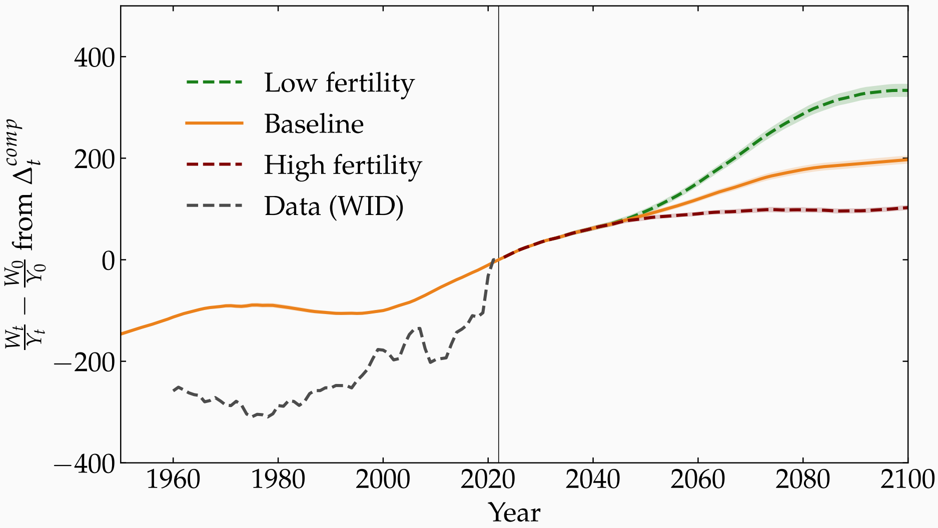
Contribution of mortality to aging in 21st century

Back



Δ^{comp} in the United States: 1950-2100 (base year: 2022)

◂ Back



Robustness to base year for age profiles (past)

Back

Change in W/Y: 1950 to 2016

Age-wealth profile (SCF) \ Age-labor income profile (LIS)	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	2019	2021
1989	85	87	87	85	83	81	82	79	78	75	74	75	75	76
1998	94	96	95	93	91	89	91	87	86	83	83	84	83	84
2004	100	102	101	99	97	95	97	93	92	89	89	90	89	91
2010	109	111	110	108	106	105	106	103	102	99	99	100	99	100
2016	109	111	111	109	107	105	106	103	102	99	99	100	99	100
2019	106	108	107	105	103	101	103	100	99	96	96	97	96	97
2022	107	109	109	107	105	103	104	101	100	97	97	98	97	98

Robustness to base year for age profiles (future)

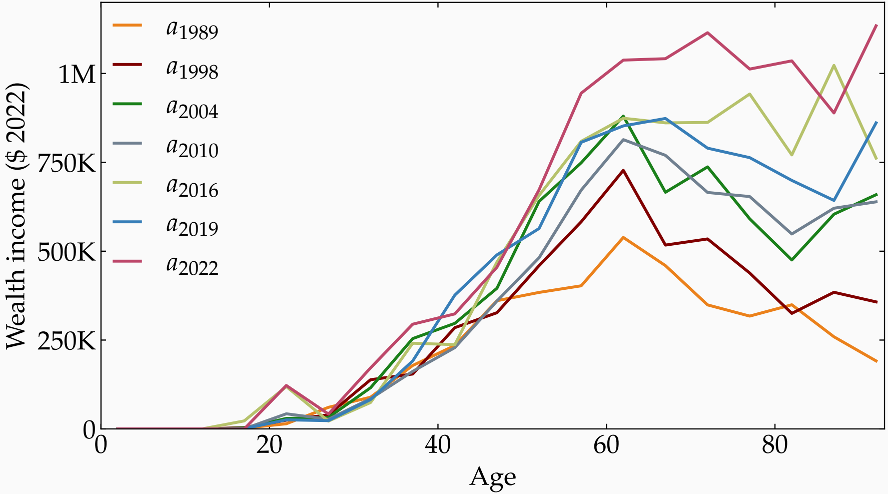
Back

Change in W/Y: 2016 to 2100

Age-wealth profile (SCF) \ Age-labor income profile (LIS)	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	2019	2021
1989	69	72	71	70	70	68	68	66	65	64	62	62	62	62
1998	76	79	78	77	76	75	75	73	71	70	68	68	68	69
2004	87	91	90	88	88	87	87	85	83	82	80	80	80	80
2010	100	103	102	101	101	99	99	97	95	94	92	92	92	93
2016	108	112	111	109	109	107	108	105	103	102	100	100	100	101
2019	101	104	103	102	102	100	100	98	96	95	93	93	93	94
2022	110	113	112	111	111	109	109	107	105	104	102	101	102	102

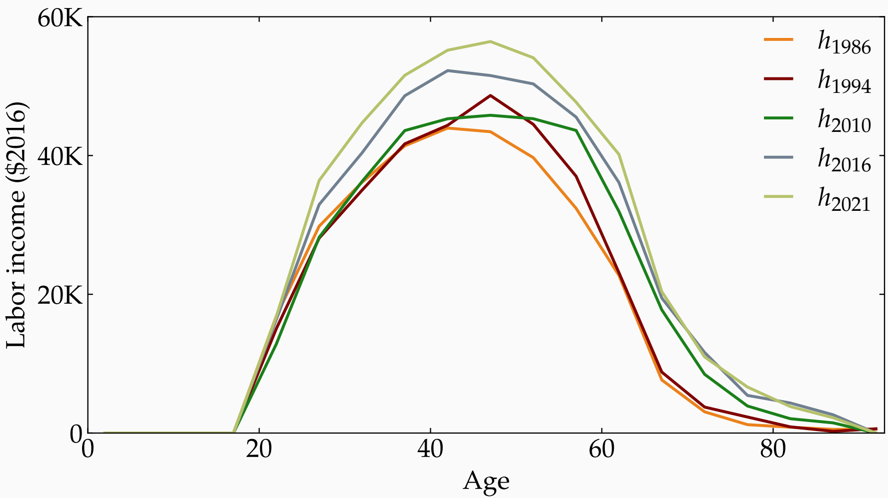
Age-wealth profiles in the U.S.

Back



Age-labor income profiles in the U.S.

Back



Semielasticities of asset demand and supply

Back

- **Asset supply elasticity** $\epsilon^s \equiv \frac{\partial \log(A^s/Y)}{\partial r}$:
  "how will bonds and capital change, relative to GDP, if steady-state r changes?"
  - Given common capital-labor substitution elasticity η, average elasticity is

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \overline{\left(\frac{K_0}{W_0} \right)}$$

$\rightarrow$ Measurable from observables and knowledge of η

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- **Asset demand semielasticity** $\epsilon^d \equiv \frac{\partial \log(W/Y)}{\partial r}$:
  "how will households change average wealth, relative to GDP, if s.s. r changes?"
  - Hard to measure [Saez and Stantcheva 2018: "paucity of empirical estimates"]
  - **Result**: dropping idiosyncratic risk and borrowing constraint from model, **exact formula** for ϵ^d in terms of σ,η, and observables [numerically similar in quantitative model]

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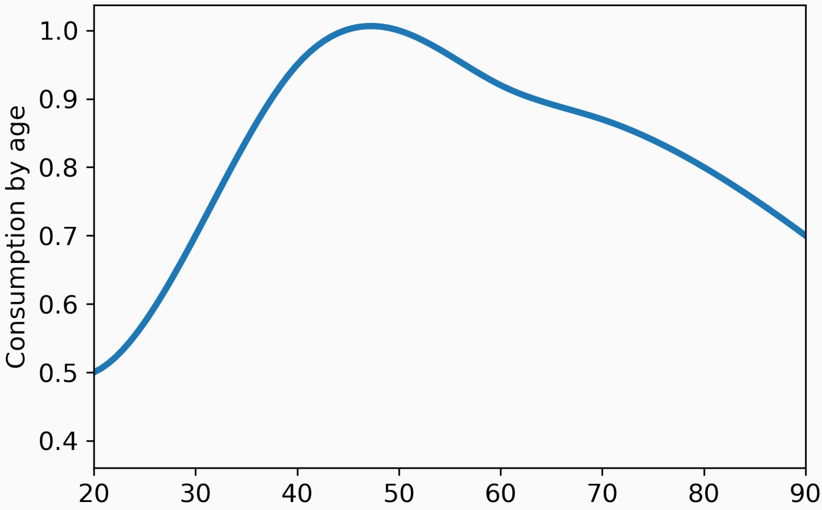
We'll separate **substitution** (via Euler equation) and **income** (via budget constraint) effects, and first derive for $r = g = 0$ and $\eta = 1$.

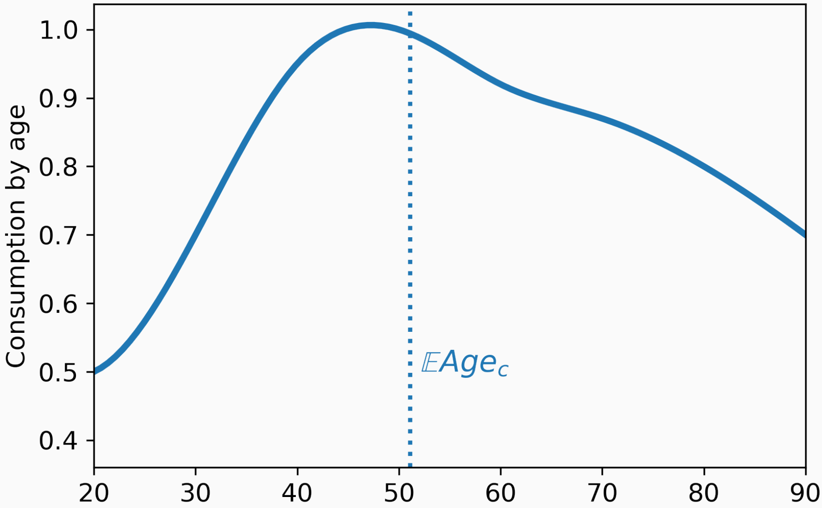
Substitution effect of *dr* on arbitrary lifecycle consumption path

Start with arbitrary lifecycle consumption profile:



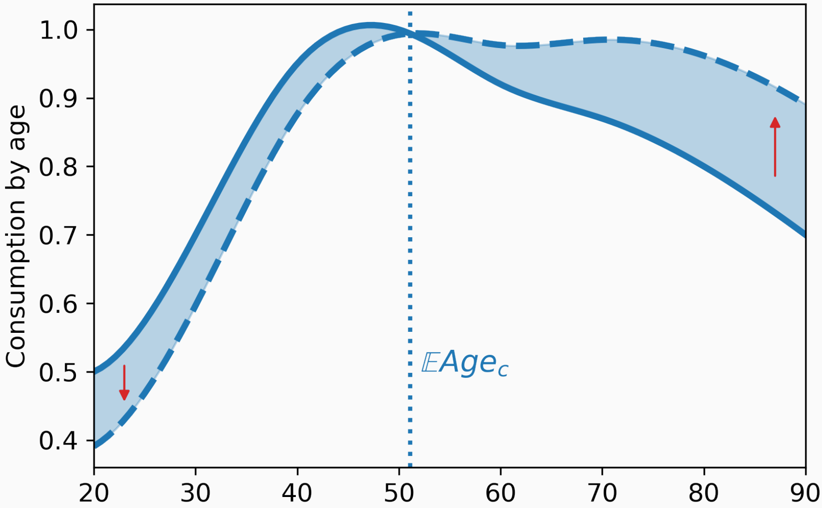
Substitution effect of *dr* on arbitrary lifecycle consumption path

Average age of consumption $\mathbb{E}Age_c \equiv \sum_j \pi_j c_j j / \sum_j \pi_j c_j \approx 51.1$



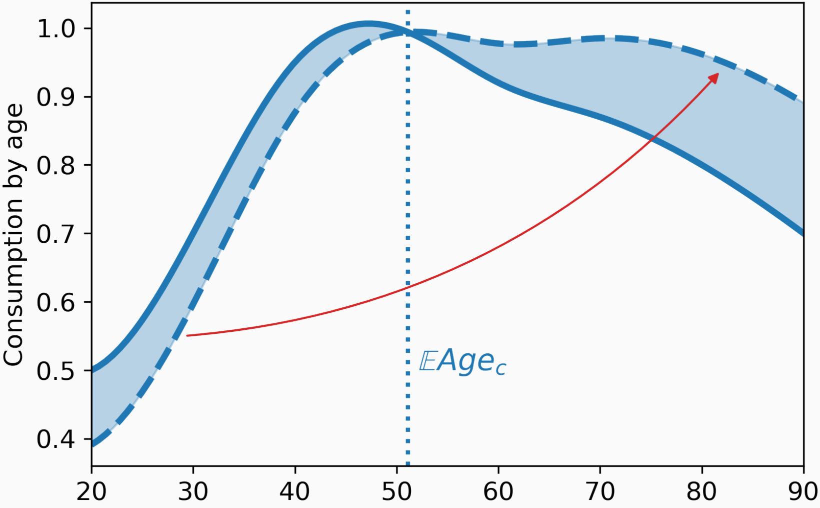
Substitution effect of *dr* on arbitrary lifecycle consumption path

Changing *r* tilts path around $\mathbb{E}Age_c$: $dc_j/c_j = -\sigma(j - \mathbb{E}Age_c)dr$



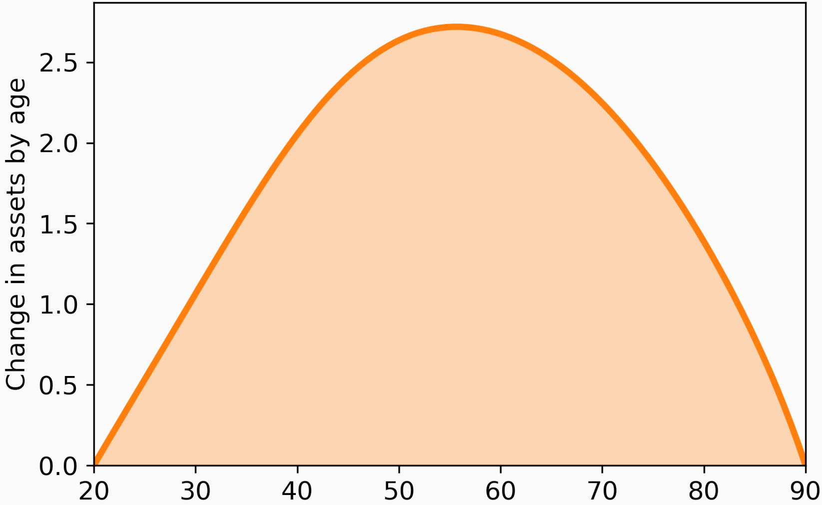
Substitution effect of *dr* on arbitrary lifecycle consumption path

Moving consumption from earlier to later in life: **more assets**



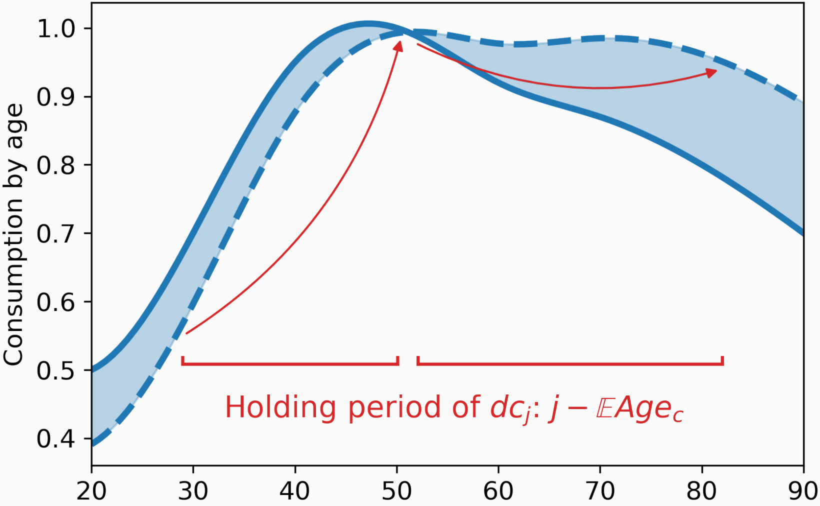
Integrating implies perturbation to asset path

$\pi_j da_j = -\int_{\underline{j}}^{j} \pi_k dc_k$, and we want $dW = \int_{\underline{j}}^{\bar{j}} \pi_j da_j$



Decomposing overall effect on wealth

Age j "contribution" to assets: $dc_j \cdot (j - \mathbb{E}Age_c)$



Putting together contributions to wealth

Aggregating dc_j (extra savings held) times $j - \mathbb{E}Age_c$ (period held):

$$dW = \sum_j \pi_j dc_j (j - \mathbb{E}Age_c) = dr \sum_j \pi_j \sigma (j - \mathbb{E}Age_c) c_j (j - \mathbb{E}Age_c)$$

$$= \sigma dr \sum_j \pi_j c_j (j - \mathbb{E}Age_c)^2$$

$$= \sigma C dr \underbrace{\sum_j \frac{\pi_j c_j}{C} (j - \mathbb{E}Age_c)^2}_{= \text{Var}Age_c}$$

Putting together contributions to wealth

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$$dW = \sum_j \pi_j dc_j (j - \mathbb{E}Age_c) = dr \sum_j \pi_j \sigma (j - \mathbb{E}Age_c) c_j (j - \mathbb{E}Age_c)$$
$$= \sigma dr \sum_j \pi_j c_j (j - \mathbb{E}Age_c)^2$$
$$= \sigma C dr \underbrace{\sum_j \frac{\pi_j c_j}{C} (j - \mathbb{E}Age_c)^2}_{= \text{Var}Age_c}$$

Log change from substitution effect therefore

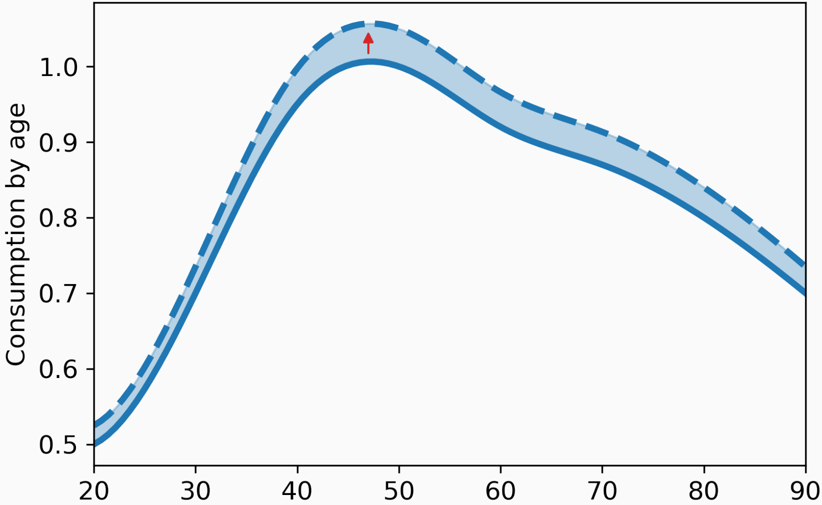
$$\frac{dW}{W} = \sigma \frac{C}{W} \text{Var}Age_c dr$$

Note **linear** in EIS σ, **quadratic** in spread of consumption

About 50σ if $C/W \approx 1/6$ and consumption uniform from ages 20 to 80 (so $\text{Var}Age_c = 300$)

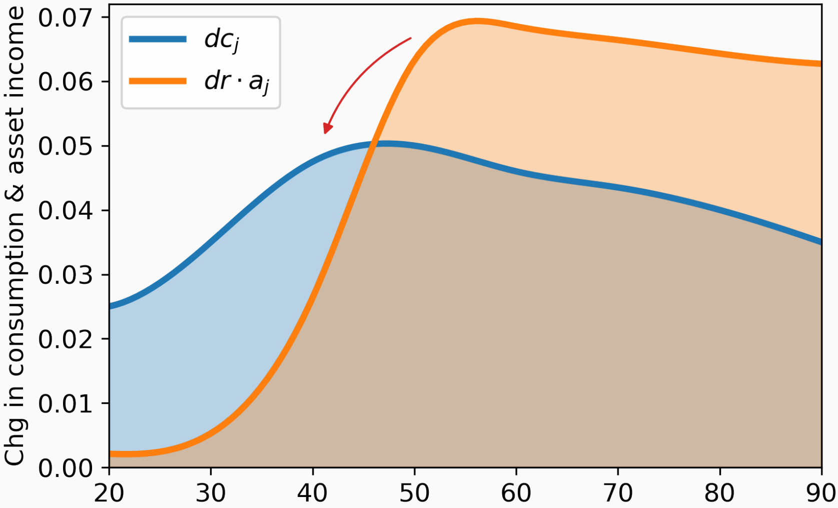
Income effect of *dr* on c_j: uniform proportional increase

Higher asset income reallocated across all ages:



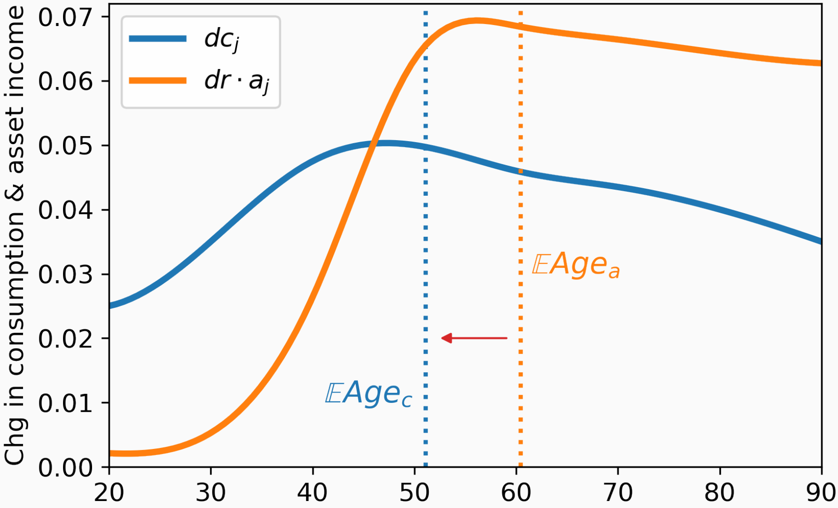
Increment to asset income vs. consumption

This involves dissaving, since a_j held later than c_j:



Increment to asset income vs. consumption

Overall effect on wealth is $dr \cdot W \cdot (\mathbb{E}Age_c - \mathbb{E}Age_a)$:



Combining substitution and income effects of dr

Back

Overall semielasticity of asset demand:

$$\epsilon^d = \frac{\partial log(W/Y)}{\partial r} = \sigma \underbrace{\frac{C}{W} \text{VarAge}_c}_{\equiv \epsilon^d_{substitution}} + \underbrace{\mathbb{E}Age_c - \mathbb{E}Age_a}_{\equiv \epsilon^d_{income}}$$

Allowing $r = g \neq 0$ identical except some $1 + r$ factors, general case close and has new term with labor share s_L:

$$\epsilon^d = \sigma \underbrace{\epsilon^d_{substitution}}_{\approx 39.5} + \underbrace{\epsilon^d_{income}}_{\approx -2} + (\eta - 1) \underbrace{\frac{(1 - s_L)/s_L}{r + \delta}}_{\approx 5.5}$$

Now: calculate GE results for reasonable σ and η

Multiple assets

- Model demand for risky assets: households now solve

$$\max \quad \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{\left(c_{jt} - a_{jt} v_j^c\left(s_{jt}\right)\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right]$$

$$\text{s.t.} \quad c_{jt} + \phi_j a_{j+1,t+1} \leq w_t((1-\tau)\ell(z_{jt}) + tr(z^{jt})) + (1 + r_t^f + s_{jt}(r_t^r - r_t^f))a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}\,(1+\gamma)^t$$

where s_{jt} is risky portfolio share of age j , and $v_j(s_{jt})$ is utility cost of bearing risk

$$v_j^c(s_{jt}) = \overline{r^r - r^f} \cdot (s_{jt} - \bar{s}_j^c) + \frac{1}{2\Psi}(s_{jt} - \bar{s}_j^c)^2$$

- New FOC is:

$$s_{jt}^c = \bar{s}_j^c + \Psi\left(r_t^r - r_t^f - \overline{r^r - r^f}\right)$$

- Now in addition to aggregate asset demand, must clear market for risky assets

$$\sum_c \sum_j N_{jt}^c \mathbb{E}\left[s_{jt}^c a_{jt}^c\right] = \sum_c K_t^c$$

Multiple assets: result

Back

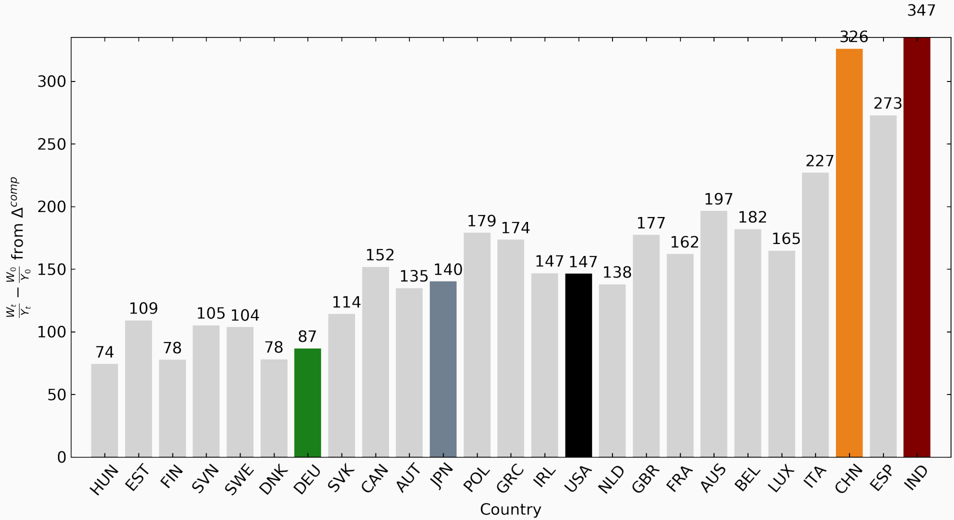
- Long-run adjustment in asset market:

$$\begin{pmatrix} \Delta r \\ \Delta r^r \end{pmatrix} = \Sigma \cdot \begin{pmatrix} \Delta \log W/Y^{comp} \\ \Delta \log W^r/Y^{comp} \end{pmatrix}$$

- New term: compositional effect on risky asset demand $\Delta \log W^r/Y^{comp}$
- Matrix of inverse elasticities Σ affected by Ψ
- Calibrate model as before + matching portfolio shares by age
- For small enough Ψ, predictions for Δr are close to baseline

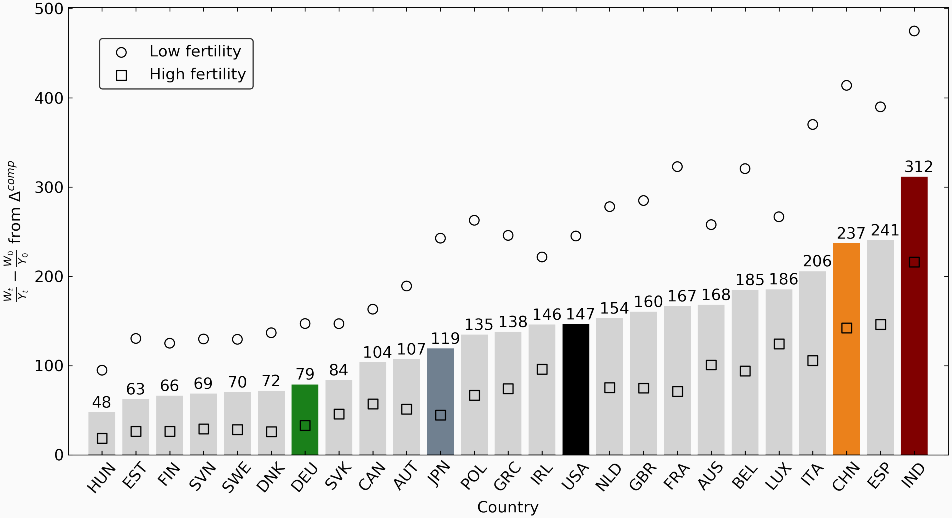
Composition effect using common US age profiles

Back



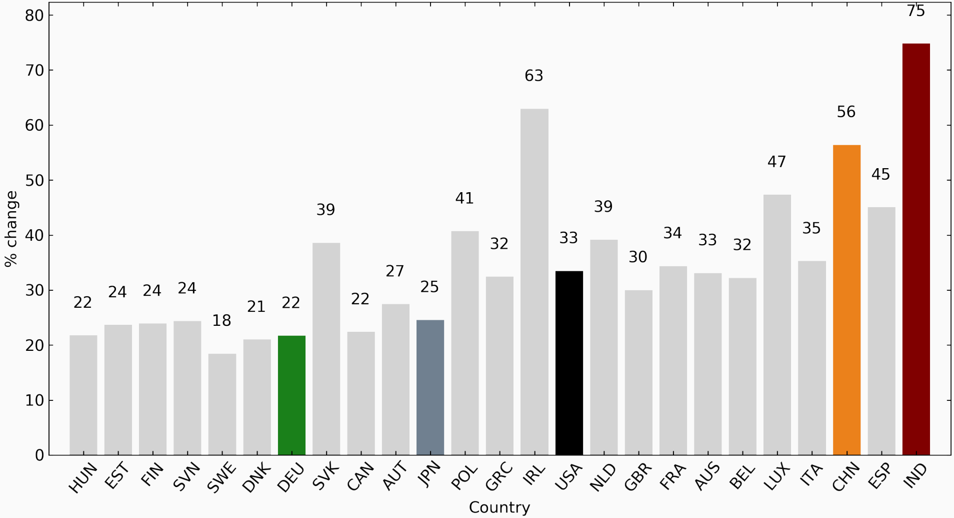
Low and high fertility scenarios

Back



Percentage change in W/Y from comp. effect

Back



Validating the model: regressing $\Delta NFA/Y$ on predictors (1970-2015)

Back

	OLS	WLS	OLS	WLS	OLS	WLS
Predicted $\Delta \frac{NFA}{Y}$	0.898	0.696	1.094	1.727	0.912	1.069
	(2.489)	(3.183)	(2.125)	(3.397)	(1.218)	(1.347)
Change in Debt-to-GDP			-0.068	-0.762	0.099	-0.730
			(-0.167)	(-2.105)	(0.132)	(-1.279)
Average TFP growth	83.029	59.880	84.846	89.511	97.551	67.197
	(2.068)	(1.564)	(1.782)	(2.139)	(1.415)	(0.978)
Average real GDP pc. growth	-78.630	3.549	-96.021	-17.656	-106.083	-4.194
	(-3.087)	(0.180)	(-3.463)	(-0.632)	(-2.479)	(-0.100)
Change in Gini					0.099	-0.730
					(0.132)	(-1.279)
Change in Top 10%					-622.507	580.584
t-stat.					(-0.329)	(0.286)
N	18	18	15	15	13	13
R^2	0.537	0.518	0.731	0.677	0.714	0.445
$\bar{R}^2$	0.437	0.415	0.623	0.548	0.427	-0.110

Validating the model: panel regression for *NFA*

Back

$$\frac{NFA_{ct}}{Y_{ct}} = \alpha_c + \delta_t + \beta \cdot \left(\frac{NFA_{ct}}{Y_{ct}}\right)^{pred} + \eta_t \cdot \text{Controls}_{ct} + \varepsilon_{ct}$$

	Baseline				Shorter sample		Alternative NFA	
Predicted NFA-to-GDP	0.404	0.413	0.545	0.808	1.517	1.238	0.438	0.836
	(3.209)	(3.664)	(2.702)	(2.846)	(3.599)	(3.229)	(1.854)	(3.229)
Debt-to-GDP			-0.231	-0.882	-0.581	-1.040	-0.213	-0.908
			(-1.106)	(-4.767)	(-2.149)	(-4.949)	(-0.960)	(-5.071)
TFP growth			-1.729	-3.409	-5.356	-6.118	-0.643	0.619
			(-1.044)	(-2.166)	(-2.643)	(-3.482)	(-0.608)	(0.770)
GDP pc. growth			-0.257	-0.281	-0.192	-0.048	0.458	0.027
			(-0.744)	(-0.438)	(-0.573)	(-0.091)	(1.068)	(0.074)
Income Gini				-2.602		-2.347		-0.766
				(-0.491)		(-0.285)		(-0.247)
Top 10% inc. share				4.657		3.335		0.829
				(0.652)		(0.313)		(0.229)
Country FE	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	NO	YES	YES	YES	YES	YES	YES	YES
N	828	828	690	470	345	299	690	470
$\bar{R}^2$	0.467	0.460	0.491	0.589	0.691	0.694	0.612	0.777

Validating model predictions: Fair-Dominguez-Higgins regressions

Back

- Fair-Dominguez (91), Higgins (98) proposed regressions of the type:

$$y_{ct} = \alpha_c + \beta \cdot D_{ct} + \gamma \cdot \text{Control}_{ct} + \delta \cdot D_{ct} \times \text{Control}_{ct} + \epsilon_{ct}$$

  where D_{ct} are 15 age group dummies (coefficients restricted to quadratic in age)
- Run this with $y_{ct} \equiv (W/Y)_c$ and r_c; compare to Δ^{comp}; control for TFP

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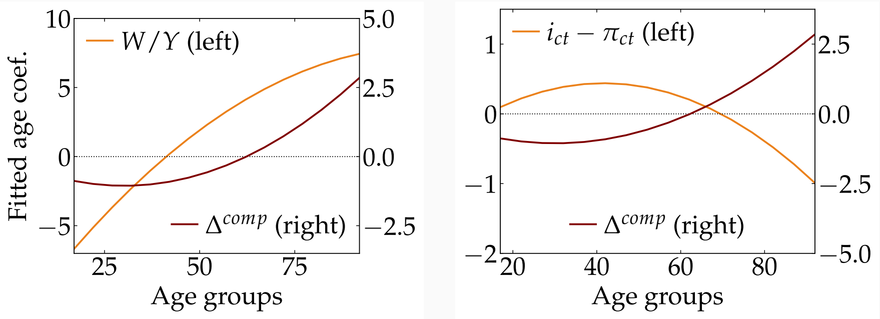
- Run this with $y_{ct} \equiv (W/Y)_c$ and r_c; compare to Δ^{comp}; control for TFP

A. W/Y and comp. effect

B. Country real returns



World economy calibration

Back

Country	$\Delta_{comp,c}$ Model	$\Delta_{comp,c}$ Data	Components of wealth $\frac{W^c}{Y^c}$	$\frac{B^c}{Y^c}$	$\frac{NFA^c}{Y^c}$	Government policy τ^c	$\frac{Ben^c}{Y^c}$
AUS	30	29	5.09	0.40	-0.46	0.29	0.04
CAN	21	20	4.63	0.92	0.20	0.31	0.04
CHN	47	45	4.20	0.44	0.25	0.30	0.04
DEU	21	20	3.64	0.69	0.58	0.50	0.10
ESP	42	37	5.33	0.99	-0.74	0.39	0.10
FRA	31	30	4.85	0.98	-0.05	0.48	0.13
GBR	27	26	5.35	0.88	0.08	0.31	0.06
IND	65	56	4.16	0.68	-0.08	0.30	0.01
ITA	34	30	5.83	1.31	-0.02	0.48	0.13
JPN	24	22	4.85	2.36	0.66	0.32	0.09
NLD	34	33	3.92	0.62	0.70	0.37	0.05
USA	32	29	4.38	1.07	-0.36	0.32	0.06

World economy calibration

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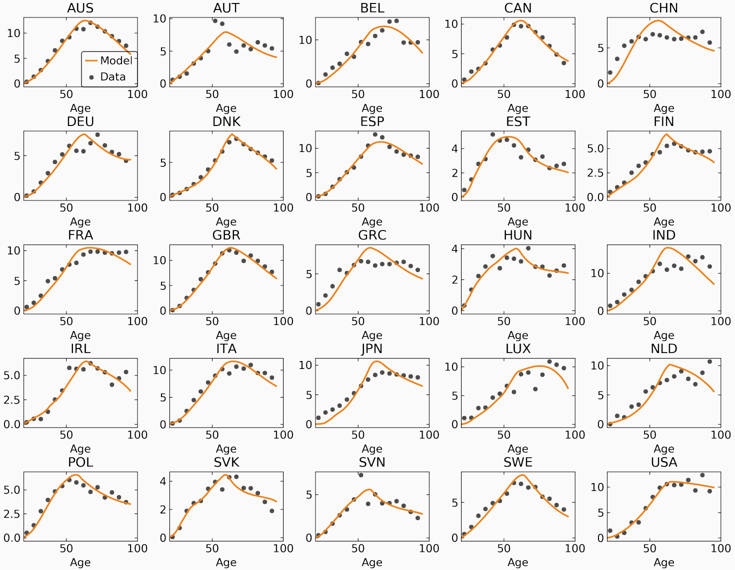

Environment: demographics

◂ Back

- Population evolves as

$$N_{jt} = \left(N_{j-1,t-1} + M_{j-1,t-1}\right)\phi_{j-1,t-1}$$

  where

  - N_{jt} denotes the numbers of individuals aged j in year t
  - $M_{j,t}$ is migration
  - $\phi_{j,t}$ are survival probabilities

- Total population is

$$N_t = \sum_j N_{jt}$$

- Population converges to stationary distribution with constant ϕ_j, $n \equiv N_{0,t}/N_{0,t-1} - 1$.

Projected survival functions

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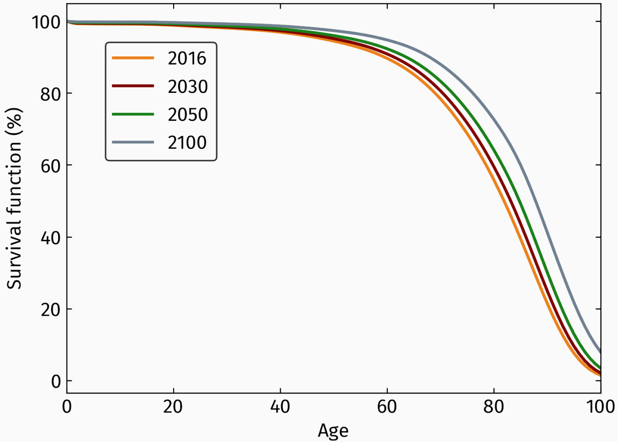

Projected population shares

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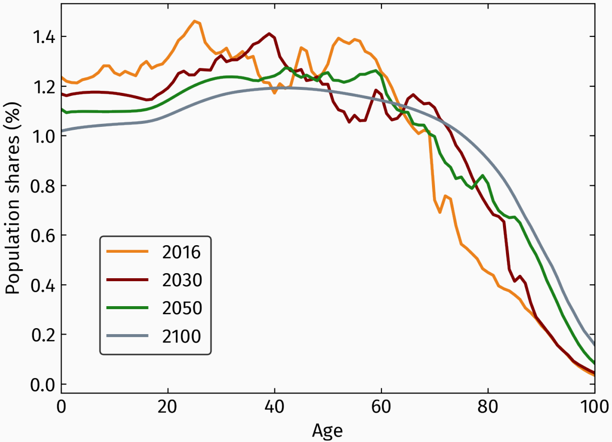

Demographics: population distributions

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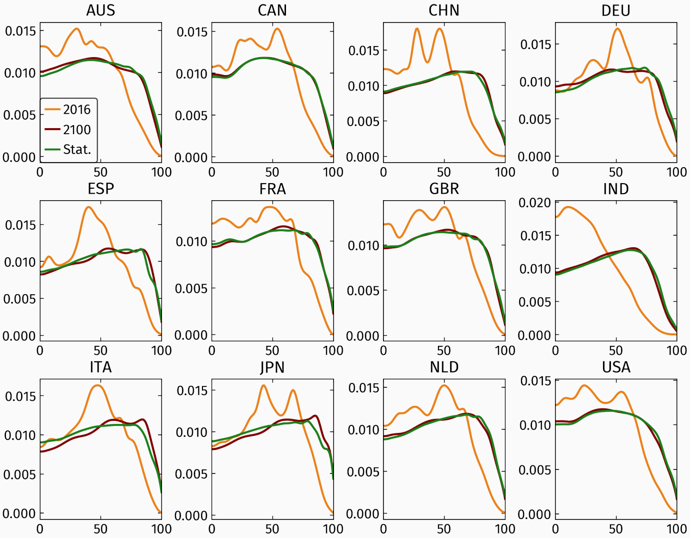

Demographics: population growth rates

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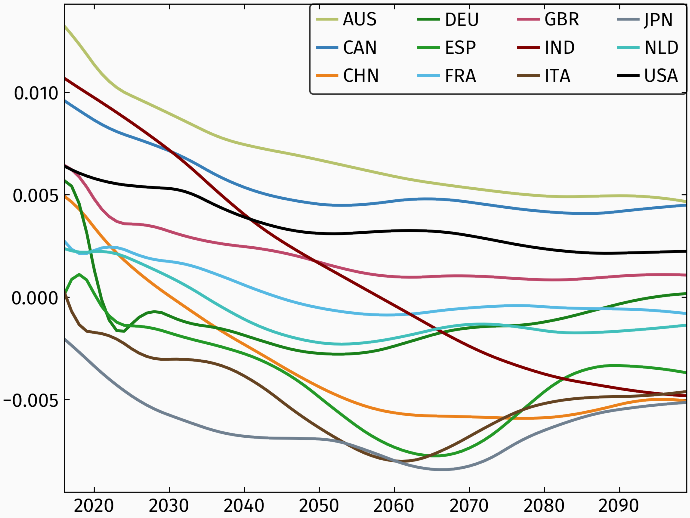

Robustness of conclusions: transitions

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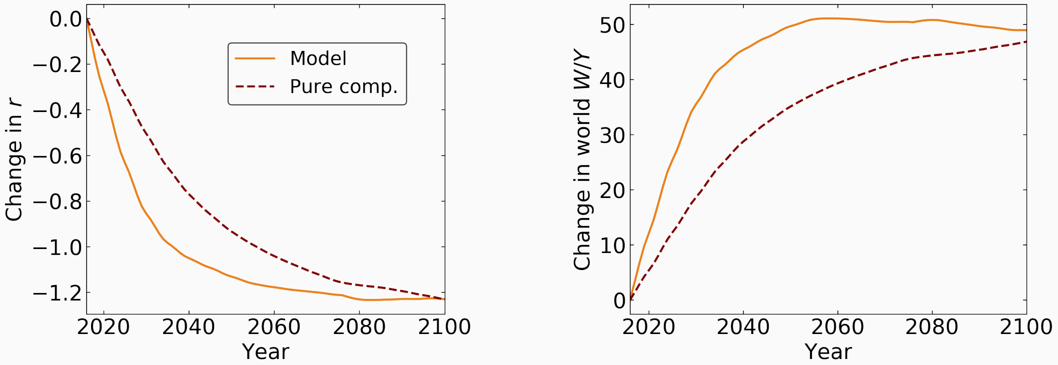
A. Change in r **B.** Change in world W/Y



Understanding differences for r^* effect in literature

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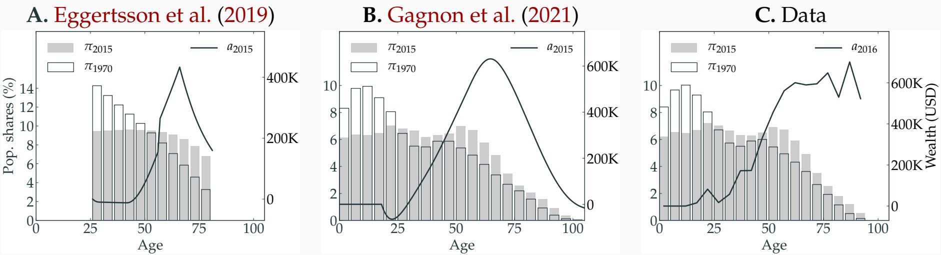

Understanding differences for r^* effect in literature

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	Eggertsson et al. (2019)	Gagnon et al. (2021)	Sufficient statistic
Time-period	1970–2015	1970–2015	1970–2015
GE transition			
Δr^{GE}	-3.44%	-0.92%	
First-order approximation $\Delta r = \frac{-\Delta^{soe}}{\epsilon^d + \epsilon^s}$			
Δr	-4.30%	-0.97%	-0.49%
Δ^{comp}	45.4%	13.4%	12.4%
$\Delta^{soe} - \Delta^{comp}$	21.1%	25.3%	0%
ϵ^s	2.8	11.1	8.0
ϵ^d	12.7	28.5	17.5
σ	0.75	0.5	0.5
η	0.6	1.0	1.0