
Lecture 9: Ramsey taxation in the sequence space

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Fiscal and Monetary Policy with Heterogeneous Agents

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Optimal Policy with heterogeneous agents?

- ❖ So far, focused on HANK, discussed lots of **positive** questions
 - ❖ e.g. effects of fiscal policy on output, monetary policy, ...
- ❖ Very little work on **normative** implications (hard!)
 - ❖ optimal capital & labor taxation? optimal level of public debt?
- ❖ **Next:** A first step ...
 - ❖ Optimal long-run fiscal policy
 - ❖ ... in a canonical HA model without NK

Ramsey steady state

- ❖ We focus on characterizing the **Ramsey steady state (RSS)**
 - ❖ long-run steady state of the full-commitment Ramsey plan
- ❖ A long literature characterizes the **RSS** in simpler models (RA, TA)
 - ❖ e.g. Chamley (1986), Judd (1985), Straub Werning (2020)
- ❖ We study the **RSS** in **neoclassical HA models**, à la Aiyagari
- ❖ Compare with “**optimal steady state**” (OSS)

Next: New “sequence-space” approach

1. Heterogeneous-agent household side, introduce **discounted elasticities**
2. Set up Ramsey problem and derive FOCs
3. Numerically evaluate FOCs, get Ramsey steady state for many specifications

Note: Generalizes to other stationary household sides (bonds in utility, OLG,...)

1. Heterogeneous-agent household side

Households

Just like before, except hours are optimally chosen by households:

$$\max_{\{c_{it}, n_{it}, a_{it}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, n_{it})$$

$$c_{it} + a_{it} = (1 + \boxed{r_t}) a_{it-1} + (1 - \boxed{\tau_t}) e_{it} n_{it} \quad a_{it} \geq 0$$

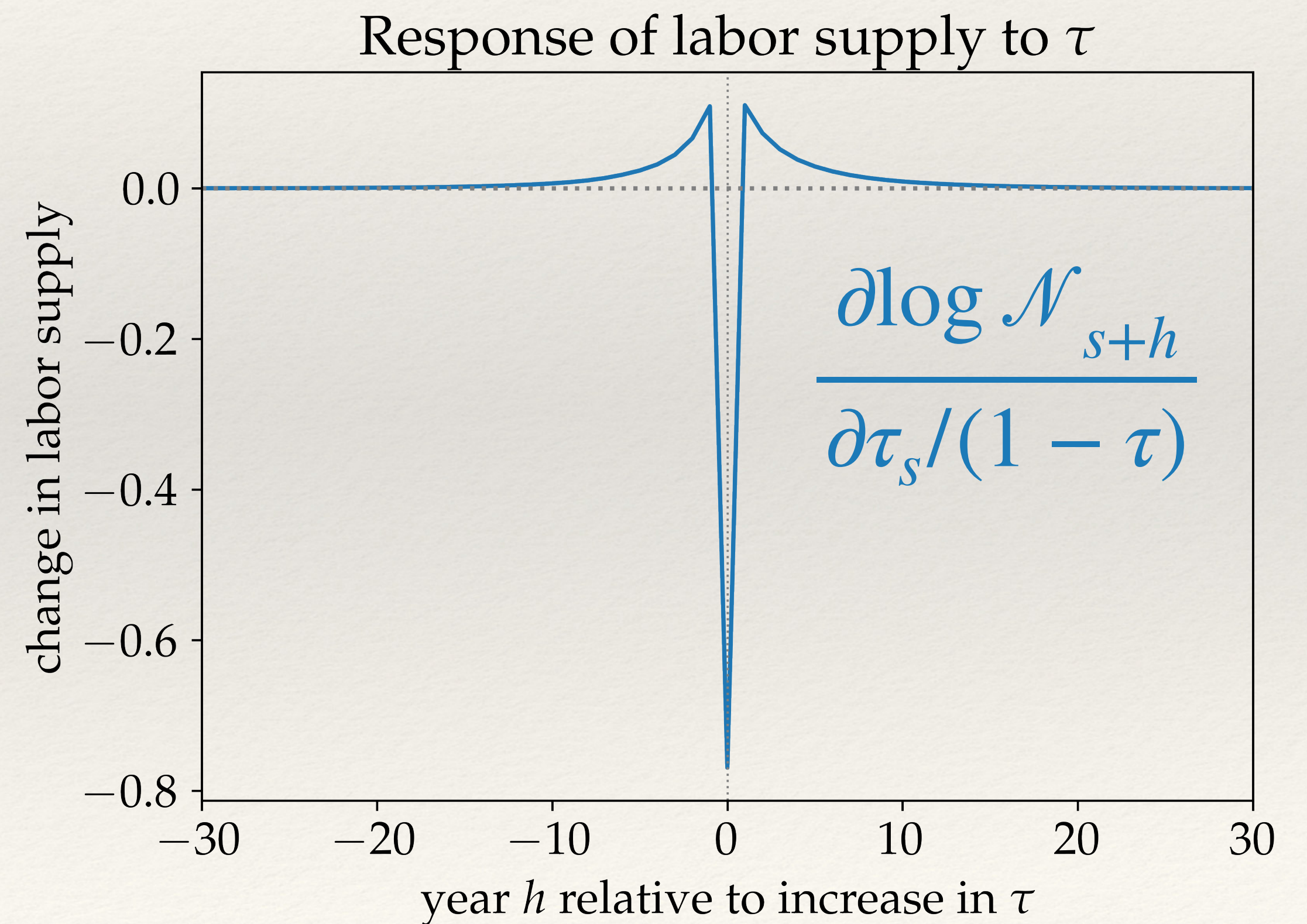
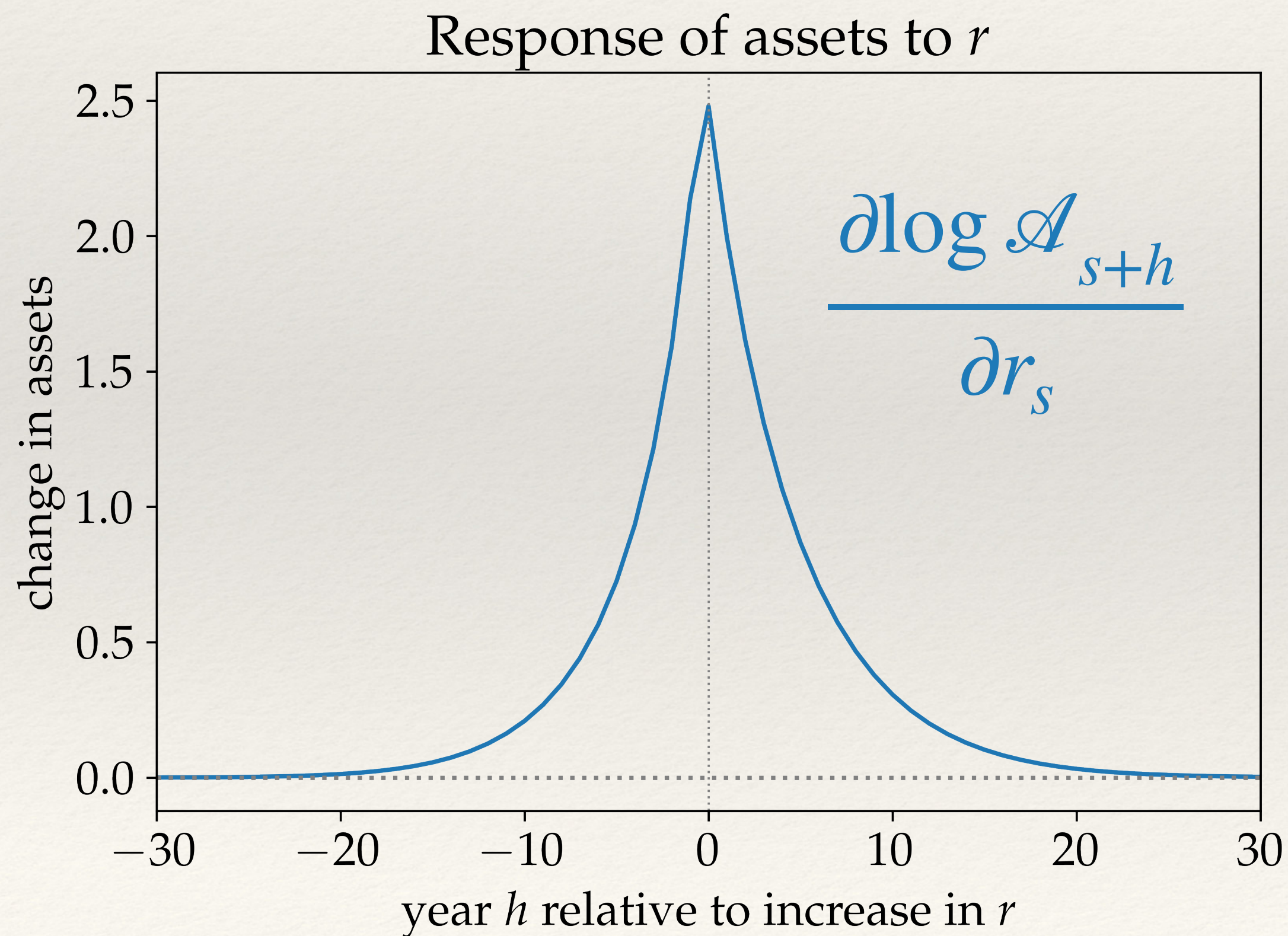
Inputs: interest rate and labor tax

Given $\{r_t\}, \{\tau_t\}$, can again aggregate household behavior using **sequence-space functions**:

Assets	$\mathcal{A}_t(\{r_s, \tau_s\}) = \int a_t dD_t$
Effective labor	$\mathcal{N}_t(\{r_s, \tau_s\}) = \int e n_t dD_t$
Utility	$\mathcal{U}_t(\{r_s, \tau_s\}) = \int u(c_t, n_t) dD_t$

Infinitely anticipated shocks

- ❖ Consider anticipated one-time shock at some far-out future date s



δ -discounted elasticities

- ❖ Define “discounted” version of these derivatives (around steady state with r, τ)

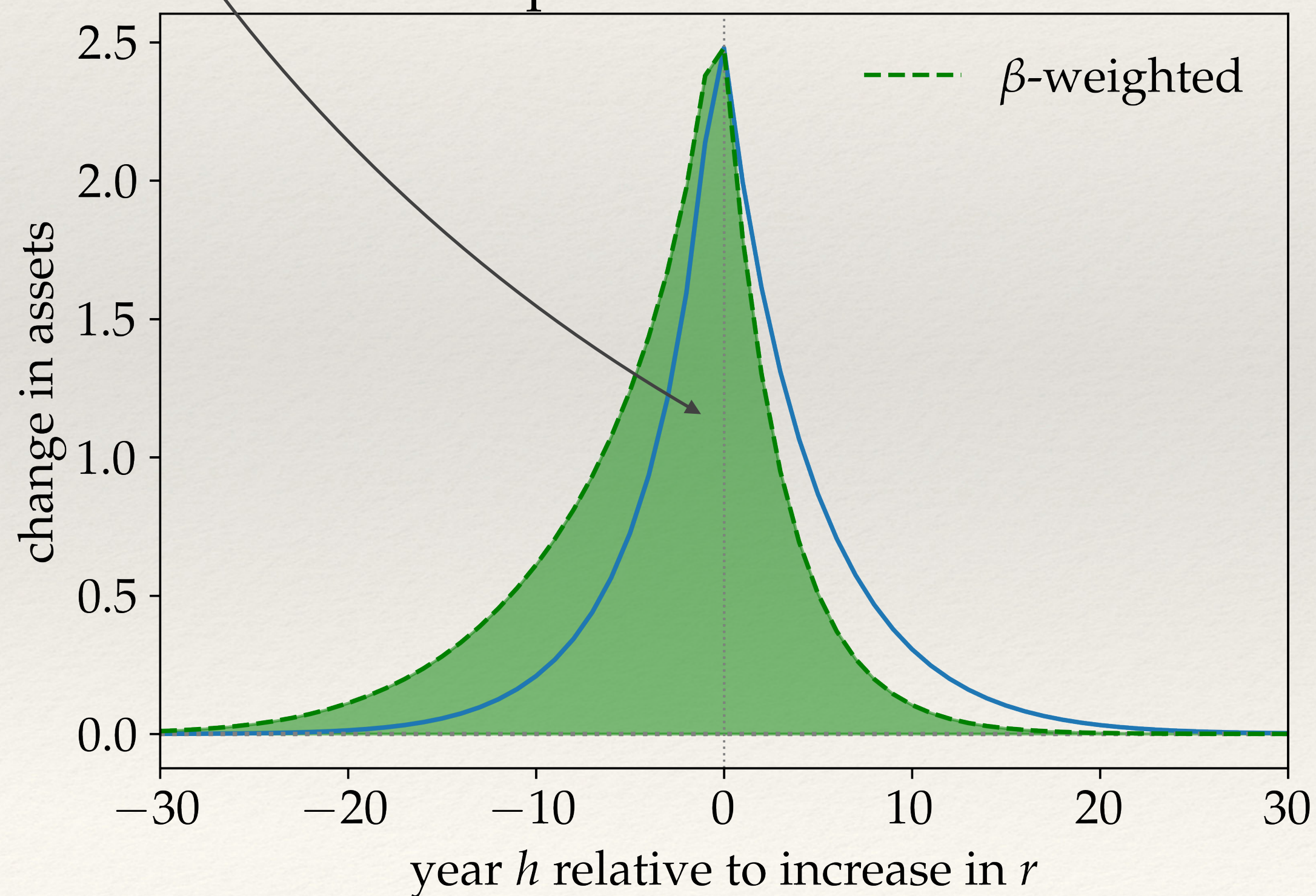
$$\epsilon^{A,r}(r, \tau) \equiv \lim_{s \rightarrow \infty} \sum_{h=-\infty}^{\infty} \delta^h \frac{\partial \log \mathcal{A}_{s+h}}{\partial r_s} \quad \epsilon^{N,\tau}(r, \tau) \equiv \lim_{s \rightarrow \infty} \sum_{h=-\infty}^{\infty} \delta^h \frac{\partial \log \mathcal{N}_{s+h}}{\partial \tau_s / (1 - \tau_s)}$$

- ❖ These elasticities are discounted with some δ (later social discount factor)
- ❖ Well-defined for $\delta \in [\beta, 1]$ precisely because the model is stationary!
- ❖ Define all the other elasticities similarly, e.g. $\epsilon^{N,r}, \epsilon^{A,\tau}, \epsilon^{U,r}$ etc

β -discounted elasticities

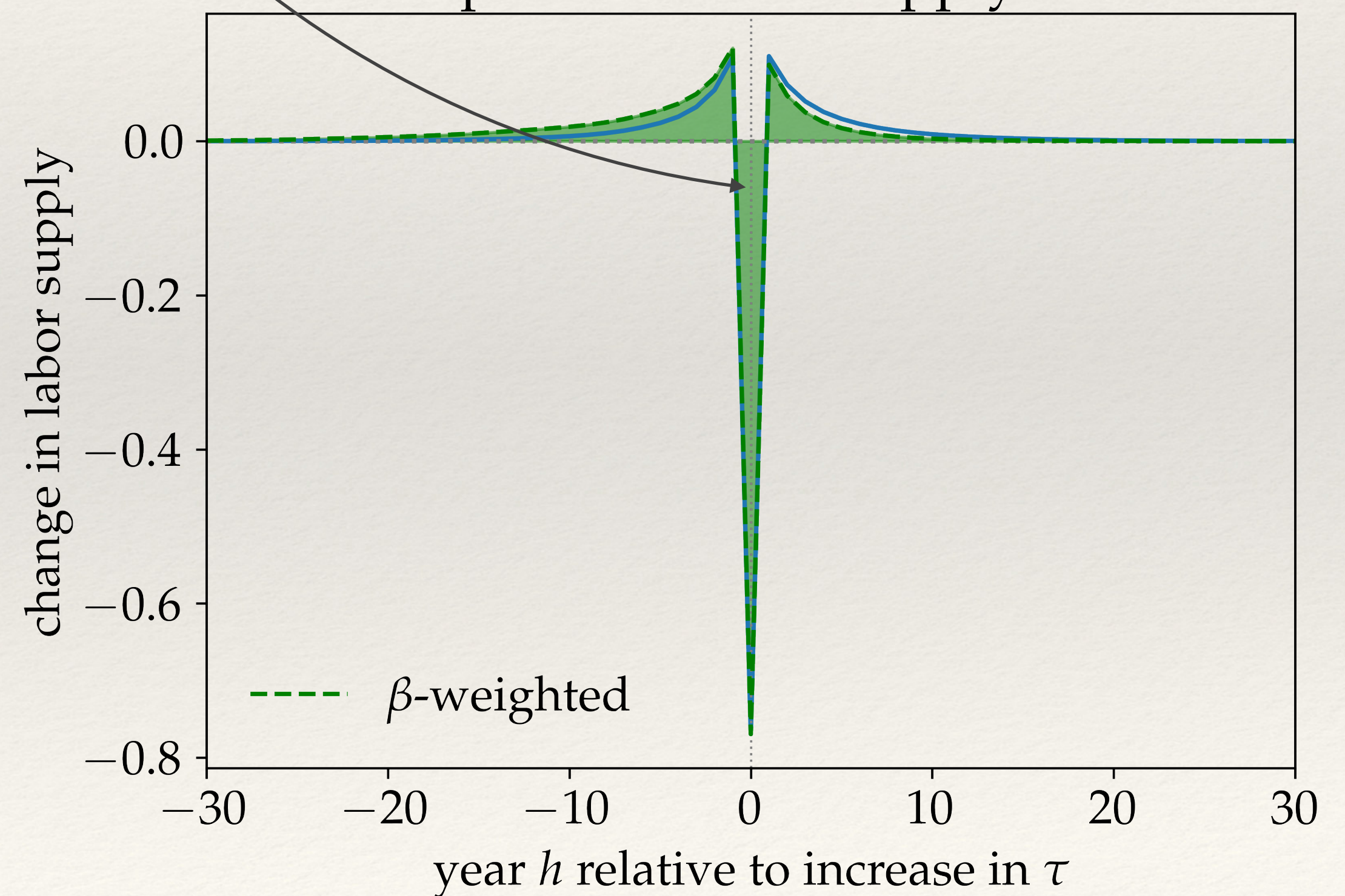
$$\epsilon^{A,r} \equiv \lim_{s \rightarrow \infty} \sum_{h=-\infty}^{\infty} \beta^h \frac{\partial \log \mathcal{A}_{s+h}}{\partial r_s} \approx 25$$

Response of assets to r



$$\epsilon^{N,\tau} \equiv \lim_{s \rightarrow \infty} \sum_{h=-\infty}^{\infty} \beta^h \frac{\partial \log \mathcal{N}_{s+h}}{\partial \tau_s / (1 - \tau)} \approx 0.15$$

Response of labor supply to τ



2. Ramsey problem

Model description

- ❖ We've seen how we can summarize household behavior using “sequence space” functions $\mathcal{A}_t, \mathcal{N}_t, \mathcal{U}_t$
- ❖ **Next:**
 - ❖ set up the rest of the model: supply side, government policies
 - ❖ derive an implementability condition
 - ❖ set up the Ramsey problem!

Production and government policy

- ❖ Representative firm: $Y_t = \mathcal{N}_t$, pre-tax wage = 1
- ❖ Government: spends fixed $G > 0$ (can relax)
- ❖ controls labor taxes $\{\tau_s\}$, budget constraint: $G + (1 + r_t) B_{t-1} = B_t + \tau_t N_t$

Implementability condition: $\{r_s\}, \{\tau_s\}$ part of an equilibrium iff

$$G + (1 + r_t) \mathcal{A}_{t-1} \left(\{r_s, \tau_s\} \right) = \mathcal{A}_t \left(\{r_s, \tau_s\} \right) + \tau_t \mathcal{N}_t \left(\{r_s, \tau_s\} \right)$$

Ramsey problem

Full-commitment Ramsey problem, with arbitrary social discount factor δ

$$\max_{\{r_s, \tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathcal{U}_t(\{r_s, \tau_s\})$$

$$G + (1 + r_t) \mathcal{A}_{t-1}(\{r_s, \tau_s\}) = \mathcal{A}_t(\{r_s, \tau_s\}) + \tau_t \mathcal{N}_t(\{r_s, \tau_s\})$$

- ❖ If solution converges to well-defined steady state ($r_s \rightarrow r < 1/\beta - 1$, $\tau_s \rightarrow \tau < 1$) we call this steady state a **Ramsey steady state (RSS)**.
- ❖ Multiplier on the constraint λ_t may or may not converge!
 - ❖ For today, assume it does, $\lambda_t \rightarrow \lambda$. Relax this in the paper.

Characterizing the Ramsey steady state

$$\max_{\{r_s, \tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathcal{U}_t(\{r_s, \tau_s\}) \quad G + (1 + r_t) \mathcal{A}_{t-1}(\{r_s, \tau_s\}) = \mathcal{A}_t(\{r_s, \tau_s\}) + \tau_t \mathcal{N}_t(\{r_s, \tau_s\})$$

❖ Begin with the FOCs with respect to r_s :

$$\sum_{h=-s}^{\infty} \delta^h \frac{\partial \mathcal{U}_{s+h}}{\partial r_s} + \sum_{h=-s}^{\infty} \delta^h \lambda_{s+h} \left(\frac{\partial \mathcal{A}_{s+h}}{\partial r_s} + \tau_{s+h} \frac{\partial \mathcal{N}_{s+h}}{\partial r_s} - (1 + r_{s+h}) \frac{\partial \mathcal{A}_{s+h-1}}{\partial r_s} \right) - \lambda_s \mathcal{A}_{s-1} = 0$$

$\epsilon^{U,r}$ as $s \rightarrow \infty$ $A\lambda \cdot \epsilon^{A,r}$ $\tau N\lambda \cdot \epsilon^{N,r}$ $A\lambda(1+r)\delta \cdot \epsilon^{A,r}$ λA

Characterizing the Ramsey steady state

$$\max_{\{r_s, \tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \mathcal{U}_t(\{r_s, \tau_s\}) \quad G + (1 + r_t) \mathcal{A}_{t-1}(\{r_s, \tau_s\}) = \mathcal{A}_t(\{r_s, \tau_s\}) + \tau_t \mathcal{N}_t(\{r_s, \tau_s\})$$

- ❖ From the r_s derivative around the (unknown) RSS:

$$\lambda^{-1} \epsilon^{U,r} = A - (1 - \delta(1 + r)) A \epsilon^{A,r} - \tau N \epsilon^{N,r}$$

- ❖ Same procedure applied to the τ_s derivative:

$$\lambda^{-1} \epsilon^{U,\tau} = (1 - \tau) N - (1 - \delta(1 + r)) A \epsilon^{A,\tau} - \tau N \epsilon^{N,\tau}$$

Two helpful objects: $\ell \equiv \frac{A}{(1 - \tau)N}$ as *liquidity*; $m \equiv -\epsilon^{U,\tau}/\epsilon^{U,r} > 0$ as *effective MRS*.

RSS optimality condition

❖ If allocation converges to a well-defined RSS with interest rate r and tax rate τ , and if λ_t converges, then (r, τ) are characterized by:

1. The steady-state government budget constraint

$$G + r\mathcal{A}(r, \tau) = \tau\mathcal{N}(r, \tau)$$

2. Optimality condition

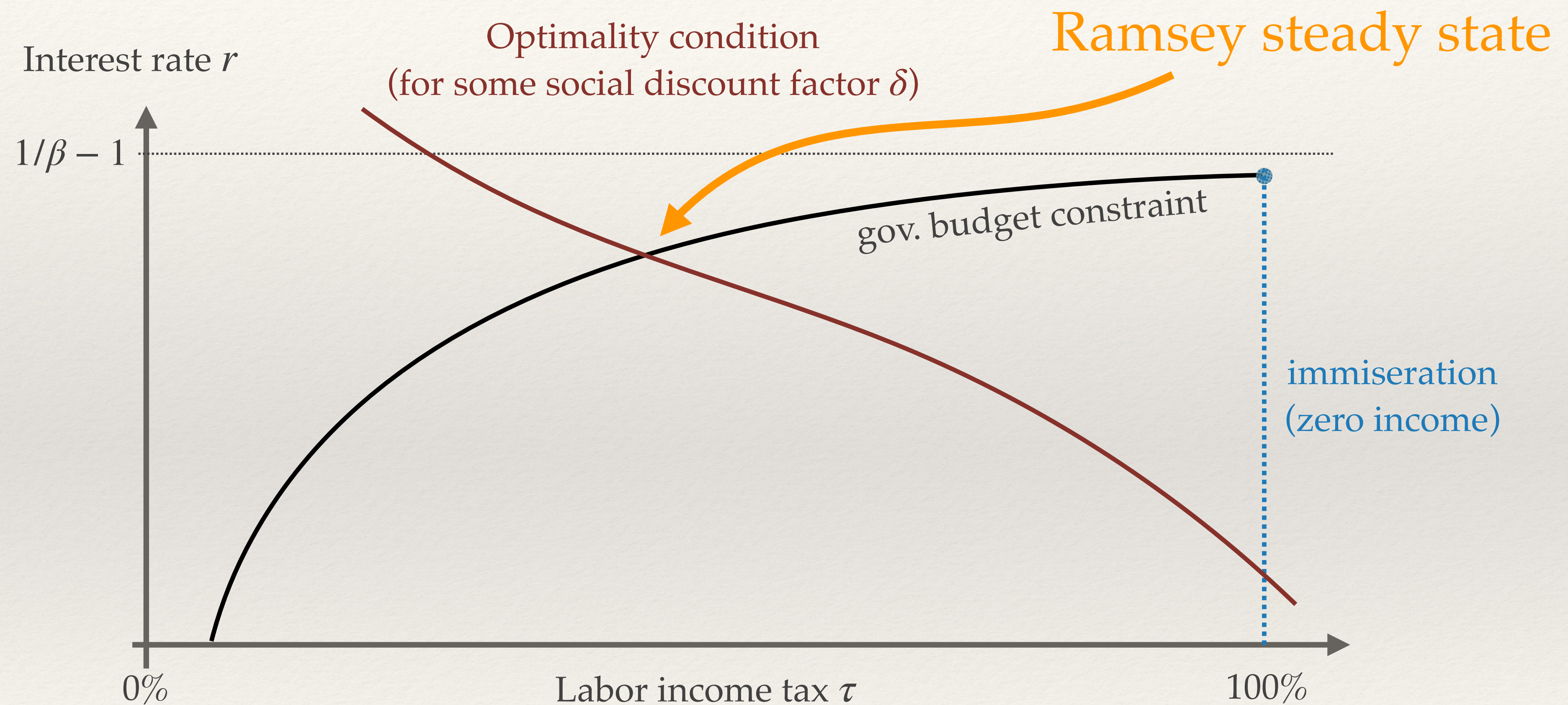
$$\underbrace{(1 - (1 + r)\delta) \ell (m\epsilon^{A,r} + \epsilon^{A,\tau})}_{\text{liquidity benefit of greater debt}} - \underbrace{\frac{\tau}{1 - \tau} (-\epsilon^{N,\tau} - m\epsilon^{N,r})}_{\text{cost (?) lower labor supply}} - \underbrace{(\ell m - 1)}_{\text{cost: redistribution from workers to savers}} = 0$$

liquidity **benefit** of greater debt

cost (?) lower labor supply

cost: redistribution from
workers to savers

The RSS first order condition



3. Searching for an RSS

Utility functions

- ❖ To solve this system of equations, need to go to the computer.
- ❖ Begin with $u(c, n) = \log c - v(n)$ with constant Frisch elasticity = 1
- ❖ Standard calibration: (details are not important)
 - ❖ AR(1) income process, initial debt = 100%, $G = 20\%$, initial $r = 2\%$
- ❖ **Idea:** For each τ , solve government budget constraint for r and evaluate FOC

The missing RSS

❖ Assume “correct” social discount factor, $\delta = \beta$. Left hand side of FOC:

liquidity **benefit** of greater debt

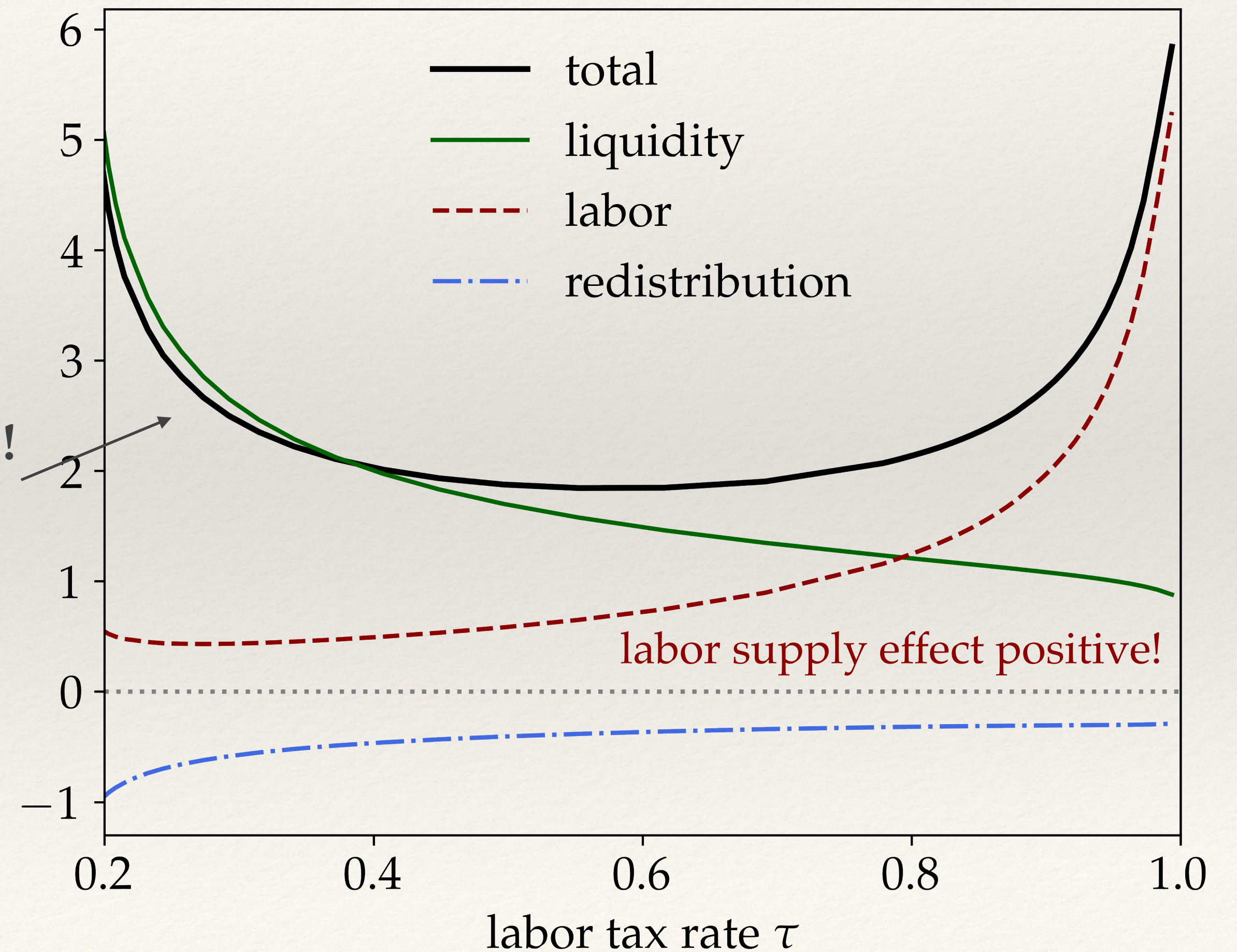
$$(1 - \beta(1 + r)) \ell (m\epsilon^{A,r} + \epsilon^{A,\tau})$$

$$-\frac{\tau}{1 - \tau} (-\epsilon^{N,\tau} - m\epsilon^{N,r}) - (\ell m - 1)$$

cost: redistribution

benefit: greater labor supply

Always > 0 !
No RSS!



Optimal steady state exists

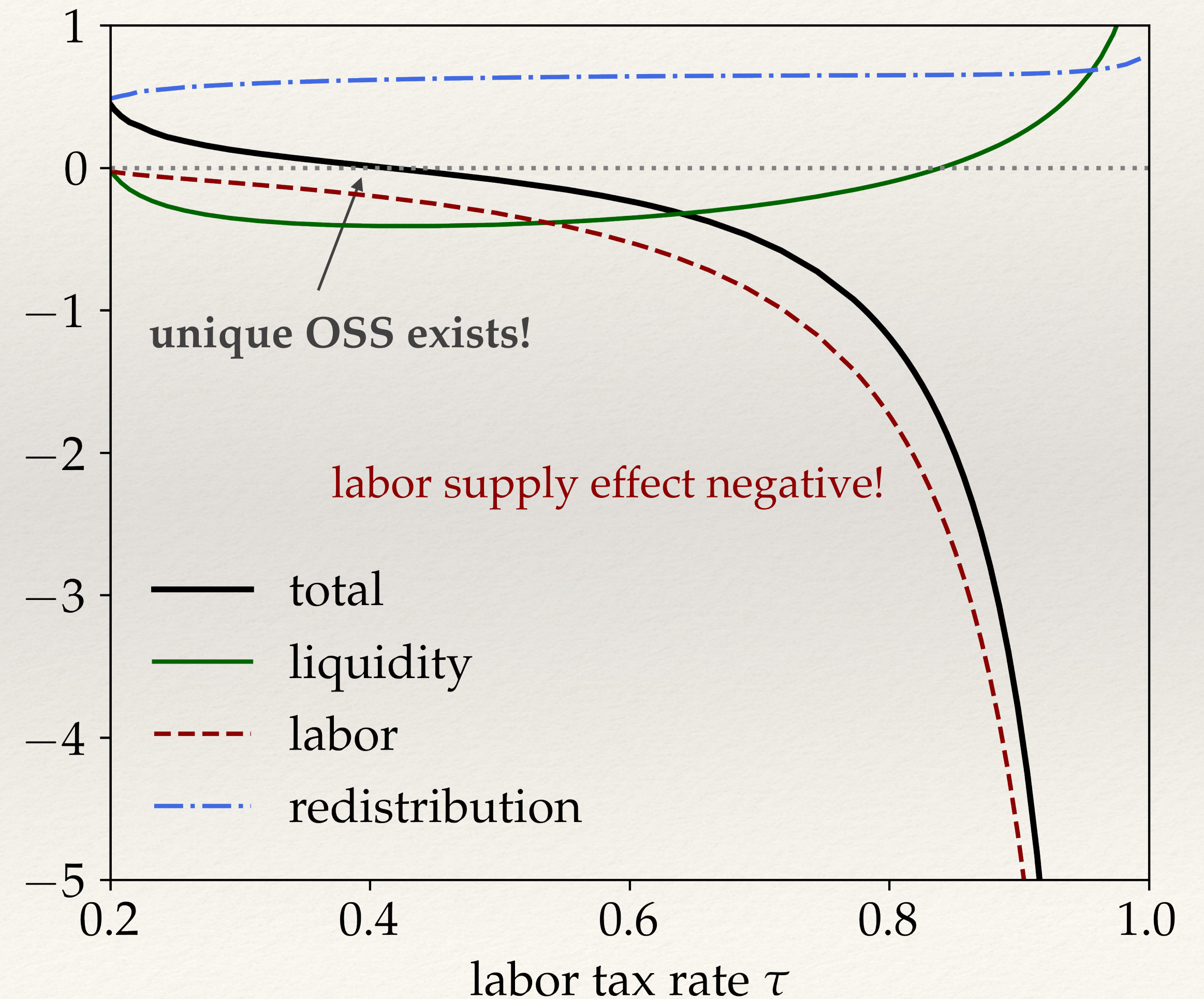
❖ Same with infinitely patient planner, $\delta = 1$:

liquidity **benefit** of greater debt

$$(1 - (1 + r)) \ell (m \epsilon^{A,r} + \epsilon^{A,\tau})$$

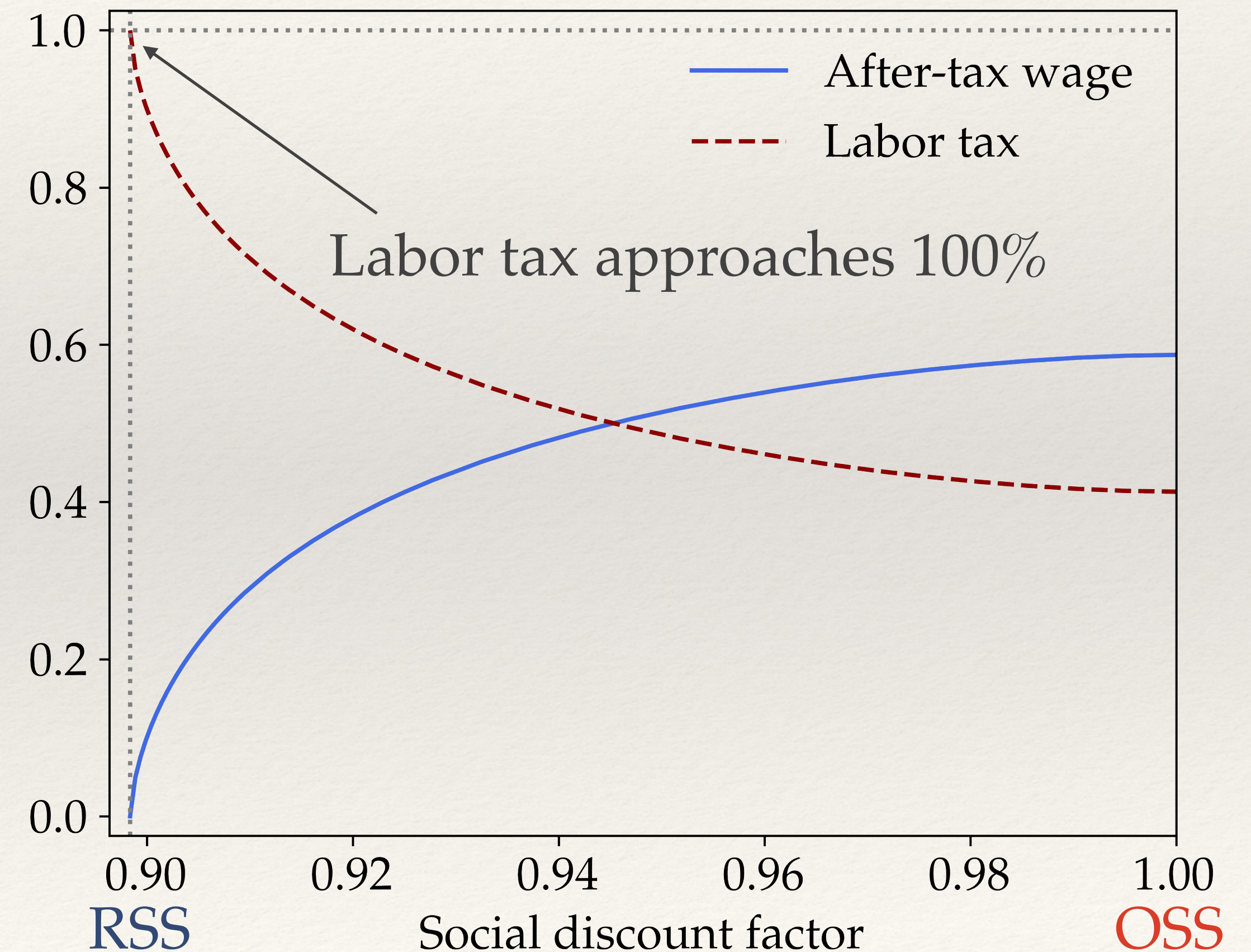
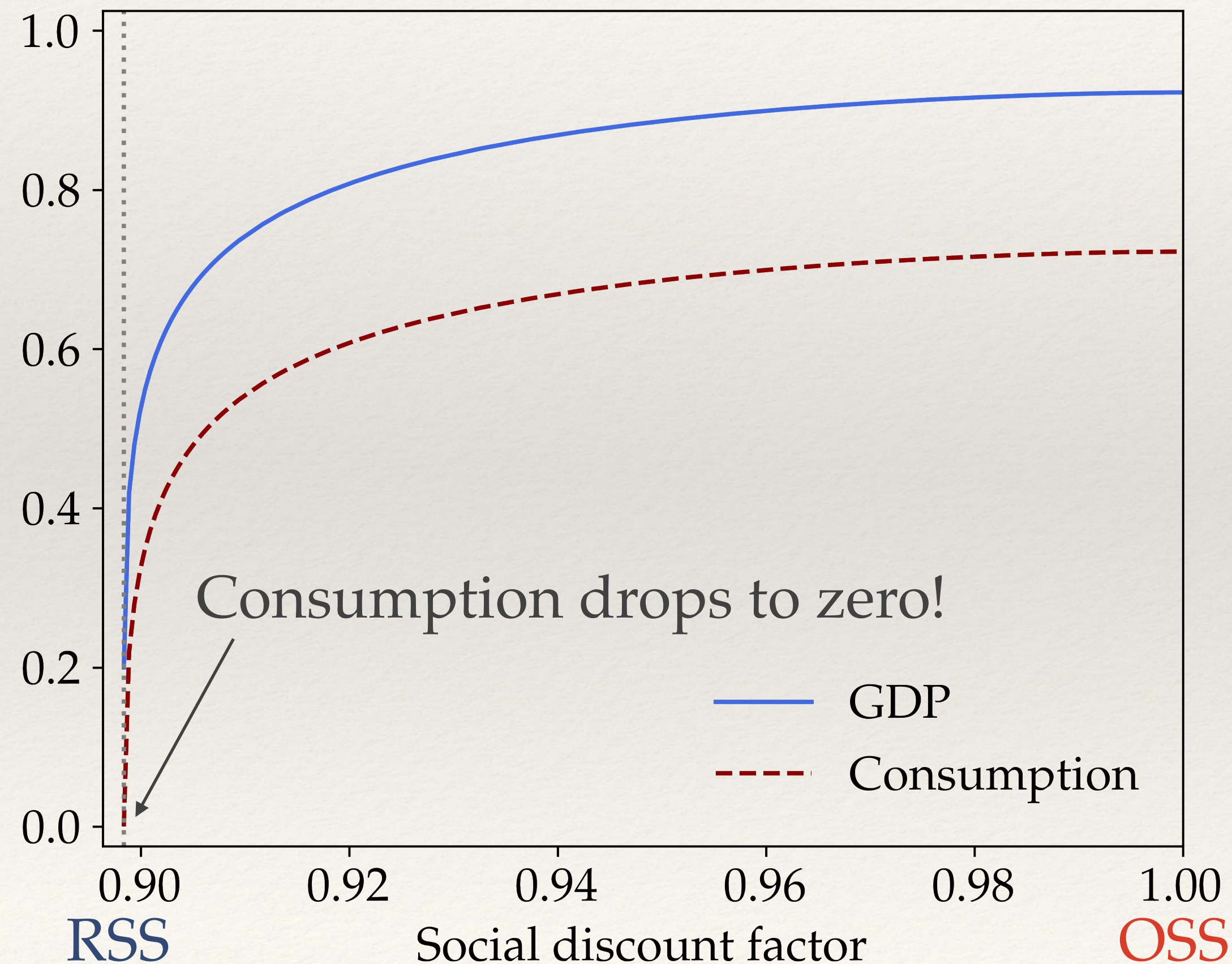
$$-\frac{\tau}{1 - \tau} (-\epsilon^{N,\tau} - m \epsilon^{N,r}) - (\ell m - 1)$$

cost: lower labor supply **cost:** redistribution



How the RSS vanishes

- ❖ Next, vary social discount factor δ between β and 1:



Standard Aiyagari economy: Why no RSS?

Benefits and costs to greater liquidity and higher labor taxes

liquidity benefit

labor supply $\uparrow$



~~labor supply~~

redistribution

cost of redistribution is quantitatively small!

Taking stock

- ❖ New method to compute Ramsey steady states in richer models than RA, TA
- ❖ Discounted elasticities of “sequence space” functions are key!
- ❖ **Insight:** RSS very extreme for standard balanced-growth Aiyagari models!
- ❖ Wide open field with many possible applications !

THANK YOU !!