# Lecture 7: Heterogeneous firms

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# Investment and firm heterogeneity

- **Workshop so far:** household heterogeneity in NK model
  - Changes transmission of monetary policy to consumption!
- **Parallel literature**: firm heterogeneity, though mostly in flexible-price models
  - **Q:** How does this change transmission of monetary policy to investment?
- Paper in preparation for Journal of Economic Literature, with Tom Winberry

# Plan

1. Canonical HANK model with **representative firm** investment
2. New transmission mechanisms for *I* with added **firm heterogeneity**
3. Complementarities from combining heterogeneous households and firms

# Adding investment to canonical HANK model

# Adding investment with rep. firm

- No investment in canonical HANK model so far. Let's change this!
- Representative firm produces $Y_t = K_{t-1}^{\alpha} N_t^{\nu}$, faces capital adjustment costs, solves

$$\max_{K_t, N_t} \sum_{t \geq 0} \prod_{s \leq t} \frac{1}{1 + r_s} D_t \qquad D_t \equiv Y_t - w_t N_t - I_t - \frac{1}{2} \phi \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} + T_t^f$$

- Flexible prices and perfect competition: $w_t = MPN_t = \nu Y_t / N_t$
- Households take $N_t$ as given, receive labor income $w_t N_t$ and transfers $T_t^h$
- Government issues debt to fund all transfers, $B_t = (1 + r_{t-1}) B_{t-1} + T_t^h + T_t^f$

# Implications of quadratic adjustment costs

- Q theory equations: $$\frac{I_t}{K_{t-1}} - \delta = \frac{1}{\phi}\left(Q_t - 1\right)$$

$$Q_t = \frac{1}{1+r_t}\left(MPK_{t+1} - I_{t+1} - \frac{\phi}{2}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2 + \frac{K_{t+1}}{K_t}Q_{t+1}\right)$$

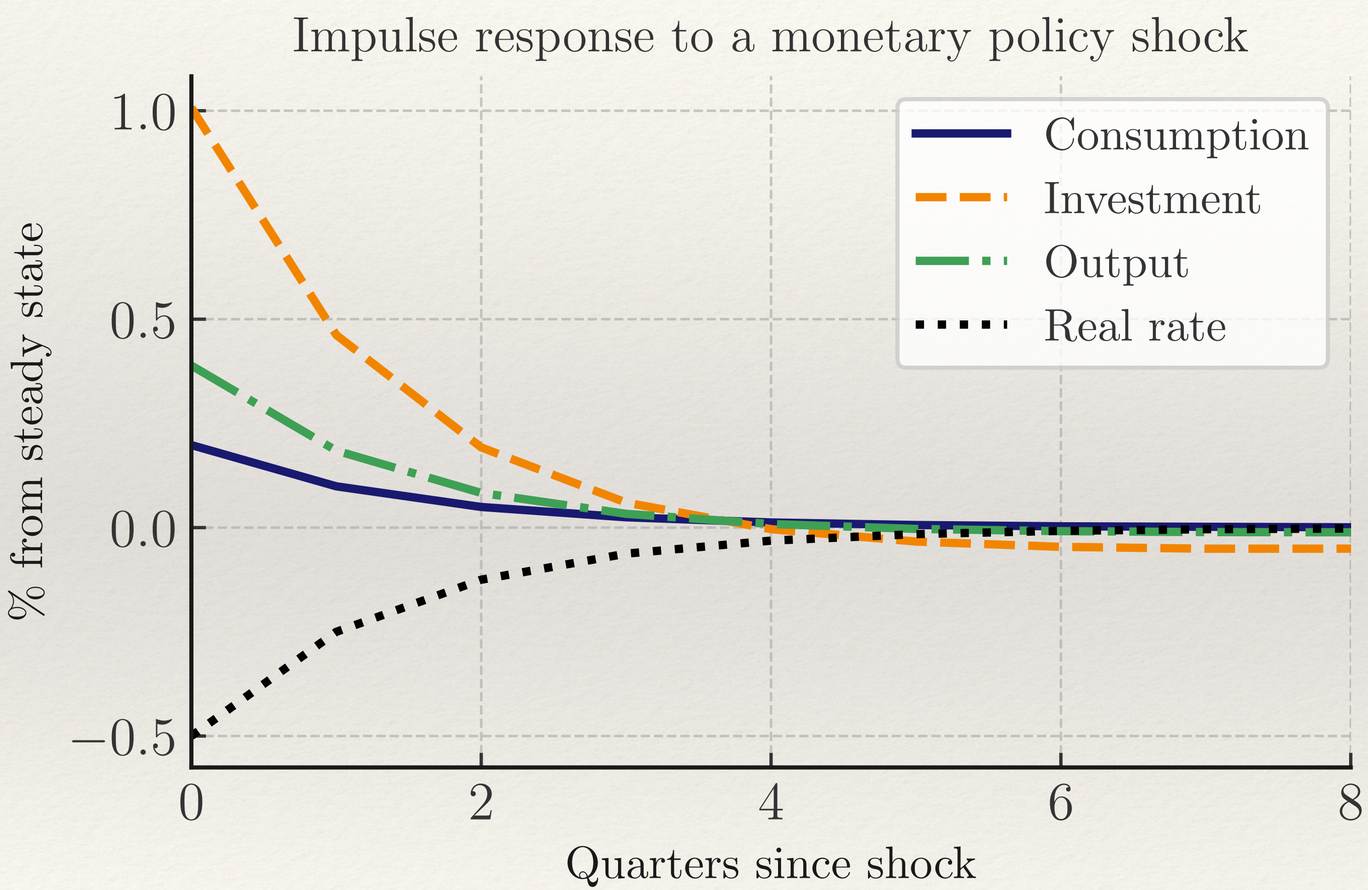
- Use mutual fund trick, assume $B = 0$ in steady state: 100% s.s. stock allocation
  - So connection to household block is $1 + r_0^p = \frac{p_0 + d_0}{p_{ss}}$
- Asset market clearing $A_t = p_t + B_t$. Goods market clearing: $C_t + I_t = Y_t = K_{t-1}^{\alpha} N_t^{\nu}$

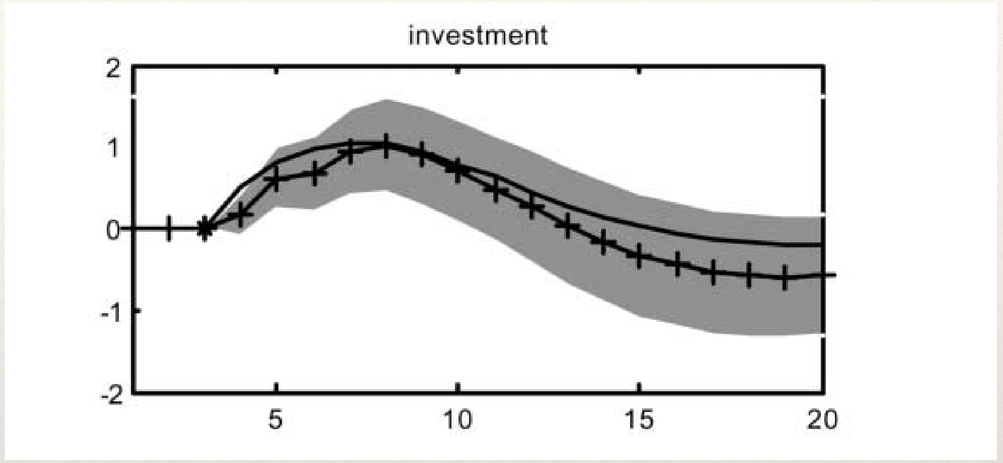
# Calibration of HANK model with investment

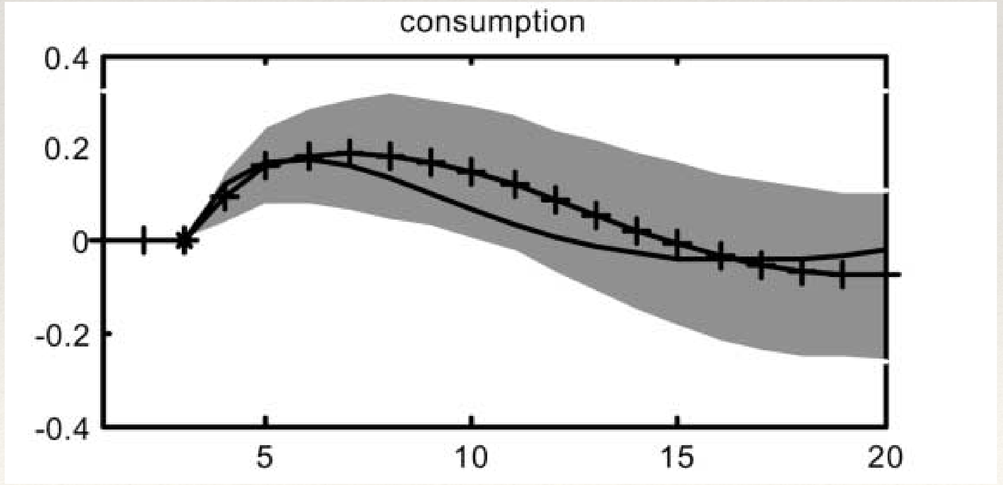
- Standard functional forms: $u(C) = \dfrac{C^{1-\sigma}}{1-\sigma}$, adj cost for firms $\dfrac{1}{2}\phi x^2$
- Set $\alpha, \nu, \delta, \psi$ exogenously (here $\alpha + \nu < 1$), $\beta$ to hit $r = 1\%$ quarterly
- Pick remaining 2 parameters to hit the **peak** impulse response to an identified monetary policy shock [in spirit of Christiano-Eichenbaum-Evans]
  - Elasticity of intertemporal substitution $1/\sigma$ controls consumption response
  - Degree of capital adjustment costs $\phi$ controls investment response

| $\alpha$ | $\nu$ | $\delta$ | $\psi$ | $\beta$ | $\sigma$ | $\phi$ |
|---|---|---|---|---|---|---|
| 0.32 | 0.6 | 2.5% | 1 | 0.99 | 1.6 | 9 |

# Impulse response to monetary policy shock




*I* accounts for ~60% of the *Y* response, *C* for ~40%

source: Christiano, Eichenbaum, Evans 2005

# Transmission mechanism to $I$

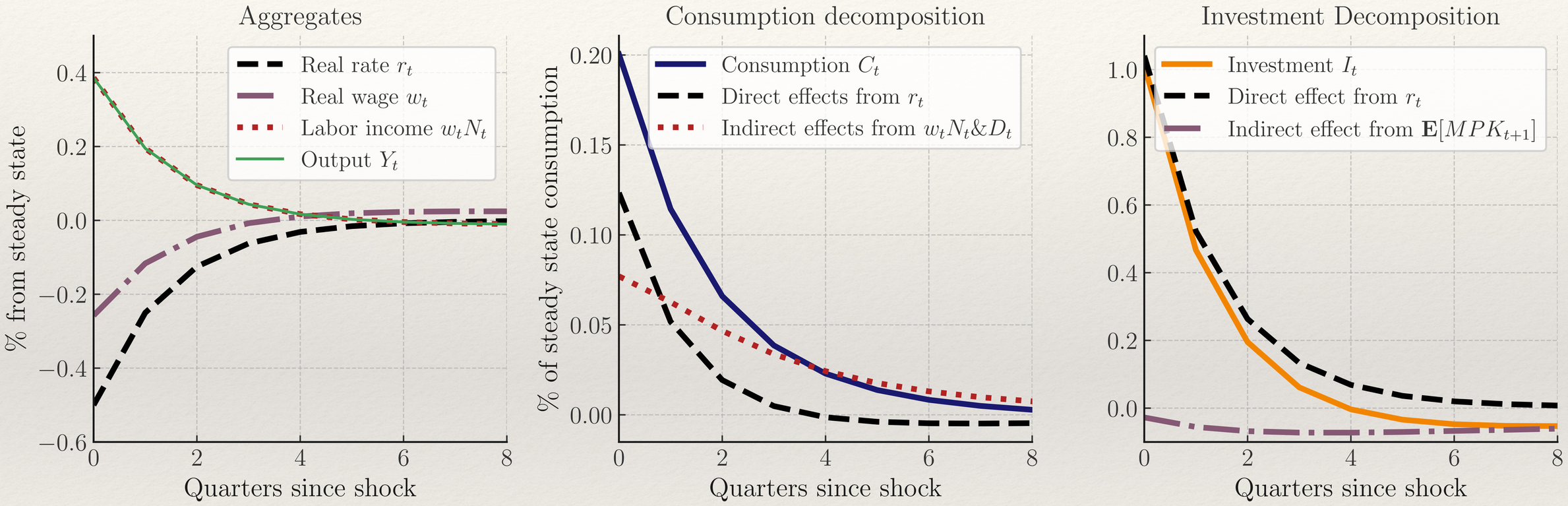
- Using $Q$ eqns, get decomposition of investment response similar to usual for $C$:

$$\frac{I_t}{K_{t-1}} - \delta = \equiv \frac{1}{\phi}\left(\frac{1}{1+r_t}\left(MPK(w_{t+1}, K_t) + 1 - \delta - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_t} - \delta\right)^2 + \phi\left(\frac{I_{t+1}}{K_t} - \delta\right)\frac{K_{t+1}}{K_t}\right) - 1\right)$$

$\rightarrow$ discounting channel ($r_t^*$) + expected $MPK$ channel ($w_{t+1}$)

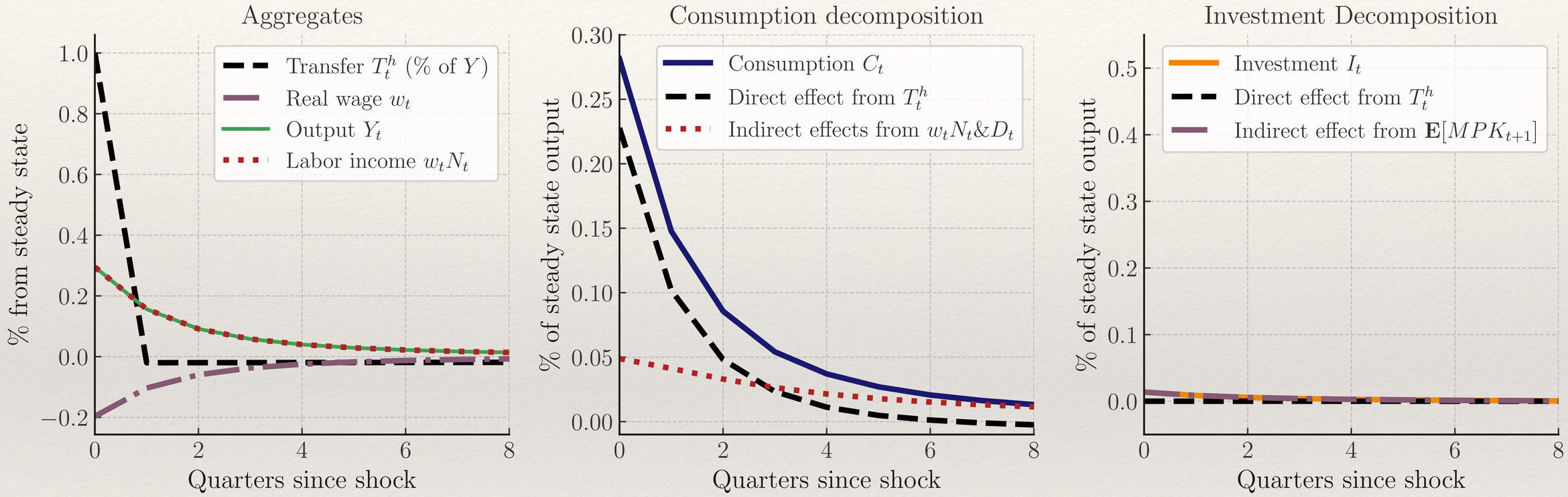
Magnitudes governed by (inverse) adjustment cost $1/\phi$

# Impulse response to monetary shock



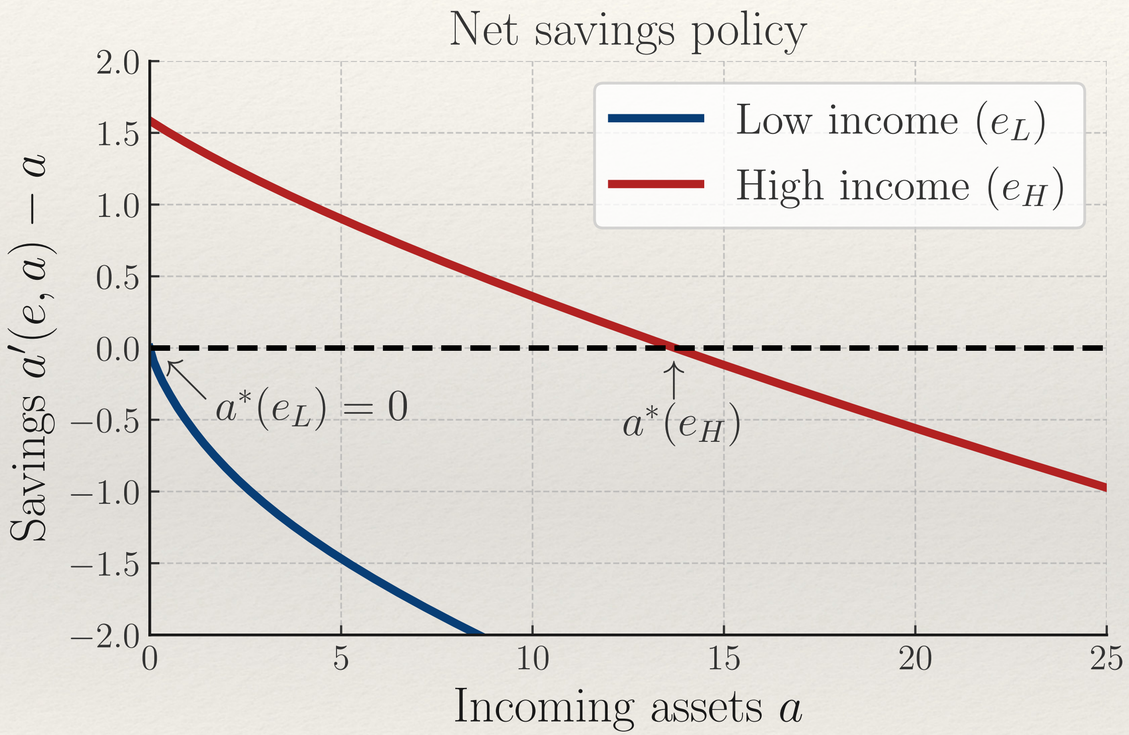

- Investment is exclusively driven by direct effects! (Here indirect effect $< 0$)
- Similar to representative household consumption.

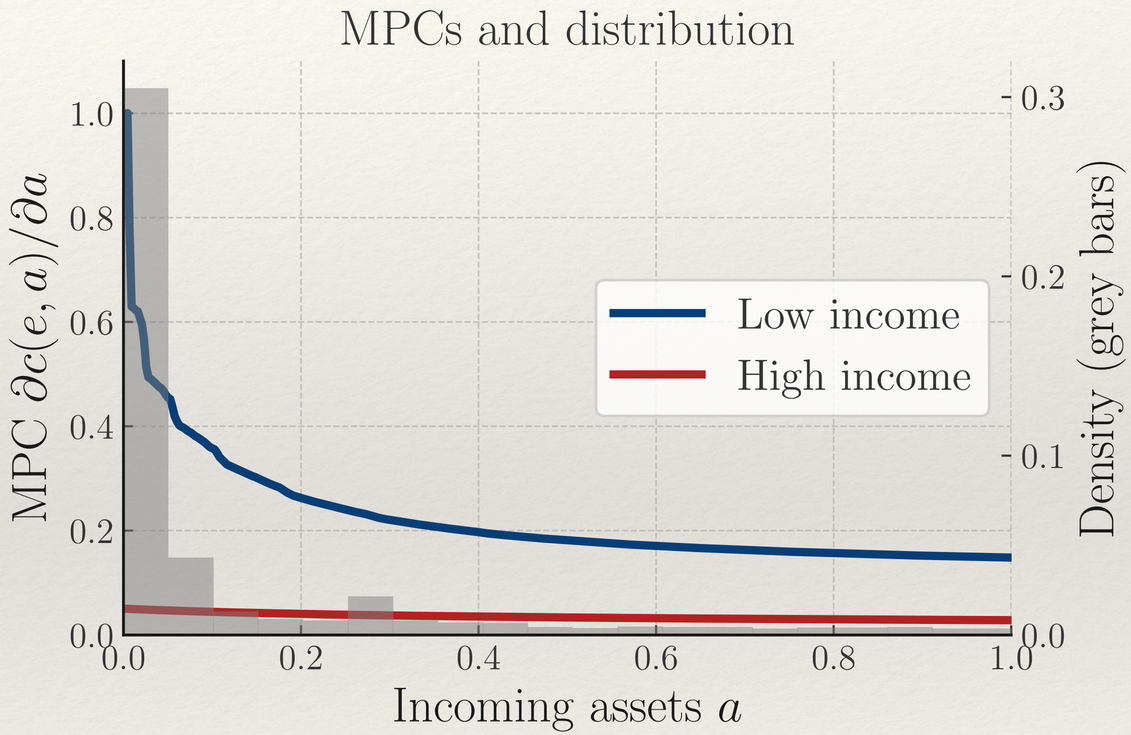
# Impulse response to household transfer shock




- Fiscal transfers to hh have large & persistent effects on $C$… but no effect on $I$
- Fiscal transfers to firms have **no effect** because inside = outside liquidity ($MPI$=0)
- **Next**: what does firm heterogeneity do?

# Under hh. hood: policies, MPCs, and distribution




- Buffer-stock behavior generates stationary distribution over assets
- Concave consumption function: MPCs negatively correlated with cash-on-hand

# Adding heterogeneous firms

# Heterogeneous firm model

- To get higher *MPI*s, build on tradition of models with idiosyncratic productivity risk + entry-exit + financial frictions [Hopenhayn; Khan-Thomas; Ottonello-Winberry…]
- Firm in idiosyncratic productivity state $z$ with incoming capital $k$ and debt $b$ solves

$$v_t\left(z,k,b\right) = \max_{n,k',b',d} d + \frac{1}{1+r_t}\mathbb{E}_t\left[\theta v_{t+1}\left(z',k',b'\right) + \left(1-\theta\right) v_{t+1}^{end}\left(z',k',b'\right)\right]$$

Exit prevents firms from growing out of constraints

$$d = zk^{\alpha}n^{\nu} - w_t n - \left(k' - \left(1-\delta\right)k\right) - \varphi\left(\frac{k'-k}{k}\right)k - b + \frac{b'}{1+r_t} + T_t^f$$

Cannot issue equity
Multiplier $\lambda$ on equity issuance

$$d \geq 0, \ [\lambda] \quad b' \leq \chi k' \ [\mu]$$

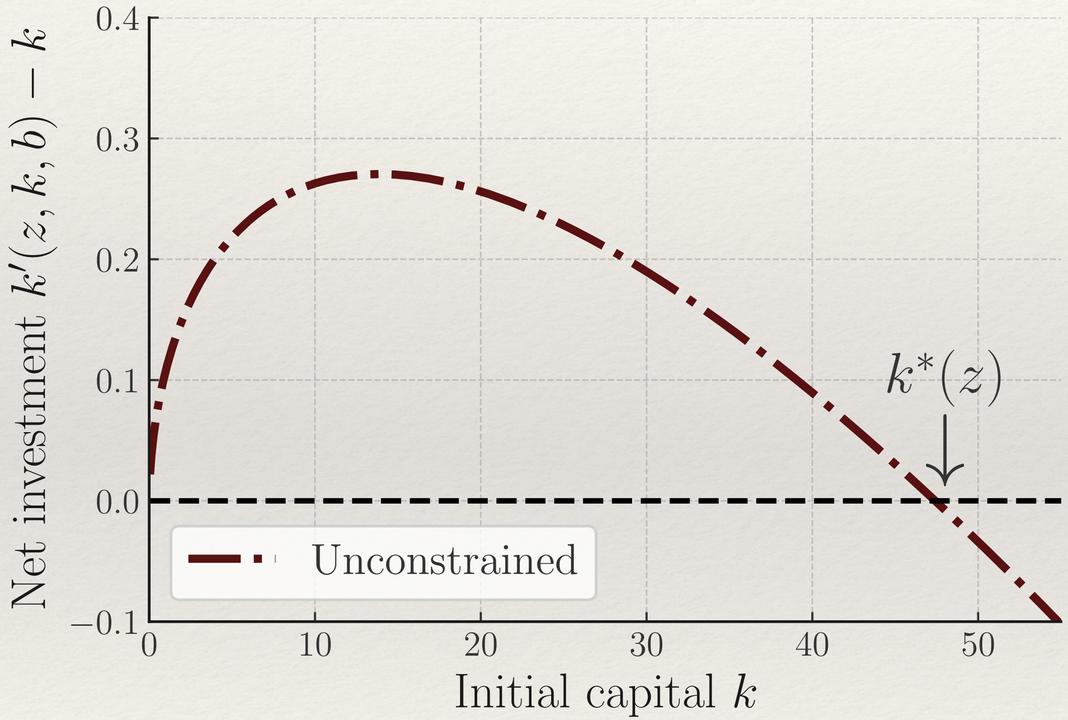
Can borrow in addition to retaining earnings subject to classic collateral constraint

$$v_t^{end}\left(z,k,b\right) = zk^{\alpha}n^{\nu} - w_t n + \left(1-\delta\right)k - b + T_t^f$$

# Calibration

- Additional parameters relative to representative firm model
  - Death probability $1-\theta = 2\,\%$ ← firm exit rate [Business Dynamics Statistics]
  - Firm productivity process ← estimated log AR(1) [Syverson 2011]
  - $\chi = 0.35$ ← ability to recover assets in bankruptcy [Kermani-Ma 2023]
  - Initial conditions $(k_0, b_0 = \chi k_0)$ ← size of new entrants << average firm
- Calibration generates average $MPI = 0.50$; ideally could use this as a data moment
- Not readily available: corporate finance *MPI* literature hasn't yet given us reliable estimates for broad set of firms (contrast with household finance *MPC* literature!)
  - [Some promising work using surveys or semi-structural estimation]

# Steady state capital policy



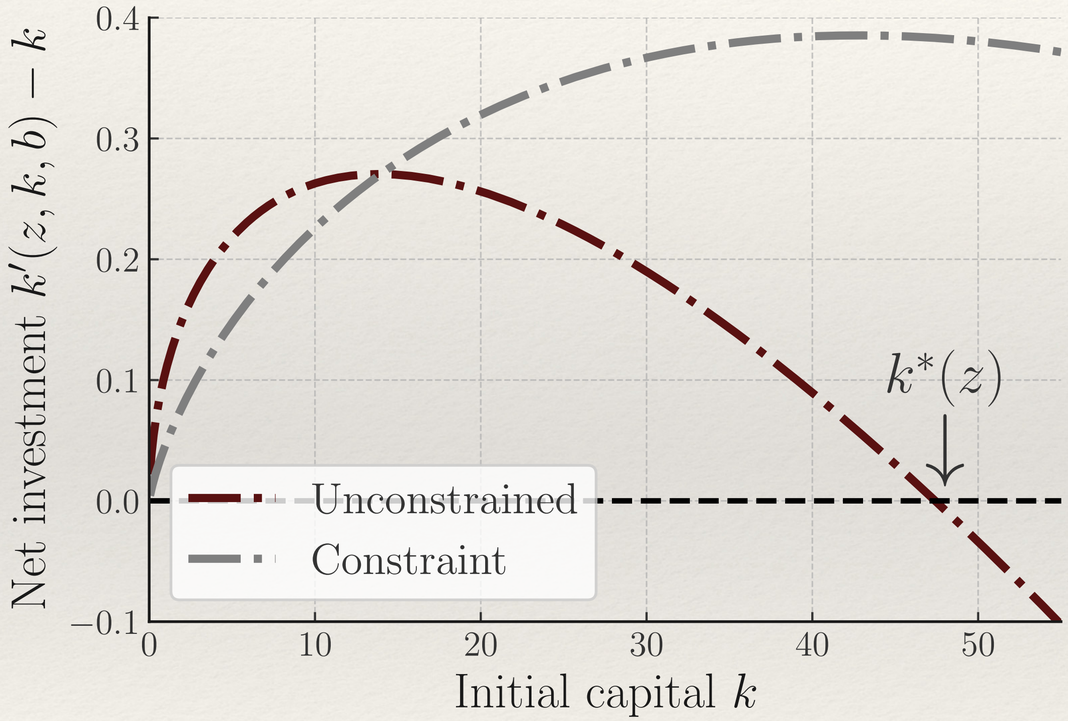

FOC for unconstrained (assuming no exit, $\theta = 1$):

$$\frac{k' - k}{k} = \frac{1}{\phi}\left(\frac{1}{1+r_t}\mathbb{E}\left[MPK(z', k', w_{t+1}) + \tilde{\varphi}(k', k'')\right] - 1\right)$$

Like rep. firm, unconstrained affected by m.p. via discounting and expected MPK effects

- Consider an unconstrained firm ($b \ll 0$)
- Diminishing returns + adjustment costs generate target $k^*(z)$

# Steady state capital policy




For borrowing and dividend constrained firm:

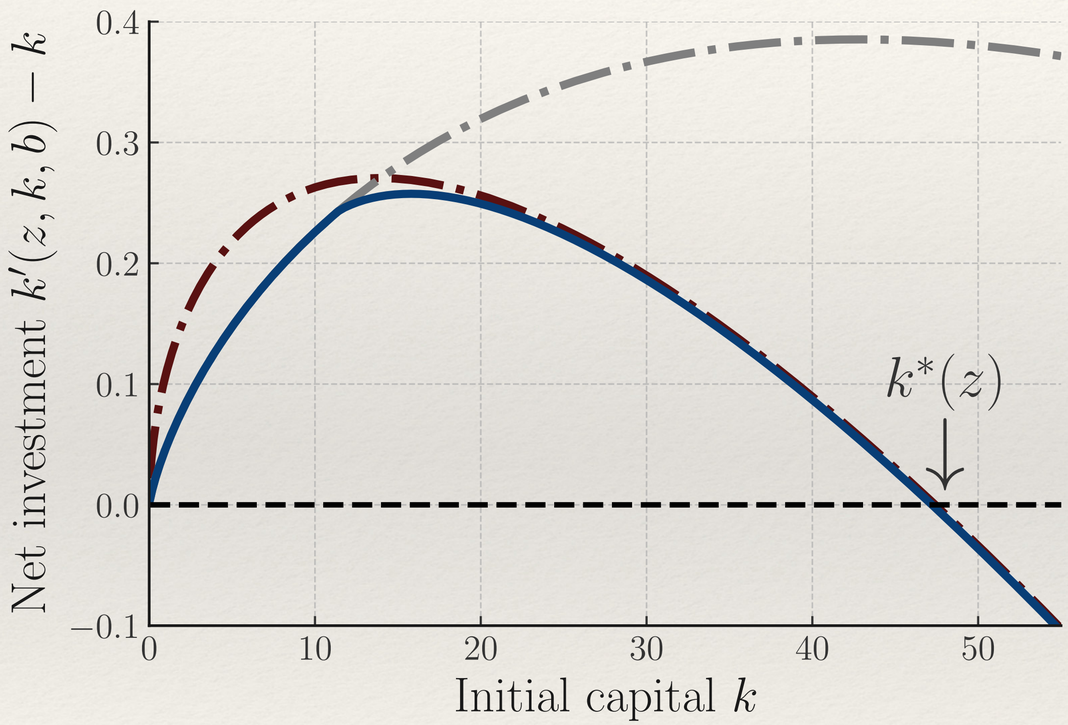
$$k' - (1-\delta)k + \varphi\left(\frac{k'-k}{k}\right)k = \pi_t(z,k) + T_t^f - b + \frac{\chi k'}{1+r_t}$$

where $\pi_t$ is profits, so $k' = \bar{k}\left(z, k, b; r_t, w_t, T_t^f\right)$

Constrained firms affected by a cash flow effect: lower $w_t$ today $\rightarrow$ higher $\pi_t \rightarrow$ higher $I_t$

- Next consider a firm that pays $d = 0$ and borrows maximally $b' = \chi k'$

# Steady state capital policy




Multiplier dynamics (with $\theta = 1$)

$$\lambda_t(z,k,b) = (1+r_t)\mu_t(z,k,b) + \mathbb{E}[\lambda_{t+1}(z',k',b')]$$

pay divs only if can avoid constraint forever

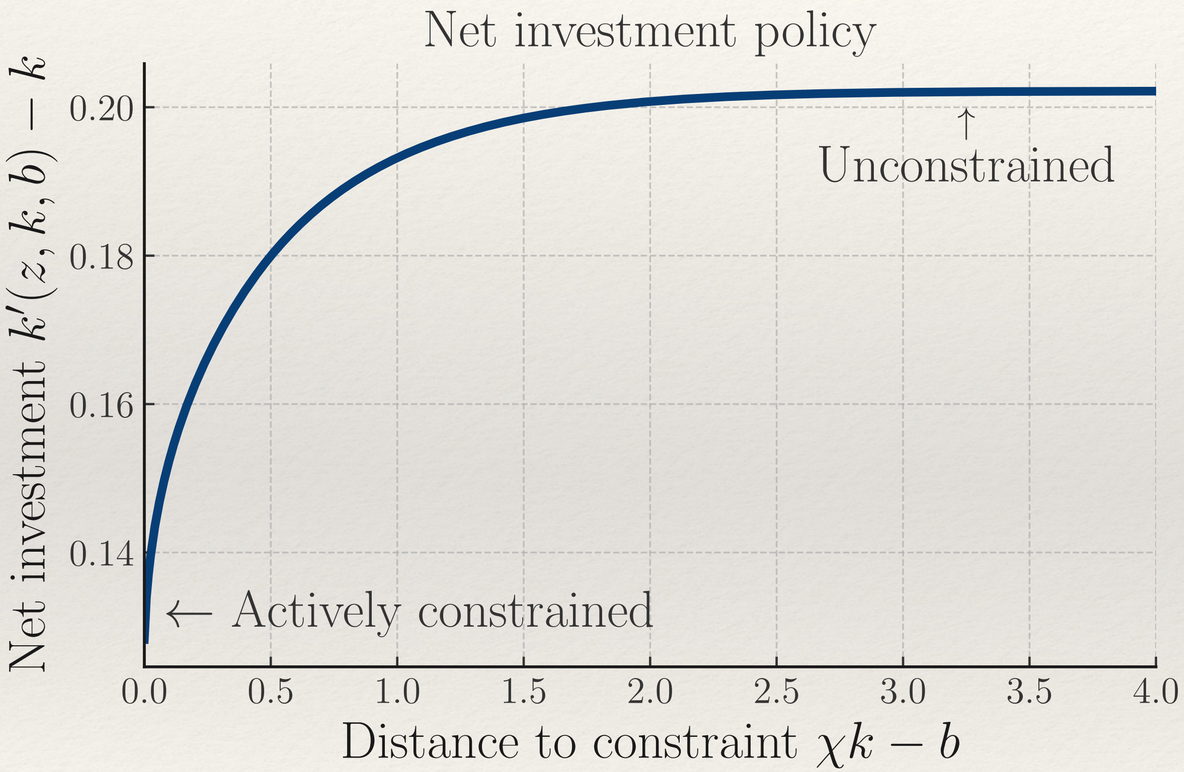
General investment FOC:

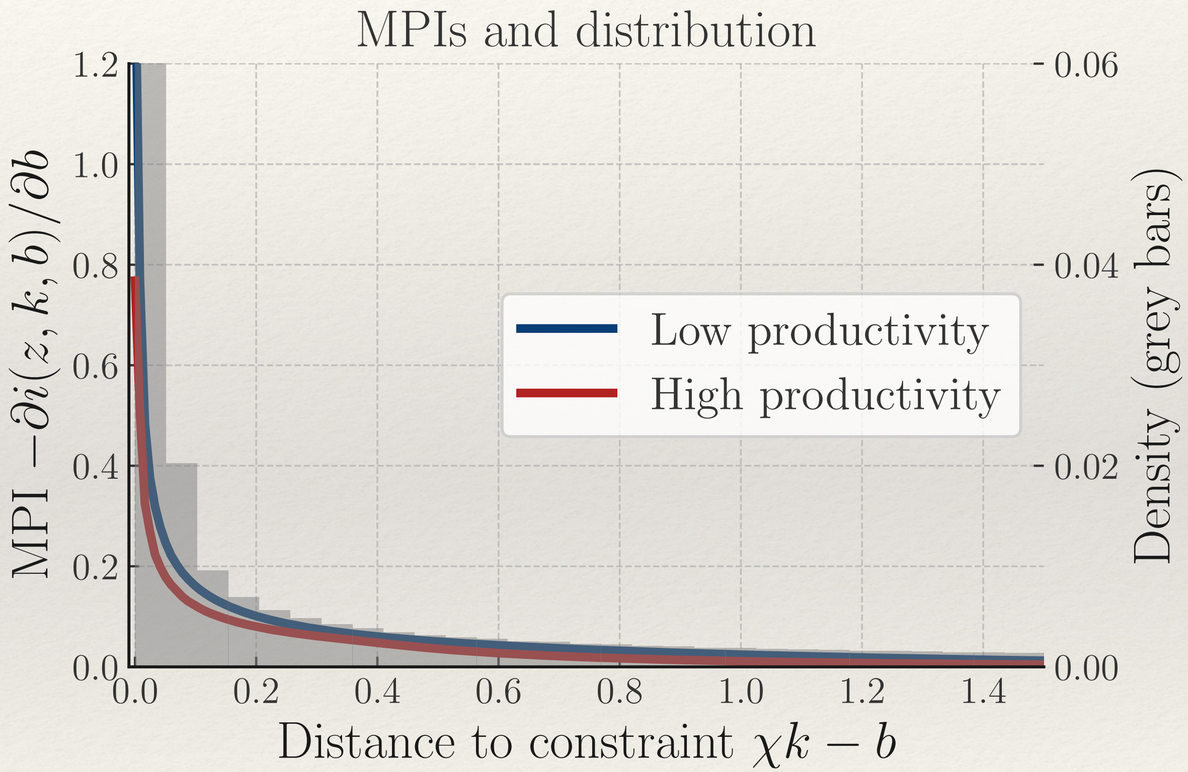
$$\frac{k'-k}{k} = \frac{1}{\phi}\frac{1}{1+r_t}\mathbb{E}\left[\frac{\lambda_{t+1}(z',k',b')}{\lambda_t(z,k,b)}\left(MPK(z',k',w_{t+1}) + \tilde{\varphi}(k',k'')\right)\right]$$

Variation in $\lambda$ acts like effective risk aversion

- Full policy pastes the two, with additional intermediate $\lambda > 0$, $b' < \chi k'$ region

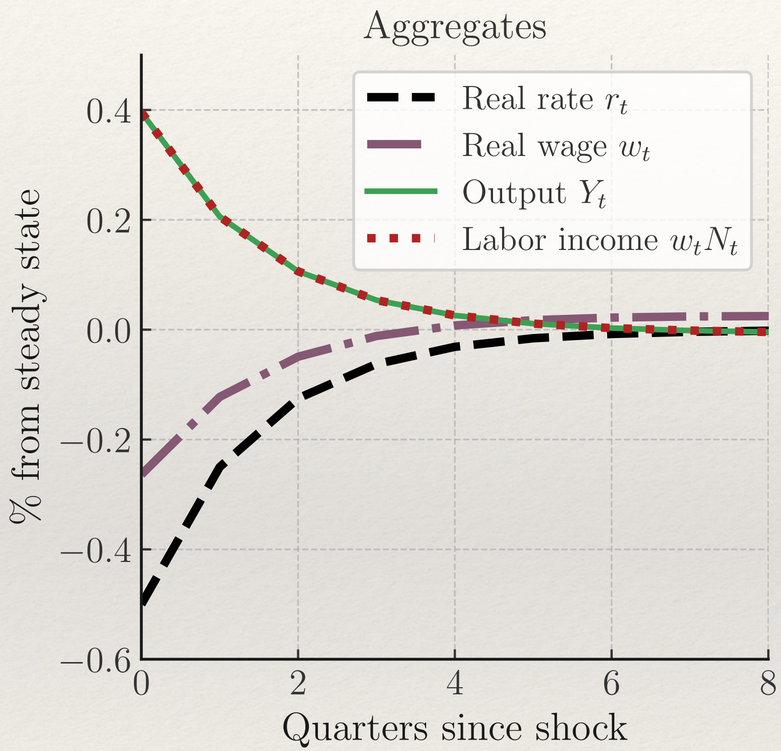
# Investment policy, MPIs and distribution

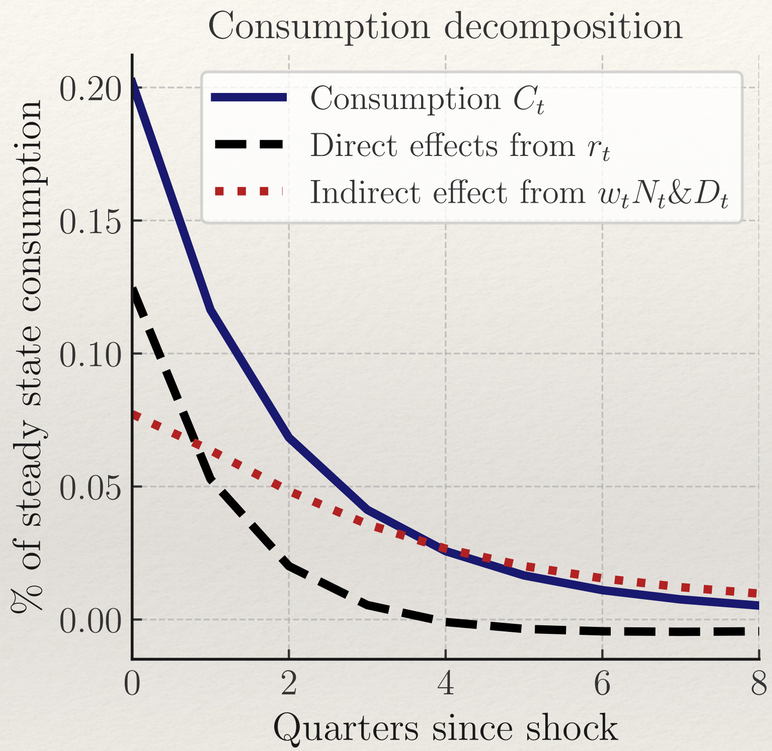


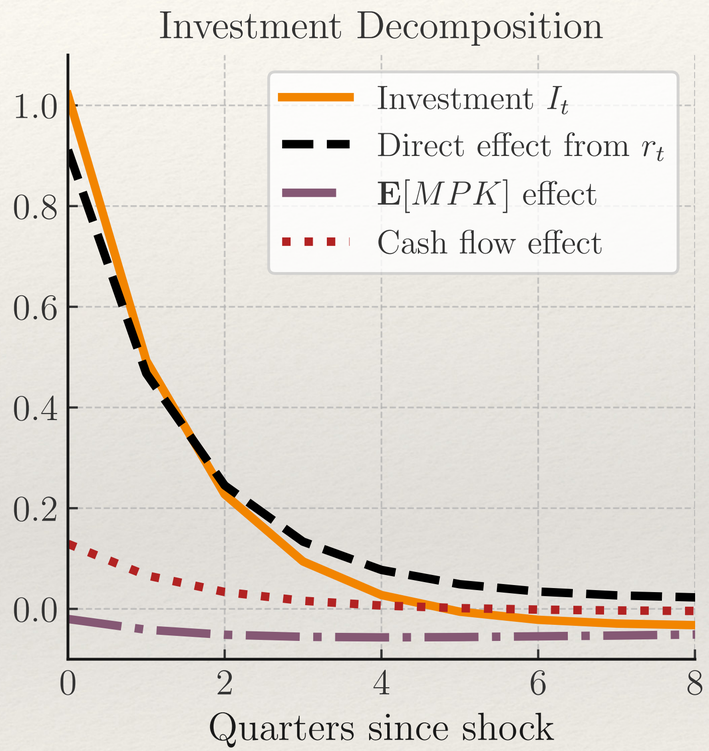




- Concave investment function: MPIs negatively correlated with dist. to constraint
- $MPI > 1$ possible due to ability to lever up $1/(1-\chi)$ - contrary to household model

# Impulse response to monetary shock

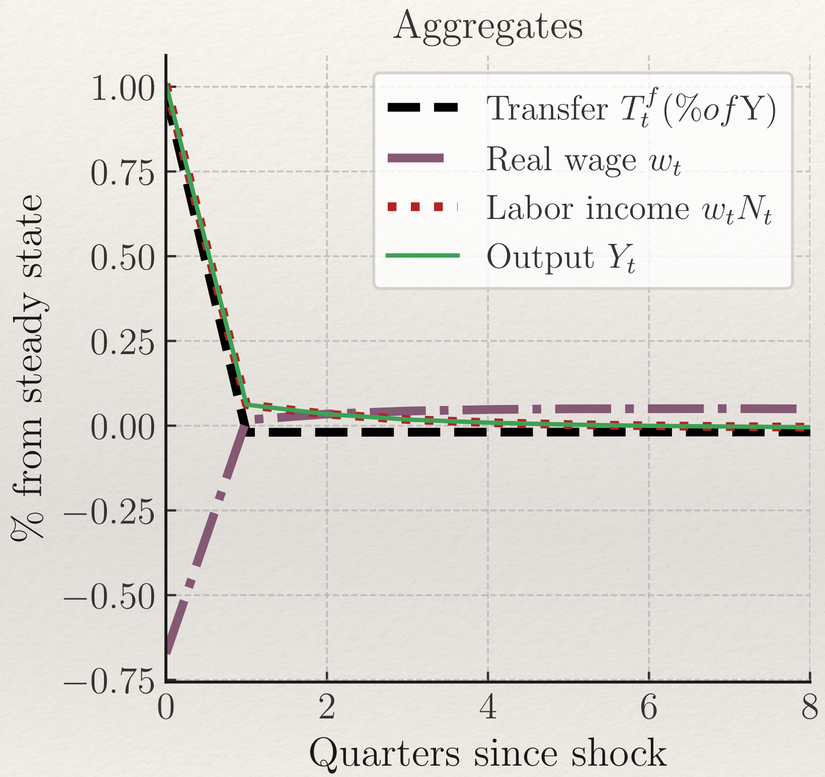


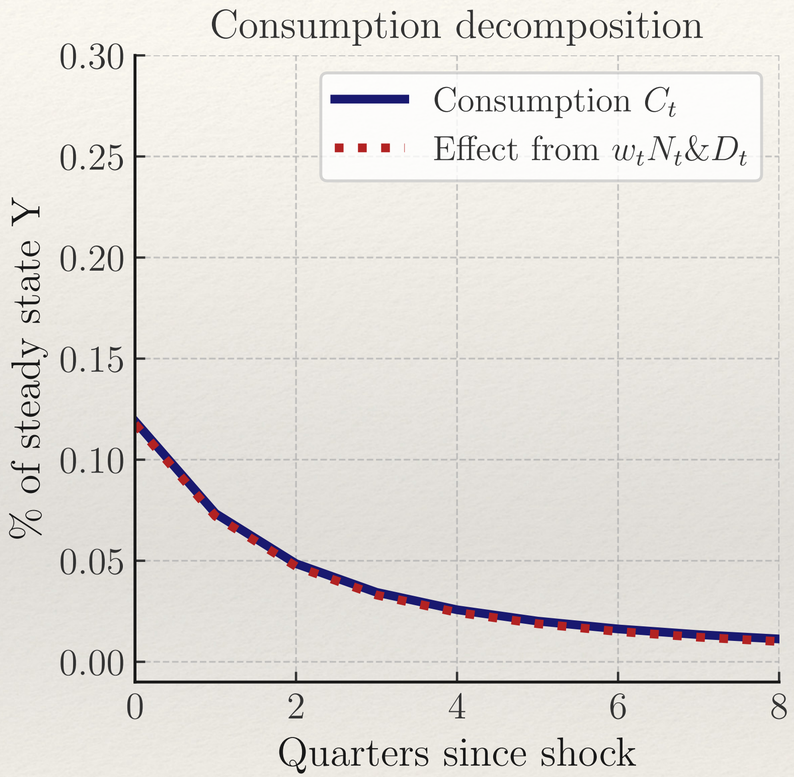



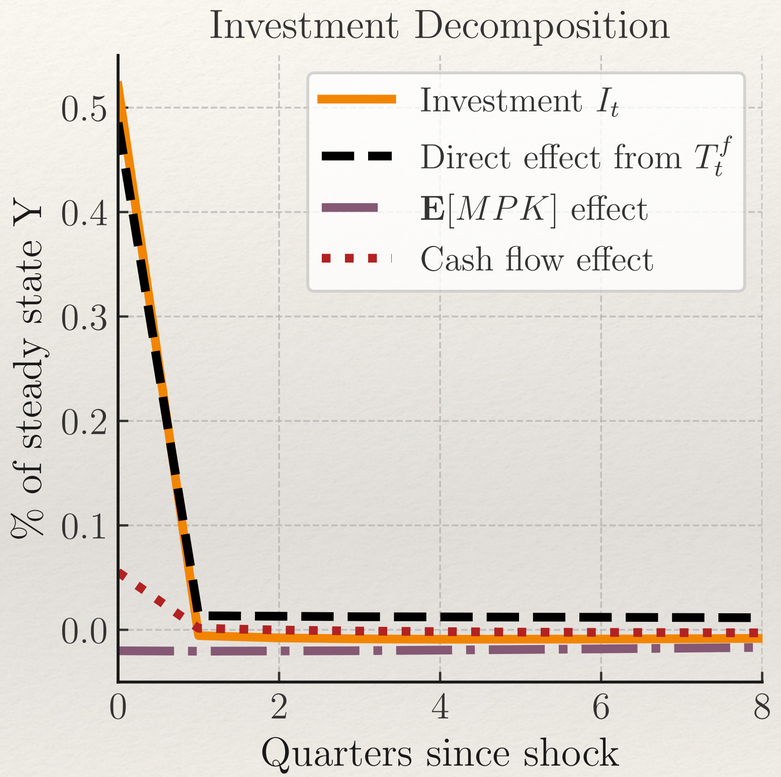




- New cash flow channel of monetary policy to investment: $w_t \downarrow \rightarrow \pi_t \uparrow \rightarrow I_t \uparrow$
- Magnitude controlled by profit-weighted *MPI* (since $\pi_{it} \propto Y_t$). Here, $\overline{MPI}$ small.

# Impulse response to firm transfer shock




- Inside ≠ outside liquidity: PV-0 transfers to firms boost output today!
- Large direct effect from firm transfer (large *MPI*), small multiplier (small $\overline{MPI}$)

# Analogies…

| | Heterogeneous households | Heterogeneous firms |
|---|---|---|
| Steady state | MPCs / iMPCs | MPIs / iMPIs |
| | Concave consumption function out of liquidity | Concave investment function out of free cash flow |
| | Prudence | Concern about hitting constraints |
| | Target (buffer) stock of assets | Target stock of capital (diminishing returns) |
| | Impatience | Life cycle (entry-exit) |
| Response to shocks | Income vs substitution effects | Income vs substitution effects |
| | Indirect effects of monetary policy | Cash flow effects of monetary policy |
| | Departure from Ricardian equivalence | Departure from inside = outside liquidity |
| | Income-weighted MPC governs multiplier | Profit-weighted MPI governs multiplier |

# How much do we get from the combination?

# Households vs firm transfers

Impact on GDP (net of adj. cost) of 1% of GDP transfer…

To households

| | Rep firm | Het firm |
|---|---|---|
| Rep household | 0 | 0 |
| Het household | 0.29 | 0.31 |
| | $\frac{MPC}{1-\overline{MPC}}$ | $\frac{MPC}{1-(\overline{MPC}+\overline{MPI})}$ |

To firms

| | Rep firm | Het firm |
|---|---|---|
| Rep household | 0 | 0.52 $\frac{MPI}{1-\overline{MPI}}$ |
| Het household | 0 | 0.64 $\frac{MPI}{1-(\overline{MPC}+\overline{MPI})}$ |

- Implications for stimulus policy ultimately depend on our views of *MPC*s vs *MPI*s !

# Bringing firm heterogeneity to HANK

- Firms have been left out but have their rightful place in HANK !
- With non-zero *MPI*s as in the data:
  - … cash flow transmission channel to *I*, paralleling indirect effect on *C*
  - … PV-0 transfers to firms have large and persistent effects on $Y$
  - … activates Keynesian feedback effects: $Y \to I \to Y \to C$ and $Y \to C \to Y \to I$
- More empirical work needed to measure MPIs & covariance with firm observables