# Lecture 8: Endogenous portfolios and risk premia

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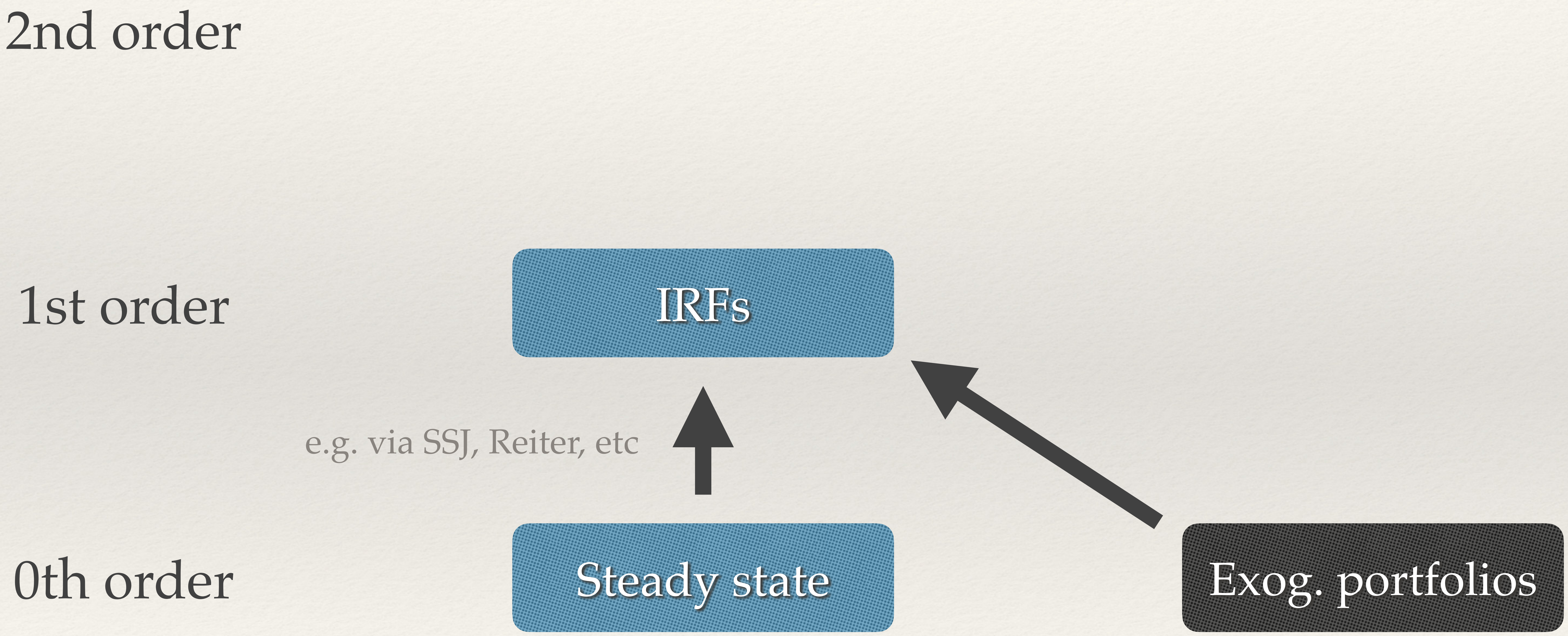
# Portfolios in heterogeneous-agent macro

- **So far:** single asset models (from the perspective of households)
- In models with several assets available, used "mutual fund trick"
  - see stocks + bonds model, but could also do this with nominal and real bonds, short and long term bonds, home and foreign bonds…
  - from this we got no-arbitrage conditions + initial $r_0^p$
- **Now**: what if we let households trade these different assets directly?
  - MIT shock world: portfolios are **indeterminate**
    - can feed in distribution of portfolios from the data
  - Expected shocks world: portfolios are **determinate**!

# What does second order perturbation give us?

Orders in aggregate risk



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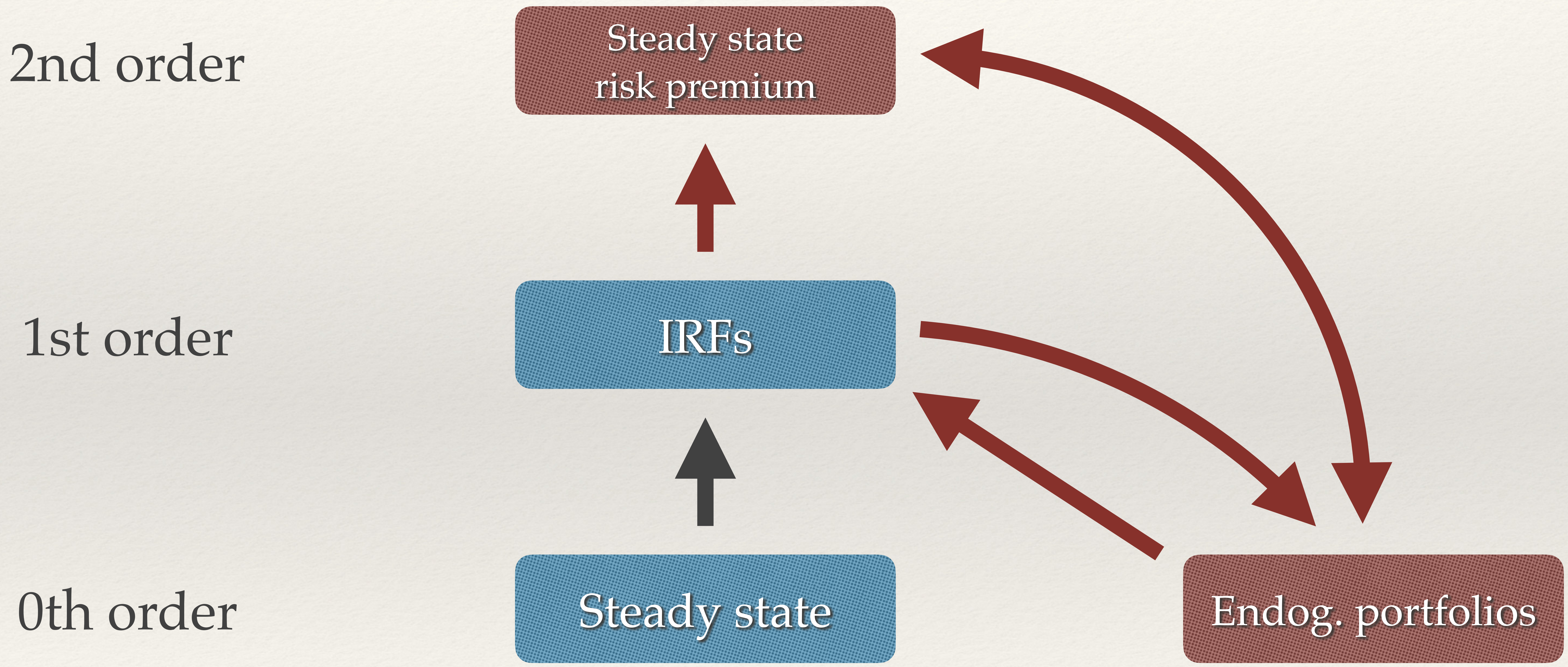

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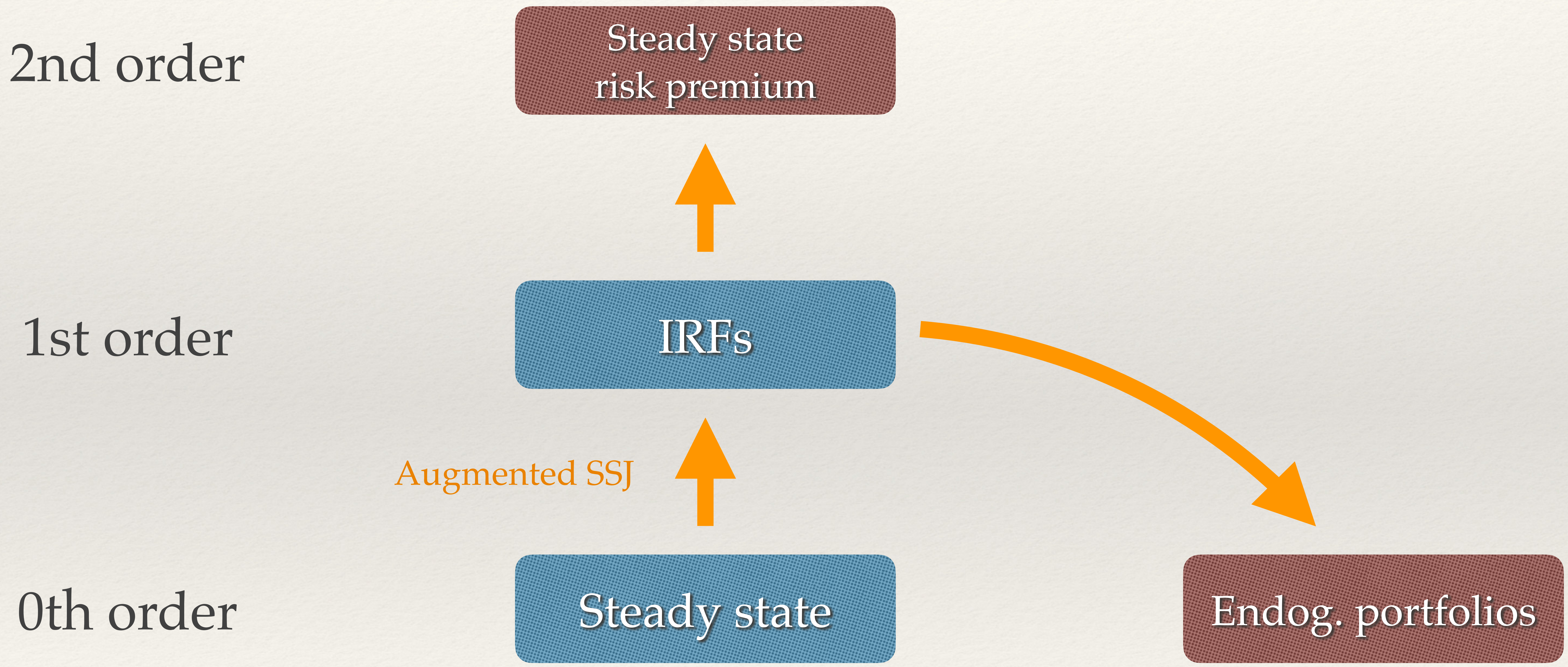


# Canonical model with locally complete markets

# General setting (one account, many assets)

- Heterogeneous households $i$ can allocate wealth $a_i$ to $K+1$ assets
- Aggregate risk: random variable $\epsilon$; asset $k$ has stochastic return $R^k(\epsilon)$
- Given value function $V_i$, problem of household $i$ is:

$$\max_{\{a_i^k\}} \mathbb{E}_\epsilon \left[ V_i \left( \sum_{k=0}^{K} R^k(\epsilon) a_i^k, \epsilon \right) \right] \text{ s.t. } \sum_{k=0}^{K} a_i^k = a_i$$

- Classic first-order condition, letting $\gamma_i \equiv$ multiplier on $i$'s budget constraint

$$\mathbb{E}_\epsilon \left[ R^k(\epsilon) \frac{\partial V_i}{\partial a}(\epsilon) \right] = \gamma_i \quad \forall i, k$$

# Implications of asset spanning

- Suppose finite $\epsilon$'s (label one $\epsilon_{ss}$). With enough assets, we get **complete markets**:

$$\frac{\partial V_i}{\partial a}(\epsilon) / \frac{\partial V_i}{\partial a}(\epsilon_{ss}) = \lambda(\epsilon) \quad \forall \epsilon$$

- $\lambda$ is (cross-sectional) stochastic discount factor, gives us **risk premia**

$$\mathbb{E}_\epsilon \left[ R^k(\epsilon) - R^0(\epsilon) \right] = -\operatorname{Cov}_\epsilon \left( R^k(\epsilon) - R^0(\epsilon), \lambda(\epsilon) / \mathbb{E}_\epsilon \left[ \lambda(\epsilon) \right] \right) \quad \forall k$$

- With continuous $\epsilon$'s, but finite $K$, can still get this "locally" (ie to 1st-order)
  - 2nd-order perturbation to $\epsilon$ shows we need $K \geq \dim \epsilon$ + spanning condition
  - "0th order portfolios" $a_i^k$ ; well-defined in the limit as $\epsilon \to \epsilon_{ss}$

  [eg Tille-van Wincoop 2010, Devereux-Sutherland 2011, Coeurdacier-Rey 2013, etc.]

# Application to one-account, many-asset model

- Consider now standard one-account model with idiosyncratic risk $e$ + aggregate risk

$$V_t\left(a_0, \ldots, a_K, e\right) = \max_{c, a_k'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V_{t+1}\left(a_0', \ldots, a_K', e'\right)\right]$$

$$c + \sum_{k=0}^{K} a^{k\prime} = \sum_{k=0}^{K} R_t^k a^k + W_t e \, ; \quad \sum_{k=0}^{K} a^{k\prime} \geq \underline{a}$$

- Say all agg. risk realized at date 0. For $t \geq 1$: same return $R_t$, only $a \equiv \sum a_k$ matters

$$V_t(a, e) = \max_{c, a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V_{t+1}(a', e')\right]$$

$$c + a' = R_t a + W_t e \qquad a' \geq \underline{a}$$

# Complete markets in the one-account model

- At $t=0$, $\epsilon$ realizes, so paths $R_t^k(\epsilon)$ and $W_t(\epsilon)$ for $t \geq 0$ become known
- At $t=-1$, using envelope $\partial V/\partial a = c^{-\sigma}$, the risk-sharing condition is

$$\frac{\mathbb{E}_e\left[c_0^{-\sigma}\left(a_{ss}+t_0(a_{ss},e_-,\epsilon),e,\epsilon\right) \Big| e_-\right]}{\mathbb{E}_e\left[c_{ss}^{-\sigma}\left(a_{ss},e\right) \Big| e_-\right]} = \lambda(\epsilon) \quad \forall \epsilon$$

- Can use this to **test** for portfolio optimality and **solve** for optimal portfolios
- Equivalently: solve for "transfers" $t_0(a_{ss}, e_-, \epsilon)$ relative to baseline portfolios
  - Later, figure out what portfolios decentralize these transfers

# First-order complete markets heuristic

- Write $\partial c_{i0}$ for date-0 consumption response to shock under baseline portfolios
- To first-order, risk sharing condition says (heuristically):

$$\frac{dc_{i0}}{c_{i0}} = \frac{\partial c_{i0} + mpc_{i0} dt_{i0}}{c_{i0}} = \frac{-d\lambda}{\sigma}$$

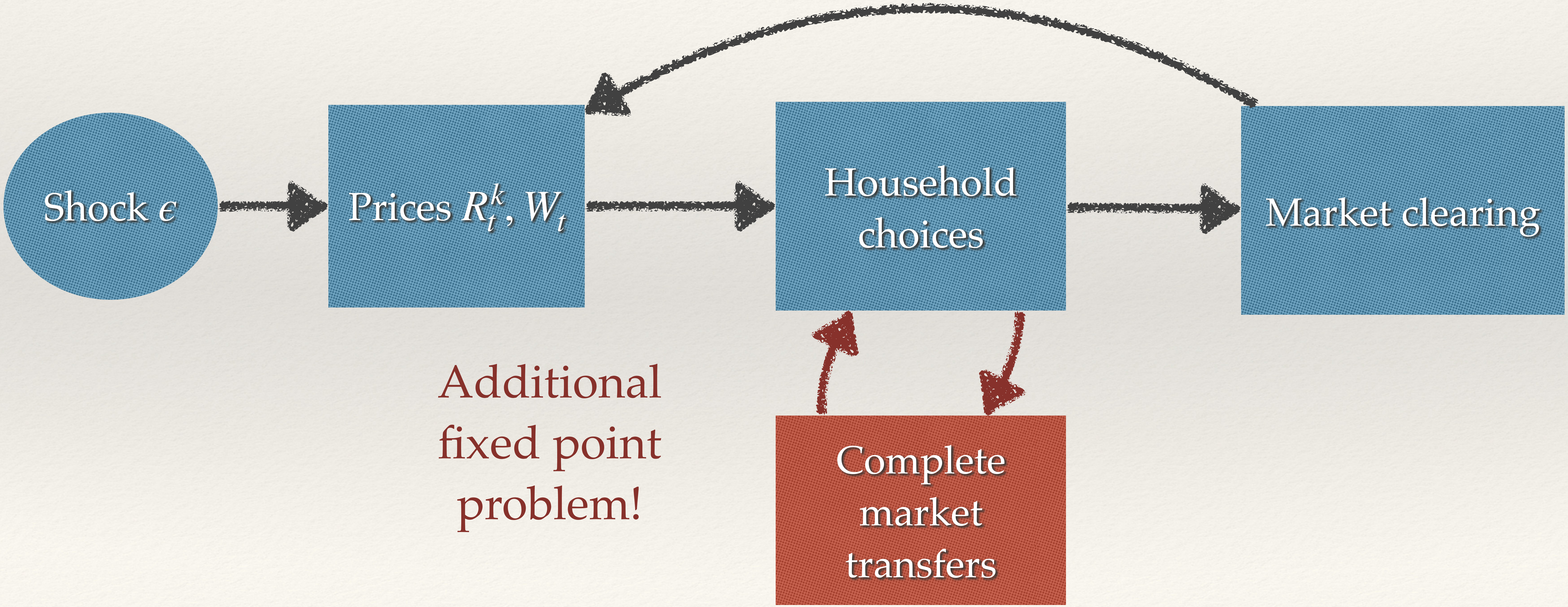
- Impose $\mathbb{E}_I[dt_{i0}] = 0$ to solve for sdf $d\lambda$ and equilibrium transfer $dt_{i0}$:

$$\frac{-d\lambda}{\sigma} = \frac{dc_{i0}}{c_{i0}} = \mathbb{E}_I\left[\frac{\frac{c_{i0}}{mpc_{i0}}}{\mathbb{E}_I\left[\frac{c_{i0}}{mpc_{i0}}\right]}\frac{\partial c_{i0}}{c_{i0}}\right] \qquad dt_{i0} = \frac{\frac{-d\lambda}{\sigma}c_{i0} - \partial c_{i0}}{mpc_{i0}}$$

➡ Complete markets ~**undo the heterogeneous consumption effects of shocks**

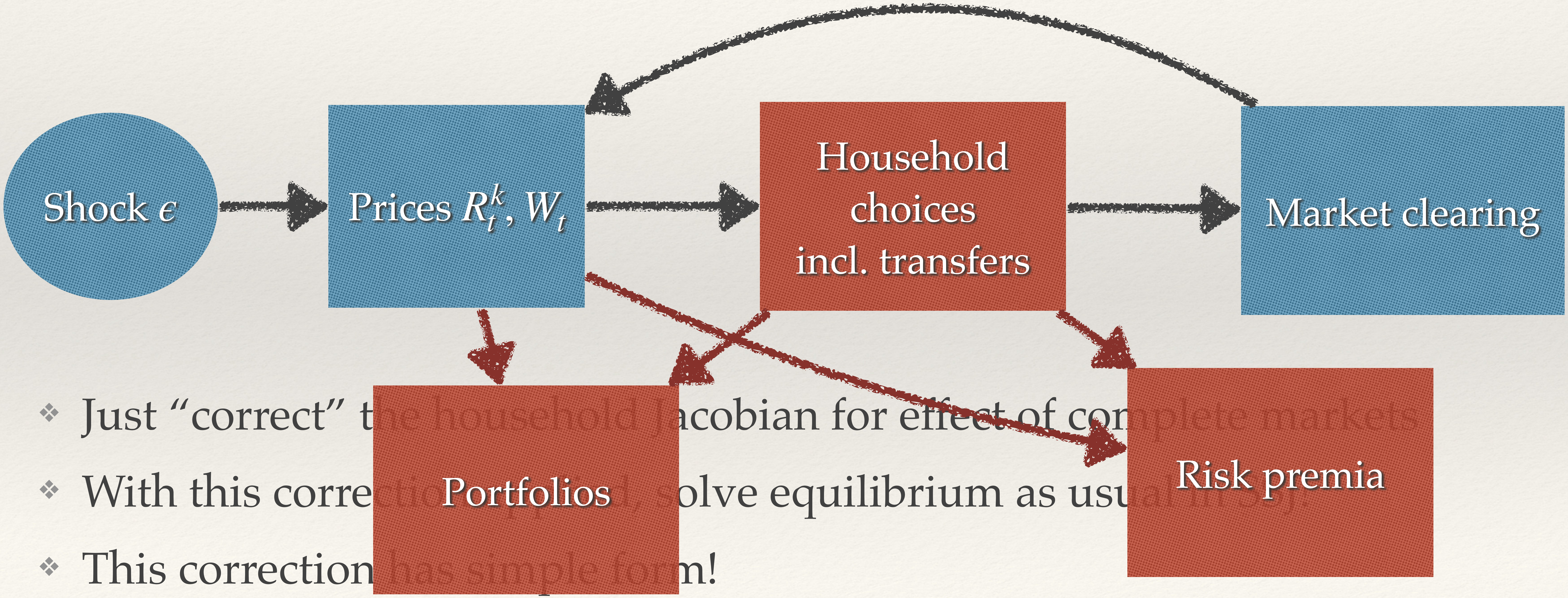
# General equilibrium

- Consider now general equilibrium:



# General equilibrium solution with SSJ

- Tackle this with SSJ as follows




- Just "correct" the household Jacobian for effect of complete markets
- With this correction applied, solve equilibrium as usual in SSJ.
- This correction has simple form!

# Implementation of correction in SSJ

- Intertemporal MPC matrix **M**: $M_{ts} = \partial C_t / \partial W_s$
- Wage change at $s$ has direct effect on policies $\partial c_{0i}^s$
- From optimal risk-sharing, get date-0 transfer $dt_{0i}^s$
- Together, these imply change in distribution of assets coming into date 0 $\mathscr{D}_s$
- In turn, this affects consumption at $t$ by $\mathscr{E}_t' \mathscr{D}_s$ ($\mathscr{E}_t$ is expectation function for $C$)
- Hence, **simple correction** to sequence space Jacobian gives us effect under CM:

$$M_{t,s}^{CM} \equiv M_{t,s}^{exo} + \mathscr{E}_t' \mathscr{D}_s$$

- See notebook for additional details!

# Sanity check: replicates known state-space results!

- Consider original Krusell-Smith (97) model: TFP shock, 2 assets capital+bond
- Portfolio constraints $k \geq 0, b \geq \underline{b}$ [needs modified, iterative algorithm, cf paper]

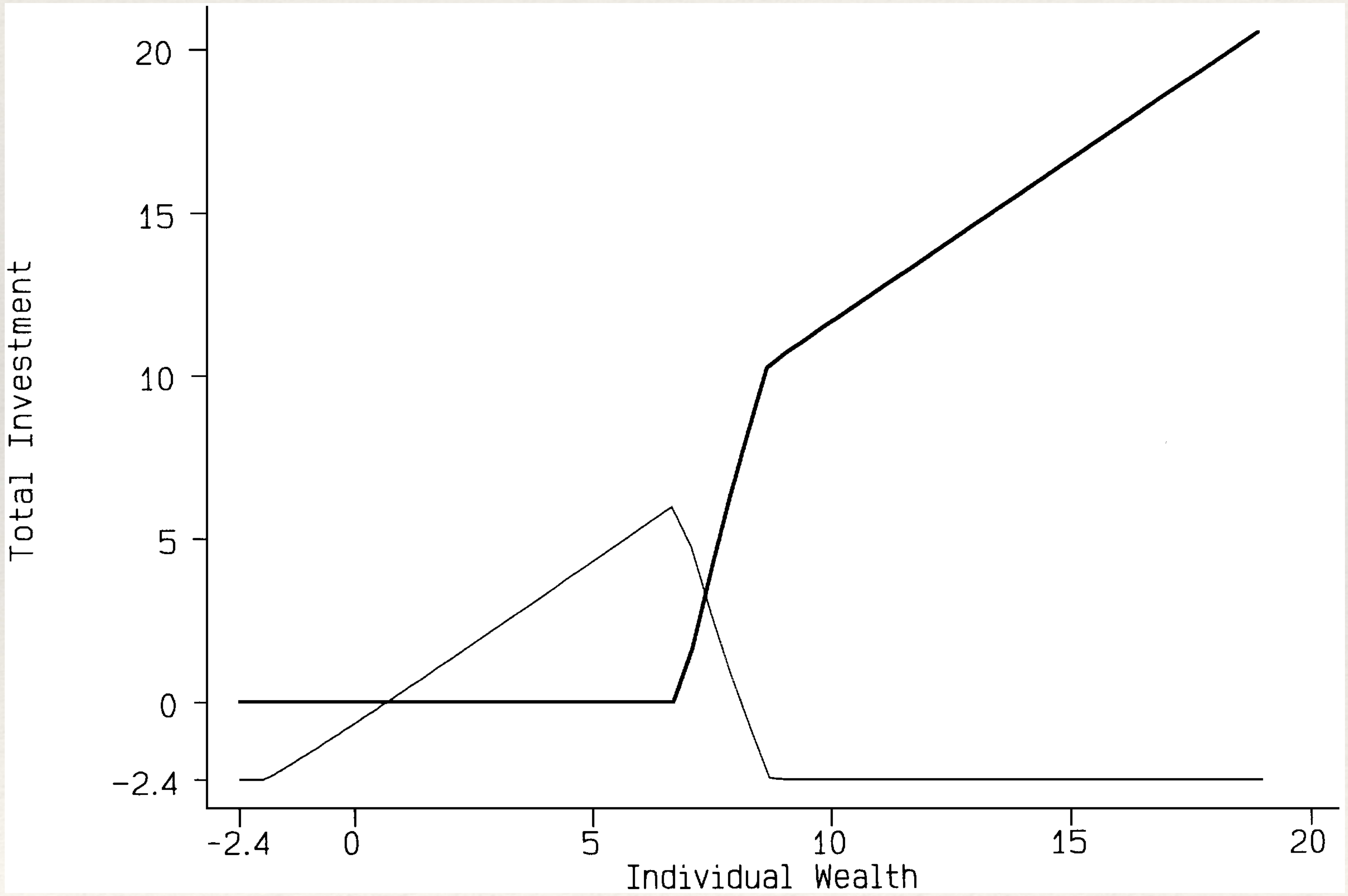
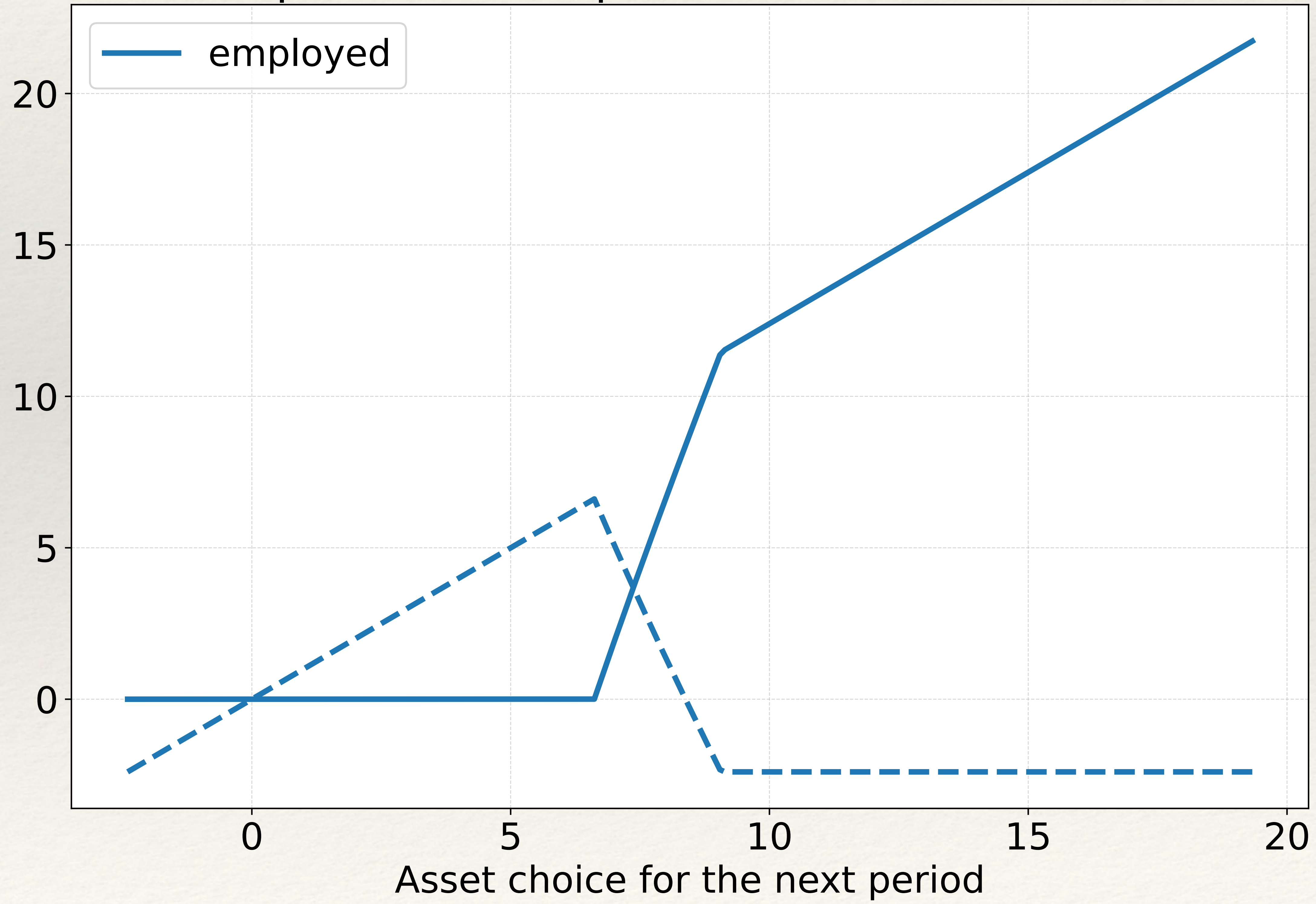



**FIGURE 3.** Portfolio decision rules of an employed agent: thick line = capital, thin line = bonds



# Application to canonical HANK model with traded stocks

# Canonical HANK model

- Application to canonical HANK model
- We now let households trade stocks and bonds directly:

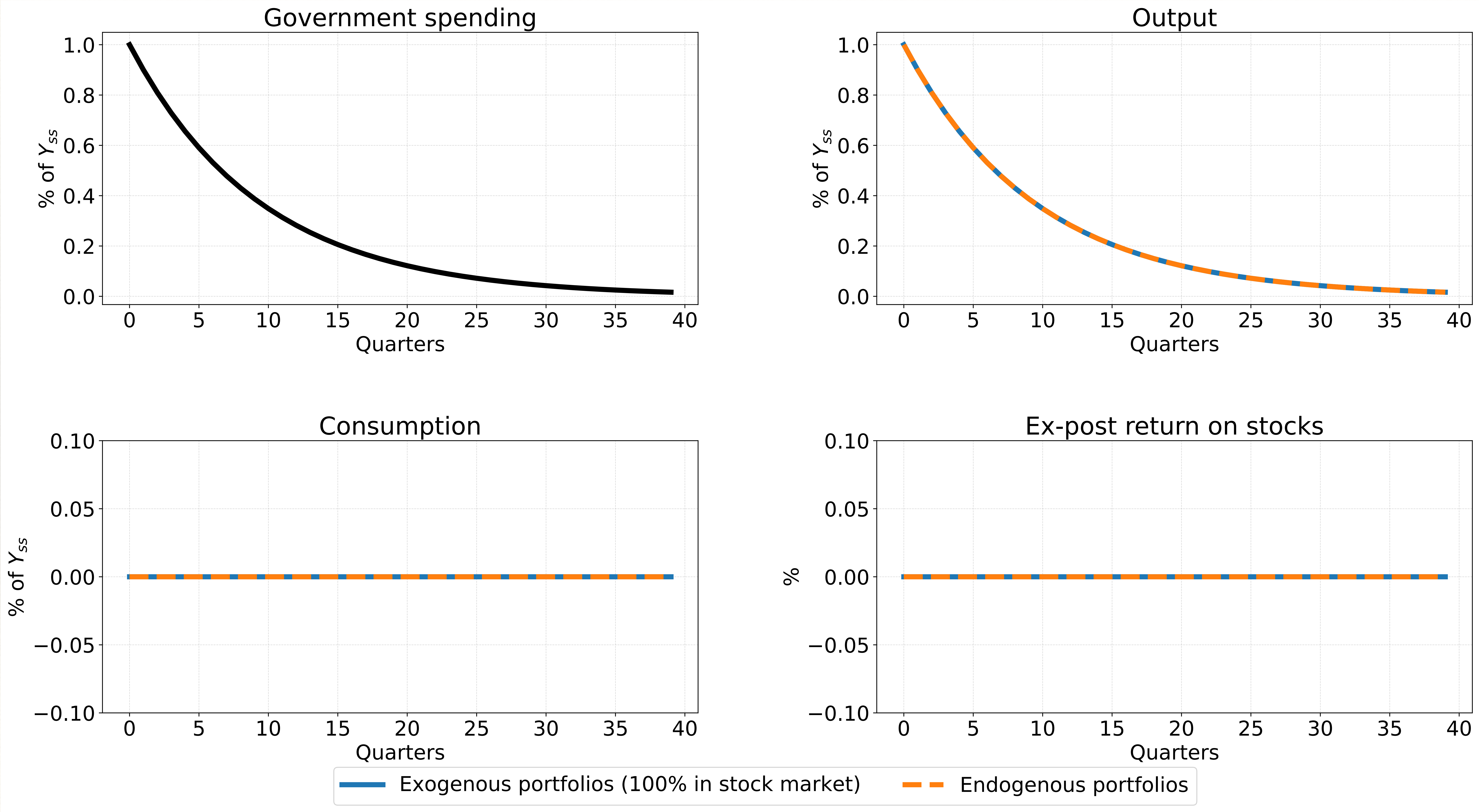
$$\max_{c_{it}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{it} \left( u(c_{it}) - v\left(N_t\right) \right)$$

$$c_{it} + p_t s_{it} + b_{it} \leq (p_t + d_t) s_{it-1} + (1 + r_{t-1}) b_{it-1} + (1 - \tau_t) \frac{W_t}{P_t} e_{it}$$

$$p_t s_{it} + b_{it} \geq 0$$

  - $s$ stocks (price $p_t$, dividends $d_t$), $b$ bonds, assume $\sigma = 1$
  - Rest of the model is as usual, $B_{ss} = 0$ (so Werning equivalence result applies)

# Example 1: balanced-budget $\{G_t\}$ shock



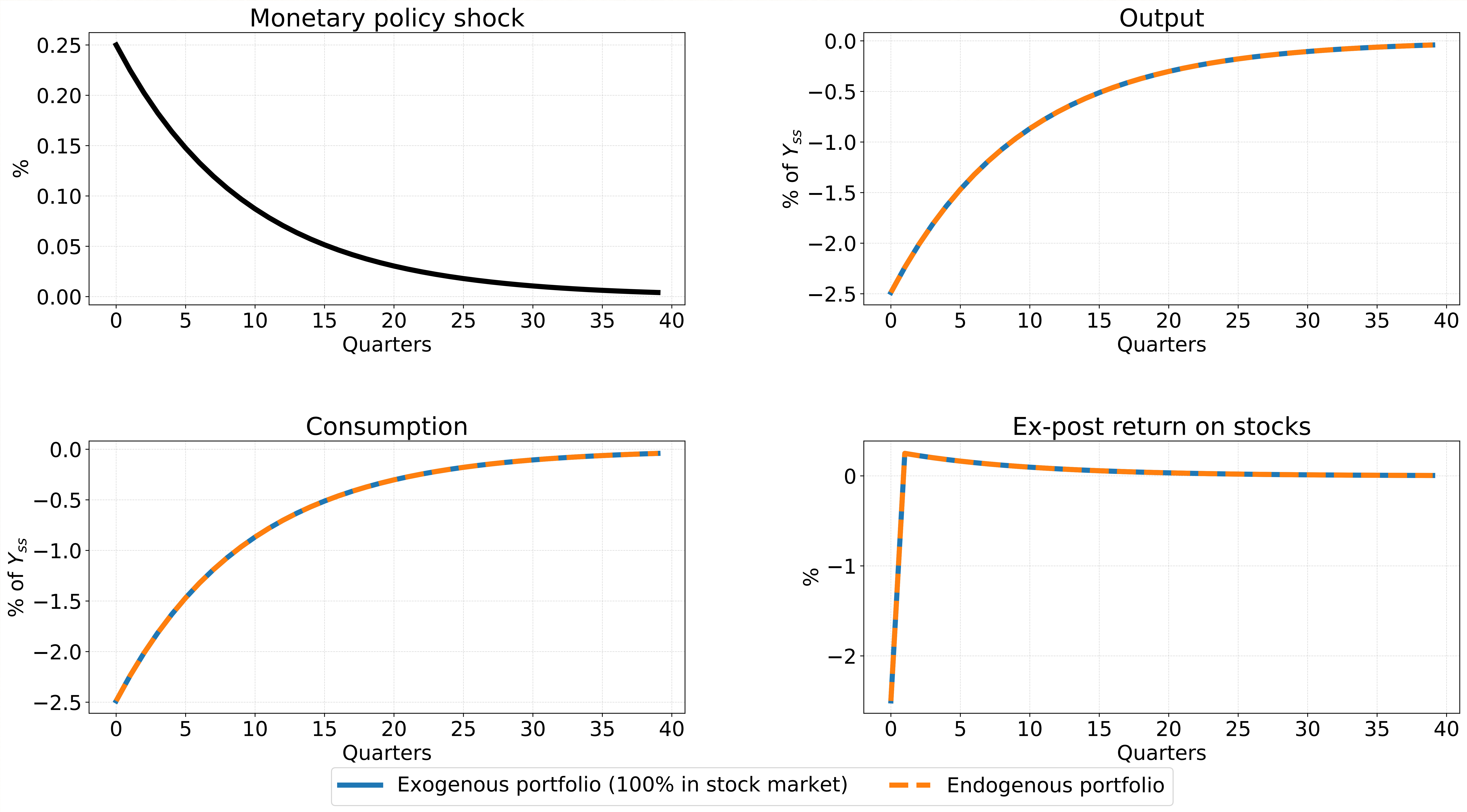

No effect from portfolio choice!

Why? Homogeneous consumption effects at exogenous portfolios, $dc_{i0} = 0$

**For endog. portfolios to matter, need unequal consumption effects of shocks**

Also, here stock prices constant -> no difference between bonds and stocks

# Example 2: monetary policy $\{r_t\}$ shock

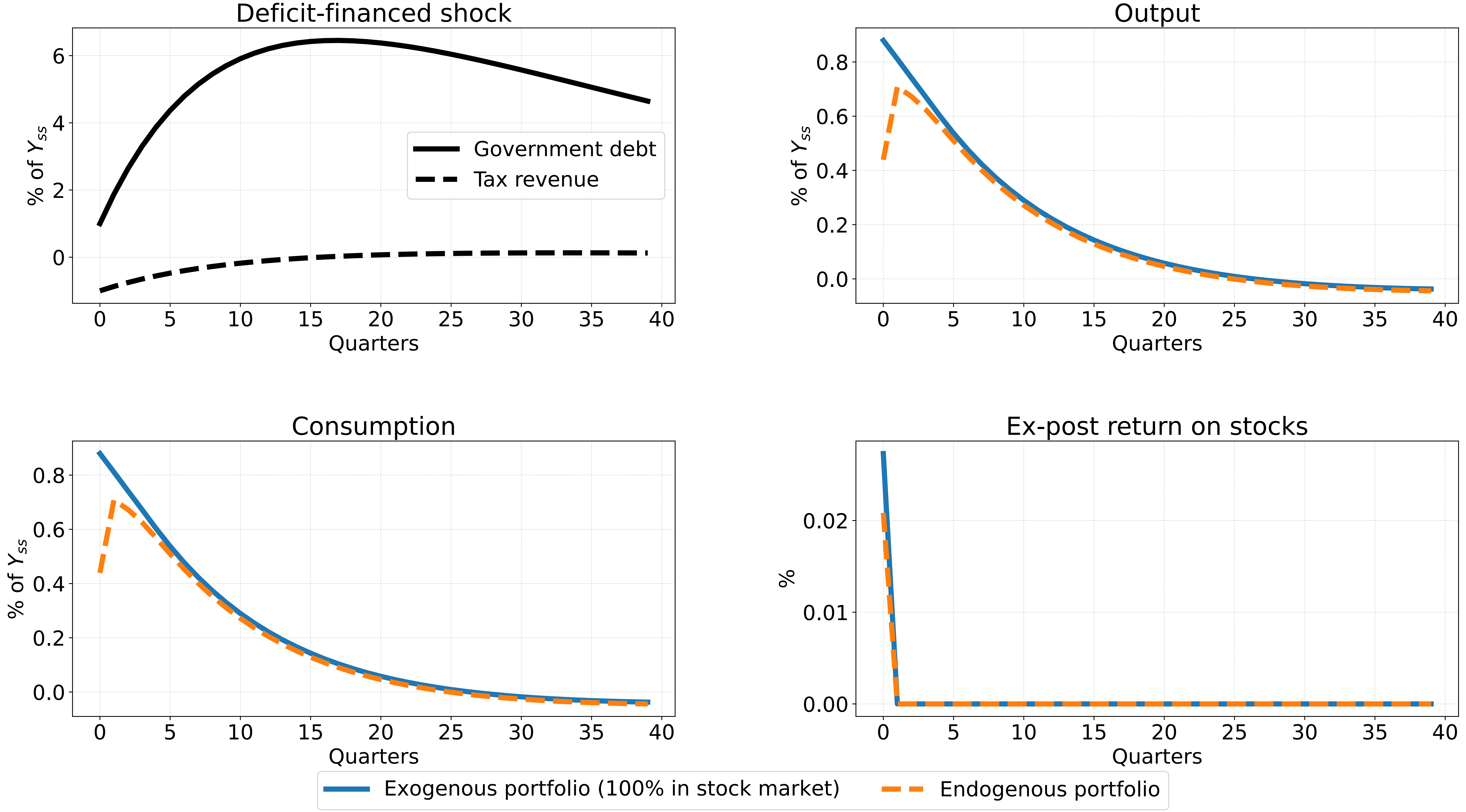



Still no effect from portfolio choice!

100% stock portfolios + log utility: homogeneous effects of monetary policy (Werning result!)

100% stocks are only optimal portfolios here

# Example 3: deficit-financed transfer $\{B_t\}$ shock



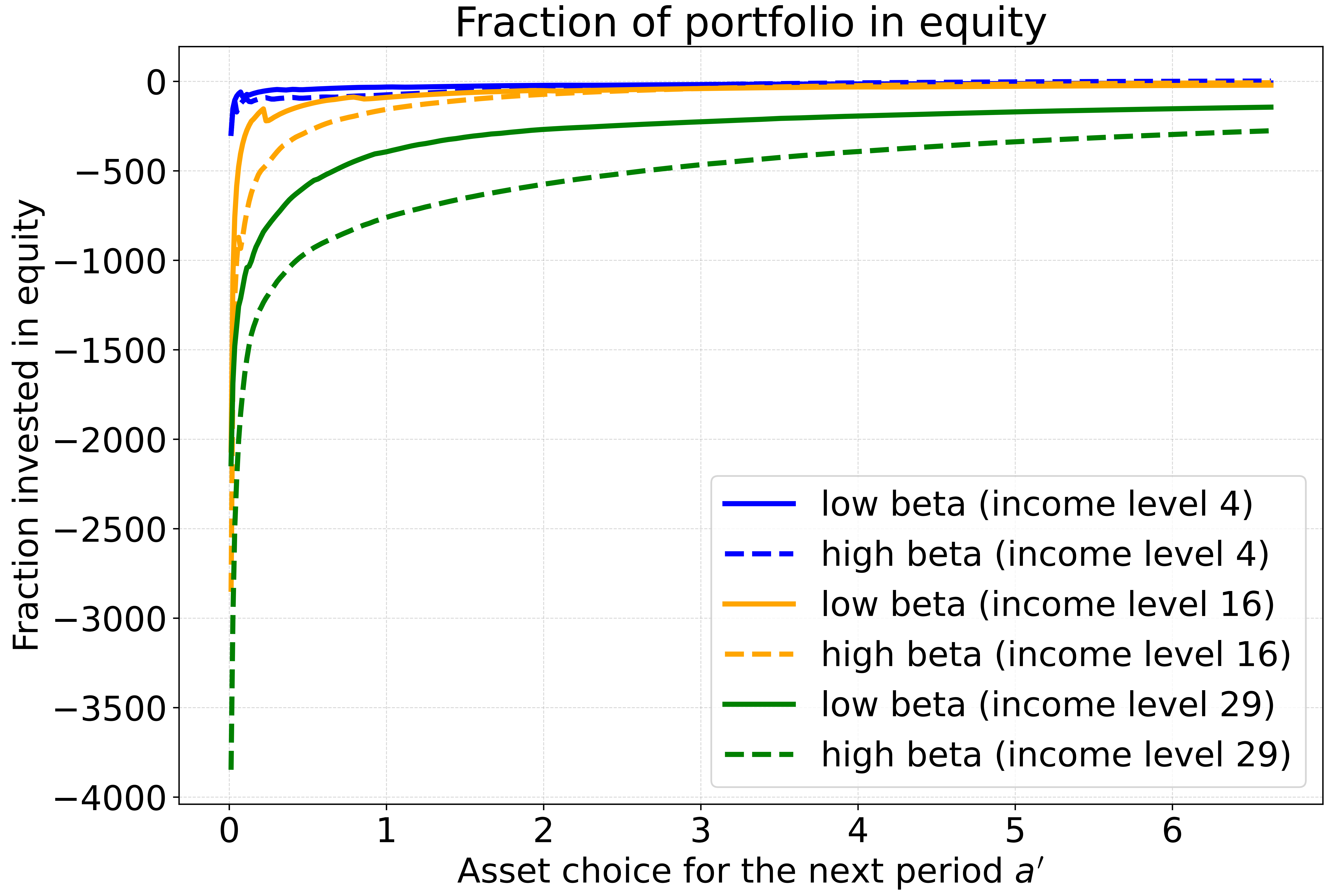

Large baseline output effects of deficit financed fiscal policy….

Largely undone by endogenous portfolios!

Poor disproportionately affected by shock, reduce stock exposure

But… how?

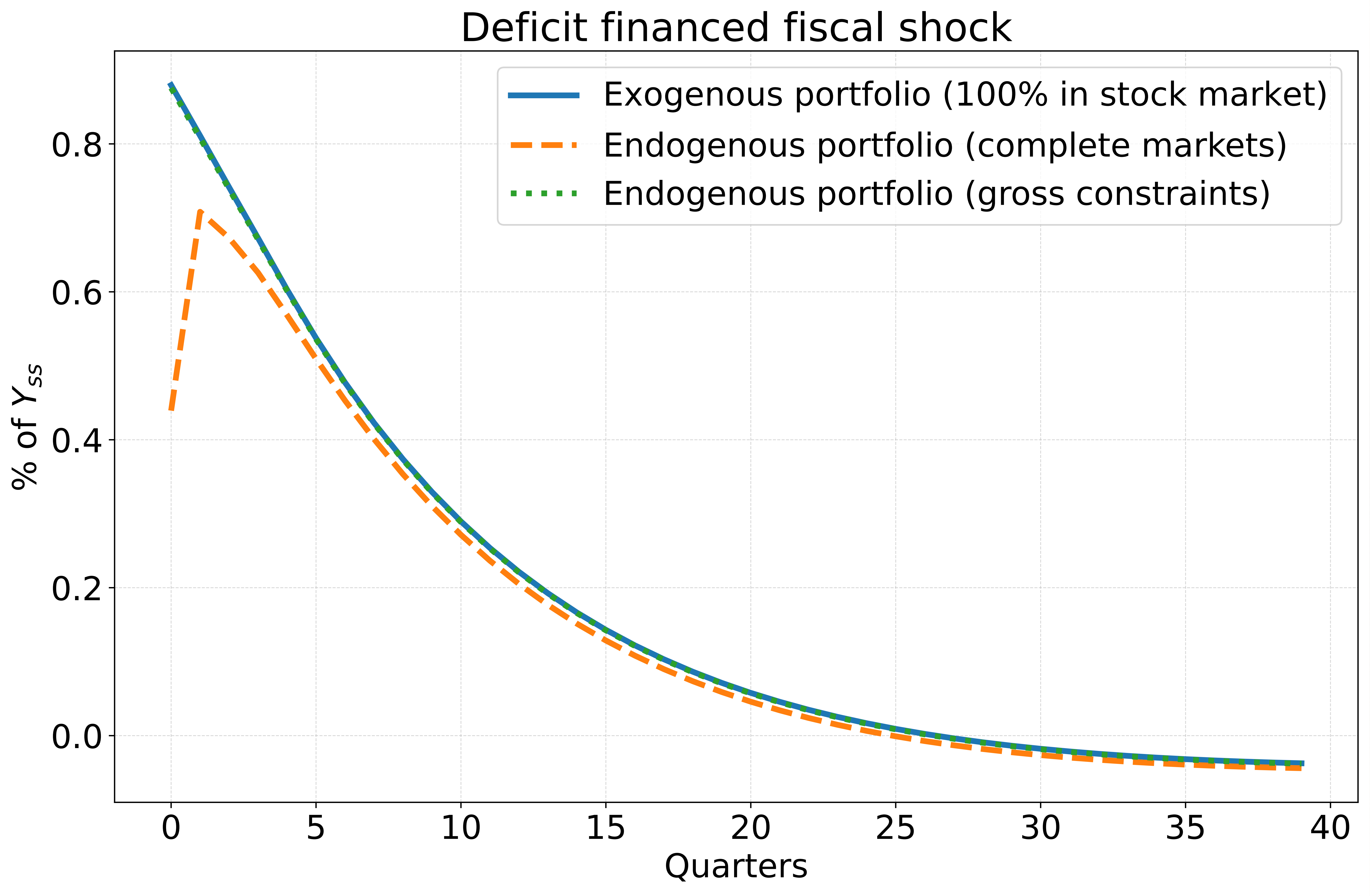
# Under the hood: implausible portfolio shares!




We didn't restrict gross positions in assets: borrowing constraint applied only to net position!

So "complete markets" transfers achieved with ultra-levered short-selling by the poor.

# With portfolio constraints… (no short sales, 1.5x leverage limit)




Constrained endogenous portfolio now ~same as exogenous portfolio: no effect!

**For endog. portfolios to matter, need high-MPC agents to be able to take large gross positions**

# Conclusion

- Simple modification of sequence-space Jacobian algorithm gives us:
  - impulses with endogenous portfolios and second-order risk premia
  - can add portfolio constraints, incomplete markets (see paper)
  - new directions for asset pricing + heterogeneous agent macro
  - new directions for sequence-space macro
- When do endogenous portfolios matter in canonical HANK model?
  - When shocks have heterogeneous consumption effects
  - When high MPC agents are able to take highly leveraged positions