

# Consumer Bankruptcy as Aggregate Demand Management

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Preliminary, Comments Welcome

## Abstract

We study the role of consumer bankruptcy policy in macroeconomic stabilization. Our economy features nominal rigidities, incomplete financial markets, and heterogeneous households with access to unsecured defaultable debt. We derive sufficient statistics for quantifying the contribution of automatic stabilizers to dampening output fluctuations. Bankruptcy is an automatic stabilizer if the average consumption effect of default, or “ACED” (the causal effect of default on consumption) is larger than the marginal propensity to consume of savers. Quantitatively, for the United States, we show that the current bankruptcy code reduces the amplitude of output fluctuations by 6%, and that bankruptcy rules that systematically respond to the business cycle could increase this number to 13%. By comparison, countercyclical government spending and deficits reduce output fluctuations by 19% and 11%, respectively.

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# 1 Introduction

Households in the United States have the possibility to discharge unsecured debts by declaring personal bankruptcy. This option to default confers them with insurance against idiosyncratic risk but drives up the cost of credit. A large literature beginning with [Zame \(1993\)](#) analyzes this trade-off and its consequences for the optimal degree of bankruptcy leniency (for example [Chatterjee, Corbae, Nakajima and Ríos-Rull 2007](#) and [Livshits, MacGee and Tertilt 2007](#)).

Since the initial Bankruptcy Act in 1898, the U.S. bankruptcy code has been subject to regular overhauls that may in part reflect the changing value society places on insurance against idiosyncratic risk. However, the policy debates regarding the benefits of bankruptcy also often stress its role as an *automatic stabilizer*. Defaulting provides an opportunity for highly constrained households to increase their level of consumption in the midst of a recession. This may, in turn, prop up aggregate demand and mitigate the severity of the downturn.

Indeed, the transfers provided by the bankruptcy system are large and countercyclical. [Figure 1a](#) shows that bankruptcy filings per household are negatively correlated with output and almost 10 times as large in magnitude: when GDP falls by 1%, the number of bankruptcy filings tends to increase by around 10%. [Figure 1b](#) shows the magnitude of the implied transfers, calculated as the amount of unsecured consumer credit charged off in each year, and compares them to total payments from the unemployment insurance (UI) system. In the past twenty years, unsecured credit chargeoffs have been just as countercyclical, and typically larger in magnitude, than UI payments. Yet, while the role of UI as an automatic stabilizer has been studied by a vast literature (see e.g. [Nakajima 2012](#), [Mitman and Rabinovich 2015](#), [McKay and Reis 2016](#), [Kekre 2019](#), [McKay and Reis 2020](#)), these equally-large bankruptcy transfers have received virtually no attention.

This paper attempts to fill this gap by studying consumer bankruptcy as an aggregate demand management tool. We introduce nominal rigidities to standard models of consumer default. This generates a role for aggregate demand in economic fluctuations. We proceed in three steps. First, motivated by the absence of a clear definition of an automatic stabilizer in the literature, we provide a systematic definition in a two-period context. Second, we show in this context that, under a certain condition that we specify, consumer bankruptcy satisfies the requirement of our definition. Finally, we build a quantitative model that captures the main features of the U.S. bankruptcy code (both Chapter 7 and Chapter 13), along the lines of those studied in the quantitative consumer default literature. We calibrate our model economy to replicate the magnitude and the heterogeneity in unsecured borrowing, chargeoffs and bankruptcy rates in the United States. We study the extent to which the current bankruptcy code (as well as other automatic stabilizers) reduces the amplitude of the output fluctuations, and the extent to which explicitly indexing the bankruptcy code to the state of the business cycle could help reduce this amplitude even further.

In order to frame our findings, we start by providing a definition of an automatic stabilizer in the context of a two-period model. This exercise serves two purposes. First, while automatic stabilizers in the sense [Musgrave and Miller \(1948\)](#) and [Christiano \(1984\)](#) have been studied previously, the literature lacks a systematic definition of an automatic stabilizer. We fill that gap. Second, our

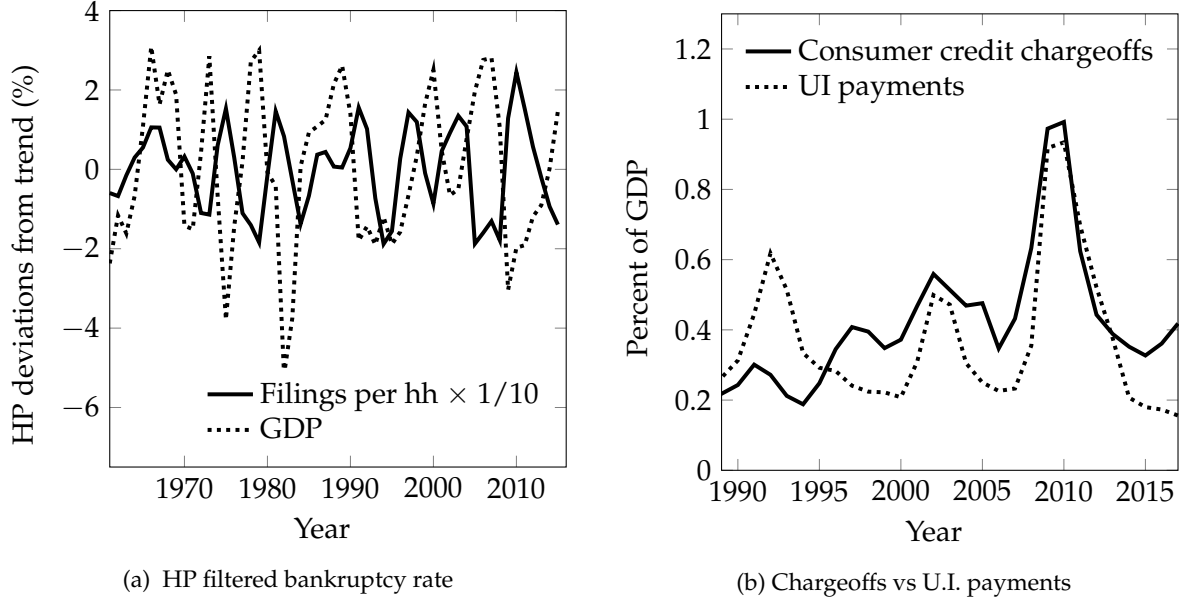


Figure 1: Cyclicity of bankruptcy and chargeoffs

definition facilitates a direct comparison of bankruptcy to other prominent automatic stabilizers (e.g., countercyclical government spending and the income tax), so that we can evaluate its importance in stabilizing the business cycle *relative* to what other automatic stabilizers do.

The two-period framework includes household heterogeneity and an aggregate demand management role for policy. Let  $\epsilon_s \equiv \frac{\partial s}{\partial y}$  be the sensitivity of some aggregate  $s$  to output  $y$ , and  $\alpha_s \equiv \frac{\partial AD}{\partial s}$  be the sensitivity of aggregate demand to  $s$ . Then we say that  $s$  is an **automatic stabilizer** if  $\epsilon_s \cdot \alpha_s < 0$ . Examples of stabilizers include government spending  $g$  ( $\epsilon_g < 0, \alpha_g > 0$ ), the income tax  $t$  ( $\epsilon_t > 0, \alpha_t < 0$ ), and monetary policy (the real interest rate  $r$ , with  $\epsilon_r > 0, \alpha_r < 0$ ). If, on the other hand,  $\epsilon_s \cdot \alpha_s > 0$ , then  $s$  is an **automatic destabilizer**. An example of such a destabilizer is Fisherian debt deflation, since the price level  $P$  satisfies  $\epsilon_P > 0$  and  $\alpha_P > 0$ . Under this definition, in the simple framework that follows we can characterize the contribution of automatic stabilizers to output stabilization. Consider an economy with stabilizers  $S_1, \dots, S_k$ , whose equilibrium satisfies a simple Keynesian cross equation:

$$AD(y, S_1(y), \dots, S_k(y), \theta) = y$$

that says that aggregate demand has to equal output. Let  $dy_0$  be output fluctuations under the status quo with all  $k$  stabilizers, and  $dy_0^*$  be output fluctuations in a counterfactual world with all stabilizers turned off (in a sense to be made precise below). We prove that counterfactual fluctuations in the absence of multipliers are higher by

$$\frac{\text{std}(dy^*)}{\text{std}(dy)} = 1 - M^* \cdot \sum_{s \in S} \epsilon_s \cdot \alpha_s, \quad (1)$$

where  $M^* = \frac{1}{1-MPC}$  is the economy's multiplier when all stabilizers are turned off. Automatic stabilizers can reduce output fluctuations by effectively lowering the multiplier, leading to smaller amplification of shocks. Intuitively, turning off stabilizers steepens the aggregate demand curve, as shown in Figure 2. These results elucidate the main forces relevant to understand the properties of an automatic stabilizer and their role for aggregate demand management. Hence, we provide both a practical definition of an automatic stabilizer and a sufficient statistic to quantify its impact on fluctuations. Our sufficient statistic formula implies that cyclical government spending and deficits reduce output fluctuations by 20% and 9%, respectively. In the appendix, we show that the sufficient statistics perform remarkably well in predicting the stabilization ratios in a full fledged HANK model (the fully dynamic extension of the simple framework), as Table 1 illustrates. Next, we turn to show that under a simple condition that consumer bankruptcy satisfies the above definition of an automatic stabilizer.

In order to build intuition for our results, we adapt our simple two-period model to capture the essence of our quantitative model. There are two types of consumers with equal mass, borrowers and savers, with a banking sector intermediating loans from the latter to the former in period 0. Borrowers come into period 0 with an initial legacy debt level  $b_0$ , and face idiosyncratic shocks to income in both periods. They have an option to default on their debts in both periods. This provides them with partial insurance, which they pay for in the form of higher interest rates to savers. A government sets the degree of bankruptcy leniency, which we model in the simplest possible form, as set of utility penalties  $(K_0, K_1)$  that the government imposes on households that declare bankruptcy in either period. Conditional on a state, bankruptcy is affected by income shocks, so that the average probability that debts are repaid in each period is a smooth function  $d_t \in (0, 1)$  of fundamentals.

We begin by defining the Consumption Effect of Default at  $e_0$ , or *CED*, as  $\frac{c_0^d(e_0) - c_0^r(e_0)}{b_0}$ . The *CED* captures the propensity to consume out of defaulted debt, which is conceptually related to, but distinct from the marginal propensity to consume. Next, we show that default is characterized by an income threshold  $\bar{e}_0$ , whereby all agents with income above the threshold repay, and those below default. We prove that the *CED* for the marginal defaulter, with  $e_0 = \bar{e}_0$ , is positive, and we denote that as the Average Consumption Effect of Default, or *ACED*. The sign of the *ACED* has the opposite sign of the cyclicity of the default rate  $\frac{\partial d_0}{\partial y_0}$ . An individual is more likely to default when their income falls if and only if their *ACED* is positive. The intuition for this result is as follows. What prevents individuals from defaulting in the first place is the utility cost of defaulting, which is independent of their level of income. By the envelope theorem, when income goes down, the value of defaulting falls by  $u'(c^d)$ , while the value of repaying falls by  $u'(c^r)$ , where  $c^d$  and  $c^r$  indicate consumption when defaulting and repaying, respectively. Hence, a positive *CED* ( $c^d > c^r$ ) and concave utility imply that the value of repaying falls faster than the value of defaulting, and the overall default rate rises.<sup>1</sup> At the macroeconomic level, since the bankruptcy rate is

<sup>1</sup>This result is related to Arellano (2008)'s Proposition 3, showing that a country's default incentives are stronger when its endowment is larger, and to Chatterjee et al. (2007) Theorem 3, showing that the set of idiosyncratic states in which households default is a closed interval.

countercyclical overall, this result suggests that the ACED is positive.

We next provide a condition under which bankruptcy satisfies our definition of an automatic stabilizer. An increase in default boosts aggregate demand,  $\frac{\partial AD}{\partial d_0} > 0$ , if and only if  $ACED > MPC^S$ , that is, if the consumption effect of default at the indifference threshold is higher than the saver MPC. This result is intuitive: if the increase in consumption of the marginal defaulter is higher than the reduction in consumption by the saver (the counterparty to that defaulted debt), aggregate demand rises. The sensitivity of aggregate demand to bankruptcy is thus  $\alpha_{Bank} = (ACED - MPC^S) b_0/2$ . The sensitivity of default to output is  $\epsilon_{Bank} = -\partial d_0/\partial y_0$ . As discussed above, the cyclicity of the bankruptcy rate has the opposite sign of the  $ACED$ . Therefore, whenever  $ACED > MPC^S$ ,  $\alpha_{Bank} \cdot \epsilon_{Bank} < 0$  and bankruptcy satisfies our definition of an automatic stabilizer. The causal effect of time-variation in defaults on the cyclicity of aggregate output is therefore given by:

$$\frac{\text{std}(dy_0^*)}{\text{std}(dy_0)} = 1 - M^* \cdot \epsilon_{Bank} \alpha_{Bank} = 1 + M^* \cdot (ACED - MPC^S) \frac{b_0}{2y_0} \left( -\frac{\partial d_0}{\partial \log y_0} \right). \quad (2)$$

Equation (2) says that consumer bankruptcy mitigates the magnitude of amplitude of economic fluctuations to the extent that bankruptcy is more countercyclical (the semielasticity of the default rate to output  $\frac{\partial d_0}{\partial \log y_0}$  is larger), the consumer-credit-to-GDP ratio is larger, and the difference between the  $ACED$  and savers' MPC is larger. The intuition is as follows. Consider a shock that pushes down on aggregate demand, such as a rise in idiosyncratic income risk. Suppose first that households cannot default. Since monetary policy does not cut interest rates to offset the impact of this shock, this shock makes output fall by an extent that depends on the Keynesian multiplier  $M^*$ . Suppose now that households are allowed to default. Since their  $ACED$  is positive, they will default more, which will create an automatic transfer of wealth from savers to borrowers. In turn, to the extent that the  $ACED$  is above the MPC of savers, this will mitigate the decline in output. Hence, in this case, bankruptcy acts as an automatic stabilizer in the sense of [Musgrave and Miller \(1948\)](#) and [Christiano \(1984\)](#).

Equation (2) provides a simple framework for measurement. From the data, we see that the credit-to-GDP ratio is around  $\frac{b_0}{2y_0} \sim 10\%$  and the semielasticity of the repayment rate to output is around 0.5. The key question is what  $ACED$  and the relevant  $MPC^S$  are. Direct measurement of the  $ACED$  at the micro level is challenging, since it would require detailed panel data on both consumption, debt and default and a way to identify the counterfactual consumption of a defaulting (repaying) household, had they chosen to default (repay) instead. [Indarte \(2020\)](#) addresses this challenge by showing that the  $CED$  for borrowers at the margin of defaulting can be obtained by following a sufficient statistic approach, comparing the relative size of what she calls the “moral hazard” and the “liquidity” effects of debt relief—the effect on the default probability of giving a household more income irrespective of their default decision, vs. only if they default. Her result can be adapted to our setting, so that we can use her empirical estimates to gauge the magnitude of the  $CED$  that is relevant for our model. Given her range of empirical estimates, and a standard range for CRRA in macro of 1 to 5, we find that the  $CED$  likely lies between 0.09 to 0.56. We

conclude that the data supports a positive  $CED$  that is plausibly larger than the  $MPC^S$  of savers. Assume a zero  $MPC^S$  and a Keynesian multiplier of 2, we arrive at an upper bound for a stabilization ratio of bankruptcy of 1.13—that is, a world where the bankruptcy rate does not respond to fluctuations would have standard deviation of output fluctuations that is 13% higher, comparable to government spending and deficits, as displayed in Table 1.

Finally, we ask how the planner can use its instruments—which in the baseline model are just the utility penalties  $K_0, K_1$  in both periods—to reduce the magnitude of fluctuations. We consider policy rules that get around the commitment issue that governments face in our environment of always being tempted to forgive legacy debts. We find a surprising result: a policy rule that lowers  $K_0$  but *increases*  $K_1$  in recessions delivers the maximal reduction in fluctuations. The intuition is as follows. Lowering  $K_0$  in a recession will create an ex-post transfer of wealth between borrowers and savers that will increase output. Raising  $K_1$  will crowd in credit supply and increase output even further.<sup>2</sup> Hence, a planner concerned with the variance of fluctuations wants to *loosen bankruptcy rules* on legacy debts and to *tighten bankruptcy rules* going forward. This conclusion enriches those of [Farhi and Werning \(2016\)](#) and [Korinek and Simsek \(2016\)](#) on the use of borrowing restrictions for macroprudential policy, by taking into account the endogenous response of credit supply.

Our simple model elucidates the main forces relevant to understand the role of the consumer bankruptcy system for aggregate demand management, but it is too stylized to provide a clear quantitative evaluation. We therefore turn to our quantitative model, which takes account the rich heterogeneity across U.S. households and the idiosyncratic and aggregate risks they face.

Our quantitative model captures the main features of the U.S. consumer bankruptcy system. As in [Livshits et al. \(2007\)](#), households experience income shocks over their life cycle, as well as occasional expenditure shocks, capturing rare events such as medical or divorce expenses that are known to be important to explain consumer bankruptcy. For a fee, household can declare bankruptcy under either chapter 7—which resets their debts to 0 but precludes them from borrowing for a number of periods—or chapter 13, which lowers their debt level and imposes an income-based repayment plan. As per the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), an income threshold determines eligibility for chapter 7. Rather than the flow utility penalty  $K$ , the government now controls four parameters of the bankruptcy system that we can map to the data: 1) the exclusion period for bankrupt debtors, captured by a stochastic probability of re-access  $\nu$ , 2) an income penalty  $\zeta^P$  3) the one-time fee  $F$  for declaring bankruptcy, and 4) the fraction of income devoted to chapter 13 repayment  $\zeta^{13}$ .

Our economy is closed, so that savers invest in both capital and borrowers’ liabilities, intermediated via the domestic banking sector. We assume that prices and wages are sticky and that monetary policy follows a Taylor rule (in an extension, we study the zero lower bound as well).

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<sup>2</sup>Raising  $K_1$  creates offsetting income, substitution, and precautionary savings effects on spending at date 0, but we can show that the net effect is to increase borrower consumption. Saver consumption is unaffected by  $K_1$ , since the saver is a permanent income consumer and that credit spreads are set so that the saver breaks even in present value terms.

Following a negative aggregate shock, households default by more than expected, and the banking sector experiences losses that it passes on to savers via lower dividends. Given the nominal rigidities and imperfectly-responsive monetary policy, this redistribution of wealth matters not only directly, but also indirectly through its effects on aggregate demand.

We calibrate the steady state of our economy to replicate key facts about unsecured borrowing and bankruptcy rates in the United States. Our model is successful, in particular, in matching the life-cycle profiles of income, consumption, credit and bankruptcy. We then estimate the remaining parameters on aggregate data using a simulated method of moments. We allow for ten aggregate shocks (to total factor productivity, price markup, wage markup, risk-premium, lending spread, the discount factor of agents, the non-pecuniary cost of default, government spending, taxes, and monetary policy) and determine the moments that get the model closest to the data. Overall, our current model is able to replicate relatively well most salient features of the business cycle, including the cyclical nature of the bankruptcy rate and chargeoffs.

With this model in place, we study two counterfactuals to quantify the role that consumer bankruptcy plays in macroeconomic stabilization. For our first counterfactual, we turn off our benchmark automatic stabilizers (countercyclical spending and deficits) as well as implement a policy rule that generates an acyclical bankruptcy rate. We shut down each stabilizer in isolation and then consider an environment where all are turned off. The results of this experiment are the first four rows of the last column of Table 1. The results from the quantitative model are well predicted by the sufficient statistic formula. In the case of bankruptcy, the stabilization ratio is lower, because the sufficient statistic omits the effects on credit supply as discussed above.<sup>3</sup> When we hit the counterfactual acyclical default economy with our estimated shocks, we find that the standard deviation of output increases by six percent relative to the benchmark. Thus, the option to smooth consumption across aggregate states via bankruptcy significantly stabilizes output and consumption fluctuations. The second counterfactual that we consider is one with an “active bankruptcy policy” where the parameters of the bankruptcy code vary systematically with the business cycle. Studying this type of policy is motivated by counter-cyclical social insurance policies, such as unemployment insurance, whose generosity is indexed to aggregates. Following the insights of our simple model, we study policies that make it easier for households when the economy falls into a recession—to provide ex-post debt relief—but commits to tighter bankruptcy policy as the economy recovers—to encourage credit supply. A simple policy that varies the time in exclusion and the filing fee as a function of the growth rate of output can further reduce the standard deviation of output fluctuations by 7 percent. As an automatic stabilizer, we find that an active bankruptcy policy would be achieve comparable stabilization to systematic increases in the generosity of unemployment insurance, estimated at 8 percent by [Kekre \(2021\)](#). Going from a world with acyclical bankruptcy rates to one with active bankruptcy policy would reduce output fluctuations by almost 15 percent.

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<sup>3</sup>In equilibrium in this framework, outlawing bankruptcy would implement the natural borrowing limit. With a log-normal income process, this implies that there would be zero borrowing in equilibrium, since there is a positive probability of having a lifetime income sequence arbitrarily close to zero.

	Stabilization ratio		
	Sufficient statistic	Simple HANK	Quantitative Model
Acyclical $G$	1.20	1.21	1.19
Acyclical deficits	1.09	1.09	1.11
Acyclical bankruptcy	1.13	—	1.06
All three acyclical	1.42	—	1.37
Active bankruptcy policy	—	—	0.93

Table 1: Automatic stabilizers quantified

## 2 Automatic stabilizers in a two-period framework

We first write down a very simple model with an aggregate demand management role for policy. In the context of this simple model with propose a practical definition of automatic stabilizers. Let:

1.  $\epsilon_s \equiv \frac{\partial s}{\partial y}$  be the sensitivity of some aggregate  $s$  to output  $y$
2.  $\alpha_s \equiv \frac{\partial AD}{\partial s}$  be the sensitivity of aggregate demand to  $s$

Then we say that  $s$  is an **automatic stabilizer** if  $\epsilon_s \cdot \alpha_s < 0$ . Examples of stabilizers include government spending  $g$  ( $\epsilon_g < 0, \alpha_g > 0$ ), income taxes  $t$  ( $\epsilon_t > 0, \alpha_t < 0$ ), and monetary policy (real interest rate  $r, \epsilon_r > 0, \alpha_r < 0$ ). If, on the other hand,  $\epsilon_s \cdot \alpha_s > 0$ , it's an **automatic destabilizer**. For example Fisher debt deflation (price level  $P, \epsilon_P > 0, \alpha_P > 0$ ). Under this definition, in the simple framework that follows we can characterize the contribution of automatic stabilizers to output stabilization. Consider an economy with stabilizers  $S_1, \dots, S_k$ , whose equilibrium satisfies a simple Keynesian cross equation:

$$AD(y, S_1(y), \dots, S_k(y), \theta) = y$$

that says that aggregate demand has to equal output. Let  $dy_0$  be output fluctuations under the status quo with all  $k$  stabilizers, and  $dy_0^*$  be output fluctuations in a counterfactual world with all stabilizers turned off (in a sense to be made precise below). We prove the following proposition.

**Proposition 1.** *Contribution of automatic stabilizers to fluctuations. Counterfactual fluctuations in the absence of multipliers are higher by*

$$\frac{std(dy^*)}{std(dy)} = 1 - M^* \cdot \sum_{s \in S} \epsilon_s \cdot \alpha_s,$$

where  $M^* = \frac{1}{1-MPC}$  is the no-stablizer multiplier. Automatic stabilizers can reduce output fluctuations by effectively lowering the multiplier, leading to smaller amplification of shocks. Intuitively, turning off stabilizers steepens the aggregate demand curve, as shown in Figure 2. These results elucidate the main forces relevant to understand the properties of an automatic stabilizer and their role for aggregate demand management. Hence, we provide a practical definition of an



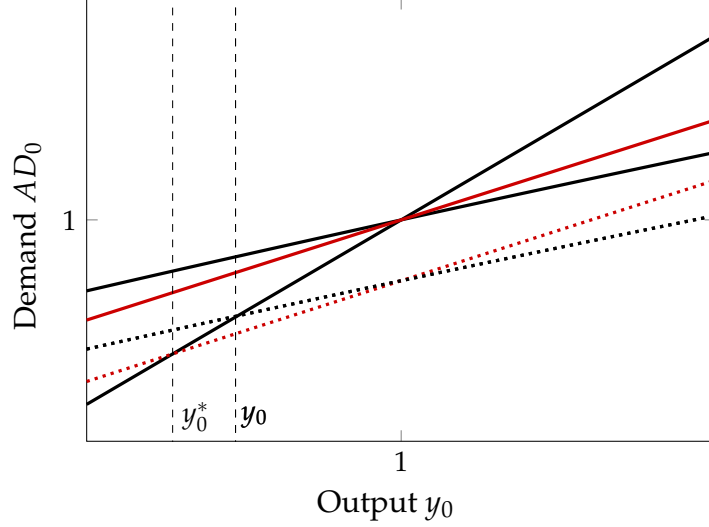


Figure 2: Automatic stabilizers

automatic stabilizer in the sense of [Musgrave and Miller \(1948\)](#) and [Christiano \(1984\)](#) and a significant statistic to quantifying its impact on fluctuations. We next turn to show that under a simple condition that consumer bankruptcy satisfies the above definition of an automatic stabilizer.

## 2.1 Fundamentals

The model features two periods  $t = 0, 1$ . We think of period 0 as the short run, where shocks will realize and nominal wages are partially rigid, whereas period 1 is the long run. There are  $I$  groups of agents each with mass  $\mu^i$ .

**Preferences and income.** Households have standard separable preferences over consumption in periods 0,1 and maximize expected discounted utility according to the following function:

$$U^i = u(c_0^i) - v(n_0^i) + \beta^i \mathbb{E} [u(c_1^i)],$$

where we allow for heterogeneity in the rate of time preference  $\beta^i$  across types.

All agents begin with zero initial asset position. In Period 0, household productivity is given by  $e_0^i$ , the real wage per effective hour is  $w_0$  and households work  $n_0^i$  hours, so pretax labor income is  $y_0^i = w_0 e_0^i n_0^i$ . In period 1, an agent of type  $i$  receives a stochastic endowment  $e_1^i$  drawn from a distribution  $F^i$ . There are no aggregate shocks in period 1, so  $\sum \mu^i \mathbb{E} [e_1^i] = y_1 = 1$ . Agents are taxed in each period according to linear tax schedule with time varying intercept  $\tau_t$ , such that individual after-tax income is given by  $z_t^i = (1 - \tau_t) y_t^i$ . Aggregate post-tax income is  $z_t = \mathbb{E} [z_t^i]$ .

**Asset market structure.** Households can borrow and save in a real risk-free bond  $b$  with a bond price given by  $\frac{1}{R}$ , where  $R$  is the real rate pinned down from monetary policy. We adopt the nota-

tion that positive holdings of  $b$  indicate borrowing. Agents are subject to type-specific borrowing limits  $\bar{b}_1^i$ . The heterogeneity in the model can thus be summarized by the vector of parameters  $\Theta \equiv (\beta^i, \bar{b}_1^i, e_0^i, F^i)$ . An agent of type  $i$  solves the following optimization problem:

$$\begin{aligned} \max_{c_0^i, c_1^i} \quad & u(c_0^i) - v(n_0^i) + \beta^i \mathbb{E} [u(c_1^i)] \\ \text{s.t.} \quad & c_0^i = (1 - \tau_0) w_0 e_0^i n_0^i + \frac{b_1^i}{R} \\ & c_1^i = (1 - \tau_1) e_1^i - b_1^i \\ & b_1^i \leq \bar{b}_1^i \end{aligned}$$

The household problem can be characterized by solving backwards. Since the economy ends at the end of the period, household repay debt  $b_1^i$  out of  $z_1^i$  and consume the rest,  $c_1^i = z_1^i - b_1^i = e_1^i z_1 - b_1^i$ .

**Production structure, price and wage setting.** At  $t = 0$ , firms produce out of labor

$$y_0 = A_0 n_0.$$

Price are flexible, so the real wage is

$$\frac{W_0}{P_0} = w_0 = A_0.$$

Wages are sticky in period 0 with equal rationing of all agents, so  $n_0^i = n_0$ . These assumptions imply that household post-tax income is  $z_0^i = (1 - \tau_0) e_0^i w_0 n_0 = (1 - \tau_0) e_0^i y_0$ . Letting aggregate post-tax income be

$$z_0 = (1 - \tau_0) \mathbb{E} [e_0^i] y_0 = (1 - \tau_0) y_0 \quad (3)$$

we have

$$z_0^i = e_0^i z_0.$$

We model period 0 wage stickiness a la Calvo: a fraction  $1 - \theta$  of infinitesimal unions, employing all workers, sets its wage  $W_0^*$  to maximize agent welfare with weight  $\mu^i$  on agent  $i$

$$W = \sum \mu^i U^i \quad (4)$$

A fraction  $\theta$  of unions cannot reset their wage. Those who can maximize welfare in (4). It is simple to show that the union reset real wage is

$$\frac{W_0^*}{P_0} = \frac{\epsilon / (\epsilon - 1)}{1 + \tau^w} \cdot \frac{\sum \mu^i v'(n_0)}{\sum \mu^i (1 - \tau_0) A_0 u'(c_0^i)} \quad (5)$$

where the term  $(1 - \tau_0) A_0$  reflects the distortionary effects of taxation and is common across individuals. Given an initial wage level of  $W_{-1}$ , the aggregate nominal wage is then

$$W_0^{1-\epsilon} = \theta (W_{-1})^{1-\epsilon} + (1 - \theta) (W_0^*)^{1-\epsilon} \quad (6)$$

Wage inflation is given by the standard wage Phillips curve,

$$1 = \theta \left( \frac{W_{-1}}{W_0} \right)^{1-\epsilon} + (1 - \theta) \left( \frac{W_0^*}{P_0} \frac{1}{A_0} \right)^{1-\epsilon}. \quad (7)$$

### Monetary policy.

We consider a monetary authority that follows strict inflation targeting, such that  $P_1 = P_0$ . The monetary authority sets the nominal rate  $i$ , so  $R = 1 + i$  given  $P_0 = P_1$ . Our benchmark assumes a constant real (and nominal) interest rate to mimic a scenario where the policy rate is constrained (e.g., the ZLB). Later we consider a real interest rate rule.

**Fiscal policy.** Consistent with the initial asset position of households, the fiscal authority starts with no initial debt. The fiscal authority spends on unvalued government consumption  $g_t$ . We assume in period 0 that spending follows an output-dependent rule  $g_0(y_0)$ , but is constant in period 1. It levies taxes in period 0 with tax rate  $\tau_0$ . In period one, the government levies taxes with tax rate  $\tau_1$  to repay any debt issued in period 0. Letting tax receipts in period  $t$  be given by  $t_t = y_t - z_t$ , we can write the government budget constraints in the two periods as

$$t_0 \equiv y_0 - z_0 = g_0(y_0) - \frac{1}{R} b_1 \quad (8)$$

$$t_1 \equiv 1 - z_1 = g_1 + b_1. \quad (9)$$

Given the rule for government spending  $g_0(y_0)$ , we consider two period-0 fiscal rules for financing it. First, we consider a tax revenue rule: the government sets a rule for  $t_0(y_0)$ . It adjusts  $\tau_0$  to raise enough to get  $t_0$ , with the baseline being constant tax revenue  $t_0$ . Second, we consider a tax rate rule: The government specifies a rule for  $\tau_0(y_0)$  directly. Our baseline here is a constant tax rate  $\tau_0$ . Under both rules, since spending in period 1 is given,  $g_1$ , period 1 taxes are the residual: the government adjusts the tax rate  $\tau_1$  to make sure it has enough to pay for its debt and spending in period 1.

**Effect of tax rule on cyclicity of taxes, post-tax income, and deficits** Consider a baseline scenario where the tax rate is adjusted such that tax revenue,  $t_0$ , is kept constant. Thus, the tax rate is given by  $\tau_0 = \frac{t_0}{y_0}$ , so the tax rate goes up in recessions and down in booms. Under this assumption,  $t'_0(y_0) = 0$  and post-tax income,  $z_0 = y_0 - t_0$  moves one for one with income. Note that this implies that the elasticity of post-tax income to output is larger than one. We also consider a constant tax rate  $\tau_0$ . Now, tax revenue is given by  $t_0(y_0) = \tau_0 y_0$ , so the derivative of tax revenue to GDP is simply  $t'_0(y_0) = \tau_0$ , the average tax rate. Under a constant tax rate, marginal and average tax

rates are the same, so the elasticity of post-tax income to output is 1. In both cases tax revenues are weakly procyclical. If we assume countercyclical government spending, , then the deficit is always countercyclical, and therefore taxes in period 1 are countercyclical,  $\frac{dt_1}{dy_0} = R(g'_0 - t'_0) < 0$ .

### Overall consumption function and general equilibrium

- Taking as given aggregate post-tax income  $z_0, z_1$  in both periods (which summarize the fiscal rule), as well as the constant  $R$ , which summarizes the monetary rule  $R$ , agent  $i$  solves

$$\begin{aligned} \max U^i &= u(c_0^i) + \beta^i \mathbb{E} [u(c_1^i)] \\ \text{s.t. } c_0^i &= e_0^i z_0 + \frac{1}{R} b_1^i \\ c_1^i &= e_1^i z_1 - b_1^i \\ b_1^i &\leq \bar{b}_1^i \end{aligned}$$

Collecting the shocks to discount factors, borrowing constraints, initial endowments (inequality) and income risk into a vector  $\Theta \equiv (\beta^i, \bar{b}_1^i, e_0^i, F^i)$  into an aggregate consumption function, we obtain

$$c_0(z_0, z_1, \Theta) = \sum \pi^i c_0^i(z_0, z_1, \Theta) \quad (10)$$

which enforces budget constraints of all agents at dates.

## 2.2 Equilibrium analysis

Equilibrium in our framework is characterized by two equations. The first is goods market clearing, which equates aggregate demand  $AD = c_0 + g_0$  to output:

$$c_0(z_0, z_1, \theta) + g_0(y_0) = y_0 \quad (11)$$

where  $z_0, z_1$  is aggregate posttax income in period 0 and 1,  $z_0 = y_0 - t_0$ ,  $z_1 = y_1 - t_1$ . The second is that the intertemporal government budget constraint holds:

$$t_0 + \frac{t_1}{R} = g_0 + \frac{g_1}{R} \quad (12)$$

In order to understand how the economy responds to shocks, we can totally differentiate (11), yielding

$$\left( \frac{\partial c_0}{\partial z_0} \frac{\partial z_0}{\partial y_0} + \frac{\partial c_0}{\partial z_1} \frac{\partial z_1}{\partial y_0} + \frac{\partial g_0}{\partial y_0} \right) dy_0 + \frac{\partial c_0}{\partial \theta} = dy_0$$

given  $z_0 = y_0 - t_0$ ,  $z_1 = 1 - t_1$ , this implies

$$\left( \underbrace{\frac{\partial c_0}{\partial z_0}}_{MPC_0} \left( 1 - \frac{\partial t_0}{\partial y_0} \right) + \underbrace{\frac{\partial c_0}{\partial z_1}}_{MPC_1} \left( -\frac{\partial t_1}{\partial y_0} \right) + \frac{\partial g_0}{\partial y_0} \right) dy_0 + \frac{\partial c_0}{\partial \theta} = dy_0 \quad (13)$$

here  $MPC_0$  is the date-0 aggregate spending response to an increase in aggregate income at date 0, and  $MPC_1$  is the date-0 response to an increase in aggregate income at date 1. Given (10), we have

$$MPC_0 = \sum \pi^i e_0^i mpc_{0i} = \sum \pi^i \frac{z_0^i}{z_0} mpc_{0i}$$

ie the post-tax-income weighted MPC. Similarly,  $MPC_1$  is the anticipatory response to an increase of a dollar of aggregate income at date 1 distributed in proportion to date-1 post-tax incomes. Next, differentiating the government budget constraint (12), we have that

$$\frac{1}{R} \frac{\partial t_1}{\partial y_0} = \frac{\partial g_0}{\partial y_0} - \frac{\partial t_0}{\partial y_0} \quad (14)$$

Substituting (14) into (13), we obtain:

**Proposition 2.** Consider a shock perturbing consumption by  $\frac{\partial c_0}{\partial \theta}$ . The equilibrium output response to this shock is:

$$dy_0 = \frac{\frac{\partial c_0}{\partial \theta}}{1 - MPC_0 + \left\{ \left( \frac{\partial t_0}{\partial y_0} \right) (MPC_0 - R \cdot MPC_1) + \left( -\frac{\partial g_0}{\partial y_0} \right) (1 - R \cdot MPC_1) \right\}} \quad (15)$$

To arrive at this result, we note that under tax revenue and government spending rule, we have  $\frac{\partial t_0}{\partial y_0} = \frac{\partial g_0}{\partial y_0} = 0$ . This gives us a baseline multiplier

$$dy_0 = \frac{\frac{\partial c_0}{\partial \theta}}{1 - MPC_0}$$

Maintaining acyclical government spending, but assuming now a tax rate rule, we have  $-\frac{\partial t_0}{\partial y_0} = -\tau_0 < 0$ . Here, the stabilizer is the tax revenue shortfall: goes up when  $y$  does down, induces redistribution from the “givers” (taxpayers tomorrow) to “receivers” (taxpayers today, who are no longer paying these taxes). This implies a redistribution between date 0 and date 1, and so

$$dy_0 = \frac{\frac{\partial c_0}{\partial \theta}}{1 - MPC_0 + \tau_0 (MPC_0 - R \cdot MPC_1)}$$

Next, turning on countercyclical government spending as well, we have (15). Again the idea is that at the margin taxes will be levied tomorrow, so the effect on spending today is  $R \cdot MPC_1$

from reduced consumption, and 1 from increased government spending. With  $t'_0 = 0$  this yields:

$$dy_0 = \frac{\frac{\partial c_0}{\partial \theta}}{1 - MPC_0 + \left(-\frac{\partial g_0}{\partial y_0}\right) (1 - R \cdot MPC_1)}.$$

We can now state a corollary to the proposition which shows how the automatic stabilizers reduce output fluctuations, and show that it maps into our general automatic stabilizer formula in Equation 1.

**Corollary 1.** *std( $dy_0^*$ ) / std( $dy_0$ ) formula for counterfactual fluctuations in the absence of stabilizer. If  $y_0^*$  is output in counterfactual with constant  $t_0, g_0$  (ie, removing the automatic stabilizers), then*

$$\frac{sd(dy_0^*)}{sd(dy_0)} = 1 + \frac{\left\{ \left(\frac{\partial t_0}{\partial y_0}\right) (MPC_0 - R \cdot MPC_1) + \left(-\frac{\partial g_0}{\partial y_0}\right) (1 - R \cdot MPC_1) \right\}}{1 - MPC_0} = 1 - M^* (\alpha_t \cdot \epsilon_t + \alpha_g \cdot \epsilon_g)$$

where  $\alpha_t = \left(\frac{\partial t_0}{\partial y_0}\right)$  is the sensitivity of tax revenue to output,  $\epsilon_t = (MPC_0 - R \cdot MPC_1)$  is the sensitivity of aggregate demand to tax revenue,  $\alpha_g = -\frac{\partial g_0}{\partial y_0}$  is the sensitivity of government spending to output, and  $\epsilon_g = (1 - R \cdot MPC_1)$  is the sensitivity of aggregate demand to government spending.

### 3 Consumer default as an automatic stabilizer

We now show that under a simple condition bankruptcy satisfies our definition of an automatic stabilizer. If the average consumption effect of default (*ACED*) exceeds the saver marginal propensity to consume ( $MPC^S$ ), we obtain four main results:

1. Bankruptcy is countercyclical (Proposition 3)
2. Bankruptcy is an automatic stabilizer (it reduces the variance of output fluctuations) (Proposition 4)
3. Changes in bankruptcy policy affects aggregate output, but leniency today boosts output today, while leniency tomorrow reduces output today due to a negative effect on credit supply (Proposition 5)
4. A systematic rule that makes policy more lenient when entering downturns, and harsher in recoveries, further reduces output variance (Proposition 6)

These results elucidate the main forces relevant to understand the role of the consumer bankruptcy system for aggregate demand management. We then turn to our quantitative model to quantify how much the role that the current U.S. bankruptcy system plays in aggregate stabilization.

### 3.1 Updated environment

We modify the two-period framework from the previous section in three ways. First, we assume that there are only two types of agents: a mass 1/2 of borrowers (generically denoted by  $B$ ) and a mass 1/2 of savers ( $S$ ). Second, we assume that borrowers begin period 0 with some legacy debt  $b_0 > 0$  that is owed to the savers. Third, we allow households to default on their debt in each period, subject to penalties specified by the bankruptcy code, and we introduce a bank to intermediate the debt between borrowers and savers. For expositional clarity, we abstract from government spending or taxes. Production and price and wage setting are identical to the previous section.

**Asset market structure with defaultable debt.** In periods  $t = 0, 1$ , borrowers can default on their debt. Defaulting at date 0 entails paying a utility cost  $K_0$  and exclusion from financial markets, while defaulting at date 1 entails paying a utility cost  $K_1$ . Absent utility costs in date 1 all agents would default; utility costs at date 0 are also required to be able to quantitatively explain the magnitude of debt repayment in the data, given that financial market exclusion alone has been known since at least [Eaton and Gersovitz \(1981\)](#) to provide a very weak incentive for repayment. At this stage, we think of  $K_0$  and  $K_1$  as instruments of policy. Our quantitative model in section 4 will provide alternative policy instruments mimicking those in the U.S bankruptcy code.

The borrower problem can be solved backwards. In period 1, after the choice of  $b_1$  and realization of  $e_1$ , households choose

$$d_1^*(b_1, e_1) = \arg \max_{d_1} \{(1 - d_1) u(e_1 - b_1) + d_1 (u(e_1) - K_1)\}$$

where  $u(e_1 - b_1)$  is their utility from repaying and  $u(e_1) - K_1$  their utility from defaulting. With standard assumptions on the period utility function  $u$ , the default decision can be characterized by an income threshold  $\bar{e}_1(b_1)$ . The household defaults if and only if  $e_1 < \bar{e}_1(b_1)$ , where  $u(\bar{e}_1(b_1) - b_1) = u(\bar{e}_1(b_1)) - K_1$ . We can write the value functions before the default decision as

$$V_1(e_1, b_1) = \begin{cases} u(e_1) - K_1 & e_1 \leq \bar{e}_1(b_1) \\ u(e_1 - b_1) & e_1 > \bar{e}_1(b_1) \end{cases}$$

and the probability of default given a debt choice  $b_1$  is simply given by  $d_1(b_1) = F^B(\bar{e}_1(b_1))$ . The utility from borrowing  $b_1$  as of period 0 in the repayment state when aggregate income is  $y_0$  is

$$U^B(b_1; y_0, R) \equiv u(e_0 y_0 - b_0 + Q(b_1, R)) + \beta^B \mathbb{E}_{e_1} [V_1(b_1, e_1)] \quad (16)$$

where  $Q(b_1; R)$  is a bond price schedule taken as given by the borrower. Denote by  $V_0^r(e_0; y_0, R)$  the maximum over all  $b_1$ 's of  $U^B(b_1; y_0, R)$ .

In period 0, after realization of aggregate  $y_0$ , households draw their initial productivity  $e_0$  and

choose

$$d_0(e_0; y_0, R, K_0) = \arg \max_{d_0} \left\{ (1 - d_0) V_0^r(e_0; y_0, R) + d_0 V_0^d(e_0; y_0, R) \right\} \quad (17)$$

where  $V_0^d(e_0; y_0, K_0) = u(e_0 y_0) + \beta^B \mathbb{E}_{e_1} [u(e_1)] - K_0$  is the value of defaulting. At this level of generality about the processes for  $e_0, e_1$  the default region is generically characterized by a closed interval, characterized by lower and upper income thresholds.<sup>4</sup> In the forgoing analysis, we make assumptions on the endowment processes to guarantee that the default decision in period 0 is again characterized by a single income threshold,  $\bar{e}_0(b_0)$  that depends on the legacy debt that the borrower starts with. The probability of default given legacy debt  $b_0$  is simply given by  $d_0(b_0) = F^B(\bar{e}_0(b_0))$ .

**Banks.** A continuum of intermediaries owned by savers make defaultable loans. They face a cost of fund  $R$ , and can diversify idiosyncratic risk across loans. We assume that banks compete loan by loan (Chatterjee et al. (2007)). This implies that they price any loan at the discounted expected probability of repayment, and make zero profits on average. Therefore, the amount of funds that the period 0 borrower can get by promising to repay  $b_1$  in period 1 is:

$$Q(b_1) = \frac{b_1}{R} \mathbb{E}_{e_1} [p_1(b_1, e_1)] = \frac{b_1}{R} \left( 1 - F^B(\bar{e}_1(b_1)) \right) \quad (18)$$

This is the schedule that the borrower faces when it optimally chooses  $b_1$ . Thus, agents internalize that borrowing more increases the effective interest rate on borrowing.

**General equilibrium.** Given  $(A_0, F^B, \beta^B, \beta^S)$ , legacy debt  $b_0$ , bankruptcy policy  $(K_0, K_1)$ , and a monetary policy rule  $R(y_0)$ , a sticky price equilibrium is default probabilities  $\{d_0(b_0, e_0), d_1(b_1, e_1)\}$ , quantities  $\{y_0, c_0^S, c_1^S, b_1\}$  and a bond schedule satisfying (18).

### 3.2 Cyclicity of bankruptcy

We start by characterizing the decision problem of households. Consider the optimal default decision  $d_0(y_0; R, K_0)$  defined in equation (17). We denote  $c_0^r(e_0)$  for the policy function in case of repayment, and  $c_0^d(e_0)$  for the policy function in case of default. We further define the Consumption Effect of Default at  $e_0$ , or *CED*, as:

$$CED(e_0) = \frac{c_0^d(e_0) - c_0^r(e_0)}{b_0} = \frac{b_0 - Q(b_1(e_0))}{b_0} \quad (19)$$

The *CED* is the additional consumption that a given household enjoys in a period if he defaults instead of repays, normalized by the level of debt outstanding. Since savings is a normal good, the *CED* is increasing in income  $e_0$ . In this simple framework, we focus on the case where the *CED* is

<sup>4</sup>See Chatterjee et al. (2007) for the theoretical characterization of the default set.



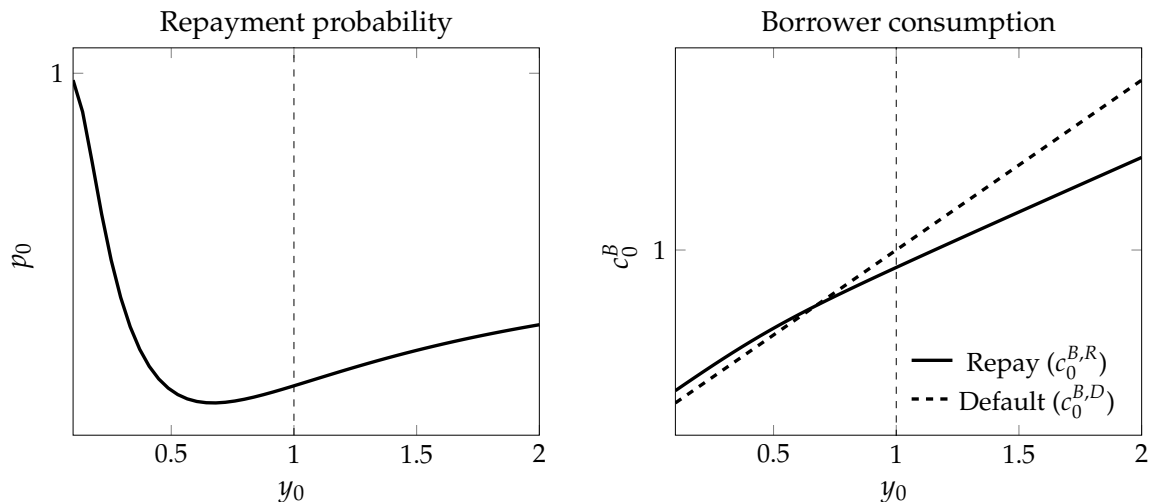


Figure 3: Repayment probability and borrower consumption

positive at the lowest level of income.<sup>5</sup> By monotonicity, it is then positive at all levels of income. With a positive *CED* we prove the following.

**Proposition 3** (Countercyclicity of bankruptcy.). *The default rate is countercyclical:  $\frac{\partial d_0}{\partial y_0} < 0$*

1.

*Proof.* See appendix A.1. □

The proposition is especially useful since it helps us interpret the cyclicity of the aggregate bankruptcy rate. The result following from a positive *CED* is intuitive. What prevents individuals from defaulting in the first place is the utility cost of defaulting, which is independent of their level of income. By the envelope theorem, when income goes down, the value of defaulting falls by  $u'(c_0^d)$ , while the value of repaying falls by  $u'(c_0^r)$ . Hence, a positive *CED* and concave utility implies that the value of repaying falls faster than the value of defaulting, and the overall repayment rate falls. This result is related to [Arellano \(2008\)](#)'s Proposition 3, showing that a country's default incentives are stronger when its endowment is larger, and to [Chatterjee et al. \(2007\)](#) Theorem 3, showing that the set of idiosyncratic states in which households default is a closed interval.

Another way to read the condition  $c_0^d > c_0^r$  is  $Q(b_1) < b_0$ , ie, borrowers are repaying debts on net. If, instead,  $Q(b_1) > b_0$  then borrowers are rolling over debt on net. Then, a decline in demand  $y_0$  actually leads household to default less. The intuition is that they need access to borrowing to support their consumption in the bad state.

Figure 3 illustrates Proposition 3. The figure is drawn for a parameterization of the model in which, when  $y_0 = 1$ , borrower consumption in repayment in default is higher than in repayment.

<sup>5</sup>In the quantitative version of the model we allow for cases where the *CED* is negative for the lowest levels of income.

Locally, then, the repayment probability is increasing in  $y_0$ , per Proposition 3. Note, however, that if  $y_0$  falls sufficiently, repayment consumption goes above default consumption, reflecting the fact that borrowers are now using the bankruptcy system to sustain their current consumption in the fact of the transitory drop in income. From that point on, a further fall in income  $y_0$  actually *increases* the repayment probability.<sup>6</sup>

### 3.3 Equilibrium analysis

The equilibrium analysis proceeds in proving three propositions. We first define aggregate demand as the sum of spending from all agents. We let this object explicitly depend on  $y_0$ , aggregate income, as well as the repayment probability  $p_0$ :

$$AD(y_0, p_0) \equiv \frac{1}{2} p_0 c_0^r(y_0) + \frac{1}{2} (1 - p_0) c_0^d(y_0) + \frac{1}{2} c_0^s(y_0)$$

We first have the following lemma.

**Lemma 1.** *An increase in default boosts aggregate demand,  $\frac{\partial AD}{\partial d_0} > 0$ , if and only if  $ACED \equiv \frac{c_0^d(\bar{e}_0) - c_0^r(\bar{e}_0)}{b_0} > MPC^S$ , ie, if the consumption effect of default at the indifference threshold is higher than the saver MPC.*

*Proof.* See Appendix A.2. □

The intuition behind this lemma is straightforward. An exogenous increase in the default probability of  $dd_0$  lowers the saver's payoff by  $b_0$ , which he spends according to his MPC. At the same time, it transfers consumption from the marginal repaying consumers to the marginal defaulting consumers, which boosts aggregate consumption by  $ACED \cdot b_0$ . The balance between the two effects determine how much additional defaults affect aggregate consumption. Using the nomenclature on automatic stabilizers, the sensitivity of demand to bankruptcy is  $\alpha_{Bank} = (ACED - MPC^S) b_0/2$ .

Of course, in equilibrium,  $d_0$  endogenously depends on  $y_0$  according to Proposition 3. The sensitivity of default to output is  $\epsilon_{Bank} = -\partial d_0 / \partial y_0$ . We can now state the conditions under which bankruptcy satisfies the definition of an automatic stabilizer as defined in. The following proposition summarizes this effect, which summarizes the causal effect of time-variation in defaults on the cyclicity of aggregate output.

**Proposition 4** (Bankruptcy as an automatic stabilizer.). *Provided that  $ACED > MPC^S$ , bankruptcy is an automatic stabilizer. The amplitude of fluctuations, measured as the standard deviation of output  $std(dy_0^*)$  in a world where consumers cannot default more in recessions ( $d_0$  is fixed), relative to the baseline  $std(dy_0)$  in which the default rate has semielasticity with respect to output  $\frac{\partial d_0}{\partial \log y_0}$ . This is given by:*

$$\frac{std(dy_0^*)}{std(dy_0)} = 1 - M^* \cdot \epsilon_{Bank} \alpha_{Bank} = 1 + M^* \cdot (ACED - MPC^S) \frac{b_0}{2y_0} \left( -\frac{\partial d_0}{\partial \log y_0} \right) \quad (20)$$

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<sup>6</sup>This mechanism may explain why default rates stayed relatively moderate or even declined during the COVID recession, which was a very acute but very transitory negative shock.

where  $M^*$  is the no-stabilizer multiplier defined above.

*Proof.* See Appendix A.2. □

Equation (2) says that consumer bankruptcy mitigates the magnitude of amplitude of economic fluctuations to the extent that bankruptcy is more countercyclical (the semielasticity of the default rate to output  $-\frac{\partial d_0}{\partial \log y_0}$  is larger), the consumer-credit-to-GDP ratio is larger, and the difference between the *ACED* and savers' *MPC* is larger. The intuition is as follows. Consider a shock that pushes down on aggregate demand, such as a rise in idiosyncratic income risk. Suppose first that households cannot default. This corresponds to shifting down the red upward sloping line in Figure 2 to the dotted line. Since monetary policy does not cut interest rates to offset the impact of this shock, this shock makes output fall by an extent that depends on the keynesian multiplier  $M^*$ . Suppose now that households are allowed to default. Since their *ACED* is positive, they will default more, which will create an automatic transfer of wealth from savers to borrowers. In turn, to the extent that the *ACED* is above the *MPC* of savers, this will mitigate the decline in output. This corresponds to the black solid line in Figure 2, which is tilted relative to the red solid line because of this effect. As is clear from the graph, for the same initial shock, equilibrium output  $y_0^*$  arrives is above  $\bar{y}_0$ . The same argument applies on the other side, when the shock to idiosyncratic risk is positive.

Equation (2) provides a simple framework for measurement. From the data, we see that the credit-to-GDP ratio is around  $\frac{b_0}{2y_0} \sim 10\%$  and the semielasticity of the repayment rate to output is around 0.5. The key question is what *ACED* and the relevant  $MPC^S$  are. It is difficult to measure *ACED* directly in the data, so we rely on estimates of our structural model. With a literal interpretation of the model,  $MPC^S$  is small. This suggests that the amplitude of fluctuations is significantly dampened by consumer bankruptcy. Of course, in a more complex model in which bank balance sheets are important,  $MPC^S$  would be larger and could even be larger than *ACED*. Our structural model makes progress in quantifying both the *ACED* and the  $MPC^S$ .

### 3.4 Effects of changes in bankruptcy policy

We next consider the effects of exogenous changes in bankruptcy policy, as summarized by the parameters  $K_0$  and  $K_1$ .

**Proposition 5** (Bankruptcy as an aggregate demand management tool). *Irrespective of the monetary policy in place,  $\frac{\partial y_0}{\partial K_0} < 0$  (harsher bankruptcy today lowers output) and, if bankruptcy is set optimally  $\frac{\partial V_0^*}{\partial K_1}$ , also  $\frac{\partial y_0}{\partial K_1} > 0$  (harsher bankruptcy tomorrow raises output)*

*Proof.* See Appendix A.3. □

The first part follows from Propositions 3 and 4. The second part is more complex, as in general there are three effects from changing  $K_1$ : an income effect, a substitution effect, and a precautionary saving effect. Our proof shows that the first two cancel each other out, so that only the income

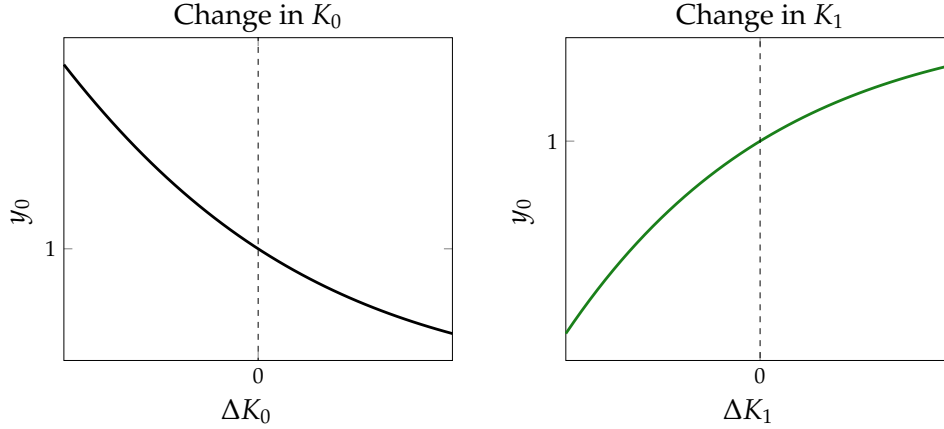


Figure 4: Output effect of a change in  $K_0$  vs  $K_1$

effect remains, and it is negative for borrowers: if  $K_1$  increases, borrowers reduce their consumption through that channel. The intuition for this is that the “credit supply” shifts in and leads borrowers to reduce their consumption. Meanwhile, savers are unaffected by the change in  $K_1$ , since they break even in present values. Overall, then, aggregate demand and therefore output increase when the severity of tomorrow’s bankruptcy rules  $K_1$  increases. Figure 4 illustrates this proposition in the context of a calibration of our simple model. We now look for the effects of bankruptcy rules  $K_0(y_0)$ ,  $K_1(y_0)$  that respond systematically to the state of the cycle. The idea behind these rules is that it would be time inconsistent to set bankruptcy policy in any other way, for instance, to always forgive debts ex-post after consumers have taken them. Instead, we consider what a policymaker can do to minimize the variance of fluctuations when it is committed to setting certain rules. Combining the insights from Propositions 3–5, we obtain the following proposition.

**Proposition 6** (Bankruptcy as an aggregate demand management tool). . *Irrespective of the monetary policy in place,  $\frac{\partial y_0}{\partial K_0} < 0$  (harsher bankruptcy today lowers output) and, if bankruptcy is set optimally  $\frac{\partial V_0^i}{\partial K_1}$ , also  $\frac{\partial y_0}{\partial K_1} > 0$  (harsher bankruptcy tomorrow raises output)*

*Proof.* See Appendix A.4. □

We find a surprising result: a policy rule that lowers  $K_0$  but *increases*  $K_1$  in recessions delivers the maximal reduction in fluctuations. The intuition is as follows. Lowering  $K_0$  in a recession will create an ex-post transfer of wealth between borrowers and savers that will increase output. Raising  $K_1$  will crowd in credit supply and increase output even further.<sup>7</sup> Hence, a planner concerned with the variance of fluctuations wants to *loosen bankruptcy rules* on legacy debts and to

<sup>7</sup>Raising  $K_1$  creates offsetting income, substitution, and precautionary savings effects on spending at date 0, but we can show that the net effect is to increase borrower consumption. Saver consumption is unaffected by  $K_1$ , since the saver is a permanent income consumer and that credit spreads are set so that the saver breaks even in present value terms.

*tighten bankruptcy rules* going forward. This conclusion enriches those of [Farhi and Werning \(2016\)](#) and [Korinek and Simsek \(2016\)](#) on the use of borrowing restrictions for macroprudential policy, by taking into account the endogenous response of credit supply.

## 4 Quantitative model

We now turn to the description of our quantitative model.

### 4.1 Overview

Our quantitative model is the natural extension of the simple model to capture rich household heterogeneity and a realistic bankruptcy code. The economy set in discrete time and is populated by overlapping generations of households who deterministically are born, work, retire and then die. Households have preferences over consumption, leisure and bequests. They are subject to idiosyncratic expenditure risk and, during working life, are subject to uninsurable idiosyncratic productivity risk. Households can save in mutual fund shares and borrow in defaultable unsecured credit. Defaulting entails forfeiture of shares, but forgiveness of debt and expenditure shocks. Consistent with U.S. bankruptcy code, households that file have to pay a filing fee, and households that file under Chapter 13 have to enter a repayment plan. After default, households are in an exclusion state and stochastically return to having good credit. Loans are supplied by banks who price loans competitively reflecting the expected losses from default as in Section 2. The supply side of the model is relatively standard and follows the frontier HANK literature with both nominal rigidities on prices and wages (e.g., [Auclert and Rognlie \(2018\)](#), [Hagedorn, Manovskii and Mitman \(2019\)](#)).<sup>8</sup>

The basic model environment shares many elements from the literature on consumer default (e.g., [Livshits et al. \(2007\)](#), [Chatterjee et al. \(2007\)](#)). The OLG structure generates natural borrowers and savers in the model due to life-cycle reasons, in addition to precautionary motives because of income and expenditure risk. Relative to the simple model, our quantitative framework provides a theory for the joint distribution of debt, assets, the MPC and the ACED that we discipline with both life-cycle and cross-sectional evidence. The model differs from most of the literature in two dimensions. First, our economy is subject to aggregate shocks which we estimate to match time-series moments in the data.<sup>9</sup> Second, we incorporate nominal rigidities, creating an aggregate demand channel and providing a potential role for bankruptcy to play in aggregate stabilization.

Four different aggregate shocks may hit our economy every period, generating fluctuations in prices, output and default rates: i) total factor productivity; ii) monetary policy; iii) demand

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<sup>8</sup>See [Auclert, Bardóczy and Rognlie \(2020\)](#) and [Broer, Harbo Hansen, Krusell and Öberg \(2020\)](#) for a discussion on the importance of including nominal wage rigidities in HANK models.

<sup>9</sup>Two notable exceptions are [Nakajima and Ríos-Rull \(2014\)](#) and [Gordon \(2015\)](#) who extend the basic consumer default environment to aggregate productivity risk to evaluate the extent to which that framework can match the cyclicity of bankruptcy and credit. Our model builds on the insights of those two papers (on the importance of countercyclical income risk) and considers more sources of aggregate risk.

(preference); and iv) government spending. We defer the specification and calibration of these shocks until Section 6.1, and first focus on the steady-state properties of the model.

## 4.2 Households

There are  $J$  cohorts of households of the same size 1. Households in a cohort born at time  $t$  maximize preferences over consumption  $c$ , leisure  $n$ , and bequests  $b$ :

$$\mathbb{E} \left[ \sum_{j=1}^J \beta_t^j (u(c_{ijt+j}) - v(n_{ijt+j})) + \mathbf{1}_{\{j=J\}} w(b_{ij+1t+j+1}) \right]$$

where  $\beta_t^j$  can include a preference shock for time  $t$  and age  $j$ .

Household fundamental productivity is  $h_{ijt} = h_j(z_{it})$ , evolving deterministically with age  $j$  and stochastically with state  $z_{it}$ . The exogenous state  $z$  follows a Markov process with transition matrix  $\Pi$ . All households initially draw from the stationary distribution of  $z$ , then their productivity evolves as

$$\log h_j(z_{it}) = \chi_j + \log z_{it}$$

We parametrize  $z_{it}$  with a persistent-transitory income process over the labor force years  $j = 1 \dots J^{work}$

$$\begin{aligned} \log z_{ijt} &= \log v_{ijt} + \epsilon_{ijt} \\ \log v_{ijt} &= \log v_{ij-1t-1} + \eta_{ijt} \\ \epsilon_{ijt} &\sim \mathcal{N}(-\sigma_\epsilon^2/2, \sigma_\epsilon^2) \\ \eta_{ijt} &\sim \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2) \end{aligned}$$

In retirement periods  $j = J^{work} + 1 \dots J$ , there are no further shocks to labor productivity, so  $\log z_{ijt}$  remains constant.

Households may also experience expenditure shocks  $\kappa_{ijt} \geq 0$  in each period, drawn iid each period from an age-dependent distribution  $(\pi_j, \kappa_j)$ . Expenditure shocks reduce the level of assets for next period, unless the household defaults. Expenditure shocks are known to be important for household default decisions (see Livshits et al. 2007). Households also receive bequests that depend on their age and productivity state  $beq_t^r(j, z)$ .

Households have access to unsecured borrowing  $b \geq 0$ , and can invest in mutual fund shares  $v \geq 0$ . Any choice of asset position  $b \geq 0$  for next period results in a current inflow  $Q_j(b, z) \geq 0$ . We abuse notation slightly here by denoting saving by negative debt positions,  $b < 0$ . The budget constraint of household  $i$  in period  $t$  is therefore given by

$$c_{it} - Q(b_{it+1}, z_{it}) + p_t v_{it+1} = y_{jt}(z_{it}) - \tilde{b} + beq_t^r(j, z_{it}). \quad (21)$$

where  $\tilde{b}_{it}$  is their net asset level after expenditure shocks and potential writeoffs from consumer bankruptcy, to be specified below,  $b_{it+1}$  is their choice of unsecured credit position for the following period,  $v_{it+1}$  their choice of mutual fund shares, and  $y_{jt}(z)$  is total after-tax income from labor earnings and pensions. Specifically,  $y_{jt}(z)$  is equal to

$$y_{jt}(z_{it}) = \kappa_t \left\{ (1 - \tau^{ss}) w_t h_j(z_{it}) n_{it} \mathbf{1}_{\{j \leq J^{work}\}} + \rho_t^{ss} h_j(z_{it}) \mathbf{1}_{\{j > J^{work}\}} \right\}^{1-\lambda} \quad (22)$$

where  $n_{it}$  is the number of hours worked by the household (determined in equilibrium by aggregate labor demand  $n_t$ ),  $\kappa_t$  the retention rate on labor income and pensions,  $\lambda$  captures the tax progressivity following HSV (2017),  $\tau^{ss}$  a supplementary income tax rate used to fund the pension system, and  $\rho_t^{ss}$  is the earnings replacement rate in retirement.

In the last period of life, households face a choice between consumption and bequests. Household leave bequests in the form of shares, which they value in utility at their market value  $b_{i,t+1} = p_t v_{i,t+1}$ .

The pension scheme is a pay-as-you go system in which the replacement rate is set to ensure a constant supplemental income tax rate of  $\tau^{ss}$ . Given stationary population with equal size  $\frac{1}{j}$  per cohort, we therefore have

$$\rho_t^{ss} = \tau^{ss} w_t \frac{\mathbb{E}^{work} [h_{it} n_{it}]}{\mathbb{E}^{ret} [h_{it}]} \frac{J^{work}}{J - J^{work}} \quad (23)$$

Hence  $\rho_t^{ss}$  is an increasing function of the inverse dependency ratio  $\frac{J^{work}}{J - J^{work}}$  and of the current level of aggregate wages  $w_t$  and employment  $n_t$ .

Households enter the period with assets  $-b_{it} + (p_t + d_t) v_{it}$ . Households must pay for the additional expenditures  $\kappa_{it}$  unless they default. If they do so, their net assets gets either reset to 0 in chapter 7, or they are crammed down to a new level under chapter 13. If they file for either chapter 7 or chapter 13, they have to pay for filing fees. Hence, their income plus net asset position  $y_{ijt} + \tilde{b}_{it}$  is

$$y_{ijt} - \tilde{b} = \begin{cases} y_{ijt} - b_{it} - \kappa + T(j, b, z, \kappa) & \text{repays or is in exclusion} \\ (1 - \gamma) y_{ijt} - F & 7 \\ \left(1 - \frac{\tilde{\zeta}}{v}\right) y_{ijt} - F & 13 \end{cases} \quad (24)$$

where  $T(j, b, z, \kappa)$  is a transfer provided by the government to guarantee a consumption floor to households in the exclusion state that cannot default. In bankruptcy, households have to maintain their net assets  $b_{it+1} + (p_t + d_t) v_{it+1}$  above a certain limit. Those limits vary depending on whether the household has declared chapter 7 or chapter 13, as described by the following policy functions.

Decisions to repay or default are subject to additive taste shocks  $(\epsilon^R, \epsilon^D)$ , which take on a Type I extreme value distribution with parameter  $\alpha$ , as in the empirical discrete choice literature dating back to [Luce \(1959\)](#) and [McFadden \(1974\)](#). This modeling assumption makes the choice of ex-ante indentical households probabilistic, allowing us to think about the causal effect of choosing

to default.<sup>10</sup> Let  $\mathcal{Y}_t = \{w_t, d_t, n_t, p_t, R_t\}$  represent the aggregate state from the perspective of the household. For a household that enters the period with the option to default,

$$V_j(b, v, z, \kappa; \mathcal{Y}) = \max \left\{ D_j(z; \mathcal{Y}) + \epsilon^D, W_j(b, v, z, \kappa; \mathcal{Y}) + \epsilon^W \right\}. \quad (25)$$

In (25), the value of continuing is

$$\begin{aligned} W_j(b, v, z, \kappa; \mathcal{Y}) &= \max_{c, b' \geq 0} u(c) - v(n) + 1_{\{j=J\}} w(b') + \beta \mathbb{E} [V_{j+1}(b', v', z', \kappa'; \mathcal{Y}')] \\ &\text{s.t.} \\ c + Q_j(b', z; \mathcal{Y}) &= b + (p + d)v - \kappa + y_j(z, n) \end{aligned}$$

the value of defaulting is independent of  $b$  or  $\kappa$  since it avoids paying the expenditure shock. This value depends on the bankruptcy chapter allowed by court

$$D_j(z; \mathcal{Y}) = \begin{cases} X_j(-F, z; \mathcal{Y}) & y_j(z, n) \leq \bar{y}_j \\ X_j(\bar{b}_j(z) - F, z; \mathcal{Y}) & \text{otherwise} \end{cases}$$

If the household meets the income test, he has access to chapter 7 where he can reset his debt to 0. Otherwise, he has to take chapter 13, which is a reduction of debt to  $\bar{b}_j(z)$  that comes together with a payment plan. We specifically assume that debt is reset to the to the present discounted value of the payments expected to be made under Chapter 13, which is

$$\bar{b}_j(z) = -\frac{\bar{\zeta} y_j(z, n)}{\nu}$$

Bankruptcy (whether 7 or 13) incurs a fee  $F$ , which is therefore subtracted from the household new asset position. Households enter an exclusion state, which they have an iid probability  $\nu$  of exiting each period. In this exclusion state, households with a liability (only possible under chapter 7) are forced to contribute a fraction  $\bar{\zeta}$  of their income towards debt repayment until they reach 0 debt, consistent with the form that payments plans take under chapter 13 of the U.S. bankruptcy code. All households in exclusion face a price schedule  $Q_j^X(b', v', z; \mathcal{Y})$

$$\begin{aligned} X_j(b, z, \kappa; \mathcal{Y}) &= \max_c u(c) - v(n) - K + 1_{\{j=T\}} w(b') \\ &\quad + \beta (\nu \mathbb{E} [V_{j+1}(b', v', z', \kappa'; \mathcal{Y}')] + (1 - \nu) \mathbb{E} [X_{j+1}(b', v', z', \kappa'; \mathcal{Y}')] ) \\ &\text{s.t.} \\ c + Q_j^X(b', v', z; \mathcal{Y}) &= b + pv + y_j(z, n) \\ b' + pv' &\geq \min \{0, b + \bar{\zeta} y_j(z, n)\} \end{aligned}$$

In the first period of life, all households draw an initial asset position  $b$  from a distribution

<sup>10</sup>It also creates smooth default decision functions, which will have important computational benefits in our quantitative model (e.g. [Iskhakov, Jørgensen, Rust and Schjerning 2017](#)).



$F(b)$  whose mean is such that total bequests given and received are the same (see appendix B.1 for details). They do not experience either expenditure or extreme value shocks in that period.

### 4.3 Production and nominal rigidities

**Final good.** The final good  $Y_t$  is produced by a competitive, representative final-good producer that aggregates a continuum of intermediate goods  $y_{jt}$  using the following technology:

$$Y_t = \left( \int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where  $\epsilon$  is the elasticity of substitution between intermediate goods.

**Intermediate goods.** Intermediate goods producers have access to a Cobb-Douglas production technology which transforms labor and capital services into intermediate goods with aggregate productivity  $A_t$ , so

$$y_{jt} = A_t k_{jt}^\alpha n_{jt}^{1-\alpha}$$

We allow for a subsidy to costs  $\tau^f$  the government uses to offset the monopolist distortion (in steady state), and make firms pay for it lump sum. Real per period firm profits are

$$d_{jt}^f = y_{jt} - (1 - \tau^f) (w_t n_{jt} + r_t^k k_{jt}) - T_t^f$$

where the real wage is  $w_t = \frac{W_t}{P_t}$ , and  $r_t^k$  is the rental rate on capital. Firms compete under monopolistic competition and set their prices subject to Calvo price setting frictions. Denote firms' real marginal cost to be  $s_{jt}$ . First order conditions for optimal input choice are

$$w_t = s_{jt} (1 - \alpha) A_t k_{jt}^\alpha n_{jt}^{-\alpha} \tag{26}$$

$$r_t^k = s_t \alpha A_t k_{jt}^{\alpha-1} n_{jt}^{1-\alpha}. \tag{27}$$

These assumptions give rise to the standard New Keynesian Phillips curve:

$$\pi_t = \kappa \left( s_t - \frac{1 - \frac{1}{\epsilon_t}}{1 - \frac{1}{\epsilon}} \right) + \beta \pi_{t+1} \tag{28}$$

where we allow for shocks to  $\epsilon_t$ , so-called price markup shocks.

**Capital firms.** Capital firms own the capital stock, rent it out to intermediate goods firms, and face quadratic adjustment costs. Their objective function is

$$\max_{k_{t+1}} d_t^k(k_{t+1}) + p_t^k(k_{t+1})$$

where

$$d_t^k(k_{t+1}) = r_t^k k_t - k_{t+1} + (1 - \delta) k_t - \frac{\Psi}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 k_t \tag{29}$$

with investment FOC

$$1 + \Psi \left( \frac{i_t}{k_t} - \delta \right) = q_t^k \quad (30)$$

and  $q$  dynamics given by

$$q_t^k = \frac{1}{1 + r_t + \zeta_t^k} \mathbb{E}_t \left[ r_{t+1}^k - \frac{i_{t+1}}{k_{t+1}} - \frac{\Psi}{2} \left( \frac{i_{t+1}}{k_{t+1}} - \delta \right)^2 + \frac{k_{t+2}}{k_{t+1}} q_{t+1}^k \right] \quad (31)$$

*shock 3*: shock to the discount rate of the capital firms  $\zeta_t^k$ . Note that the value of the firm  $p_t^k$  inclusive of the wedge is:

$$p_t^k = \frac{1}{1 + r_t + \zeta_t^k} \left\{ d_{t+1}^k + p_{t+1}^k \right\}$$

while

$$q_t^k k_{t+1} = \frac{1}{1 + r_t + \zeta_t^k} \mathbb{E}_t \left[ \underbrace{r_{t+1}^k k_{t+1} - i_{t+1} - \frac{\Psi}{2} \left( \frac{i_{t+1}}{k_{t+1}} - \delta \right)^2 k_{t+1}}_{d_{t+1}^k} + q_{t+1}^k k_{t+2} \right]$$

so if we compute  $p_t^k$  from (29), then we should have:

$$p_t^k = q_t^k k_{t+1}$$

**Labor firms, unions and wage setting.** A continuum of labor firms aggregate the labor from different unions into a composite  $n_t$  that is sold competitively to the intermediate goods firms. The  $n_t$  is a CES of each union-provided labor,

$$n_t = \left[ \int (n_t^u)^{\frac{\epsilon_w - 1}{\epsilon_w}} du \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

so the nominal wage is a CES index of each union wage,  $W_t = \left[ \int (W_t^u)^{1 - \epsilon_w} du \right]^{\frac{1}{1 - \epsilon_w}}$ , and they have demand

$$n_t^u = \left( \frac{W_t^u}{W_t} \right)^{-\epsilon_w} n_t$$

Nominal wages are also rigid, and are reset by unions a la Rotemberg. Each **union** hires a mix of households and pays them the wage  $W_t^u$ , which they set to maximize the average household utility in its membership. They understand the relationship between the wage they set and the demand for hours for union labor  $n_t^u$  above, then allocate  $n_t^u$  across its membership using an aggregation rule

$$n_t^i = \Gamma^i (n_t^u) \quad (32)$$

in the steady state this function includes life-cycle variation. In addition, if  $n_t \neq n_{ss}$ , it can capture unequal rationing across  $i$ 's (including across the age and skill dimension). In our benchmark, we

assume that

$$n_{ijt} = n_t$$

for every household. As shown in [Auclert, Rognlie and Straub \(2018\)](#), this implies a Phillips curve for nominal wage inflation  $\pi_t^w = \frac{W_t}{W_{t-1}}$  of the form

$$\pi_t^w = \kappa^w \left\{ \frac{v'(n_t)}{u'(c_t) (1 - \lambda) \kappa_t (w_t)^{1-\lambda} n_t^{-\lambda}} - \left( \frac{\epsilon_t^w - 1}{\epsilon_t^w} \right) \right\} + \beta \pi_{t+1}^w$$

where  $\phi$  is the Frisch elasticity of labor supply and  $c_t = \int c_{it} di$  is aggregate consumption, and we allow for wage markup shocks, i.e. shocks to  $\epsilon_t^w$ .<sup>11</sup> In words, inflation is a function of the gap between the average marginal rate of substitution between consumption and hours for households and the after-tax real wage.

Unions distribute aggregate employment  $n_t$  according to a rule  $n_{ijt} = \Gamma_j (\epsilon_{ijt}, z_{ijt}, n_t)$ .

#### 4.4 Asset market: banks and mutual fund

**Banks.** There is a continuum of banks that can issue loans to households, and have a unit share outstanding. Banks compete loan by loan to make a return that is given by  $r_t + \zeta_t^m$ . We interpret the wedge in the return to loans over the risk-free rate as a common rent that the banking sector makes.

To capture movements in the lending spread above and beyond movements in default risk, we allow for shocks to  $\zeta_t^m$ .

The mutual fund sector collectively has a unit share outstanding.

**Mutual funds.** The mutual fund sector collectively has a unit share outstanding. Mutual funds own shares from final goods firms and capital firms in the economy, government bonds, and a generic portfolio of loans issued to households. Any given mutual fund can issue shares at price  $p_t$  and short government bonds. No arbitrage by mutual funds means that the rate of return on shares must be equal to the gross real interest rate

$$\frac{p_{t+1} + d_{t+1}}{p_t} = R_t$$

and that each loan is priced competitively. It's useful to define chargeoffs for the banks, that is, the losses that the banks make on their loan portfolios. For households in good credit standing, this

<sup>11</sup>The objective function for the union is implicitly assumed to be the welfare of the "as if" representative agent in the economy following [Hagedorn et al. \(2019\)](#). Our results are robust to using alternative moments of the consumption distribution.

implies that

$$\begin{aligned}
Q_{jt}(b, v, z) &= \frac{1}{R_t} \int \mathbf{1}_{\{(b, v, y, \kappa, \epsilon^D, \epsilon^W) \text{ pays}\}} (b + (p' + d') v - \kappa) \\
&\quad + \frac{1}{R_t} \int \mathbf{1}_{\{(b, v, y, \kappa, \epsilon^D, \epsilon^W) \text{ in chapter 7}\}} (-F) \\
&\quad + \frac{1}{R_t} \int \mathbf{1}_{\{(b, v, y, \kappa, \epsilon^D, \epsilon^W) \text{ in chapter 13}\}} \left( -\frac{\bar{\zeta} w \epsilon z e^{\chi_j n}}{\nu} - F \right)
\end{aligned} \tag{33}$$

while for households in exclusion, who have to repay their debts unless they return to good credit standing,

$$Q_{jt}^X(b, v, z) = \nu Q_{jt}(b, v, z) + (1 - \nu) \frac{b + (p' + d') v}{R_t}$$

The aggregate dividend for the mutual fund then corresponds to total inflows (firm dividends, new deposits and government debt payments) net of total outflows (new loans and government debt purchases)

$$d_t = - \int_i \tilde{b}_{it} di + \int_i Q_{jt}(b_{it+1}, z_{ijt}) di + B_t - \frac{B_{t+1}}{R_t} + d_t^f \tag{34}$$

where  $\tilde{b}_{it}$  is given in (24).

Aggregate wealth in this economy is defined as  $W_t = \int_i \tilde{b}_{it} di$ . In stationary equilibrium aggregate asset demand by households  $W$  must equal aggregate asset supply  $B + \frac{D^f}{1 - \frac{1}{R}}$ .

## 4.5 Government

The government runs the pay-as-you go pension system and receives the receipts from the expenditure shocks and running the bankruptcy court,  $\tau_t^d$ , and follows a set of monetary and fiscal rules.

Fiscal policy specifies rules for the income  $\kappa_t$ ,  $\lambda$ , government spending  $g_t$  and debt  $B_t$  such that the government budget constraint is satisfied. Since social security taxes are earmarked for the pay-as-you-go system, the government budget constraint simply reads

$$B_{t-1}^g + G_t = \frac{B_t^g}{1 + r_t} + w_t n_t - \int \kappa_t (w_t n_{ijt} e_{ijt})^{1-\lambda} + \tau_t^d \tag{35}$$

We specify the fiscal rules as follows:

$$\begin{aligned}
g_t - g^* &= -\phi_{gy} (y_t - y^*) - \phi_{gB} (B_t - B^*) \\
\kappa_t - \kappa^* &= \phi_{\kappa y} (y_t - y^*) + \phi_{\kappa B} (B_t - B^*)
\end{aligned}$$

so that government spending and taxes are countercyclical on average but respond to deviations of debt from steady state to maintain a constant average long-run level of debt.

Monetary policy follows a Taylor rule with a constant intercept equal to the steady state interest

rate  $R_{ss}$ ,

$$1 + i_t = R_{ss} (1 + \pi_t)^\phi$$

The Fisher equation, combined with the monetary policy implies

$$R_t = \frac{1 + i_t}{1 + \pi_{t+1}} = \frac{R_{ss} (1 + \pi_t)^\phi}{1 + \pi_{t+1}}$$

- In default split up total resources to households, government and bank as follows.
  - The government always gets

$$\tau^d = \begin{cases} \kappa & \text{if } i \text{ repays} \\ F + \min \{ \kappa, \gamma atw_t \cdot \Gamma_j \cdot z \} - \min \{ b_{it}, 0 \} & \text{if } i \text{ declares chapter 7} \\ F + \min \left\{ \kappa, \frac{\bar{\zeta} atw_t \cdot \Gamma_j \cdot z}{v} \right\} - \min \{ b_{it}, 0 \} & \text{if } i \text{ declares chapter 13} \\ \kappa - T(j, b, z, \kappa) & \text{if in exclusion} \end{cases}$$

ie if you have assets, the government gets to take those.

- The bank always gets

$$\tilde{b} = \begin{cases} b_{it} & \text{if } i \text{ repays or is in exclusion, or defaults with } b < 0 \\ \max \{ 0, \gamma atw_t \cdot \Gamma_j \cdot z - \kappa \} & \text{if } i \text{ declares chapter 7 and } b > 0 \\ \max \left\{ 0, \frac{\bar{\zeta} atw_t \cdot \Gamma_j \cdot z}{v} - \kappa \right\} & \text{if } i \text{ declares chapter 13 and } b > 0 \end{cases}$$

ie the bank lets you access your assets (to pay the government) if you default with positive assets. If you default with debt, but there is enough from the garnishment/chapter 13 plan to pay the expenditure shock, then the bank gets the remainder in compensation.

- \* To conclude, the bank prices loans understanding that it will have to allow households to access all their assets in bankruptcy, and any debt is reset to either 0 or the amount that the bank can garnish after the expenditure shock is paid off.

## 4.6 Timeline

Time is discrete, with subtiming as follows:

1. At the beginning of time  $t$ , the household state is  $(j, z_{-1}, b, v, x)$ : age  $j$ , previous-period income  $z_{-1}$ , an amount of defaultable debt  $b$ , shares  $v$ , and credit exclusion state  $x \in \{0, 1\}$ .
2. An income shock  $z$  and an expenditure shock  $\kappa$  realize.
3. For non-excluded ( $x = 1$ ) households extreme-value shocks  $\epsilon^D, \epsilon^W$  realize and they choose between default and repayment.

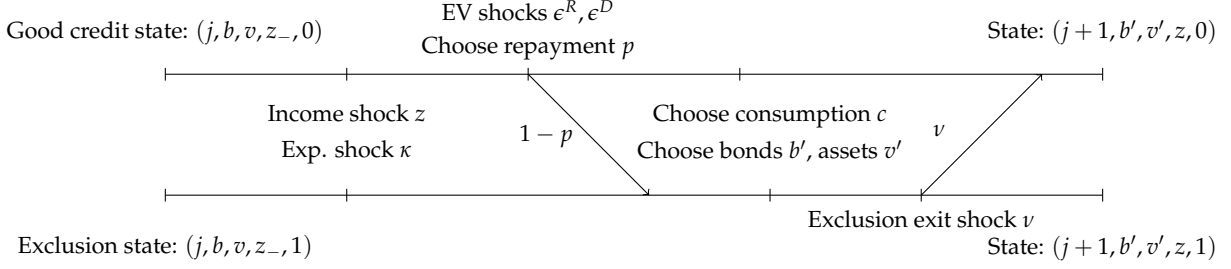


Figure 5: Within-period timing for households

4. Households that repay have consolidated debt position  $\tilde{b} = b + \kappa - vp$ .
5. If households default:
  - (a) If they are below median income,  $\Gamma_j z \leq \text{median}(\Gamma_j z)$ , they can choose between Chapter 7 and Chapter 13, those above must file Chapter 13.
  - (b) If they file Chapter 7 they must pay the bankruptcy fee  $F$  a fraction  $\gamma$  of their earnings, but have their debts are reset to 0, so their post-fee debt level is  $\tilde{b}^7 = F + \gamma atw_t \cdot \Gamma_j \cdot z$ .
  - (c) If they file Chapter 13 they enter into a repayment plan. Their debt is reset to  $\tilde{b}^{13} = F + \frac{\zeta atw_t \cdot \Gamma_j \cdot z}{v}$ , reflecting the present value of the payments that will be made by the household under the chapter 13 payment plan.
6. Excluded households have to repay any debts and any expenditure shocks. They may receive a means-tested transfer  $T$  from the government.
7. Households choose their consumption and their new debt level  $b'$  and purchase shares  $v'$  at price  $p$ , given the constraints they face from their current exclusion state.
8. An exclusion exit shock  $v$  realizes. With probability  $v$ , households's bad credit report gets wiped out.
9. Banks determine their level of dividend  $d$ .
10. Netting and settlement takes place: dividends are paid, income distributed, debts repaid, production and physical goods exchange occurs.

Figure 5 illustrates our timing assumptions for households.

## 4.7 Equilibrium

As in the simple model of section 2, a stationary equilibrium given policy  $\{F, \bar{y}_j, K\}$  is a set of household decision rules  $c(\tilde{b}, z)$ ,  $b'(\tilde{b}, z)$ ,  $v'(\tilde{b}, z)$ ,  $d(\tilde{b}, z)$ , and a bond price schedule  $Q(b, z)$  such that monetary policy and fiscal policy follow their rules, and the goods and asset markets clear. An equilibrium given aggregate shocks is formulated in the same way.

Aggregating up budget constraints (21) and using (22), (23) and (34), we have

$$\begin{aligned}
\int c_{it} &= \int \kappa_t (w_t n_{ijt} e_{ijt})^{1-\lambda} + \int \tilde{b}_{it} - \int Q_{jt}(b_{it+1}, z_{ijt}) + d_t \\
c_t &= \int \kappa_t (w_t n_{ijt} e_{ijt})^{1-\lambda} + d_t^f + B_t - \frac{B_{t+1}}{R_t} \\
c_t &= \int \kappa_t (w_t n_{ijt} e_{ijt})^{1-\lambda} + d_t^f + \left( w_t n_t - \int \kappa_t (w_t n_{ijt} e_{ijt})^{1-\lambda} - g_t \right) \\
c_t + g_t &= w_t n_t + d_t^f = y_t
\end{aligned}$$

## 5 Calibration and steady-state outcomes

We bring our model to the data in two steps. In the first step, described in this section, we calibrate preference and technological parameters to match steady-state, cross-sectional and life-cycle moments from the data. In Section 6, we introduce aggregate risk and detail the estimation procedure for the cyclical properties of the model.

Our time period is a quarter. Table 2 shows our calibration parameters, and figure 6 shows the steady state profiles of income, consumption, assets and bankruptcy rates.

**Preferences** Households have CRRA preferences, and we set the intertemporal elasticity of substitution  $1/\gamma = 1/2$ . The disutility of labor is parameterized in a standard power function  $v(n) = \psi n^{1+1/\phi}$ . We set the frisch elasticity  $\phi = 1$  following evidence in Chetty, Guren, Manoli and Weber (2011) and we calibrate the disutility scale parameter so that aggregate hours are equal to  $n = 1$ .

We now explore features of our calibration in more detail. We first look at the predictions of the model relative to data that we could in principle observe.

Consumption has a hump shape, following the typical pattern from life cycle model with precautionary motive: it rises early in life due to combination of borrowing constraints and precautionary motive, and declines late in life because households are impatient and face limited risk. In Figure 6 we plot the probability of repayment as a function of debt for a 45-year old household with various levels of skills (left panel) and the effective interest rate that that household would face if it were to take on new loans (right panel). As expected, the probability of repayment is monotonically decreasing in the amount of debt the household has taken on, and similarly the effective interest rate that the households would face on borrowing is monotonically increasing in the amount borrowed. The interest rate plots are truncated at the peaks of the debt “laffer curve”, i.e. the promised amount of debt that maximizes the resources that the intermediary will give to the household today. No household would optimally choose to borrow beyond that point, since she could be strictly better off by reducing the debt promised, thereby raising contemporaneous consumption and having weakly greater continuation value.

Parameter		Value	Target
Discount factor	$\beta$	0.99403	SCF financial wealth
Risk aversion	$\gamma$	2	Standard
Frisch elasticity	$\phi$	1	Chetty et al. (2011)
Elasticity of subst.	$\varepsilon$	8	Markup
Elasticity of subst.	$\varepsilon^w$	16	Wage markdown
Consumption floor	$\underline{c}$	0.0051	SSDI
Working life	$J^{work}$	160	160 quarters work life
Life span	$J$	80	40 quarters retirement
Deterministic age profile	$\chi_j$	Cubic	PSID
Level of free bequest	$beq_l$	0.01	hh with $beq > 0$
Bequest util. curvature	$beq_c$	0.25	PSID NW profile
Bequest util. scale	$beq_s$	2	PSID NW profile
Children util. weight	$\lambda_{child}$	0.1	PSID Cons profile
Children util. curvature	$\phi_{child}$	0.807	PSID Cons profile
Payroll tax	$\tau_{ss}$	0.153	Social security tax
Income retention	$\kappa$	0.854	Budget residual
Tax progressivity	$\lambda$	0.181	HSV (2017)
Persistent innov var	$\sigma_{\eta}^2$	0.0384	Krueger, Mitman and Perri (2016)
Persistence of skills	$\varphi$	0.97	Krueger et al. (2016)
Expenditure shock	$\kappa$	0.8wh	Bankruptcy in Ret
Extreme value inverse s.d.	$\alpha$	2	Taste shock scale
Discount factor shocks	$\beta_{shock}$	[1;0.7;1]	
Utility cost of default	$K$	2	Bankruptcy profile
Flow default penalty	$K_{flow}$	0.00375	Mean bankruptcy rate
Filing fee	$F$	\$2000	US Courts
Reaccess prob	$\nu$	0.05	5 year plan
Income threshold	$\bar{y}_j$	Median	BAPCPA 2005
Outcomes	Data	Model	Source (for 2011)
Total debt (%GDP)	26.5	30.6	New York Fed CCP
Bankruptcy rate (%hh)	0.30	0.42	U.S. Courts
Chargeoffs (%GDP)	0.24	0.30	Federal Reserve
Net worth (%GDP)	1029	1032	PSID
Consumption (%GDP)	69	69	

Table 2: Calibration parameters and outcomes



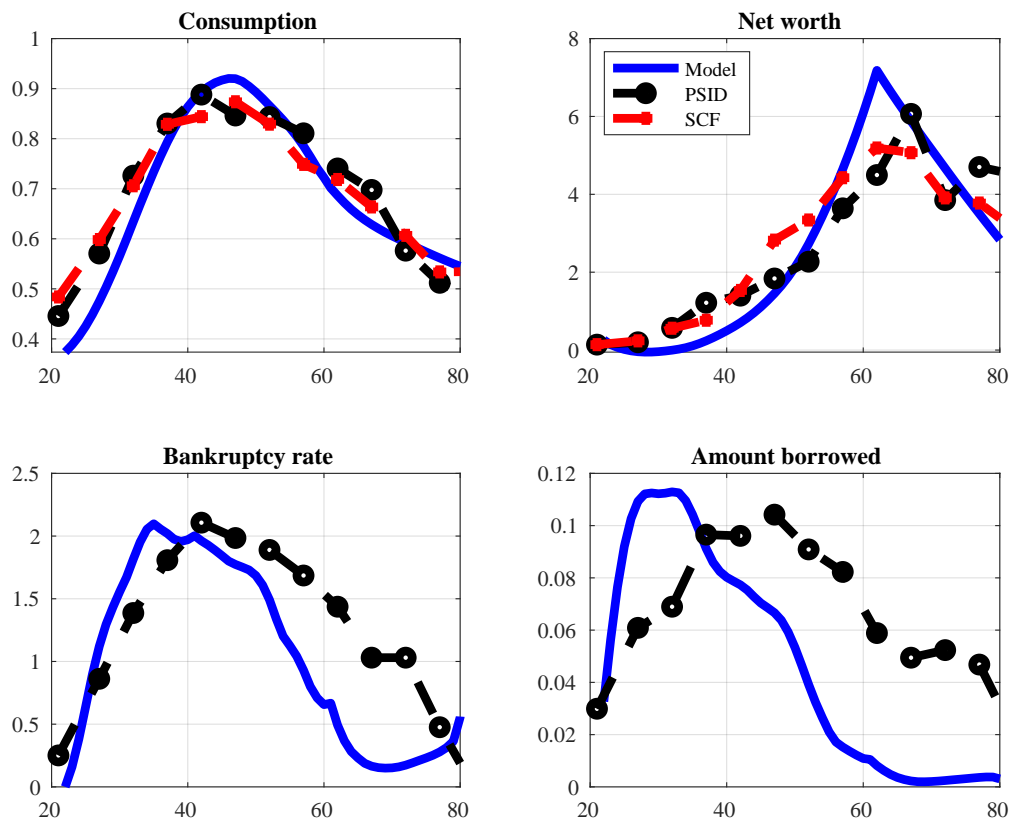


Figure 6: Steady state profiles

## 6 Quantifying the role of the bankruptcy system in aggregate stabilization

### 6.1 Aggregate shocks and solution method

1. Include seven aggregate shocks:
  - Monetary policy shock  $\epsilon^{mp}$
  - Demand shock  $\epsilon^\beta$
  - Government spending shock  $\epsilon^G$
  - Tax shock  $\epsilon^{tax}$
  - Price markup shock  $\epsilon^p$
  - Wage markup shock  $\epsilon^w$
  - TFP shock  $\epsilon^A$
2. We estimate fiscal rules directly in the data:  $\phi_{ty} = 0.34$ ,  $\phi_{gy} = -0.15$
3. Aggregate shocks and transition parameters estimated via SMM to match standard deviations and covariances of standard aggregates, in addition to the cyclicity of bankruptcy, chargeoffs and debt.
  - Shock s.d. and persistence  $\sigma^Z, \rho^Z$  for  $Z \in \{mp, \beta, G, tax, p, w, A\}$
  - Slopes of Phillips curves,  $\kappa^p, \kappa^w$
  - Adjustment cost on capital,  $\Psi$
  -
4. Transitions computed with small MIT shocks
  - Assumption: certainty equivalence for aggregate shocks
  - Equivalent to “Reiter method” [Auclert et al 2020, Boppart et al 2018]

The results of the SMM exercise are displayed in Table 3. The model performs well in capturing business cycle co-movements and unconditional variances. In addition, the model can explain the strongly countercyclical bankruptcy and chargeoff rates. One current limitation of the calibration is that it generates countercyclical household debt, whereas in the data debt is pro-cyclical. Additional model elements, such as counter-cyclical income risk or countercyclical borrowing spreads are needed to overcome the counter-cyclical debt dynamics induced by consumption-smoothing in response to mean-reverting shocks.

We present the estimated parameters and the variance decomposition for our seven shocks in Table 4.

## 6.2 Simulated Method of Moments

		Model		Data		
Var	Std Dev	Cor( $y, x$ )	Cor( $x, x_{-1}$ )	Std Dev	Cor( $y, x$ )	Cor( $x, x_{-1}$ )
Y	0.020	1	0.57	0.020	1	0.58
C	0.019	0.78	0.65	0.018	0.9	0.66
G	0.035	-0.25	0.79	0.028	0.27	0.8
I	0.082	0.80	0.45	0.081	0.84	0.48
N	0.026	0.93	0.62	0.018	0.83	0.63
BK	0.128	-0.29	0.88	0.109	-0.38	0.53
CO	0.186	-0.35	0.93	0.202	-0.38	0.83
D	0.058	-0.20	0.96	0.046	0.71	0.9
w	0.018	0.15	0.73	0.015	0.36	0.37
$\pi$	0.030	0.09	0.75	0.022	0.035	0.87
i	0.027	0.05	0.97	0.036	0.14	0.87
tax	0.061	0.73	0.47	0.060	0.58	0.57
$B^s/Y$	0.055	-0.44	0.92	0.048	-0.56	0.90

Table 3: SMM outcomes

Shock	SMM Estimator		Contribution to variance of					Parameter	Value	SMM Estimated?
	$\sigma$	$\rho$	Y	C	bk	CO	$\pi$			
$A$	0.001	0.98	1%	2%	3%	3%	2%	$\kappa^w$	0.005	Y
$mp$	0.053	0.04	29%	4%	17%	12%	4%	$\kappa^p$	2.69	Y
$G$	0.04	0.52	48%	33%	33%	38%	49%	$\Psi$	1.89	Y
$\beta$	0.011	0.83	7%	31%	1%	1%	1%	$\phi_\pi$	1	N
$p$	1.05	0.5	0%	0%	0%	0%	0%	$\phi_{g,y}$	-0.155	N
$w$	0.64	0.32	8%	12%	7%	8%	26%	$\phi_{\tau,y}$	0.3357	N
$tax$	0.007	0.30	7%	18%	40%	38%	18%	$\phi_{g,b}$	0	N
								$\phi_{\tau,b}$	0.024	Y

Table 4: SMM Parameter Estimates

## 6.3 Automatic Stabilizers Quantified

We now perform our main quantitative experiment. We turn off each of our automatic stabilizers in turn and compute the stabilization ratio, measure as the ratio of the standard deviation of output fluctuations in the model without the automatic stabilizer, relative to the benchmark economy. We then compute the stabilization ratio with all stabilizers turned off. The results are displayed in the right-most column of Table 5. We find that bankruptcy dampens output fluctuations by 6 percent, a similar order of magnitude to government spending and income taxes. Further, the simple statistic formulas developed in Section 2 perform very well in predicting the stabilization ratios in the full quantitative model.

	Stabilization ratio		
	Sufficient statistic	Simple HANK	Quantitative Model
Acyclical G	1.20	1.21	1.19
Acyclical deficits	1.09	1.09	1.11
Acyclical bankruptcy	1.13	—	1.06
All three acyclical	1.42	—	1.37
Active bankruptcy policy	—	—	0.93

Table 5: Automatic stabilizers quantified

## 6.4 Active policy counterfactual

For our last counterfactual, we ask how much more stabilization could we achieve by indexing the bankruptcy code to business cycle indicators, much like unemployment insurance generosity is index to the unemployment rate. In other words, a natural simplified objective is to minimize the amplitude of fluctuations, captured in proposition 2 by  $\text{std}(dy_0)$ . The proposition suggests that this can be achieved by enhancing the countercyclicality of the default rate, ie raising  $\left(-\frac{\partial d_0}{\partial \log y_0}\right)$ . More generally, any systematic change in bankruptcy rules that succeeds in boosting aggregate demand when output is low would serve this objective. Proposition 5 suggests lowering  $K_0$  and raising  $K_1$  when  $y_0$  is low. A simple way to achieve these objectives is to put in place a policy rule  $\mathfrak{B}_t(y_t, y_{t-1})$  that responds to the state and evolution of the business cycle. We assume that the government cannot directly control the non-pecuniary cost of default  $K$ , but can include the fee, the exclusion period, and repayment plan. Thus,  $\mathfrak{B}_t$  is vector valued and includes all of the punishment levers that the government controls. If GDP tends to mean revert, one simple way to achieve this is a policy rule that responds to changes in the level of GDP,

$$\mathfrak{B}_t(y_t, y_{t-1}) = \mathfrak{B}^* + \phi_{y, \mathfrak{B}}(y_t - y_{t-1})$$

For  $\phi_{y, \mathfrak{B}} > 0$ , this rule tightens bankruptcy rules when the economy is currently growing, and relaxes them when the economy is currently shrinking. When  $y_t$  has declined relative to the its level, punishments are low, achieving ex-post debt relief, and when GDP is expected to recover, default costs are expected to be high, crowding in credit supply. Since debt relief becomes predictable as a function of the state of the business cycle, these rules also avoid the credibility issues that plague typical discussions of debt restructuring.

We pick a value of  $\phi_{y, \mathfrak{B}}$  such that increases the countercyclicality of the default rate  $\left(-\frac{\partial d_0}{\partial \log y_0}\right)$  by a factor of three relative to the status quo. The factor of three is motivated by the increase in relative generosity of the duration of unemployment insurance. We find that implementing such a policy would reduce output fluctuations by 7 percent. Thus, a systematic debt relief policy compared to a world with acyclical bankruptcy would reduce output fluctuations by almost 15 percent.

## Conclusion

We provide the first quantitative evaluation of the aggregate demand management role of the consumer bankruptcy system. In a simple model, we highlight the automatic stabilizer role that bankruptcy plays. We then illustrated that there is a tension between providing ex-post debt relief via the bankruptcy code, which is expansionary, vs ex-ante laxer bankruptcy penalties which are contractionary. We then study quantitatively the effect of consumer bankruptcy on business cycle stabilization and revisit the level and state dependence of optimal bankruptcy penalties. We find that the option to default reduces output and consumption volatility by around 6 percent, and that a simple counter-cyclical bankruptcy code could further reduce output fluctuations by 7 percent.

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# Appendix to “Consumer Bankruptcy as Aggregate Demand Management”

## A Proofs for section 2

### A.1 Proof of Proposition 3

*Proof.* Let

$$\mathcal{P}(x) \equiv \frac{1}{1 + \exp\{-ax\}}$$

then  $p$  increases in  $x$  and we have

$$\mathcal{P}'(x) = a\mathcal{P}(x)(1 - \mathcal{P}(x)) > 0$$

Here,

$$p_0 = \mathcal{P}\left(V_0^r(y_0, R) - u(y_0) - \beta^B \mathbb{E}_e[u(e)] + K_0\right)$$

so

$$\begin{aligned} \frac{\partial p_0}{\partial K_0} &= \mathcal{P}'(\cdot) > 0 \\ \frac{\partial p_0}{\partial R} &= \mathcal{P}'(\cdot) \frac{\partial V_0^r}{\partial R} \\ \frac{\partial p_0}{\partial y_0} &= \mathcal{P}'(\cdot) \frac{\partial (V_0^r(y_0, R) - u(y_0))}{\partial y_0} \end{aligned}$$

Next, from the envelope theorem, we have that

$$\frac{\partial V_0^r}{\partial R} = u'(\cdot) \frac{\partial Q}{\partial R} = u'(\cdot) \left(-\frac{Q}{R}\right)$$

since  $R$  does not affect the repayment probability in period 1 conditional on  $b_1$ . Hence,  $\frac{\partial p_0}{\partial R} < 0$  if and only if  $\frac{\partial V_0^r}{\partial R} < 0$ , which is equivalent to  $Q(b_1) > 0$ , which in turn requires  $b_1 > 0$ .

Finally, from the envelope theorem

$$\frac{\partial V_0^r}{\partial y_0} = u'(y_0 - b_0 + Q(b_1))$$

so

$$\frac{\partial p_0}{\partial y_0} = \mathcal{P}'(\cdot) \cdot (u'(y_0 - b_0 + Q(b_1)) - u'(y_0)) = \mathcal{P}'(\cdot) \cdot (u'(c_0^R) - u'(c_0^D))$$

From concavity of  $u$ , this is positive provided that  $c_0^d > c_0^r$ , or equivalently  $Q(b_1) < b_0$ , ie provided that borrowers repay debts on net.  $\square$



## A.2 Proof of Lemma 1 and Proposition 1

TO BE ADDED

## A.3 Proof of Proposition 5

TO BE ADDED

## A.4 Proof of Proposition 6

TO BE ADDED

# B Quantitative model solution

## B.1 Details of the household problem

Households enter the beginning of a period with their new age  $j$ , their previous choice of bond  $b$ , previous income state  $z_{-}$ , and their previous exclusion status  $x_{-}$ . A new expenditure shock  $\kappa$  and income shock  $z$  realizes, so that if they repay, households know that their new asset position would be  $\tilde{b} = b - \kappa$ . Next, extreme-value shocks realize, based on which households decide whether to repay and owe  $b - \kappa$ , or default and owe the amount allowed by court (either  $F$  in chapter 7, or  $\bar{b}_j(z) - F$  in chapter 13). Given this decision, households choose consumption  $c$  and bonds for next period  $b'$  faced with the bond price schedule price schedule  $Q_j(b', z, x)$ . Finally, an exclusion exit shock realizes, which can move back their exclusion status to  $x = 0$  from  $x = 1$ . This determines their state for the following period.

**Beginning of life.** In the first period of their life,  $j = 0$ , households draw an initial asset position  $b$  from a distribution  $F$  defined as:

$$b = \begin{cases} 0 & \text{w.p. } p \\ B^+ & \text{w.p. } 1 - p \end{cases}$$

where  $\log B^+ \sim N\left(-\frac{\sigma^2}{2} + \log\left(\frac{B_t}{1-p}\right), \sigma\right)$ . Note that the mean of this distribution is such that the average bequest received is  $E[b] = (1-p)e^{\mu + \frac{\sigma^2}{2}} = B_t$ , the total amount of bequests distributed in that period by age- $J$  agents. Households do not experience any shock in that first period, so their probability of repayment is  $p = 1$ .

**End of life.** In the last period of life, when  $j = J$ , we let all households declare Chapter 7 bankruptcy, so that the income threshold requirement is waived.<sup>12</sup> They then choose bequests

---

<sup>12</sup>Practically, this corresponds to setting  $\bar{y}_J = \infty$  and  $v_J = 1$ . In turn, this affects the pricing of bonds and the chapter 13 debt cram-down level for households in default in period  $J - 1$ .

$b'$  rather than bonds for the next period, to maximize their objective  $u(c) + w(b')$  under the constraint that

$$c + \frac{b'}{R_t} = \tilde{b} + y$$

These bequests aggregate to  $B_t$ , which are received by the age-0 cohort in the following period.

## **B.2 Solution to the household problem**

We make use of the endogenous grid method together with the first order conditions of the household problem (which, under extreme-value shocks, are always necessary for an optimum) to construct a series of candidate policy functions. We then make use of the information contained in the value functions to find the policy function solving the household Bellman equation.

### B.3 Consumption and income dynamics around default

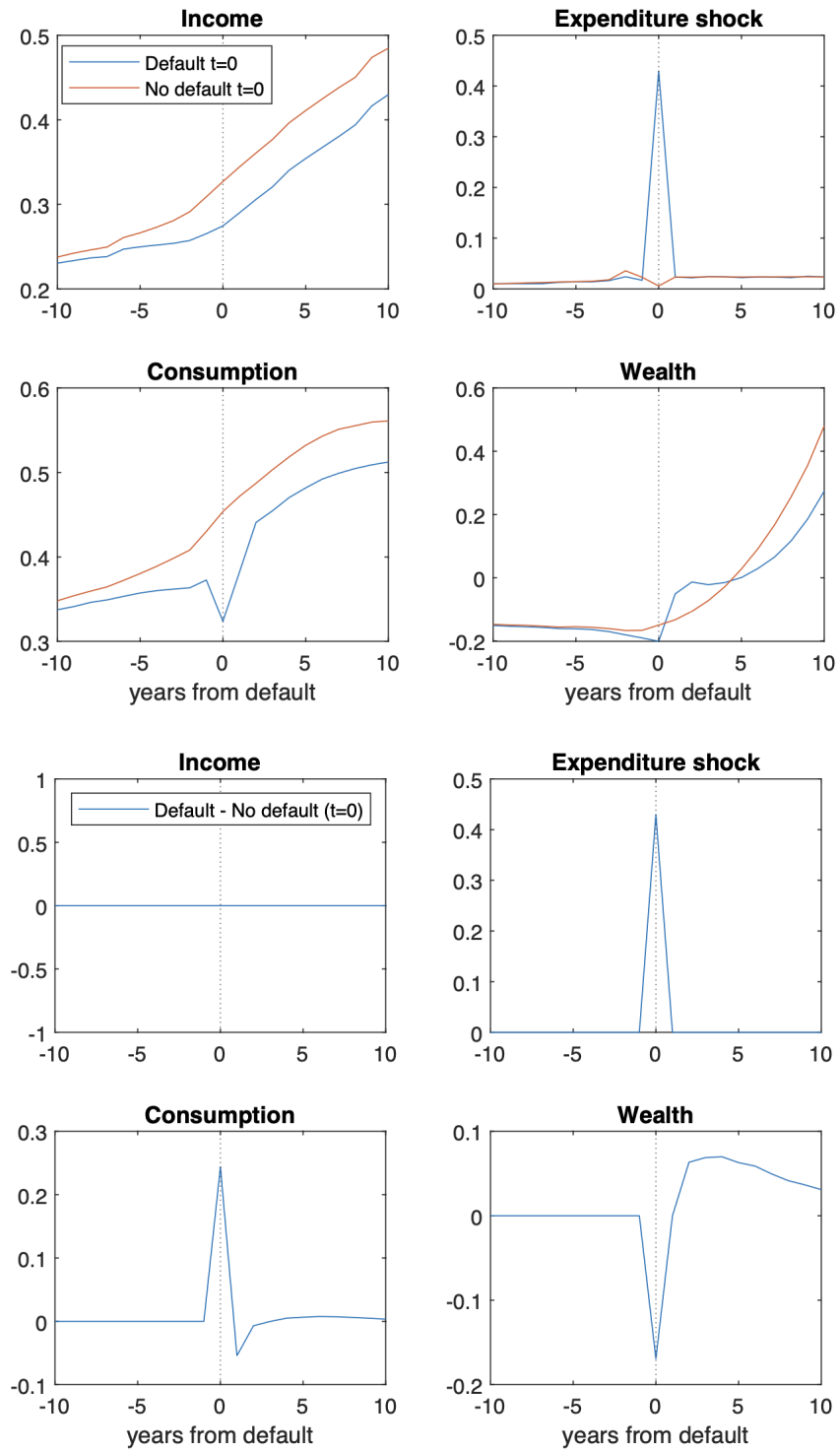


Figure 7: Behavior of defaulters around default event: income, expenditure shocks, consumption and debt

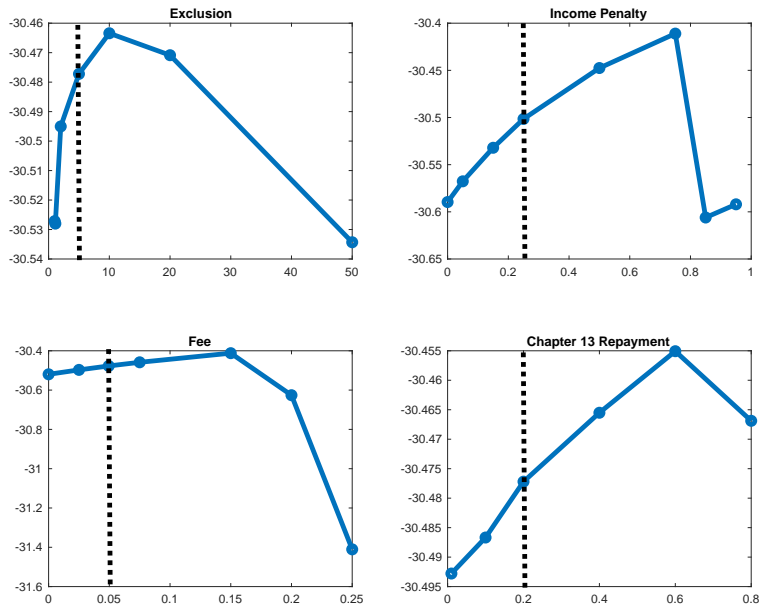


Figure 8: Steady state optimal policy

#### B.4 Steady state optimal policy

#### B.5 Full set of impulse responses

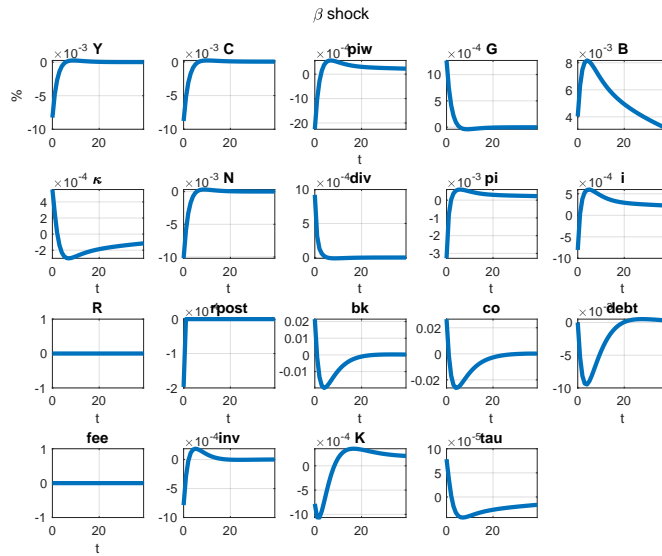


Figure 9: Discount factor shock

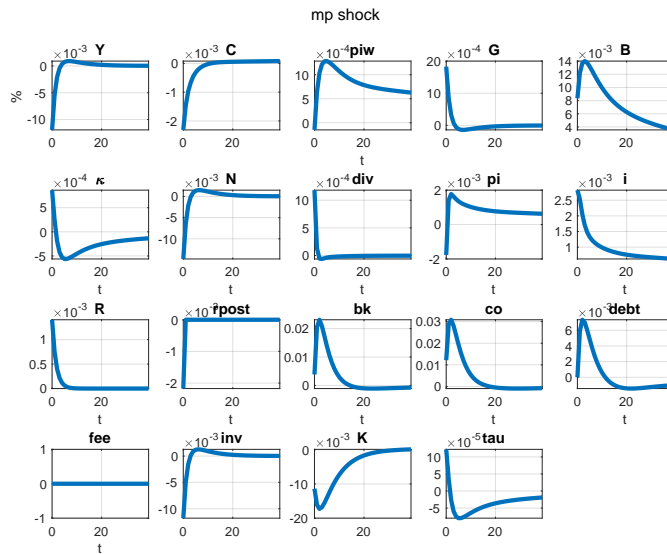


Figure 10: Monetary policy shock

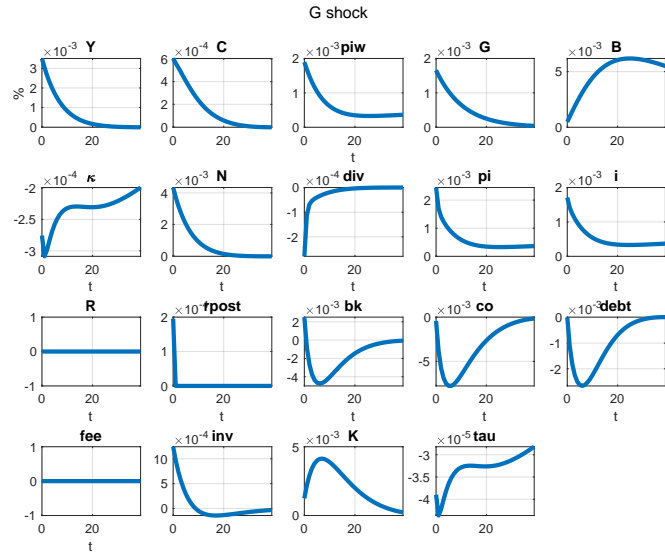


Figure 11: Government spending shock

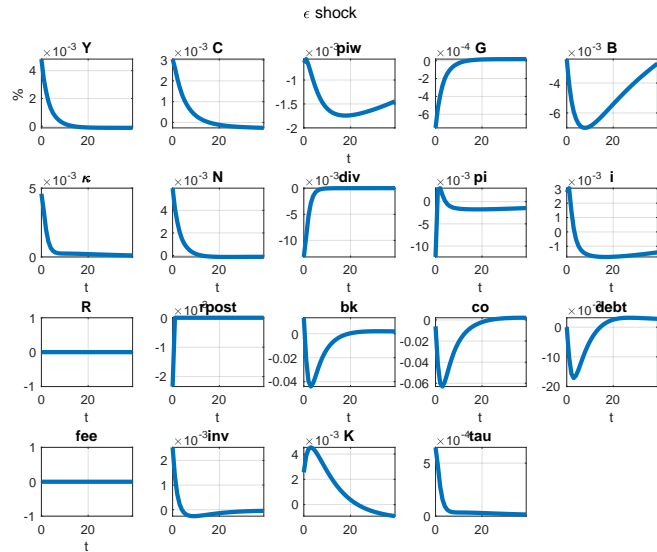


Figure 12: Price markup shock

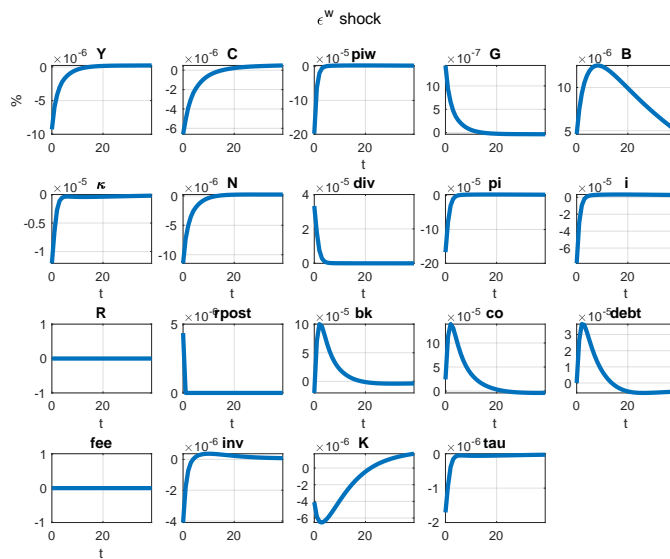


Figure 13: Wage markup shock

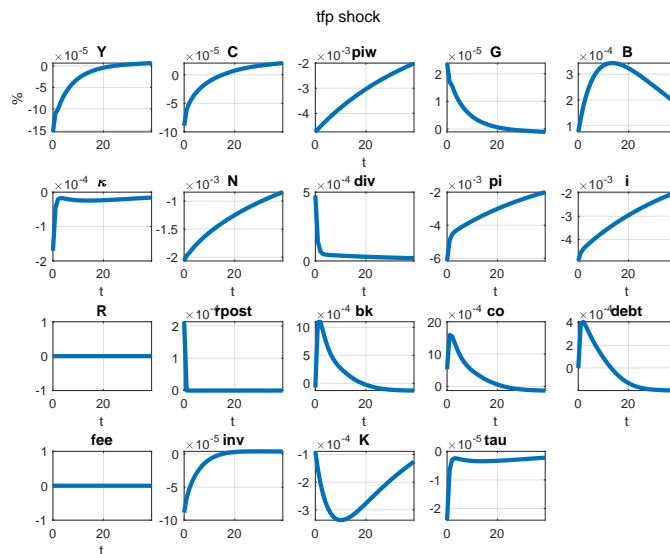


Figure 14: Productivity shock

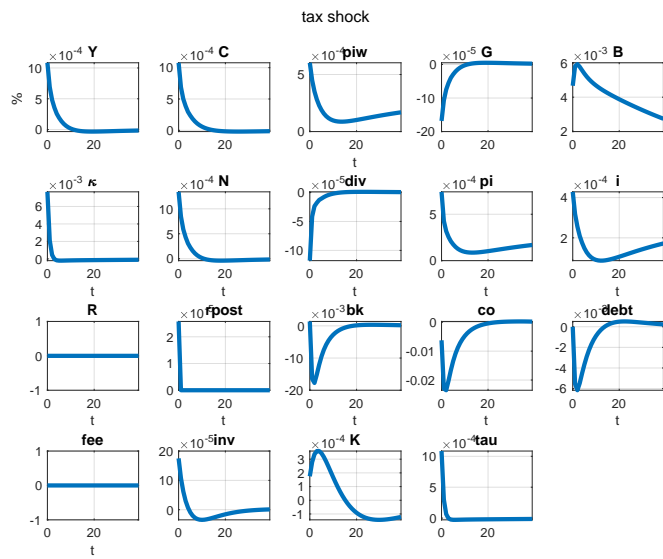


Figure 15: Tax shock