Lecture 3
Fiscal policy in the canonical HANK model

Adrien Auclert (Stanford University)

Fiscal and Monetary Policy with Heterogeneous Agents Minicourse
Bank of Portugal
June 2026

This lecture

We just introduced the canonical HANK model.

Now: Focus on fiscal policy!

- Hold monetary policy at $r_t = r$ constant
- Focus on **first order** shocks to fiscal policy: $d\mathbf{G} = \{dG_t\}, d\mathbf{T} = \{dT_t\}$ such that

$$\sum_{t=0}^{\infty}(1+r)^{-t}(dG_t - dT_t) = 0$$

- Main reference for this lecture is Auclert et al. (2024)

Government budget arithmetic in sequence space

- Writing $d\mathbf{B} = \{dB_t\}$, government budget constraint is

$$d\mathbf{T} = d\mathbf{G} + ((1+r)\,\mathbf{L} - \mathbf{I})\,d\mathbf{B} \tag{1}$$

  where $\mathbf{L}$ is lag operator.

- Let $\mathbf{F}$ be forward operator, which satisfies $\mathbf{FL} = \mathbf{I}$. We have also

$$d\mathbf{B} = \underbrace{\frac{-1}{1+r}\sum_{k\geq 0}\left(\frac{1}{1+r}\right)^{k}\mathbf{F}^{k+1}}_{\equiv\mathbf{K}}(d\mathbf{G} - d\mathbf{T}) \tag{2}$$

- We can specify $d\mathbf{G}$, $d\mathbf{B}$ and get $d\mathbf{T}$ from (1).
- Alternatively, we can specify $d\mathbf{T}$, $d\mathbf{G}$ and then get $d\mathbf{B}$ from (2). Will be useful.

Roadmap

1. The intertemporal Keynesian cross
2. Intertemporal MPCs in models and data
3. Fiscal policy in general equilibrium
4. Solving the model using blocks and DAGs
5. Takeaway

The intertemporal Keynesian cross

The aggregate consumption function

- Recall household "block": mapping sequences $\{r_t^p, Z_t\}$ into $\{C_t, A_t\}$
- With constant $r_t^p = r$, this means that date-t consumption can be written as

$$C_t = \mathcal{C}_t(Z_0, Z_1, Z_2, \ldots) = \mathcal{C}_t(\{Z_s\})$$

- We call this the intertemporal consumption function
- Similarly, there is an intertemporal asset function $A_t = \mathcal{A}_t(\{Z_s\})$
- Since in equilibrium we also have $Z_t = Y_t - T_t$, market clearing writes

$$Y_t = G_t + \mathcal{C}_t\left(\{Y_s - T_s\}\right) \qquad \mathcal{A}_t(\{Y_s - T_s\}) = B_t$$

Intertemporal MPCs

$$Y_t = G_t + \mathcal{C}_t\left(\{Y_s - T_s\}\right)$$

- Given shocks $\{dG_t, dT_t\}$, must have

$$dY_t = dG_t + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial Z_s} \cdot (dY_s - dT_s) \tag{3}$$

→ Response dY_t **entirely** characterized by the **Jacobian** of the $\mathcal{C}$ function:

$$M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial Z_s} \qquad \left(= \mathcal{J}_{t,s}^{C,Z}\right)$$

We call these **intertemporal MPCs**

- $M_{t,s}$ = how much of an income change at date s is spent at date t
- Note: All income is spent at some point, so $\sum_{t=0}^{\infty}(1+r)^{s-t}M_{t,s} = 1$

Visualizing intertemporal MPCs

$$\mathbf{M} = \begin{pmatrix} M_{0,0} & M_{0,1} & M_{0,2} & M_{0,3} & \cdots \\ M_{1,0} & M_{1,1} & M_{1,2} & M_{1,3} & \cdots \\ M_{2,0} & M_{2,1} & M_{2,2} & M_{2,3} & \cdots \\ M_{3,0} & M_{3,1} & M_{3,2} & M_{3,3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

First column: response to income increase at $s = 0$

Second column: response to income increase at $s = 1$...

Intertemporal budget constraint implies $\mathbf{q}'\mathbf{M} = \mathbf{q}'$; $\mathbf{q} \equiv (1, (1+r)^{-1}, (1+r)^{-2}, \ldots)'$

- Rewrite equation (3) in vector / matrix notation:

$$d\mathbf{Y} = d\mathbf{G} - \mathbf{M}d\mathbf{T} + \mathbf{M}d\mathbf{Y} \tag{4}$$

- This is an **intertemporal Keynesian cross**
  - entire complexity of model is in **M**
  - with **M** from data, could get $d\mathbf{Y}$ without model!
    ("sufficient statistic", though hard to measure in entirety...)

Connecting to the standard Keynesian cross...

- Standard IS-LM theory postulates $C_t = \mathcal{C}(Y_t - T_t)$ plus market clearing, so

$$\mathcal{C}(Y_t - T_t) + G_t = Y_t$$

  Differentiate around steady state with constant Y, T, G:

$$dY_t = dG_t - mpc \cdot dT_t + mpc \cdot dY_t$$

  where $mpc = \mathcal{C}'(Y - T)$. This is the static Keynesian cross.

- The intertemporal Keynesian cross is a vector-valued version of this
- HANK models tend to revive & microfound IS-LM logic

Solving the intertemporal Keynesian cross

- How can we solve (4)? Rewrite as

$$(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = d\mathbf{G} - \mathbf{M} d\mathbf{T} \tag{5}$$

- Standard Keynesian cross solution:

$$dY_t = \left(1 + mpc + mpc^2 + \cdots\right)\left(dG_t - mpc \cdot dT_t\right) = \frac{dG_t - mpc \cdot dT_t}{1 - mpc}$$

  Can we do the same, inverting $(\mathbf{I}-\mathbf{M})$? **Need to be careful!**

- Why? Multiply both sides of (5) by: $\mathbf{q} \equiv \left(1, (1+r)^{-1}, (1+r)^{-2}, \ldots\right)'$

$$\mathbf{q}'(\mathbf{I}-\mathbf{M})\, d\mathbf{Y} = 0 \qquad \mathbf{q}' d\mathbf{G} - \mathbf{q}'\mathbf{M} d\mathbf{T} = \mathbf{q}' d\mathbf{G} - \mathbf{q}' d\mathbf{T} = 0$$

  both left and right hand side have **zero NPV** !

- Intuition: present value of *mpc* is 1, dY is 0/0... What to do?

Solving in the asset market

- Consider instead the asset market clearing condition:

$$\mathcal{A}_t\left(\{Y_s - T_s\}\right) = B_t \quad \forall t$$

- By Walras's law, equivalent to goods market clearing. Linearize:

$$\mathbf{A}\left(d\mathbf{Y} - d\mathbf{T}\right) = d\mathbf{B} \tag{6}$$

  This doesn't have the same obvious problem as (5) !

- Assuming that $\mathbf{A}$ is invertible (can check using a "winding number criterion"), using $d\mathbf{B} = \mathbf{K}\left(d\mathbf{G} - d\mathbf{T}\right)$ in (2) and similarly $\mathbf{A} = \mathbf{K}\left(\mathbf{I} - \mathbf{M}\right)$, we have:

$$\begin{aligned} d\mathbf{Y} &= d\mathbf{T} + \mathbf{A}^{-1}\mathbf{K}\left(d\mathbf{G} - d\mathbf{T}\right) \\ &= \mathbf{A}^{-1}\mathbf{K}\left(\left(\mathbf{I} - \mathbf{M}\right)d\mathbf{T} + d\mathbf{G} - d\mathbf{T}\right) \\ &= \mathbf{A}^{-1}\mathbf{K}\left(d\mathbf{G} - \mathbf{M}d\mathbf{T}\right) \end{aligned}$$

- That is: $\mathcal{M} \equiv \mathbf{A}^{-1}\mathbf{K}$ is the left inverse of $\mathbf{I} - \mathbf{M}$ we need to solve (5)!

The balanced budget multiplier

- Suppose $d\mathbf{G} = d\mathbf{T}$ (balanced budget) and $\mathbf{A}$ is invertible
- **Result**: We always have $d\mathbf{Y} = d\mathbf{G}$!
- Irrespective of **all** household heterogeneity, holds for any path of spending
- IS-LM antecedents: Gelting (1941), Haavelmo (1945)
- Proof is immediate: $d\mathbf{Y} = d\mathbf{G}$ is unique solution to

$$d\mathbf{Y} = (\mathbf{I} - \mathbf{M}) \cdot d\mathbf{G} + \mathbf{M} \cdot d\mathbf{Y}$$

Deficit financed fiscal policy

- With deficit financing $d\mathbf{G} \neq d\mathbf{T}$ we have, using $\mathcal{M}(\mathbf{I}-\mathbf{M}) \equiv \mathbf{I}$,

$$d\mathbf{Y} = d\mathbf{G} + \underbrace{\mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T})}_{d\mathbf{C}} \tag{7}$$

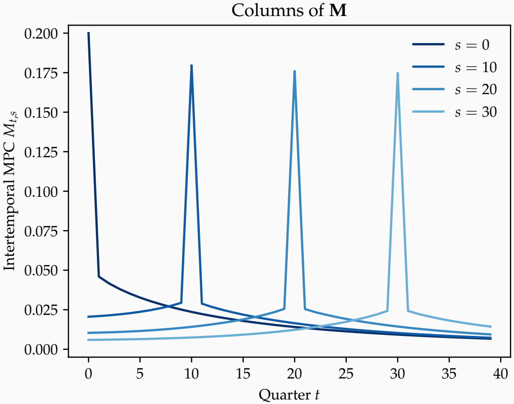
  Consumption $d\mathbf{C}$ depends on **primary deficits** $d\mathbf{G} - d\mathbf{T}$

- Interaction term: Deficits matter precisely when $\mathbf{M}$ is "large" (in practice "large" will mean very different from RA model)
- **Next:**
  - iMPCs $\mathbf{M}$ in models and data
  - Implications for deficit-financed fiscal policy

Intertemporal MPCs in models and data

Intertemporal MPCs in baseline model

- Use calibration with $A/Y = 1$. Get $\mathbf{M}$ quickly using fake-news algorithm:



- What about other models?

Representative-agent model

Last lecture we derived consumption function for RA model when $\beta(1+r)=1$

$$C_t = (1-\beta)\sum_{s\geq 0}\beta^s Z_s + ra_{-1}$$

In particular

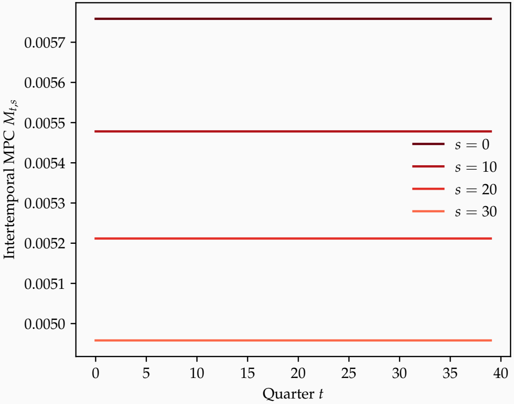
$$M_{t,s} = \frac{\partial C_t}{\partial Z_s} = (1-\beta)\beta^s$$

Thus iMPC matrix is given by

$$\mathbf{M}^{RA} = \begin{pmatrix} 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ 1-\beta & (1-\beta)\beta & (1-\beta)\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \frac{\mathbf{1q}'}{\mathbf{1}'\mathbf{q}}$$

Easy to verify that $\mathbf{q}'\mathbf{M} = \mathbf{q}'$, and also that $\mathbf{Mw} = 0$ for any zero NPV $\mathbf{w}$

Representative-agent model iMPCs



- Low and flat pattern, as in the PIH

Two agent model

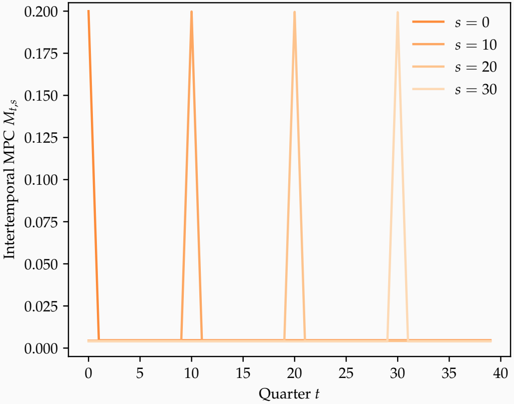
- $1-\mu$ share of agents behave like RA agent, μ are hand to mouth

  $\Rightarrow$ **M** matrix is simple linear combination

$$\mathbf{M}^{TA} = (1-\mu)\mathbf{M}^{RA} + \mu\mathbf{I}$$

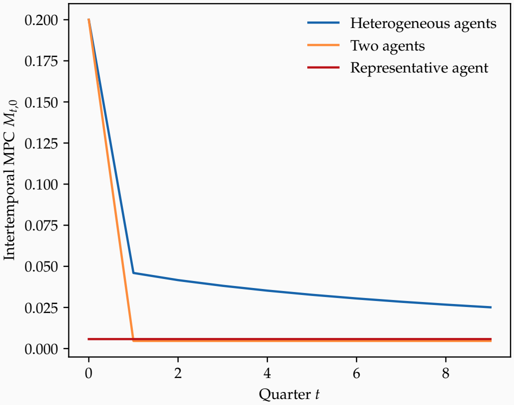
- Only strong **contemporaneous** spending effect

iMPCs in TA model



- Spiky tents, reflecting division into savers and spenders

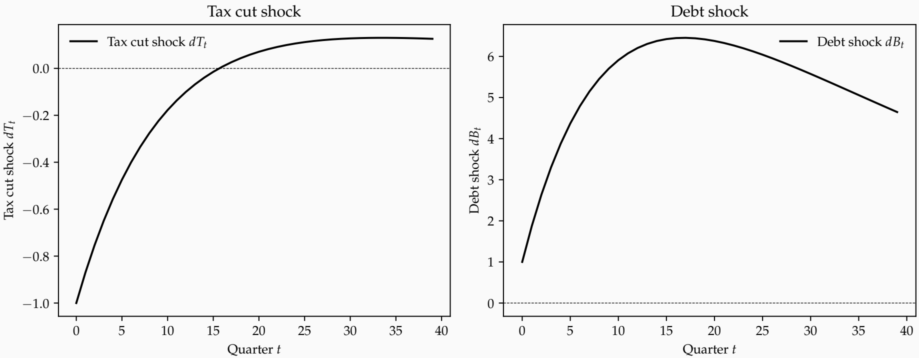
Comparing first columns in all three models



- Substantial delayed C response in HA, supported by several recent studies: e.g. Fagereng et al. (2021), Colarieti et al. (2024), Eichenbaum et al. (2026)
- Others find noisy zero for M_{10}, e.g. Boehm et al. (2025)

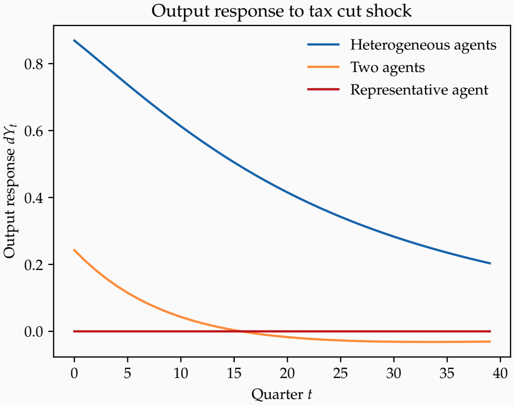
Fiscal policy in general equilibrium

A deficit-financed tax cut



- Parameterize a tax shock: $d\tilde{T}_t = \rho_T d\tilde{T}_{t-1} + \epsilon_t$, with debt $dB_t = \rho dB_{t-1} + d\tilde{T}_t$
- (Note actual $dT_t \neq d\tilde{T}_t$ since need to raise taxes to bring B_t back)

General equilibrium effects in HA, TA, and RA



- Sustained boom in HA, RA is Ricardian
- Temporary boom in TA that turns to bust. Why?

- Let's solve the IKC for the RA model
- Calculate:

$$
\begin{aligned}
d\mathbf{C} &= \mathcal{M} \cdot \mathbf{M} \cdot (d\mathbf{G} - d\mathbf{T}) \\
&= \mathcal{M} \cdot (1 - \beta)\, \mathbf{1q}' \,(d\mathbf{G} - d\mathbf{T})
\end{aligned}
$$

  But government budget balance implies $\mathbf{q}'\,(d\mathbf{G} - d\mathbf{T}) = 0$! So:

$$
d\mathbf{Y} = d\mathbf{G}
$$

- Can of course prove this directly, too (eg Woodford 2011)
- **Deficits are irrelevant in RA!**

- In Keynesian cross:

$$\left(\mathbf{I}-\mathbf{M}^{TA}\right) d\mathbf{Y} = d\mathbf{G} - \mathbf{M}^{TA} d\mathbf{T} \quad \Leftrightarrow \quad \left(\mathbf{I}-\mathbf{M}^{RA}\right) d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right] - \mathbf{M}^{RA} d\mathbf{T}$$

  This equation has same form as for RA, hence:

$$d\mathbf{Y} = \frac{1}{1-\mu}\left[d\mathbf{G} - \mu d\mathbf{T}\right]$$

- cf classic Keynesian cross: spending multiplier $1/(1-\mu)$ and transfer multiplier $\mu/(1-\mu)$. So: μ is "effective" MPC, ignoring RA

- Can also write:

$$d\mathbf{Y} = d\mathbf{G} + \frac{\mu}{1-\mu} \underbrace{\left[d\mathbf{G} - d\mathbf{T}\right]}_{\text{primary deficit}}$$

- Only **current** deficit matters. Transfer multiplier is initially $\in \frac{1}{1-\mu}$, but effect flips sign as soon as taxes rise. Cumulative multiplier is 1: $\mathbf{q}' d\mathbf{Y} = \mathbf{q}' d\mathbf{G}$.

Solving the model using blocks and DAGs

Blocks

- We just solved a GE heterogeneous agent model using “blocks”
- We can do this much more generally
- The SSJ toolbox exploits this systematically
  - See Auclert et al. (2021) for details

Defining blocks and models

Block: mapping from sequences of *inputs* to sequences of *outputs*.

For the IKC model:

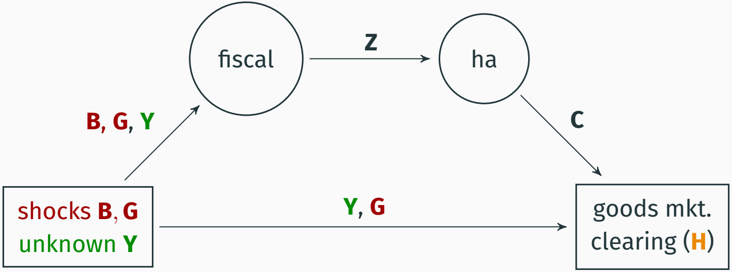
- Household block: $\mathbf{Z} \to \mathbf{C}, \mathbf{A}$
- Fiscal policy block: $\mathbf{B}, \mathbf{G}, \mathbf{Y} \to \mathbf{T}, \mathbf{Z}$
- Goods market clearing block: $\mathbf{Y}, \mathbf{C}, \mathbf{G} \to \mathbf{H} \equiv \mathbf{C} + \mathbf{G} - \mathbf{Y}$

Model: combination of blocks

- some inputs are exogenous **shocks**, e.g. $\mathbf{B}, \mathbf{G}$
- some inputs are endogenous **unknowns**, e.g. $\mathbf{Y}$
- some outputs are **targets** that must be zero in GE, e.g. $\mathbf{H}$ [#targets = #unknowns]

Many macro models can be written this way. Will help us solve them efficiently!

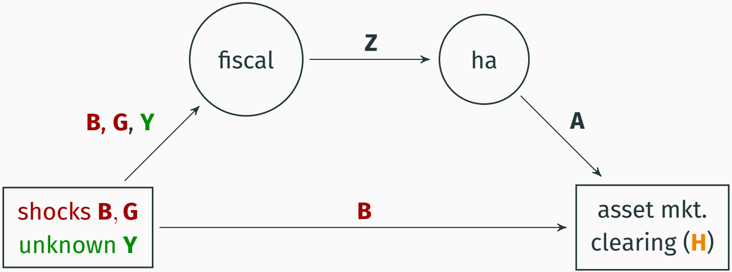
Require that models have no cycles $\rightarrow$ draw as directed acyclic graphs (DAGs).



- Model is composite mapping: $(\mathbf{Y}, \mathbf{B}, \mathbf{G}) \rightarrow \mathbf{H}$.
- GE response of $\mathbf{Y}$ to shocks satisfies $\mathbf{H} = 0$.

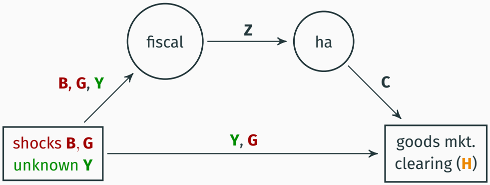
Side note: DAGs are not unique

- Can use asset market rather than goods market clearing



- This is usually how we solve the model in practice

Solving for output response to shocks



- Imagine we change the path of government spending $\mathbf{G}$. How is $\mathbf{Y}$ affected?
- We need to find $\mathbf{Y}$ such that

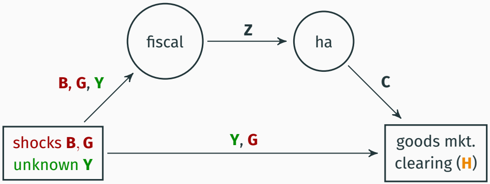
$$\mathbf{H}(\mathbf{Y}, \mathbf{G}) = 0$$

- First order shock $d\mathbf{G} \Rightarrow$ use implicit function theorem:

$$d\mathbf{Y} = -\left(\mathbf{H}_{\mathbf{Y}}\right)^{-1} \cdot \mathbf{H}_{\mathbf{G}} \cdot d\mathbf{G}$$

All we need is $\mathbf{H}$'s Jacobians $\mathbf{H}_{\mathbf{Y}}$ and $\mathbf{H}_{\mathbf{G}}$ ...

How do we get the Jacobians?



First step is to compute **individual blocks' Jacobians**, e.g. $\mathcal{J}^{\mathbf{Z},\mathbf{Y}}$, $\mathcal{J}^{\mathbf{C},\mathbf{Z}}$, $\mathcal{J}^{\mathbf{H},\mathbf{Y}}$, $\mathcal{J}^{\mathbf{H},\mathbf{G}}$

- If block is analytical, its derivative is analytical too
  - e.g. $\mathcal{J}^{\mathbf{Z},\mathbf{Y}} = \mathbf{I}$ or $\mathcal{J}^{\mathbf{H},\mathbf{G}} = \mathbf{I}$
- If block has heterogeneous agents, solve Jacobian numerically
  - e.g. solve $\mathcal{J}^{\mathbf{C},\mathbf{Z}}$ using fake news algorithm

Then "chain" the Jacobians together:

$$\mathbf{H}_{\mathbf{Y}} = \mathcal{J}^{\mathbf{H},\mathbf{Y}} + \mathcal{J}^{\mathbf{H},\mathbf{C}} \cdot \mathcal{J}^{\mathbf{C},\mathbf{Z}} \cdot \mathcal{J}^{\mathbf{Z},\mathbf{Y}} \qquad \mathbf{H}_{\mathbf{G}} = \mathcal{J}^{\mathbf{H},\mathbf{G}} + \mathcal{J}^{\mathbf{H},\mathbf{C}} \cdot \mathcal{J}^{\mathbf{C},\mathbf{Z}} \cdot \mathcal{J}^{\mathbf{Z},\mathbf{G}}$$

On the computer: Sequence-Space Jacobian workflow

Workflow in our Sequence-Space Jacobian (SSJ) toolbox:

1. Define individual blocks: `SimpleBlock`, `HetBlock`, etc
    - More complex `SolvedBlock` to solve out recursions, e.g. an Euler equation
2. Combine the blocks into a model
3. Set steady state parameters and solve the model at the steady state.
4. Solve for the responses of the model directly, code handles all Jacobians.
    - e.g. `solve_impulse_linear` automatically computes $d\mathbf{Y} = -(\mathbf{H_Y})^{-1} \cdot \mathbf{H_G} \cdot d\mathbf{G}$
    - but can also compute $\mathbf{H_Y}$, $\mathbf{H_G}$ individually, or even $\mathcal{J}^{\mathbf{C},\mathbf{Z}}, \mathcal{J}^{\mathbf{Z},\mathbf{Y}}$ etc
    - we'll see this is helpful to inspect the model's mechanics!

Takeaway

- Fiscal policy in HANK is very different from RANK and TANK
- Why? Large **off-diagonal iMPCs**: households spend down past savings
- Consequence: deficit financing has persistent effects on output
  - Both initial **and** cumulative fiscal multipliers are large

References

Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models. *Econometrica*, 89(5):2375–2408.

Auclert, A., Rognlie, M., and Straub, L. (2024). The Intertemporal Keynesian Cross. *Journal of Political Economy*, 132(12):4068–4121.

Boehm, J., Fize, E., and Jaravel, X. (2025). Five Facts about MPCs: Evidence from a Randomized Experiment. *American Economic Review*, 115(1):1–42.

Colarieti, R., Mei, P., and Stantcheva, S. (2024). The How and Why of Household Reactions to Income Shocks.

Eichenbaum, M. S., Guerreiro, J., and Obradovic, J. (2026). Ricardian Non-Equivalence. *Manuscript*.

Fagereng, A., Holm, M. B., and Natvik, G. J. (2021). MPC Heterogeneity and Household Balance Sheets. *American Economic Journal: Macroeconomics*, 13(4):1–54.

Gelting, J. (1941). Nogle Bemærkninger Om Finansieringen Af Offentlig Virksomhed. *Nationaløkonomisk Tidsskrift*, 3.

Haavelmo, T. (1945). Multiplier Effects of a Balanced Budget. *Econometrica*, 13(4):311–318.

Woodford, M. (2011). Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.