Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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The world population is aging...

Source: 2019 United Nations World Population Prospects
...wealth-to-GDP ratios are increasing...

...rates of return on wealth are falling...

Source: National Accounts, Flow of Funds, WID.
...and “global imbalances” are rising

Source: International Monetary Fund (IMF), Penn World Table (PWT) 9.1
How will demographics shape these trends in the 21st century?

- Broad agreement that demographics has contributed to historical trends in $W/Y$, real returns ($r$), and $NFA$ imbalances
  - Older population saves more, unevenly across countries
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  \( > -100 \text{bp} \) in Gagnon-Johannsen-Lopez-Salido 2021
  \( < -300 \text{bp} \) in Eggertsson-Mehrotra-Robbins 2019
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- Critical Q for monetary policy: what will happen going forward?
- Influential view that these trends will revert:
  “While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.”
  [Lane 2020]
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  [Lane 2020]
- "asset market meltdown" hypothesis [Poterba 2001]
- "great demographic reversal" hypothesis [Goodhart-Pradhan 2020]
In a baseline multi-country GE OLG model, the effect of demographic change on $\frac{W}{Y}, r$ and $NFA$ depends only on:

1. Age profiles of wealth, labor income, and consumption
2. Demographic projections
3. The elasticity of intertemporal substitution $\sigma$
4. The elasticity of substitution between capital and labor $\eta$

This provides a framework for measurement, which we implement
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→ Soundly reject the great demographic reversal hypothesis
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Conclusions are robust to quantitative simulations of richer model
A bridge between reduced-form and structural approaches

• Existing literature follows two broad approaches:

1. **Reduced-form**, based on shift-share exercises
   - Projected labor supply [Cutler et al 1990], demographic dividend literature [Bloom-Canning-Sevilla 2003...]

2. **Structural**, based on fully specified GE OLG models

• **Sufficient statistic approach** bridges the gap between both
Baseline model
OLG model, **demographic change + multiple countries** facing \{r_t\}

**Demographics** [drop country subscripts]
- Exogenous, **time-varying sequence of births** \(N_{ot}\)
- Exogenous, constant sequence of mortality rates \(\phi_j\)
- No migration

**Production**
- Aggregate production function with capital and effective labor, with elasticity of substitution \(\eta\)
- Constant growth rate of labor-augmenting technology \(\gamma\)
- Perfect competition, free capital adjustment

**Government**
- Flow budget constraint
  \[
  G_t + w_t \sum_{j=0}^{T} N_{jt}Etr_j + (1 + r_t)B_t = \tau w_t \sum_{j=0}^{T} N_{jt}E\ell_j + B_{t+1},
  \]
- Balance budget by changing \(G_t\), not \(\tau_t\) or \(tr_{jt}\), to keep \(B_t/Y_t \equiv \text{cst}\)
OLG model, demographic change + multiple countries facing \( \{r_t\} \)

### Demographics [drop country subscripts]
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Problem for **heterogeneous agents** of cohort $k$ (age $j \equiv t - k$)

\[
\max_k \mathbb{E}_k \left[ \sum_j \beta_j \Phi_j \frac{c_{jt}^{1 - \frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right]
\]

s.t. $c_{jt} + \phi_j a_{j+1,t+1} \leq w_t \left((1 - \tau)\ell(z_j) + tr(z_j^j)\right) + (1 + r_t)a_{jt}$

\[
a_{j+1,t+1} \geq -a
\]

- $\sigma \equiv$ elasticity of intertemporal substitution
- $\beta_j$: age-specific discount rate
- $\Phi_j$: survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_t)$: risky labor supply driven by arbitrary stochastic process $z_t$
- $\tau, tr(z_j^j)$: taxes and (state-contingent) government transfers
- $a_{jt}$: annuity holdings
Problem for heterogeneous agents of cohort $k$ (age $j \equiv t - k$)

$$\max \mathbb{E}_k \left[ \sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right]$$

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Given demographics and policy, in an integrated world equilibrium:

- Individuals optimize
- Firms optimize
- Global asset markets clear

\[ \sum_c W_t^c = \sum_c (K_t^c + B_t^c) \quad \forall t \]
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Next: consider small country aging alone, with world at steady state
\[ \rightarrow r \text{ constant (will adjust later) } \]
Compositional effects as sufficient statistics

Proposition

The wealth-to-GDP ratio of a small country aging alone with constant \( r \) and \( \gamma \) follows

\[
\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}}
\]

where \( a_{jo} \equiv \mathbb{E}a_{j,o} \) and \( h_{jo} = \mathbb{E}w_0\ell_{j,o} \) are average initial asset holdings and pretax labor income by age, and \( \pi_{jt} = N_{jt}/N_t \) is the share of the population of age \( j \).
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\( \Rightarrow \) change in log wealth to GDP ratio:

\[
\log \left( \frac{W_t}{Y_t} \right) - \log \left( \frac{W_0}{Y_0} \right) = \log \left( \frac{\sum_j \pi_{jt} a_{jo}}{\sum_j \pi_{jt} h_{jo}} \right) - \log \left( \frac{\sum_j \pi_{jo} a_{jo}}{\sum_j \pi_{jo} h_{jo}} \right) = \Delta_t^{comp}
\]

measurable from demographic projections and hh. surveys

Why? Demographics do not affect (normalized) individual decisions
Measuring compositional effects
Measuring $\Delta^{comp}$

- Calculate $\Delta_t^{comp}$ for 25 countries:

$$\Delta_t^{comp} \equiv \log \left( \frac{\sum \pi_{jt}a_{jo}}{\sum \pi_{jt}h_{jo}} \right) - \log \left( \frac{\sum \pi_{jo}a_{jo}}{\sum \pi_{jo}h_{jo}} \right)$$

- Data:
  - $\pi_{jt}$: projections of age distributions over individuals
    2019 UN World Population Prospects
  - $a_{jo}, h_{jo}$ age-wealth and labor income profiles in base year
    $a_{jo}$ includes funded part of DB pensions
    Household $\rightarrow$ individual ($j$) by splitting wealth among adults

- Report implied level change

$$\frac{W_t}{Y_t} - \frac{W_o}{Y_o} = \frac{W_o}{Y_o} \left( \exp \{ \Delta_t^{comp} \} - 1 \right)$$
\( \Delta^{comp} \) in the United States: 1950-2100

- **Base year**
- **Historical**

![Graph showing population growth projections with different fertility scenarios from 1960 to 2100. The graph compares low fertility, baseline, high fertility, and data (WID) scenarios.](image-url)
Where do these large effects come from?

In paper: separate contribution of numerator and denominator.

Going forward: $W$ contributes $\sim \pi_{2016}/\alpha_{2016}/\rho_{2016}$, $Y$ contributes $\sim \gamma_{2016}/\rho_{2016}$.

Historically, demographic dividend pushed $Y$ up, reversed in $\pi_{2016}/\rho_{2016}/\alpha_{2016}/\gamma_{2016}/\rho_{2016}$. 

**Diagram:**
- **Numerator ($W$)**
  - $\pi_{2016}$
  - $\alpha_{2016}$

- **Population shares (%)**
- **Wealth (USD)**

- **Age**
Where do these large effects come from?

In paper: separate contribution of numerator and denominator.

Going forward: \( W \) contributes \( \sim \) two.osf /three.osf, \( Y \) contributes \( \sim \) one.osf /three.osf.

Historically demographic dividend pushed \( Y \) up, reversed in \( \sim \) zero.osf /one.osf/five.osf.
Where do these large effects come from?

- In paper: separate contribution of numerator and denominator
- Going forward: \( W \) contributes \( \sim \frac{\pi_{2016}}{\pi_{2100}} \), \( Y \) contributes \( \sim \frac{\pi_{2016}}{\pi_{2100}} \)
- Historically demographic dividend pushed \( Y \) up, reversed in \( \frac{\pi_{2016}}{\pi_{2100}} \)
Where do these large effects come from?

- In paper: separate contribution of numerator and denominator
  - Going forward: $W$ contributes $\sim \frac{2}{3}$, $Y$ contributes $\sim \frac{1}{3}$
  - Historically demographic dividend pushed $Y$ up, reversed in 2010
Across countries, $\Delta^{comp}$ large and heterogeneous by 2100
General equilibrium implications
General equilibrium implications

Semielasticity formulas

Semielasticity of asset demand $\bar{\epsilon}_d$: depends on $\sigma$ and observables

Semielasticity of asset supply $\bar{\epsilon}_s$: depends on $\eta$ and observables
Asset demand shift of $\bar{\Delta}^{\text{comp}}$ : wealth-weighted average of $\Delta^{\text{comp},c}$

Large and positive in the data.
General equilibrium implications

\[ \Delta r \approx -\frac{\Delta \text{comp}}{\bar{\epsilon_s} + \bar{\epsilon_d}} < 0, \quad \Delta \log \left( \frac{W}{Y} \right) \approx \frac{\bar{\epsilon_s}}{\bar{\epsilon_s} + \bar{\epsilon_d}} \Delta \text{comp} > 0 \]
Changes in $r$ and $W/Y$: 2016 to 2100

\[ \Delta r \approx - \frac{\Delta^{\text{comp}}}{\bar{\epsilon}^d + \bar{\epsilon}^s} \]

\[ \Delta \log \left( \frac{W}{Y} \right) \approx \frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s} \Delta^{\text{comp}} \]

**A. Change in world $r$**

<table>
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**B. Change in avg. $\log W/Y$**

<table>
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<tr>
<th>$\eta$</th>
<th>0.25</th>
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- Simulations of general model deliver very similar outcomes
Country-specific shifts $\Delta^{comp}$ large and heterogeneous in data
General equilibrium implications, part 2

\[ \Delta \left( \frac{NFA}{Y} \right) \approx \frac{W_0}{Y_0} \left( \Delta^{\text{comp}} - \bar{\Delta}^{\text{comp}} \right) \]
Demeaned compositional effect and NFAs

\[ \Delta \left( \frac{NFA^c}{Y^c} \right) \simeq \frac{W^c}{Y^c} \left( \Delta_t^c - \bar{\Delta}_{comp} \right) \]

A. NFA projection

Data suggests large global imbalances for the 21st century
Demeaned compositional effect and NFAs

$$\Delta \left( \frac{NFA^c}{Y^c} \right) \approx \frac{W^c}{Y^c} \left( \Delta_t^c - \bar{\Delta}_{comp} \right)$$

A. NFA projection

B. Historical performance

→ Data suggests large global imbalances for the 21st century
A great demographic reversal?
Worldwide: decreasing $S_t/Y_t$ everywhere

- Perform same exercise, but projecting $S/Y$ from composition.
Declining $r$ despite falling savings?

- Will dissaving of the old reverse the effects of demographics? [Lane 2020, Goodhart and Pradhan 2020]

- Measured $S_t/Y_t$ from composition does decline

- **But**: $r$ does not increase
Declining $r$ despite falling savings?

- Will dissaving of the old reverse the effects of demographics? [Lane 2020, Goodhart and Pradhan 2020]

- Measured $S_t/Y_t$ from composition does decline

- **But**: $r$ does not increase

- Why? Savings is misleading with declining pop. growth. In s.s.:

$$\frac{W}{Y} = \frac{S/Y}{g}$$

where $g$ is GDP growth

- With demographic change, $S/Y$ falls, but $g$ falls by more!
Conclusion

• How does population aging affect wealth-output ratios, real interest rates, and capital flows?

→ what matters is the compositional effect $\Delta^{comp}$

large and heterogeneous in the data

• For the 21st century, our approach:

  • Refutes the asset market meltdown hypothesis: $r$ definitively falls
  • Suggests the global savings glut has just begun
Thank you!
Additional slides
US Wealth-to-GDP from SCF vs World Inequality Database

Source: World Inequality Database (WID), Survey of Consumer Finances (SCF)
Countries by income group

Source: 2019 United Nations World Population Prospects
National Wealth over GDP

Source: World Inequality Database (WID)
Rates of return on wealth

• Baseline safe return $r_t^{safe}$ is 10 year constant maturity interest rate minus HP-filtered PCE deflator

• Baseline total return is

$$r_t = \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t}$$

where $(s_K Y - \delta K)_t$ is net capital income
Age-wealth profiles in the U.S.
Age-labor income profiles in the U.S.
Contribution of mortality to aging since 1950
Contribution of mortality to aging in 21st century

2016-2100 change in the share of 50+ : percentage due to mortality

Country: AUS, AUT, BEL, CAN, CHN, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, HUN, IND, IRL, ITA, JPN, LUX, NLD, POL, SVK, SVN, SWE, USA

Percentage of total change
\[ \Delta^{\text{comp}} \text{ around the world in 2100} \]
## Robustness to baseline year for age profiles (past)

### Age-wage profile (SCF)

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Robustness to baseline year for age profiles (future)

<table>
<thead>
<tr>
<th>Age-wealth profile (SCF)</th>
<th>Change in W/Y: 2016 to 2100</th>
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<tbody>
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<td>1989</td>
<td>106 107 110 111 111 110 112 109 107 105 103 102</td>
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<td>89  89  92  92  92  92  93  90  88  86  84  83</td>
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<td>1995</td>
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<td>1998</td>
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<td>2001</td>
<td>97  98 100 101 101 100 101 99  96  94  92  91</td>
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<tr>
<td>2004</td>
<td>115 116 119 120 120 119 120 118 115 113 111 110</td>
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<td>2007</td>
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<tr>
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<td>143 145 149 151 151 150 152 149 147 144 142 141</td>
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<tr>
<td>DH-t</td>
<td>128 130 133 134 135 134 136 133 130 128 125 124</td>
</tr>
</tbody>
</table>
Low and high fertility scenarios

The graph illustrates a comparison of low and high fertility scenarios across various countries. The vertical axis represents a measure of fertility difference, while the horizontal axis lists countries. The data points are colored to differentiate between low fertility circles and high fertility squares. The countries are ranked according to their fertility rates, with higher numbers indicating a higher fertility rate in the high fertility scenario.
W/Y from comp. effect in 2016 and in 2100

![Graph showing W/Y from comp. effect in 2016 and in 2100 for various countries.](image)

- Countries include: HUN, EST, FIN, SVN, SWE, DNK, DEU, SVK, CAN, AUT, JPN, POL, GRC, IRL, USA, NLD, GBR, FRA, AUS, BEL, LUX, ITA, CHN, ESP, IND.
- The graph displays the values for W/Y from comp. effect in 2016 and in 2100 for each country.

Note: The exact values are not visible in the image provided.
Percentage change in $W/Y$ from comp. effect
Compositional effect at common age profiles

![Graph showing compositional effect at common age profiles across different countries. The x-axis represents countries, and the y-axis represents the compositional effect. The bars indicate the values for each country.]
Compositional effect at common demographic change

[Bar chart showing the compositional effect at common demographic change for various countries. The x-axis represents the countries (e.g., HUN, EST, FIN), and the y-axis represents the compositional effect. Each country has a bar indicating the value.]
• Population evolves as

\[ N_{jt} = (N_{j-1,t-1} + M_{j-1,t-1}) \phi_{j-1,t-1} \]

where

• \( N_{jt} \) denotes the numbers of individuals aged \( j \) in year \( t \)
• \( M_{j,t} \) is migration
• \( \phi_{j,t} \) are survival probabilities

• Total population is

\[ N_t = \sum_j N_{jt} \]

• Population converges to a stat. distribution in the long run
Projected survival functions

![Graph showing survival functions for different years (2016, 2030, 2050, and 2100). The x-axis represents age, and the y-axis represents survival function (%). Each year has a distinct line indicating the survival function over age.]
Demographics: population distributions
Demographics: population growth rates

![Graph showing population growth rates for different countries over time.](image-url)
Semielasticities of asset supply and demand

- Assuming common capital-labor substitution elasticity $\eta$,

$$\bar{\varepsilon}^s = \frac{\eta}{r_o + \delta} \left( \frac{K_o}{W_o} \right)$$

→ Measurable from observables and knowledge of $\eta$

**Proposition**

*With no idiosyncratic risk, $a = \infty$, $\eta = 1$ and $r = \gamma = 0$:

$$\varepsilon^d = \frac{1}{1 + r} \frac{C}{W} \cdot \sigma \cdot \text{Var} \ (Age_c) - \frac{1}{1 + r} (E[Age_a] - E[Age_c])$$

- **substitution effect**
- **income effect**

→ Measurable from observables and knowledge of $\sigma$
• Using formulas from the paper:

\[
\bar{\epsilon}_d = \text{Literature Estimates, range 2–40}
\]

Updated environment

Household problem becomes

$$\max E_k \sum_j \beta_j \Phi_{jk} \left[ \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \gamma Z_t^{\nu-\frac{1}{\sigma}} (1 - \phi_{jt}) \left( \frac{a_{jt}}{1-\nu} \right)^{1-\nu} \right] \nu \geq \frac{1}{\sigma}$$

s.t. \( c_{jt} + a_{j+1,t+1} \leq w_t \left( (1 - \tau_t) \ell_{jt}(z_j)(1 - \rho_{jt}) + tr_{jt}(z_j) \right) + (1 + r_t) a_{jt} + b_{jt}^r(z_j) \)

\( a_{j+1,t+1} \geq -\bar{a}Z_t \)

- From annuities to bequests:
  - assets become bequests at death, distributed as \( b_{jt}^r(z_j) \)

- Time-variation in mortality \( \Phi_{jk} \), labor supply \( \ell_{jt} \), ret. age \( \rho_{jt} \)

- Fiscal rule with adjustments in taxes and transfers, income process with intergenerational persistence

- Migration
Robustness of conclusions: steady-state

- Assume $\sigma = 0.5, \eta = 1$

<table>
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<tr>
<th></th>
<th>$\Delta r$</th>
<th>$\Delta \frac{W}{Y}$</th>
<th>$\Delta^{\text{comp}}$</th>
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**Alternative model specifications**

- + Constant bequests: $\Delta r = -1.18$, $\Delta \frac{W}{Y} = 10.0$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 27.0$, $\zeta^d = 14.9$, $\zeta^s = 8.0$
- + Constant mortality: $\Delta r = -1.23$, $\Delta \frac{W}{Y} = 10.9$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 27.1$, $\zeta^d = 13.9$, $\zeta^s = 8.0$
- + Constant taxes and transfers: $\Delta r = -1.33$, $\Delta \frac{W}{Y} = 11.9$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 30.1$, $\zeta^d = 14.6$, $\zeta^s = 8.0$
- + Constant retirement age: $\Delta r = -1.49$, $\Delta \frac{W}{Y} = 13.4$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 34.1$, $\zeta^d = 14.7$, $\zeta^s = 8.0$
- + No income risk: $\Delta r = -1.47$, $\Delta \frac{W}{Y} = 13.2$, $\Delta^{\text{comp}} = 33.9$, $\Delta^{\text{soe}} = 33.9$, $\zeta^d = 13.8$, $\zeta^s = 8.0$
- + Annuities: $\Delta r = -1.33$, $\Delta \frac{W}{Y} = 11.5$, $\Delta^{\text{comp}} = 34.2$, $\Delta^{\text{soe}} = 34.2$, $\zeta^d = 17.2$, $\zeta^s = 8.0$

**Alternative fiscal rules**

- Only lower expenditures: $\Delta r = -1.29$, $\Delta \frac{W}{Y} = 11.0$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 32.6$, $\zeta^d = 17.9$, $\zeta^s = 8.0$
- Only higher taxes: $\Delta r = -0.88$, $\Delta \frac{W}{Y} = 6.7$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 19.4$, $\zeta^d = 14.6$, $\zeta^s = 8.0$
- Only lower benefits: $\Delta r = -1.50$, $\Delta \frac{W}{Y} = 12.9$, $\Delta^{\text{comp}} = 34.1$, $\Delta^{\text{soe}} = 39.1$, $\zeta^d = 18.4$, $\zeta^s = 8.0$

$\Delta^{\text{soe}}$ is response of $W/Y$ to demographics at fixed $r$
Robustness of conclusions: transitions

A. Change in $r$

B. Change in world $W/Y$

![Graph A: Change in r](image)

- Model
- Pure comp.

![Graph B: Change in world W/Y](image)

Year

Change in $r$

Change in world $W/Y$
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World economy calibration