Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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The world population is aging...

Source: 2019 United Nations World Population Prospects
...wealth-to-GDP ratios are increasing...

...rates of return on wealth are falling...
...and “global imbalances” are rising

Source: International Monetary Fund (IMF), Penn World Table (PWT) 9.1
How will demographics shape these trends in the 21st century?

• Broad agreement that population aging has contributed to trends in $W/Y$, real returns ($r$), and NFA positions in the past.

• Much less agreement about likely direction for the future.

• Popular view focuses on the savings rate in an aged population:
  
  • “The current phase of population ageing is contributing to the trend decline in the underlying equilibrium real interest rate [...] While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.” (Philip Lane, May 2020)

  • cf the “asset market meltdown” hypothesis [Poterba 2001]
Q: What guidance do modern GE models give on the causal effects of demographics on global wealth accumulation and returns?

• We show that a ratio of two shift-shares provides a natural starting point for forecasts:

\[
\left( \frac{W_t}{Y_t} \right)^{comp} = \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} \quad t \geq 0
\]

• \(a_{jo}, h_{jo}\) are today’s asset and labor income profiles by age \(j\)

• \(\pi_{jt}\) are projections of the population share of age \(j\) in year \(t\)

Captures the compositional effect of aging on \(W/Y\)

Disciplines general equilibrium counterfactuals

1. Sufficient statistic for \(W/Y\) in special “balanced growth” SOE case
2. Gives direction of change in \(r\) and \(W/Y\), and approx. magnitude of change in \(NFA/Y\), in integrated world general case
A bridge between reduced-form and structural approaches

- Existing literature follows two broad approaches:

1. **Reduced-form**, based on shift-share exercises
   - **Denominator**: Projected labor supply [Cutler et al 1990], demographic dividend literature [Bloom-Canning-Sevilla 2003...]

2. **Structural**, based on fully specified GE OLG models

- Our **sufficient statistic approach** bridges the gap between both
What we find

\[ \Delta_t^{\text{comp}} \equiv \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} - \frac{W_o}{Y_o} \]

1. **Measurement:**
   - \( \Delta^{\text{comp}} \) is **positive, large and heterogeneous** across countries
   [in 2100: 85pp in Germany vs 305pp in India]
   - a) Older individuals hold more wealth and earn less income
   - b) Timing of aging transition uneven across countries

2. **Quantitative GE OLG model:** across range of calibrations
   - \( \Delta^{\text{comp}} \) **closely approximates** \( W/Y \) transition of small open econ.
   - In integrated world, matching \( \Delta^{\text{comp}} \) in each country implies:
   - a) **returns on wealth** definitively fall and **wealth-GDP ratios** rise, but exact magnitudes are uncertain
   - b) **global imbalances** rise dramatically by the end of the 21st century
     [2016-2100: \( \Delta \text{NFA}/Y \) of -50pp in Germany vs 180pp in India]
1. The compositional effect of aging on $W/Y$

2. Measurement

3. General equilibrium implications
1. The compositional effect of aging on $W/Y$
Environment

- Economy with output $Y_t$ experiencing demographic change
- Population of age $j$ $N_{jt}$, total population $N_t \equiv \sum_j N_{jt}$
- Wealth
  \[ W_t = \sum_j N_{jt} A_{jt} \] (1)
- Effective labor supply
  \[ L_t = \sum_j N_{jt} h_{jt} \] (2)
- Suppose there is growth in labor productivity $Y_t/L_t$
  - We expect $A_{jt}$ to scale with $Y_t/L_t$
  - Let $a_{jt} \equiv \frac{A_{jt}}{Y_t/L_t}$ denote productivity-normalized assets by age
Wealth-to-GDP ratio

- Rewrite wealth (1)
  \[ W_t = \frac{Y_t}{L_t} \sum_j N_j t a_{jt} \]

- Wealth-to-GDP ratio using (2)
  \[ \frac{W_t}{Y_t} = \frac{\sum_j \pi_{jt} a_{jt}}{\sum_j \pi_{jt} h_{jt}} \]

  where \( \pi_{jt} \equiv \frac{N_{jt}}{N_t} \) is share of population age \( j \)

- Three reasons for changing \( W_t / Y_t \):
  1. Changing population shares: \( \pi_{jt} \)
  2. Changing age profiles of productivity-normalized assets: \( a_{jt} \)
  3. Changing age profiles of labor efficiency: \( h_{jt} \)
The compositional effect

• For any base year $o$, define

$$
\Delta_t^{comp} \equiv \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} \cdot \frac{W_o}{Y_o}
$$

• Can calculate $\Delta^{comp}$ directly from micro data and pop. projns

• Why is this a natural starting point for macro projections?

1. It can be a sufficient statistic for $W/Y$ in a demographic transition
   • Small open economy special case: $a_{jt}$ and $h_{jt}$ are constant
   • We say the economy ages without “behavioral effects”

2. It is always a component of the total change in $W/Y$:

$$
\underbrace{\frac{W_t}{Y_t} - \frac{W_o}{Y_o}}_{\equiv \Delta_t} = \Delta_t^{comp} + \frac{\sum_j \pi_{jt} a_{jt}}{\sum_j \pi_{jt} h_{jt}} - \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}
$$

$\Delta_t^{beh}$

→ Benchmark to evaluate transition dynamics in any GE model
Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$W (r, \Theta) = A^s (r, \Theta)$$

Both $W$ and $A^s$ depend on $\Theta$. Argument in the paper has 3 parts:
The compositional effect in GE: roadmap

Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$\frac{W}{Y}(r, \Theta) = \frac{A^s}{Y}(r)$$

**Part o:** $\frac{A^s}{Y}$ depends on technology and gov. policy, not $\Theta$

![Graph showing the relationship between $r$, $W/Y$, and $A^s/Y$.](image)
The compositional effect in GE: roadmap

Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$\frac{W}{Y}(r, \Theta) = \frac{A^s}{Y}(r)$$

**Part 1:** for fixed $r$, $\Delta \frac{W}{Y} \simeq \Delta^{\text{comp}} \gg 0 \text{ (ie. } \Delta^{\text{beh}|r} \simeq 0) \text{)
Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$W/Y (r, \Theta) = A^s/Y (r)$$

**Part 2:** world $r$ must fall: the opposite of an asset market meltdown!
Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$\frac{W}{Y}(r, \Theta) = \frac{A^s}{Y}(r)$$

**Part 3:** after demeaning $\Delta^{comp}$, we obtain close approx. to $\Delta NFA$
Let $\Theta \equiv$ demographics. Equilibrium in long-run world asset market:

$$\frac{W}{Y}(r, \Theta) = \frac{A^s}{Y}(r)$$

**Part 3:** after demeaning $\Delta^{comp}$, we obtain close approx. to $\Delta NFA$
2. Measurement
Measuring $\Delta^{comp}$

- Calculate shift-share $\Delta_t^{comp}$ for US and 24 other countries

Implementation:
- Normalize labor supply so that $\sum \pi_{jo} h_{jo} = 1$
- Then $a_{jo}$ is average wealth by age normalized by GDP per capita
- Can measure relative $h_{jo}$ from relative labor income

Data:
- $\pi_{jt}$: projections of age distributions over individuals
  - 2019 UN World Population Prospects, SSA and Gagnon et al. (2016)
- $a_{jo}, h_{jo}$: age-wealth and labor income profiles in base year
  - $a_{jo}$ rescaled to match total wealth from World Inequality Database
  - $a_{jo}$ includes funded part of DB pensions
  - Household $\rightarrow$ individual $j$ by attributing all wealth to hh head
\( \Delta^\text{comp} \) in the United States: 1950-2100

- **Base year**
- **Historical**

![Graph showing population changes](image-url)

- **Low fertility**
- **Baseline**
- **High fertility**
- **Data (WID)**
Where do these large effects come from?

- In paper: separate contribution of numerator and denominator
  - $W$ contributes $\sim \frac{2}{3}$, $Y$ contributes $\sim \frac{1}{3}$ going forward
  - Historically demographic dividend pushed $Y$ up, reversed in 2010
Global trends: large and heterogeneous \( \Delta^{\text{comp}} \)
3. General equilibrium implications
Environment: overview

- Standard multi-country GE OLG model featuring idiosyncratic income risk, intergenerational transmission of skills, bequests, and a social security system [eg Krueger-Ludwig 2007]
  - Output produced out of capital and effective labor
  - Perfect competition, free capital adjustment
  - Inelastic labor supply, exog. vary retirement & LFPR
  - Five reasons for savings:
    1. Life-cycle motive
    2. Bequest motive (warm-glow, nonhomothetic)
    3. Providing for children consumption (age dependent \( mu \) modifier)
    4. Precautionary motive against income risk
    5. Precautionary motive against longevity risk
  - Government follows a fiscal rule, can adjust taxes, social security benefits, spending, or debt
Behavioral responses

- Model has five forces for non-zero behavioral effects at given $r$:
  1. **Labor supply** effect (changing LFPR/retirement age)
  2. **Declining mortality** effect (mortality tables vary by cohort)
  3. **Cost of children** effect ($mu_j$ varies with # of children)
  4. **Bequest dilution** effect (changing ratio of givers to receivers)
  5. **Social security balance** effect (adjust taxes or benefits)

- **Next**: evaluate quantitative magnitude of these effects
  - Start from sufficient statistic scenario, where 1–5 shut down
  - Progressively relax using quantitative model, fitted to:
    - observed 2016 age distribution
    - our measure of $\Delta^{comp}$ for 2016-2100 (vs age-asset profile)
Part 1: in SOE, behavioral effects are small
World economy counterfactual

- Next solve for integrated world equilibrium
  - 12 countries that are at least 1% of GDP among our 25

- Country specific targets:
  - Demographics and social security
  - \( \frac{W}{Y}, \frac{NFA}{Y} \) and \( \Delta^{comp} \)

- Vary parameters that are not identified in the steady state:
  1. Elasticity of intertemporal substitution \( \sigma^{-1} \)
     - Wealth tax literature supports range between 0.5 and 2
  2. Elasticity of capital-labor substitution \( \eta \)
     - Existing literature supports range between 0.6 and 1.25
Part 2: world \( r \) falls, but magnitudes uncertain
Change in $NFA/Y$ for fast aging countries for alternative $\sigma$ and $\eta$
Part 3: demeaned $\Delta^{\text{comp}}$ predicts NFAs — model 2016-2100
Historical performance of demeaned $\Delta_{\text{comp}}$ — data 1970-2011
Dissaving of the baby boomers?

- GE framework shows that thinking about savings rates is misleading for effects of aging on equilibrium asset returns.

- In steady state:
  \[
  \frac{W}{Y} = \frac{s}{g}
  \]

- Savings rate \( s \) falls with aging, but growth rate \( g \) does too!

- Also, much harder to perform accurate shift-share on \( s \) than \( \frac{W}{Y} \).
Other extensions in paper

1. Accounting for historical movements in US $W/Y$ and $r$
2. Reconciling literature findings on $r^*$ effects of demographics
3. Multiple assets and rates of return
4. Housing
5. Population aging and wealth inequality
Conclusion

• How does population aging affect wealth-output ratios, real interest rates, and capital flows?

• Use compositional effect $\Delta^{comp}$ as starting point for forecasts

• $\Delta^{comp}$ are large and heterogeneous in the data

• For the 21st century, our approach:
  • Refutes the asset market meltdown hypothesis: $r$ definitively falls
  • Suggests the global savings glut has just begun
Thank you!
Additional slides
US Wealth-to-GDP from SCF vs World Inequality Database

Source: World Inequality Database (WID), Survey of Consumer Finances (SCF)
Share of the population aged 65+

Source: 2019 United Nations World Population Prospects
National Wealth over GDP

Source: World Inequality Database (WID)
Rates of return on wealth

• Baseline safe return \( r_t^{\text{safe}} \) is 10 year constant maturity interest rate minus HP-filtered PCE deflator

• Baseline total return is

\[
r_t = \left( s_K Y - \delta K \right)_t + r_t^{\text{safe}} B_t \left/ \frac{W_t - NFA_t}{W_t - NFA_t} \right.
\]

where \( (s_K Y - \delta K)_t \) is net capital income
Age-wealth profiles

![Graph showing age-wealth profiles with different years: a1989, a1998, a2004, a2010, a2016, and Age-eff a2016. The graph plots wealth percentage over GDP against age.]

- **a1989**: Orange line
- **a1998**: Red line
- **a2004**: Green line
- **a2010**: Gray line
- **a2016**: Light green line
- **Age-eff a2016**: Blue line
Age-labor income profiles

The diagram illustrates age-labor income profiles over different years, showing how normalized labor income changes with age. The profiles are labeled for the years 1974, 1986, 1994, 2010, and 2016, with each line representing a different year's data. The x-axis represents age, ranging from 0 to 100, and the y-axis represents normalized labor income, ranging from 0.0 to 2.55.
Contribution of fertility to aging in the 21st century

- Fixed 2016 mortality
- Fixed 1950 mortality
- Actual

Percentage of population aged 50+

Year:
- 1960
- 1980
- 2000
- 2020
- 2040
- 2060
- 2080
- 2100
Measuring income and wealth profiles

• **Measuring age-labor income profiles** $h_{jt}$
  - Data from the Luxembourg Income Study (LIS)
  - $h_{jt}$ is proportional to total labor income per person
  - In 2016: normalize aggregate effective labor per person
    \[
    1 = L_{2016} = \sum_j \pi_{j,2016} h_{j,2016}
    \]
  - In $t$: $L_t$ grows as aggregate labor input from the BLS
    \[
    \frac{L_t^{BLS}}{L_{2016}^{BLS}}
    \]

• **Measuring age-wealth profiles** $a_{jt} = \frac{A_{jt}}{Y_t/L_t}$
  - Data from the Survey of Consumer Finances (SCF)
  - Provide net worth by age at the household level
  - $A_{jt}$ is aggregate household net worth over total individuals
  - Divide by $Y_t/L_t^{BLS}$ to obtain $a_{jt}$
Retrospective U.S. exercise

- To first order:

\[
\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \sum_i \pi_{it} a_{io} - \sum_i \pi_{io} a_{io} + \sum_i \pi_{io} (a_{it} - a_{io}) - \sum_i \pi_{io} \frac{W_0}{Y_0} (h_{it} - h_{io}) + \Delta^e_t
\]
$\Delta^{comp}$ around the world in 2100
### Change in W/Y: 1950 to 2016

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**Age eff.:**
- 1974: 98
- 1979: 101
- 1986: 104
- 1991: 104
- 1994: 102
- 1997: 101
- 2000: 104
- 2004: 101
- 2007: 100
- 2010: 97
- 2013: 96
- 2016: 97
- **Age eff.: 132**
Robustness to baseline year for age profiles (future)

Change in W/Y: 2016 to 2100

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W/Y from shift-share in 2016 and in 2100
Percentage change in $W/Y$ from shift-share
Shift-share at common age profiles (rescaled)
Shift-share at common demographic change

The chart shows the shift-share impact of common demographic change across various countries. The countries are listed along the x-axis, and the Δ_comp values are indicated on the y-axis. Countries like Germany (DEU), China (CHN), and India (IND) have higher Δ_comp values, suggesting a more significant demographic impact. Each bar represents a different country, with the height of the bar corresponding to the Δ_comp value.
• Population evolves as

\[ N_{jt} = \left( N_{j-1,t-1} + M_{j-1,t-1} \right) \phi_{j-1,t-1} \]

where

• \( N_{jt} \) denotes the numbers of individuals aged \( j \) in year \( t \)
• \( M_{j,t} \) is migration
• \( \phi_{j,t} \) are survival probabilities

• Total population is

\[ N_t = \sum_j N_{jt} \]

• Population converges to a stationary distribution in the long run
• Let \( c = c^P + nc^C \) be the total cons. of parent and children

• Assume flow utility function of a parent is

\[
U(c^P, c^C) = u(c^P) + \lambda n \varphi u(c^C)
\]

• Utility maximization implies:

\[
u'(c^P) = \lambda n \varphi^{-1} u'(c^C)
\]

⇒ total value of having children

\[
W(c) = u(c^P) + \lambda n \varphi u(c^C) = \left(1 + \lambda \frac{1}{\sigma} n \frac{\varphi - 1}{\sigma}\right)^\sigma u(c)
\]

• Hence \( \psi_i = \left(1 + \lambda \frac{1}{\sigma} n_i \frac{\varphi - 1}{\sigma}\right)^\sigma \)

• Children raise the m.u.c. if \( \lambda > 0 \) and \( \varphi > 1 - \sigma \)

• \( n_i \) comes from empirical distribution of children for parent aged \( i \)
Retirement policy

• Retirement is phased at age $T'_t$

• At age $T'_t$, agents still work a fraction $\rho_t \in [0, 1]$ of total hours

• Retirement policy is therefore

$$\rho_{jt} = 1_{j < T'_t} + \rho_t 1_{j = T'_t}$$

• Effective labor supply is

$$L_t \equiv \sum_{j < T'_t} \pi_{jt} \tilde{h}_{jt} + \rho_t \pi_{T'_t T'_t} \tilde{h}_{T'_t T'_t}$$

• Effective share of retirees is

$$\mu_{t, ret} \equiv (1 - \rho_t) \pi_{T'_t T'_t} + \sum_{j \geq T'_t} \pi_{jt}$$
Government policy

- Flow budget constraint
  \[ B_t + T_t = (1 + r_{t-1}) B_{t-1} + G_t \]

  where \( B_t \) is debt, \( G_t \) are expenditures, \( T_t \) are net taxes

  \[ T_t = w_t N_t \left( (\tau_{ts}^s + \tau_t (1 - \tau_{ts}^s)) L_t - (1 - \tau_t) \bar{d}_t \mu_t^{ret} \right) \]

- Government sets retirement policy \( \{ \rho_{jt} \} \) and follows fiscal rules

  \[ \tau_{ts}^s = \bar{\tau}^s + \varphi^s (B_t/Y_t - \bar{b}) \]
  \[ \tau_t = \bar{\tau} + \varphi^\tau (B_t/Y_t - \bar{b}) \]
  \[ G_t = \bar{G} - \varphi^G (B_t/Y_t - \bar{b}) \]
  \[ \bar{d}_t = \bar{d} - \varphi^d (B_t/Y_t - \bar{b}) \]

  where \( \bar{b} \) is the 2016 debt-to-GDP ratio

- Coefficients \( \varphi \)'s regulate the aggressiveness of the adjustment
Extension 1: other sources of asset supply

- In simple cases, alternative assets just add to supply

- Allow for
  - Markups $\mu$, capitalized monopoly profits
  - Government bonds with long-run rule $\frac{B}{Y} = b(r)$

- Then

$$\frac{a(r, \theta)}{y(r)} = \frac{k(r)}{y(r)} + b(r) + \left(1 - \frac{1}{\mu}\right) \frac{1}{r - (n + \gamma)}$$

- $\theta$ directly affects both $W$ and market cap. through discounting

- Extra terms on RHS affect elasticity of asset supply $\epsilon^s$
  - Similar formula still determines $dr$
Extension 2: Housing

- Model housing by introducing Cobb-Douglas utility

\[
\frac{1}{1 - \sigma} \left( c^{1-\alpha_h} h^{\alpha_h} \right)^{1-\sigma}
\]

- All households rent to a REIT who owns
  - fixed supply of land \( L \), equilibrium price \( P^L \)
  - stock of dwellings \( H \), depreciating at \( \delta^H \), investment price = 1
  - \( \beta = \frac{p^L_L}{p^L_{L+H}} \) is s.s. share of land

- Households invest in mutual fund that owns the REIT

- Housing supply in steady state adjusts so that

\[
\frac{a(r, \theta)}{y(r)} = \frac{k(r)}{y(r)} + \frac{\alpha^h}{1 - \alpha^h} \left( \frac{\beta}{r - (n + \gamma)} + \frac{1 - \beta}{r + \delta^H} \right) \frac{\sum_i \pi_i(\theta) \frac{c_i(r, \theta)}{y(r)}}{\sum_i \pi_i(\theta) h_i}
\]
Projected survival functions

![Projected survival functions graph](image)
Projected population growth rate
Distribution of children

![Chart showing the distribution of dependent children by age for different years: 2016, 2030, 2050, and 2100. The chart peaks around age 40 for all years, with a noticeable increase in 2016, followed by a decrease towards 2100.](image)
Distribution of bequests received
Bequests distribution and consumption profile

- Model, $B_Y = 4.5\%$
- Data (Hurd and Smith; 2002)

![Graph 1: Perct. over bequests vs. Percentile](image1)

![Graph 2: Consumption-over-GDP vs. Age](image2)
Robustness

- Lower benefits only
- Higher SS taxes only
- No migration
- Baseline
Historical exercise: inputs

Graphs showing trends in:
- $r$: interest rate
- $\gamma$: growth rate
- $G/Y$: government expenditure as a percent of GDP
- $B/Y$: foreign assets as a percent of GDP
Historical trends in wealth

• We’ll use our model primarily for prospective counterfactuals
• But: can the model account for trends in wealth since 1960?
• Concurrent developments to demographics over the period:
  • Falling real rates
  • Falling productivity growth
• We feed the model with observed trends in $r$, $\gamma$, $B$ and $G$
Historical trends in wealth
Demographics: population distributions
## World economy calibration

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<th>Parameters</th>
<th>Model</th>
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World economy calibration

[Graphs showing age distribution for different countries: AUS, CAN, CHN, DEU, ESP, FRA, GBR, IND, ITA, JPN, NLD, USA]

Model
Data
Predicted NFA/Y from demographics

Historical (data)

- USA
- IND
- CHN
- DEU
- JPN

Net Foreign Assets (% GDP)

Year

1980 1990 2000 2010

Predicted from demographics (model)

Year (Model)

2020 2040 2060 2080 2100
Elasticities by country
Jakobsen et al. (2020) validation

Note: Response of wealth to a reduction in the wealth tax. We replicate the model experiments of Jakobsen et al. (2020). The first (Couples DD) analyzes a reduction of the wealth tax from 2.2% to 1.2% on the top 1%. The second (Ceiling DD) analyzes the a reduction of 1.56 percentage points on the top 0.3%.