The Macroeconomics of Household Debt Relief

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Abstract

Policymakers regularly discuss enacting debt relief programs whose apparent goals are to redistribute towards debtors and/or to stabilize the economy. We examine the macroeconomic and welfare effects of these types of programs. Unexpected, ex-post debt relief can boost aggregate demand and improve welfare, but these benefits can be offset by expectations of future debt relief, which contract credit supply. Existing empirical evidence supports the model's assumptions and conclusions. We argue that the stabilization benefits of debt relief are best realized by committing to rules that vary systematically with the state of the business cycle.

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1 Introduction

Governments are in charge of setting up the legal framework determining the penalties for not repaying one's debts, and therefore the extent to which households repay their debts in practice. This framework is regularly revised, as perceptions that these penalties are "too harsh" or "too lenient" fluctuate. From the Panic of 1797 to the Great Depression, recessions and financial crises have triggered overhauls of the bankruptcy system throughout U.S. history. In fact, the first major reform that was *not* put in place in response to a severe economic contraction was the Bankruptcy Reform Act of 1978 (Tabb 1995). Governments also have a long history of intervening in existing private debt contracts, by providing debt moratoria, forbearance, or outright forgiveness (Bolton and Rosenthal 2002). Salient recent examples of such ex-post interventions in debt contracts include the foreclosure and student debt moratoria that have followed the Covid-19 crisis, and the current Biden administration proposals for cancelling student debt.

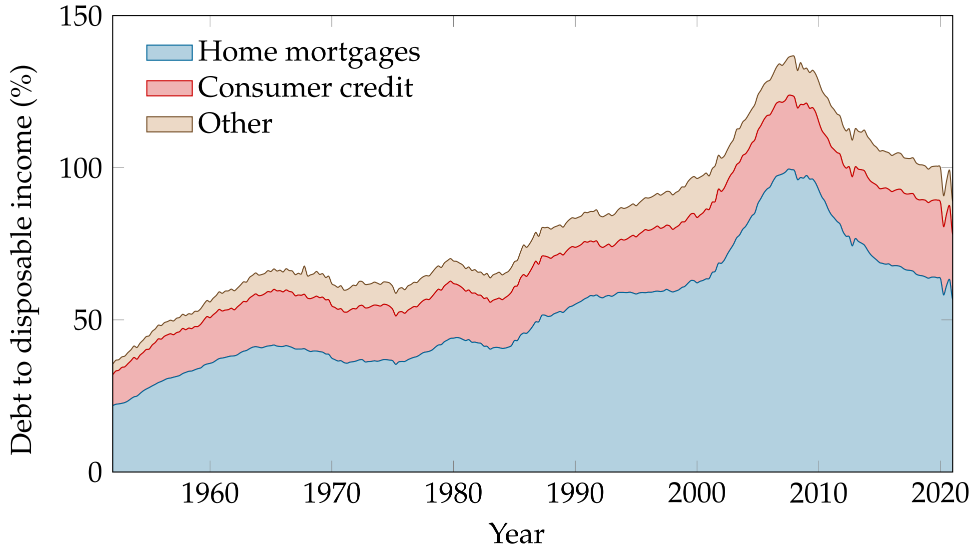
From an economic standpoint, there are two main reasons why governments may want to intervene in debt contracts. First, they may think that these interventions are needed to achieve a desired level of redistribution that cannot be achieved through other redistributive tools, such as the tax system. Second, they may believe that overly debt-burdened households are holding back aggregate demand and exacerbating recessions. Current proposals for debt relief and bankruptcy reform emphasize both of these objectives (see, e.g., the Consumer Bankruptcy Reform Act of 2020).[1] These proposals have recently taken center stage in the policy debate because household debt is so elevated. In the United States, it represents about 100% of disposable income, which is slightly lower than it was at the peak of the Great Recession, but almost double its level of fifty years ago (see figure 1).

In this paper, we provide a simple framework to organize the discussion around these two governmental objectives. The framework is a streamlined version of our ongoing work in Auclert and Mitman (2022), which primarily focuses on the stabilization objective. Here, we additionally discuss the welfare effects of government interventions in debt contracts. We hope that our findings can help contribute to the important policy debate around household debt relief.

Our framework builds on standard models of consumer bankruptcy in the literature (e.g., Chatterjee, Corbae, Nakajima and Ríos-Rull 2007, Livshits, MacGee and Tertilt 2007). A government sets the legal framework for debt repayment, in the form of (utility) penalties that households face for not repaying their debts and financial market exclusion. Households and banks internalize this legal framework, and this determines credit demand and supply.

We first consider what happens if the government changes the rules after initial borrowing and lending decisions are made. We study the impact of such changes on welfare and on aggregate demand. We show that unexpected debt relief, in the form of declines in default penalties with (a credible promise of) no change in penalties going forward, can raise welfare and stimulate aggregate demand.

[1] https://www.congress.gov/bill/116th-congress/house-bill/8902/



Source: Federal Reserve Financial Accounts of the US (Z1 release).

Figure 1: Household debt and its composition in the U.S.

Next, we consider what happens if the government pre-announces debt relief. We show that announced declines in future penalties tend to hurt aggregate demand, because banks respond to them by shifting in credit supply. We view this result as a cautionary tale for ad-hoc interventions, which are likely to trigger some changes in credit supply.

We then review the existing empirical evidence and argue that it supports both the model's assumptions and its aggregate predictions. At the micro level, the model framework is well supported by existing evidence on consumer default behavior and bank pricing decisions. At the aggregate level, there is evidence of both the positive ex-post effects on aggregate demand and the negative ex-ante effects from the change in credit supply that are the key implications of our model.

Finally, we discuss the policy implications of our framework. We first observe that it is difficult to make debt relief a credible one-time event, so that any debt relief will likely trigger a response of credit supply. We then turn to a proposal that can avoid these credibility issues: conduct systematically more debt relief in recessions, and less in booms—that is, use consumer bankruptcy as part of the aggregate demand management toolkit. Our model suggests that, if well calibrated, such a rule could dampen aggregate fluctuations. The reason is that consumer defaults, which are countercyclical, already provide some automatic stabilization benefits. Our proposal would help magnify this effect.

2 Theory

In this section, we write down a simplified version of the model in Auclert and Mitman (2022), which is a macroeconomic model of defaultable household debt. We adapt the results in that paper to this simpler context, and add two results on the welfare implications of debt relief. We consider a policymaker that can enact household debt relief to achieve an objective in terms of maximizing welfare (i.e., a redistribution motive) or boosting GDP (i.e., an aggregate demand motive).

2.1 Model setup

There are two periods $t = 0, 1$, which we think of as the "short-run" and the "long-run". Aggregate GDP in period t is y_t. In period 0, the economy may be depressed, i.e., y_0 is determined by the level of aggregate demand and may be below its potential y_0^*. In period 1, GDP is always at potential, which we normalize to $y_1 = y_1^* = 1$.[2]

The economy is populated by households, financial intermediaries and the policymaker. There are two types of households: borrowers B and savers S, with mass $1/2$ each. Financial intermediaries who lend to borrowers in period 0 and collect debts in periods 0 and 1. The savers own these financial intermediaries.

Borrowers start date 0 with some legacy debt b_0 that they owe to the financial intermediaries. This debt is defaultable: if a borrower chooses to default, he is excluded from financial markets and has to consume his income in both periods. If the borrower repays, he can continue to borrow for the next period. At date 1, he again gets to choose whether to repay b_1 or default.

In addition to financial market exclusion, defaulting in period t entails a flow utility cost K_t. Since the world ends after period 1, K_1 is the only reason why agents repay any debt in that period. We think of K_t as representing, for instance, the various non-economic cost of defaults that lead people to repay rather than default—the literature, which we review in section 3, suggests that these costs can be substantial. In addition, we think of the policymaker as being able to manipulate K_t to some extent. By lowering K_t, a policymaker makes it easier to default in period t. At the same time, if lenders anticipate this, it will be harder to borrow in period $t - 1$. Changing K_0 captures pure ex-post debt relief, since lenders have no time to change the terms of debt contracts. By contrast, the effect of changing K_1 on date-0 outcomes captures the ex-ante effect of an announcement of future debt relief. In this section, we assume that the policymaker can independently manipulate K_0 and K_1. We come back to this assumption in section 4.

We now specify the borrowers' problem in more detail. Borrowers have log utility and are ex-ante homogeneous. They experience an income shock in both periods, in the form of an i.i.d idiosyncratic draw e_t from a distribution with cumulative density function F, which we assume to have continuous density f and an increasing hazard. When aggregate income is y_t and the income shock is e_t, the borrower's income is $e_t y_t$.

[2] We can microfound the difference between potential and actual GDP by assuming nominal wage rigidities in period 0. All the details are in Auclert and Mitman (2022).

A borrower in period 0, with income shock e_0, solves the following problem:

$$\max\left\{\underbrace{\max_{b_1}\left\{\log\left(y_0e_0-b_0+Q\left(b_1\right)\right)+\beta V\left(b_1\right)\right\}}_{V^r(e_0)},\underbrace{\log\left(y_0e_0\right)+\beta\mathbb{E}\left[\log\left(e_1\right)\right]-K_0}_{V^d(e_0)}\right\} \tag{1}$$

where the value of entering period 1 with debt b_1, before seeing the period 1 income realization, is

$$V\left(b_1\right)=\mathbb{E}_{e_1}\left[\max\left\{\log\left(e_1-b_1\right),\log\left(e_1\right)-K_1\right\}\right] \tag{2}$$

In both periods, a borrower in good credit standing observes income shock realization e_t and then chooses whether to repay or default.

From equation (2), it is clear that the borrower chooses to default in period 1 whenever $e_1 < \overline{e_1} \equiv \frac{b_1}{1-e^{-K_1}}$. The default region, for any given b_1, is therefore a segment $[0,\overline{e_1}]$ that is larger when the level of debt b_1 is higher and the penalty level K_1 is lower, since higher debt and lower penalties both make default more attractive. The shaded orange region of Figure 2 illustrates the default region at a high level of K_1, and the blue region illustrates how it expands with lower penalties. Since e_1 has cumulative distribution function F, it follows that the fraction of borrowers defaulting is:

$$d_1\left(b_1\right)=F\left(\overline{e_1}\right)=F\left(\frac{b_1}{1-e^{-K_1}}\right)$$

which is increasing in b_1 and decreasing in K_1.

Financial intermediaries price loans competitively given a safe real cost of funds of R between date 0 and date 1. This implies that they price any loan at the discounted expected probability of repayment, and make no profits on average. Therefore, the amount of funds that the period 0 borrower can get by promising to repay b_1 is

$$Q\left(b_1\right)=\frac{b_1}{R}\left(1-d_1\left(b_1\right)\right)=\frac{b_1}{R}\left(1-F\left(\frac{b_1}{1-e^{-K_1}}\right)\right) \tag{3}$$

Our assumptions imply that the schedule $Q\left(b_1\right)$ has a "Laffer curve" shape in b_1, as illustrated in the right panel of Figure 2. Borrowing more raises the probability of default in period 1, and financial intermediaries pass this through to borrowers by lending them less per unit borrowed.[3] Beyond the peak of the Laffer curve, this latter effect dominates, and they lend less overall, so that it is never optimal for borrowers to choose b_1 beyond this peak. Similarly, when penalties K_1 are lower, lenders reduce lending for any given promise and the peak of the Laffer curve shifts to the left, as a comparison between the blue and orange line shows. This argument shows that the partial derivative of Q with respect to K_1 is positive, $\frac{\partial Q}{\partial K_1}>0$.

[3] In other words, they charge them higher spreads. Since the effective interest rate on the loan is $R/\left(1-F\left(\frac{b_1}{1-e^{-K_1}}\right)\right)$, the spread to the risk-free rate is $R\left(\left(1-F\left(\frac{b_1}{1-e^{-K_1}}\right)\right)^{-1}-1\right)$.

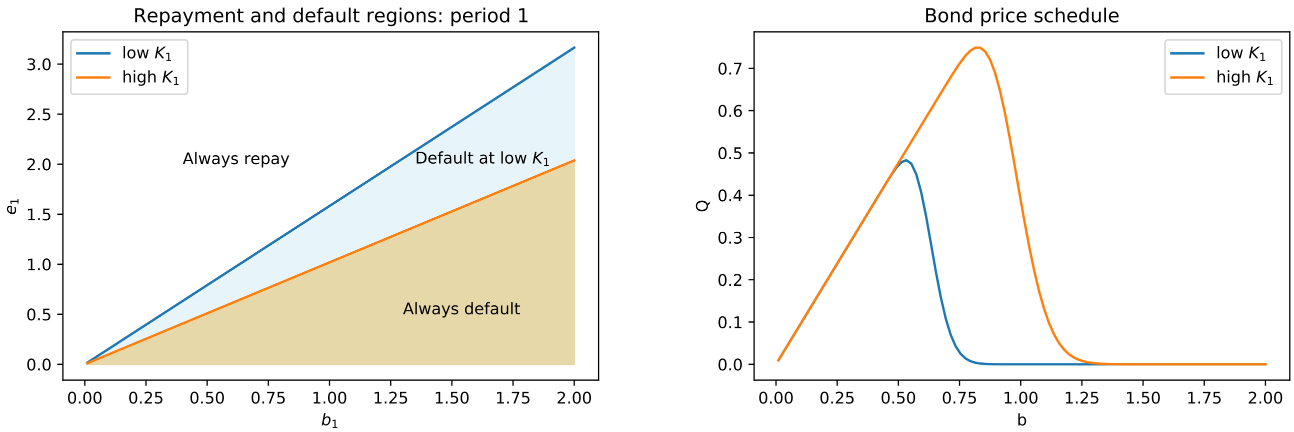


Figure 2: Repayment region and bond price schedule

In period 0, households solve the maximization problem (1) taking the loan schedule (3) as given. We write the fraction of borrowers that choose to default in period 0 as d_0. A key question for us is how d_0 varies with penalties K_0 and the level of GDP y_0. We study these questions in sections 2.2 and 4.2, respectively.

For their part, savers simply smooth consumption across the two periods. They face no income uncertainty, have log utility, and discount the future at rate β^S. They collect labor income, which is y_0 in period 0 and 1 in period 1, as well as the profits made by the intermediary. Their problem, taking into account the value Π of intermediary shares, is therefore:

$$\max \log\left(c_0^S\right) + \beta^S \log\left(c_1^S\right)$$
$$\text{s.t.} \quad c_0^S + \frac{c_1^S}{R} = y_0 + \frac{1}{R} + \Pi$$

Given that the intermediary makes no profits on any loan between date 0 and date 1, the value of the intermediary starting from date 0 is just the value of the legacy debts that are repaid:

$$\Pi = (1 - d_0)\, b_0$$

Equilibrium. Household consumption is the only source of demand for goods. Denoting by $c_0^B(e_0)$ the consumption policy of a borrower with income shock e_0, aggregate consumption in period 0 is defined as:

$$c_0 \equiv \frac{1}{2}\int c_0^B(e_0)\, dF(e_0) + \frac{1}{2}c_0^S$$

similarly, denoting by $c_1^B(e_1, b_1)$ the consumption policy of a borrower with income shock e_1 and debt b_0 in period 1, and by $b_1(e_0)$ the debt policy in period 0, aggregate consumption in period 1 is defined as:

$$c_1 \equiv \frac{1}{2}\int\int c_1^B(e_1, b_1(e_0))\, dF(e_1)\, dF(e_0) + \frac{1}{2}c_1^S$$

In a *small open economy*, R, y_0 and $y_1 = 1$ are all given. The economy starts period 0 and ends period 1 with no net claim to the rest of the world. Any excess of absorption c_0 over production y_0 in period 0 is resolved by net imports in period 0, and net exports in period 1.

In a *demand determined economy*, R and $y_1 = 1$ are also given, but y_0 adjusts to clear the goods market in period 0,[4] that is:

$$c_0 = y_0$$

By Walras's law, the goods market then also clears in period 1.

Policy-maker tools and objectives. We assume that the policymaker can choose K_0 and K_1. This policymaker may be interested in raising the level of consumption c_0, the level of equilibrium short-run output y_0, or in maximizing societal welfare, placing a certain weight λ on savers:

$$W = \int\int \left(u\left(c_0^B\left(e_0\right)\right) + \beta u\left(c_1^B\left(e_0, e_1\right)\right)\right) dF\left(e_0\right) dF\left(e_1\right) + \lambda\left(u\left(c_0^S\right) + \beta^S u\left(c_1^S\right)\right)$$

where $u\left(c\right) \equiv \log c$.

2.2 Ex-post debt relief

We first study the macroeconomic and welfare implications of changing the default laws in period 0 in a completely unexpected way, as captured in our model by changing K_0.

Period 0 default decision. A household entering period 0 with income shock e_0 chooses the action that maximizes the value in (1). We write $c_0^r\left(e_0\right)$ for the policy function in case of repayment, and $c_0^d\left(e_0\right)$ for the policy in case of default. We further define the Consumption Effect of Default at e_0, or CED, as:

$$CED\left(e_0\right) = \frac{c_0^d\left(e_0\right) - c_0^r\left(e_0\right)}{b_0} = \frac{b_0 - Q\left(b_1\left(e_0\right)\right)}{b_0} \tag{4}$$

The CED is the additional consumption that a given household enjoys over a certain period if he defaults rather than repays, normalized by his level of debt. It is also the share of that household's debt that he repays rather than roll over. Since higher income households tend to save more (borrow less), the CED is increasing in income e_0. Moreover, the CED is tightly connected to incentives to repay rather than default. Indeed, the envelope theorem and log utility together imply that, at all levels of income,

$$\frac{\left(V^r\right)'\left(e_0\right) - \left(V^d\right)'\left(e_0\right)}{u'\left(c_0^r\left(e_0\right)\right)} = \frac{c_0^d\left(e_0\right) - c_0^r\left(e_0\right)}{c_0^d\left(e_0\right)} = CED\left(e_0\right) \times \frac{c_0^d\left(e_0\right)}{b_0} \tag{5}$$

[4] We can microfound this assumption by assuming that the central bank is at the zero lower bound and and achieves no inflation.

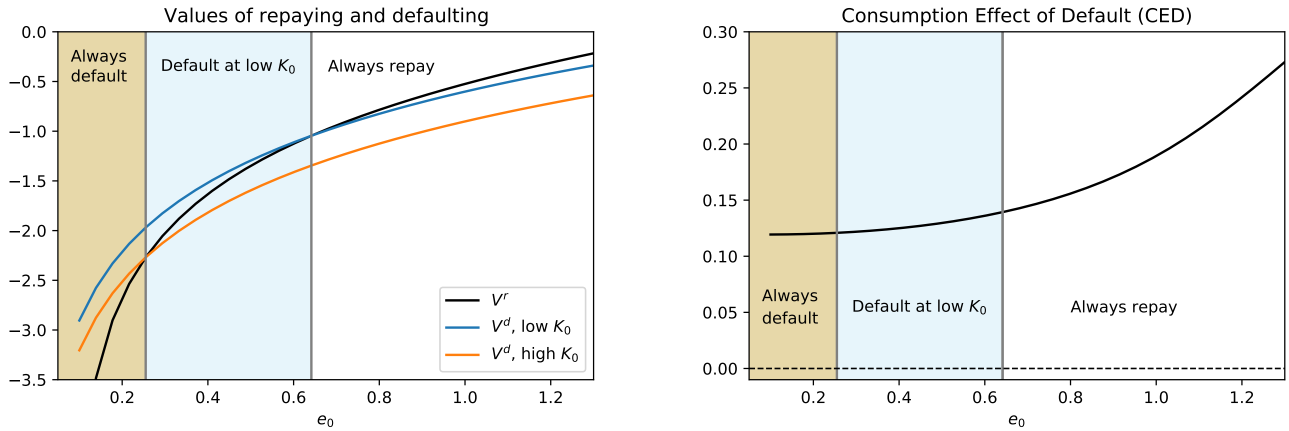


Figure 3: Values of repaying and defaulting and the Consumption Effect of Default

In this paper, we focus on the case where the CED is positive at the lowest level of income. By monotonicity, it is then positive at all levels of income. Equation (5) then implies that the difference between V^r and V^d grows with e_0, so that there is a single intersection $\overline{e_0}$ such that $V^r(\overline{e_0}) = V^d(\overline{e_0})$. Figure 3 illustrates this situation in a calibrated example. On the left, the black line V^r has steeper slope than the orange line V^d, so the two value functions intersect at a unique point $\overline{e_0}$. On the right, we see that the CED is positive throughout. Since the default region is $[0, \overline{e_0}]$,[5] the default rate is:

$$d_0 = F(\overline{e_0})$$

Consider what happens to this default rate as the policymaker lowers K_0. This raises the value of defaulting (to the blue line in Figure 3), without affecting the value of repaying. Hence, the threshold $\overline{e_0}$ moves to the right, and the default rate d_0 increases. This argument shows that the partial derivative of d_0 with respect to K_0 is negative, $\frac{\partial d_0}{\partial K_0} < 0$.

The open economy. We are ready to consider the general equilibrium implications of debt relief. We first consider debt relief in the small open economy (*soe*).

Proposition 1. *In the small open economy, a marginal change in K_0 affects aggregate consumption according to:*

$$\left(\frac{\partial c_0}{\partial K_0}\right)^{soe} = \left(ACED - MPC^S\right) \cdot \frac{b_0}{2} \cdot \frac{\partial d_0}{\partial K_0} \tag{6}$$

where $ACED = CED(\overline{e_0}) > 0$ is the consumption effect of default at the indifference threshold, and $MPC^S = \frac{1}{1+\beta^S}$ is the marginal propensity to consume of savers.

[5] While this situation can be shown to be the only one in some special cases (e.g. Arellano 2008) and is fairly generic in quantitative models, in the general case, the default region is a segment $\left[\underline{e_0}, \overline{e_0}\right]$ (e.g. Chatterjee et al. 2007, Mitman 2016), with households repaying at incomes below $\underline{e_0}$. In that case, the CED is negative in the lower repayment region, as poorer borrowers prefer to borrow to sustain their consumption above the level that they can achieve in default.

The intuition for Proposition 1 is as follows. Consider debt relief, with the policymaker lowering penalties by $dK_0 < 0$. Most borrowers do not change their decisions in terms of either defaulting or spending, but a mass $dd_0 = \frac{\partial d_0}{\partial K_0} dK_0 > 0$ of marginal borrowers with income $\overline{e_0}$ switches over from repaying to defaulting on their debt. The effect on spending from this additional consumption is $ACED \cdot \frac{b_0}{2} \cdot dd_0$. At the same time, financial intermediaries make losses $\frac{b_0}{2} \cdot dd_0$, which they pass on to savers. In turn, savers cut consumption by $MPC^S \cdot \frac{b_0}{2} \cdot dd_0$. The net effect on aggregate spending then depends on the balance between the $ACED$ and the MPC^S, scaled by the level of debt and the effect on the default rate.

This argument shows that default constitutes redistribution between savers and borrowers, and that the $ACED$ (the causal effect of defaulting on spending) is the correct way of evaluating the effect of that redistribution on aggregate borrower spending. While here we study a simple case with a single marginal type, in general there are many marginal types. What matters then is a certain average CED across these types, hence the term $ACED$.

While we saw that theory naturally predicts that the $ACED$ is positive, the MPC^S is also positive. However, in a permanent-income calibration, MPC^S would be quite low (in an infinite horizon model interpretation, $\beta^S = \frac{\beta}{1-\beta}$, and $MPC^S = 1 - \beta$), while data from defaulters suggests that the $ACED$ may be quite high (see section 3.2). Hence, Proposition 1 suggests that the effect of debt relief on aggregate spending is likely to be positive.

We next turn to the study of welfare. We prove the following:

Proposition 2. *In the small open economy, a marginal change in K_0 affects aggregate welfare according to:*

$$\left(-\frac{\partial W}{\partial K_0}\right)^{soe} = d_0 - \lambda u'\left(c_0^S\right) b_0 \left(-\frac{\partial d_0}{\partial K_0}\right)$$

The intuition is simple. The first term, d_0, is the marginal social benefit of lower default penalties. Debt relief $dK_0 < 0$ directly raises by one util the utility of the fraction d_0 of borrowers who are defaulting. Some borrowers also switch to defaulting rather than repaying, but these are marginal borrowers who were already indifferent between their two options, so this does not affect social welfare. The second term is the marginal social cost of these lower penalties: they raise the default rate by $dd_0 = \frac{\partial d_0}{\partial K_0} dK_0 > 0$, so impose losses on all savers of $b_0 \cdot dd_0$, and these are valued at $\lambda u'\left(c_0^S\right)$ by the planner. If the planner places low enough weight on savers ($\lambda u'\left(c_0^S\right)$ is low enough), then debt relief can raise welfare.

Proposition 2 shows that the benefits of the lower penalties from debt relief are perceived by all the defaulters in the same way. This contrasts with transfers, which would raise social welfare according to the average marginal borrower utility $u'\left(c_0^B\right)$. This suggests that debt relief operates in a different space than some other redistributive instruments, and may be helpful in conjunction with these instruments to raise social welfare.

Demand determined economy. We now briefly consider the case of a demand-determined economy, studied in much more detail in Auclert and Mitman (2022). This is relevant since aggregate demand externalities are often invoked in discussions of debt relief.

Proposition 3. *In the demand-determined economy, a marginal change in K_0 affects equilibrium output according to:*

$$\left(\frac{\partial y_0}{\partial K_0}\right)^{dd} = M \cdot \left(\frac{\partial c_0}{\partial K_0}\right)^{soe} \tag{7}$$

where $M = \frac{1}{1-\frac{\partial c_0}{\partial y_0}}$ is the economy's multiplier, i.e. the sensitivity of output to a change in demand.

In the small open economy, the increase in spending from debt relief $dK_0 < 0$ leads to an increase in the economy's imports. By contrast, in a demand-determined economy, it leads to more spending on domestic goods, raising domestic incomes and therefore spending according to the economy's aggregate MPC $\frac{\partial c_0}{\partial y_0}$. The equilibrium effect on output is the initial soe effect multiplied by the standard Keynesian multiplier $M = 1 + \frac{\partial c_0}{\partial y_0} + \left(\frac{\partial c_0}{\partial y_0}\right)^2 + \cdots$. In equilibrium, this amplifies the aggregate demand effect of debt relief.

Proposition 4. *In the demand-determined economy, a marginal change in K_0 affects welfare according to:*

$$\left(\frac{\partial W}{\partial K_0}\right)^{dd} = \left(\frac{\partial W}{\partial K_0}\right)^{soe} + \tau \cdot M \cdot \left(\frac{\partial c_0}{\partial K_0}\right)^{soe} \tag{8}$$

where τ is the sensitivity of welfare to an increase in aggregate demand.

When the economy is demand-determined, an increase in aggregate demand may be beneficial for welfare ($\tau > 0$). In that case, the welfare benefits of debt relief captured in proposition 2 are amplified by the presence of an aggregate demand demand externality, as captured by the second term in (8) (see e.g. Farhi and Werning 2016, Korinek and Simsek 2016).

Taking stock. Unexpected debt relief can boost aggregate spending provided that the spending effect on defaulters (the $ACED$) is larger than that on those who pay for it (the MPC^S). It can also raise social welfare if the policymaker perceives the weight on those who pay for the debt relief to be low enough (low λ), and may even be better at achieving this objective than direct transfers to defaulters. Finally, there may be additional benefits if the economy is depressed, since then the increased aggregate spending has a multiplier effect on output, and social welfare may be further helped by the resultant increase in output if there is an aggregate demand externality.

While we have made the case that the $ACED$ is likely larger than the MPC^S, we note that our model assumes a frictionless banking system. In practice, financial frictions can cause losses in the banking system to propagate to declines in aggregate output (e.g. Gertler and Kiyotaki 2010). In the language of our model, this would raise the MPC^S, possibly above the $ACED$. However, this effect can be avoided by taxing savers, or borrowing from future taxpayers, to recapitalize the banking system.

2.3 Expected debt relief

We now study the effect of an announcement at date 0 of debt relief at date 1. Recall that, as Figure 2 showed, a decline in penalties for period 1 (K_1) shifts in credit supply and lowers the amount that borrowers receive in the first period ($\frac{\partial Q}{\partial K_1} > 0$). This has important implications for the aggregate demand effects of expected debt relief.

We first observe that, contrary to the unexpected change in K_0, which affected previously-written contracts, a change in K_1 has no effect on saver spending, since banks pass through the higher expected defaults to borrowers in the form of higher spreads, to the point that they do not make losses. On the other hand, borrower spending is affected because of the direct shift in credit supply and the indirect response of borrowers to it.

Proposition 5. *In the small open economy, a marginal change in K_1 affects aggregate consumption according to:*

$$\left(\frac{\partial c_0}{\partial K_1}\right)^{soe} = \frac{1}{2}\underbrace{\mathbb{E}_{e_0}\left[\frac{\partial Q}{\partial K_1}(b_1)\right]}_{\text{direct effect}} + \frac{1}{2}\underbrace{\mathbb{E}_{e_0}\left[\frac{\partial Q}{\partial b_1}(b_1)\frac{\partial b_1}{\partial K_1}\right]}_{\text{indirect effect through change in borrowing}} > 0 \tag{9}$$

We saw in section 2.1 that the direct effect of a decline in K_1, holding borrowing decisions fixed, is always negative. In principle, borrowers could offset the effects of this shift on their consumption by borrowing more. The behavioral response involves an income effect, a substitution effect, and a precautionary saving effect. It is possible to show that these effects are never enough to undo the direct effect of the credit supply shift, so that announced future debt relief $dK_1 < 0$ always leads to a decline in aggregate spending $dc_0^{soe} < 0$.

Turning to welfare, we have the following result:

Proposition 6. *In the small open economy, a marginal change in K_1 affects welfare according to:*

$$\left(-\frac{\partial W}{\partial K_1}\right)^{soe} = \beta\mathbb{E}_{e_0}[d_1] - \mathbb{E}_{e_0}\left[u'\left(c_0^{B,r}\right)\frac{\partial Q}{\partial K_1}\right] \tag{10}$$

Here, savers are on their Euler equation, so their welfare is not affected by the change in the timing of cash flows induced by the change in K_1. The aggregate effect on welfare therefore only reflects how borrower welfare is affected. The envelope theorem implies that there are only two effects from announced future debt relief ($dK_1 < 0$). First, a direct benefit to all future defaulters, similar to Proposition 2. Second, a marginal cost from the increased difficulty in borrowing today as credit supply is reduced. See Dávila (2020) for a closely connected result.

Propositions 5 and 6 capture all the economics of expected debt relief. In the demand-determined economy, propositions 3 and 4 apply directly, but replacing $\left(\frac{\partial c_0}{\partial K_0}\right)^{soe}$ by $\left(\frac{\partial c_0}{\partial K_1}\right)^{soe}$ in (9) and $\left(\frac{dW}{dK_1}\right)^{soe}$ by $\left(\frac{dW}{dK_1}\right)^{soe}$ in (10). Hence, if expected debt relief leads to a contraction in credit supply and therefore spending, the effect on output will be aggravated because of the multiplier M, and the effect on welfare will be lower because of the aggregate demand externality τ.

3 Supporting empirical evidence

In this section, we discuss recent micro and macro evidence that supports the assumptions and the aggregate predictions of our model. For a more detailed discussion of the micro evidence on household debt relief in its various forms, see Indarte (2021) in this volume.

3.1 Legal framework in the United States

It is useful here to briefly summarize the details of the consumer bankruptcy system in the U.S. to clarify the types of variation that past empirical studies have exploited to make inference on the effect of debt relief on various economic outcomes. Bankruptcy in the U.S. is the legal procedure for discharging personal debts. A unique feature of the U.S. bankruptcy code is that it is a so-called "fresh start" system, whereby at the end of the bankruptcy procedure all debts are discharged and creditors have no claims on future income.[6] Furthermore, debtors are not required to surrender all of their property and assets when filing. Each state can determine the type and amount of property that is exempt from seizure in bankruptcy. Households have the option of filing under two different parts of the code: Chapter 7 or 13. Chapter 7 is "liquidation" whereby all non-exempt property is seized and then all unsecured debts are canceled. Chapter 13 entails a three to five year repayment plan where households can keep all of their assets, but are obliged to contribute all of their discretionary income to debt payments, after which any remaining debt is canceled. The vast majority of households file under Chapter 7 (which is analogous to default in our theoretical framework), so the remainder of this section will focus on aspects of Chapter 7 bankruptcy.[7]

When a household files for bankruptcy, it has to declare all of its assets and unsecured liabilities to the court. In theory, any non-exempt assets are seized and liquidated, and the proceeds are transferred to creditors. In practice, more than 95% of bankruptcy cases result in no assets being seized by the court. The primary asset that households can keep in bankruptcy is their primary residence. The so-called homestead exemption varies significantly across states, from a low of $5,000 in Kentucky, to an unlimited amount in states like Florida and Texas. The relative generosity of exemptions has been remarkably stable over time, and thus the variation in exemptions has been frequently used in cross-sectional analyses for inference.

After filing, households are precluded from filing again for six years. The bankruptcy "flag" remains on the credit report for 10 years after the filing date. This implies that household access to unsecured credit is limited (Han and Li 2011) and remains so persistently until the flag is removed from the credit report (Musto 2004, Herkenhoff 2019). These institutional features motivate our modeling choice of exclusion from unsecured borrowing after a default in the model period 0.

[6] By contrast, in most European countries bankruptcy serves more as debt restructuring, whereby individuals are liable for the rest of their lives until the debt is repaid in full.

[7] Throughout the rest of the paper we will slightly abuse language and refer to Chapter 7 bankruptcy as bankruptcy or default. A separate strand of the literature has focused on "informal default" where households stop making payments on debt, but do not file for bankruptcy (e.g., Chatterjee and Gordon 2012, Athreya, Sánchez, Tam and Young 2018).

3.2 Micro evidence supporting the model's assumptions

While the focus of this paper is primarily on the aggregate consequences of debt relief, as the framework for the analysis is based on a micro-founded macro model, it is important that household behavior in the model is consistent with micro evidence on debt and default. In this subsection, we discuss the evidence on whether households respond to financial incentives when they choose to default, whether banks pass through expected default costs to borrowers in the form of higher interest rates, and what we know about the CED.

Do households respond to incentives to default as in equations (1) and (2)? The key assumption of underlying our theoretical framework is that households maximize their welfare in their consumption, savings and default decisions. While the connection between theory and evidence on micro consumption behavior is well established, going back to Deaton (1992), evidence on household default decisions is more scarce. In an important early contribution, Fay, Hurst and White (2002) document that U.S. households are more likely to default if there is a greater financial benefit from doing so. They exploit variations in homestead exemptions across states to calculate the net financial benefit from defaulting. In recent innovative work, Indarte (2020) uses a regression kink design on a large loan-level data set to estimate the effect of non-exempt home equity on the probability of filing for bankruptcy. She finds a significant reduction in bankruptcy filings as seizable home equity increases, consistent with households responding to the financial incentives to default.

Fay et al. (2002) also find that a significant fraction of households that would benefit financially from defaulting choose not to do so. Their findings are consistent with earlier work by White (1998) who calculated that at least 15% of households would benefit financially from filing for bankruptcy. Taking these two pieces of evidence together—the fact that a large number of households who would benefit financially from default choose not to do so, and that the likelihood of default is increasing in the financial gain—suggests that there is a significant non-pecuniary cost of default (e.g. guilt, stigma) which motivates our use of the non-pecuniary cost K in the model (similar to previous work, e.g., inter alia Zame 1993, Athreya 2002, Dubey, Geanakoplos and Shubik 2005).

Do banks pass through expected default costs to interest rates as in equation (3)? In the model, we have assumed that the expected cost of default is perfectly passed through to borrowers in terms of a higher effective interest rate charged (3). There is ongoing debate in the literature on consumer credit on the extent of market power in the banking sector and how that gets translated into credit supply, both in terms of the prices and quantities (see, e.g. Ausubel 1991, Herkenhoff and Raveendranathan 2020, Dempsey and Ionescu 2021, Galenianos and Gavazza 2019). Recent evidence, however, suggests that changes in the expected costs of default are almost perfectly passed through to consumers. Gross, Kluender, Liu, Notowidigdo and Wang (2021) exploit the implementation of the 2005 Bankruptcy Abuse Prevention and Consumer Protection Act

(BAPCPA)—a major reform to the bankruptcy code—to measure the extent of pass through in interest rates charged on credit cards. They measure the change in default probability induced by the reform across credit scores, and find that virtually all of the reduction in expected loss given default was passed through to consumers in the form of lower interest rates, justifying the competitive pass-through benchmark in our model.

Is the CED (4) positive, and what is its magnitude? Direct measurement of the CED at the micro level is challenging, since it would require detailed panel data on both consumption, debt and default and a way to identify the counterfactual consumption of a defaulting (repaying) household, had they chosen to default (repay) instead. Indarte (2020) addresses this challenge by showing that the CED for borrowers at the margin of defaulting can be obtained by following a sufficient statistic approach, comparing the relative size of what she calls the "moral hazard" and the "liquidity" effects of debt relief—the effect on the default probability of giving a household more income irrespective of their default decision, vs. only if they default. Her result can be adapted to our setting, so that we can use her empirical estimates to gauge the magnitude of the CED that is relevant for our model. Given her range of empirical estimates, and a standard range for CRRA in macro of 1 to 5, we find that the CED likely lies between 0.09 to 0.56.[8] We conclude that the data supports a positive CED that is plausibly larger than the MPC^S of savers.

3.3 Aggregate evidence on ex-post effects

Existing macroeconomic evidence suggests that ex-post debt relief can help boost aggregate demand. For instance, Auclert, Dobbie and Goldsmith-Pinkham (2022) use the case study of the 2008-2009 Great Recession in the United States, exploiting the variation in bankruptcy exemptions across states described above. They show that states with higher exemptions experienced both a higher level of debt relief and a higher level of non-tradable employment during the recession. They then show that these cross-state patterns are those predicted by a model in which ex-post debt relief boosts aggregate demand. Their results suggest that the decline in aggregate employment would have been mitigated by around 7.5% if low bankruptcy protection states had received the same amount of debt relief as high protection states during the Great Recession.

In related work, Verner and Gyöngyösi (2020) study the effect of adverse debt revaluation (that is, the opposite of debt relief) experienced by households with foreign currency mortgages during Hungary's 2008 currency crisis. The sudden depreciation of the forint led to a significant revaluation of mortgages that were denominated in euros or Swiss francs. Exploiting plausibly exogenous cross-sectional variation in the share of households with foreign currency debt, Verner and Gyöngyösi (2020) show that regions with more foreign currency debt experienced more severe recessions and larger declines in employment, translating into an output multiplier of high debt service of 1.67.

[8] That the upper range is attained under log preferences, so is consistent with the framework here. For the full details of the derivation of our sufficient statistic see Auclert and Mitman (2022).

One shortcoming of the work in this area is that it tends to infer aggregate demand effects from measures of local sectoral employment. While these estimates include local general equilibrium effects, direct measures of spending at both the individual and the regional level would be useful, and may help refine the confidence intervals around the point estimates in the above papers, which tend to be quite wide.

3.4 Aggregate evidence on ex-ante effects

Measuring the ex-ante effects of default policy on aggregate credit supply and other macroeconomic outcomes is also challenging, as rich data on credit supply and interest rates are difficult to come by. Even when data is available, we typically do not observe the entire credit supply schedule. We only see the optimal choices of households. In response to changes in the generosity of debt relief, these optimal choices will reflect direct changes in demand due to the change in the insurance value of default, but also responses to changes in the price of credit, and households' ability to substitute into other forms of credit, such as mortgages or home-equity lines of credit. These data limitations and the dearth of quasi-natural experiments that generate significant changes in default penalties has lead researchers to take a structural approach to inference on these questions using cross-sectional data.

Several papers use the cross-state differences in the homestead exemption, as in the previous section, to make inference on the effects of default policy on credit supply. In a seminal contribution, using 1983 Survey of Consumer Finances data, Gropp, Scholz and White (1997) found that more generous bankruptcy exemptions reduce credit supply to less well-off borrowers—these borrowers were more likely to have been denied credit. In addition to being able to borrow less, they found that interest rates were higher on consumer credit in states with more generous exemptions.[9] How these estimates aggregate, however, it less clear. Mitman (2016) shows that on average, states with higher homestead exemptions have lower bankruptcy rates, but also higher foreclosure rates. These findings may seem in conflict with the evidence discussed earlier that households with a higher financial incentive are more likely to default, as *ceteris paribus*, higher bankruptcy exemptions increase the benefit from defaulting. Importantly, however, the micro findings were conditional on a given debt position, but were silent about the distribution of debt is in different states. Using a structural model, Mitman (2016) can reconcile both the micro and state-level evidence through a substitution channel, whereby household substitute towards secured credit in response to the higher interest rates charged on unsecured credit in states with high exemptions.[10] Thus, in high exemption households states have less unsecured credit, and in high exemption states households with more home equity are more likely to default. Empirical findings by Miller (2019), using evidence from a novel bankruptcy dataset and the Panel Study of

[9]Pence (2006) finds similar effects for secured credit, exploiting spatial discontinuities across state borders that have more and less defaulter-friendly foreclosure laws.

[10]Severino and Brown (2017), Dávila (2020) estimate the elasticity of the credit card interest rates to homestead exemptions both pre- and post-BAPCPA and find that higher exemptions lead to higher interest rates charged on unsecured credit.

Income Dynamics, are consistent with these findings.

Finally, a series of recent papers have used time-series variation exploiting the quasi-natural experiment of the implementation of the BAPCPA reform to identify the effects of changes in the bankruptcy code on credit supply. Albanesi and Nosal (2018) finds an increased likelihood in households having access to a new credit line post-BAPCPA, and Gross et al. (2021) find an increase in credit card offers sent to households. Thus, both in the U.S. cross-section and the time-series around BAPCPA, the evidence points to potentially significant ex-ante effects on credit supply. While these results give a sense of the magnitudes for the $\partial Q/\partial b_1$ term in equation (9), more evidence is needed to understand how those effects translate to consumption. We hope that future research can help shed light on this question.

4 Policy implications

Having established the positive properties of our model and shown that these are supported by the data, we now discuss the policy implications of our framework.

4.1 Credibility and the ex-ante/ex-post frontier

In our two-period framework, policymakers fool lenders once and for all when they change K_0, and do not fool them at all when changing K_1: the latter has no effect on lender profits, since they fully pass through the induced change in period 1 defaults into loan spreads for borrowers in period 0.

In practice, any attempt at debt relief on ex-post contracts is likely to lead to some anticipation that the government will do the same again. In our framework, we can capture this idea by assuming that with any K_0 comes some inevitable expected change in K_1, call it $\frac{\partial K_1}{\partial K_0} > 0$. The more damaged is a government's credibility, the higher is $\frac{\partial K_1}{\partial K_0}$.

Given this, it is possible to write the total change in spending in the small open economy[11] as

$$\left(\frac{dc_0}{dK_0}\right)^{soe} = \left(\frac{\partial c_0}{\partial K_0}\right)^{soe} + \left(\frac{\partial c_0}{\partial K_1}\right)^{soe} \frac{\partial K_1}{\partial K_0} \tag{11}$$

Since we argued that propositions 1 and 5 imply that $\left(\frac{dc_0}{dK_0}\right)^{soe} < 0$ and $\left(\frac{dc_0}{dK_1}\right)^{soe} > 0$, the credibility problem creates a tension tension between the spending benefits of the ex-post debt relief and the costs of the contraction from expected future debt relief.

We now turn to a proposal that can avoid this credibility problem. This proposal starts by observing that defaults already naturally increase in recessions and act as an automatic stabilizer of the business cycle.

[11] The effect on output in a demand-determined economy is then $M\left(\frac{dc_0}{dK_0}\right)^{soe}$

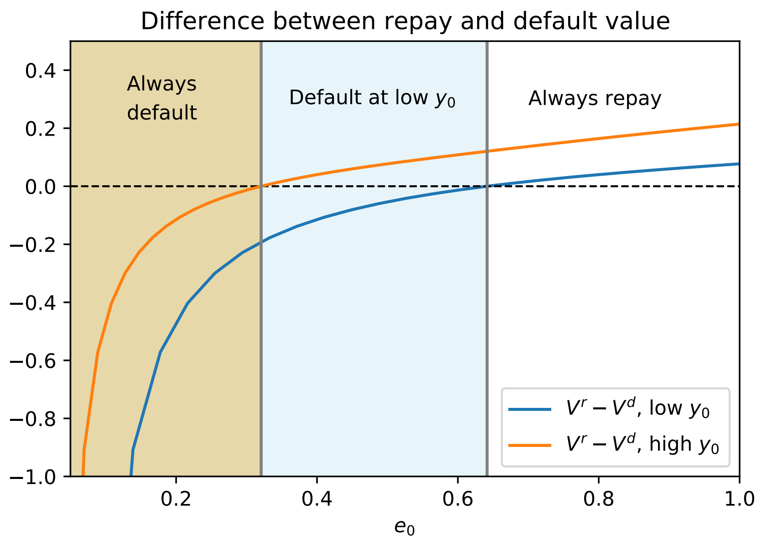


Figure 4: Difference between repay and default value: high vs low y_0

4.2 Defaults as a natural automatic stabilizer

To understand the effect of recessions on defaults, consider Figure 4. The orange line displays the difference between the value of repayment V^r and the value of default V^d from Figure 3, at a "high" level of GDP $y_0 = 1$. As discussed above, the difference $V^r - V^d$ increases with income because the *CED* is positive, and the intersection with 0 determines the initial default region $[0, \overline{e_0}]$.

Consider next what happens when aggregate GDP falls to $y_0^L < 1$. Now, agents at any given income shock e_0 have total income $e_0 y_0^L < e_0$, so they perceive the value of repayment and defaulting accordingly. This shifts the curve $V^r - V^d$ to the right, and therefore generates an increase in the number of defaulting agents, as Figure 4 illustrates. We have proved the following proposition:

Proposition 7. *The default rate is countercyclical:* $\frac{\partial d_0}{\partial y_0} < 0$.

In the data, the countercyclicality of the default rate is clearly apparent, at least on unsecured credit (e.g. Nakajima and Ríos-Rull 2019), which validates this implication of our model.

Consider now the effect of shocks, such as adverse demand shocks, that lower the level of output y_t. By proposition 7, these shocks raise the default rate d_0. But the logic of Proposition 1 tells us that this, in turn, props up the level of aggregate demand. In other words, defaults act as an automatic stabilizer of the business cycle. In Auclert and Mitman (2022), we show this formally by considering a stochastic version of the model. There, we prove the following proposition:

Proposition 8. *Consider the amplitude of fluctuations, measured as the standard deviation of output* $std\left(dy_0^*\right)$ *in a world where consumers cannot default more in recessions (d_0 is fixed), relative to the baseline* $std\left(dy_0\right)$ *in which the default rate has semielasticity with respect to output* $\frac{\partial d_0}{\partial \log y_0}$*. This is given by:*

$$\frac{std\left(dy_0^*\right)}{std\left(dy_0\right)} = 1 + M \cdot \left(ACED - MPC^S\right) \frac{1}{2} \frac{b_0}{y_0} \left(-\frac{\partial d_0}{\partial \log y_0}\right) \tag{12}$$

In other words, the natural response of defaults in the economy to falls in GDP, as captured in the data by $-\frac{\partial d_0}{\partial \log y_0}$, is already contributing to dampen output fluctuations.

4.3 Enacting systematic debt relief

Governments naturally want to boost aggregate demand in recessions and reduce it in booms. In other words, a natural simplified objective is to minimize the amplitude of fluctuations, captured in proposition 8 by std (dy_0). The proposition suggests that this can be achieved by enhancing the countercyclicality of the default rate, ie raising $\left(-\frac{\partial d_0}{\partial \log y_0}\right)$. More generally, any systematic change in bankruptcy rules that succeeds in boosting aggregate demand when output is low would serve this objective. Proposition 1 suggests lowering K_0 when y_0 is low, while Proposition 5 suggests raising K_1. A simple way to achieve these objectives is to put in place a policy *rule* $K_t\left(y_t, y_{t-1}\right)$ that respond to the state and evolution of the business cycle. If GDP tends to mean revert, one simple way to achieve this is a policy rule that responds to changes in the level of GDP,

$$K_t\left(y_t, y_{t-1}\right) = K^* + \psi\left(y_t - y_{t-1}\right)$$

For $\psi > 0$, this rule tightens bankruptcy rules when the economy is currently growing, and relaxes them when the economy is currently shrinking. When y_t has declined relative to the its level, K_t is low, achieving ex-post debt relief, and when GDP is expected to recover, K_{t+1} is expected to be high, crowding in credit supply. In Auclert and Mitman (2022), we study these types of rules and show they can be helpful with aggregate demand management. Moreover, since debt relief becomes predictable as a function of the state of the business cycle, these rules also avoid the credibility issues that plague typical discussions of debt restructuring.

5 Conclusion

Household debt levels in the U.S. and around the developed world are at or near historic highs. These extraordinary debt burdens have attracted the interest of policy makers who seek to encourage robust and inclusive growth in the face of the economic challenges of the twenty-first century. In this paper, we used a streamlined version of the framework in Auclert and Mitman (2022) to examine how policymakers can leverage the legal framework around consumer default to achieve both macroeconomic and equity policy objectives. Our empirically-grounded model predicts that unexpected, ex-post debt relief can improve welfare and boost aggregate demand, but that the extent of these benefits can be limited by expectations of future debt relief. On the other hand, systematically giving more relief in recessions and less in booms can help manage aggregate demand while avoiding damaging the government's credibility. We hope that these findings can be useful to structure the ongoing policy debate around household debt relief.

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A Proofs for section 2

We formalize the assumption stated in section 2.1 as follows.

Assumption 1. *F is a cdf with a continuous density on f on $(0,\infty)$ with an increasing hazard, ie, $\lambda(x) = \frac{f(x)}{1-F(x)}$ is increasing*

A.1 Preliminaries

The period 1 default region is a segment $[0, \overline{e_1}(b_1)]$. In period 1, a borrower with debt b_1 defaults (resolving indifference with default)[12] when:

$$\log(e_1 - b_1) \leq \log(e_1) - K_1$$

so

$$\log\left(1 - \frac{b_1}{e_1}\right) \leq -K_1$$

ie

$$1 - \frac{b_1}{e_1} \leq e^{-K_1}$$

or

$$e_1 \leq \frac{b_1}{1 - e^{-K_1}} \equiv \overline{e_1}(b_1, K_1)$$

note that, when $K_1 = \infty$, the borrower only defaults if he gets infinite negative utility from repaying, ie when $b_1 \geq e_1$.

The loan schedule has a Laffer-curve shape. The loan schedule is

$$Q(b_1) = \frac{b_1}{R}(1 - d_1(b_1)) = \frac{b_1}{R}\left(1 - F\left(\frac{b_1}{1 - e^{-K_1}}\right)\right) \tag{13}$$

Define the $\mathcal{Q}$ function as

$$\mathcal{Q}(b) = \frac{b}{R}(1 - F(b))$$

that is, $\mathcal{Q}$ is the bond schedule Q when $K_1 = \infty$. The peak, if it exists, solves:

$$\mathcal{Q}'(b) = \frac{1}{R}(1 - F(b) - bf(b)) = 0$$

so is at the point where b^*

$$1 - F(b^*) = b^* f(b^*)$$

or

$$\lambda(b^*) = \frac{f(b^*)}{1 - F(b^*)} = \frac{1}{b^*} \tag{14}$$

[12] This makes the default set a closed interval, and implies that we can define default probabilities with standard cdfs.

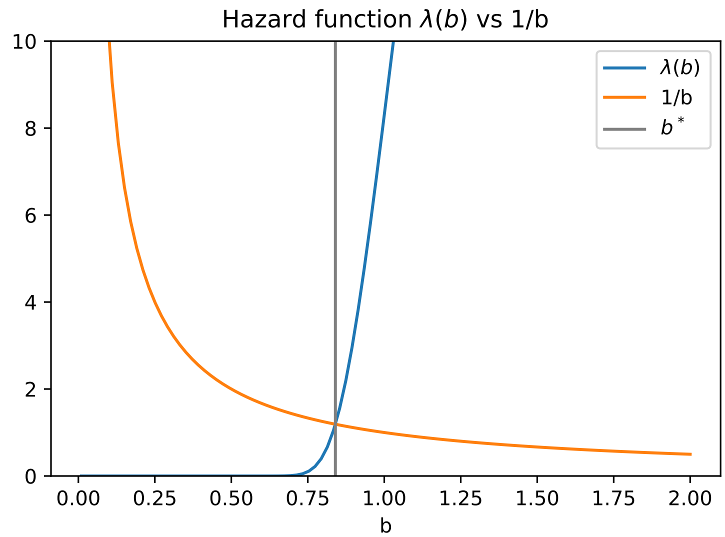

Figure 5: $\lambda(b)$, b, and b^* in our calibration

By Assumption 1, since F has increasing hazard, there is a unique value b^* that achieves (14), with $\frac{1}{b} > \lambda(b)$ for all $b < b^*$. This is illustrated in Figure 5. Summing up, the $\mathcal{Q}$ schedule satisfies:

$$
\begin{aligned}
\mathcal{Q}(0) &= 0 \\
\mathcal{Q}(b^*) &= \frac{b^*}{R}\left(1 - F(b^*)\right) = \frac{b^*}{\lambda(b^*) R} \\
\mathcal{Q}'(b) &= \frac{1 - F(b)}{R} \cdot \left(1 - b\lambda(b)\right)
\end{aligned} \tag{15}
$$

and $\mathcal{Q}'(b) > 0$ for all $b < b^*$. Note that the first term in $\mathcal{Q}'$ is the discounted default probability, and the second is the adjustment for the effect that any increase in b has on the price of inframarginal units.

Finally, for all $b < b^*$ we have that $\mathcal{Q}''(b) < 0$. To see this, note that:

$$
\mathcal{Q}''(b) = \frac{-1}{R}\left(2f(b) + bf'(b)\right)
$$

but, by the increasing hazard property,

$$
\lambda'(b) = \frac{f'(b)\left(1 - F(b)\right) + f(b)^2}{\left(1 - F(b)\right)^2} > 0
$$

which implies that

$$
f'(b) > -\frac{\left(f(b)\right)^2}{\left(1 - F(b)\right)} = -f(b)\lambda(b)
$$

now, below the peak of the Laffer curve ($b < b^*$), we have $\lambda(b) < \frac{1}{b}$, so $-\lambda(b) > -\frac{1}{b}$, so

$$bf'(b) > -f(b)$$

Hence, for $b < b^*$ we have:

$$2f(b) + bf'(b) > f(b)$$

and so

$$\mathcal{Q}''(b) < \frac{-f(b)}{R} < 0$$

as claimed.

In the general case, for any given K_1, we have:

$$Q(b_1; K_1) = \left(1 - e^{-K_1}\right) \mathcal{Q}\left(\frac{b_1}{1 - e^{-K_1}}\right) \tag{16}$$

hence, the Q schedule at penalties K_1 is just the $\mathcal{Q}$ schedule with infinite penalties, shifted to the left and down by the factor $1 - e^{-K_1}$. It follows immediately that

$$\frac{\partial Q}{\partial b_1}(b_1; K_1) = \frac{1 - e^{-K_1}}{1 - e^{-K_1}} \mathcal{Q}'\left(\frac{b_1}{1 - e^{-K_1}}\right) = \mathcal{Q}'\left(\frac{b_1}{1 - e^{-K_1}}\right) \tag{17}$$

which is positive whenever $\mathcal{Q}' > 0$ is (on the left side of the Laffer curve), and

$$\begin{aligned}
\frac{\partial Q}{\partial K_1}(b_1; K_1) &= e^{-K_1} \mathcal{Q}\left(\frac{b_1}{1 - e^{-K_1}}\right) + \left(1 - e^{-K_1}\right) \mathcal{Q}'\left(\frac{b_1}{1 - e^{-K_1}}\right) \frac{b_1}{\left(1 - e^{-K_1}\right)^2} e^{-K_1} \\
&= e^{-K_1} \left\{ \mathcal{Q}\left(\frac{b_1}{1 - e^{-K_1}}\right) + \mathcal{Q}'\left(\frac{b_1}{1 - e^{-K_1}}\right) \frac{b_1}{1 - e^{-K_1}} \right\}
\end{aligned} \tag{18}$$

which is positive since both $\mathcal{Q} > 0$ and $\mathcal{Q}' > 0$ in the relevant range.

Characterizing the borrower consumption policy $c_0^r(e_0)$ and debt policy $b_1(e_0)$. Let b_1 be the value that maximizes:

$$\max_{b_1} \left\{ \log\left(y_0 e_0 - b_0 + Q(b_1)\right) + \beta V(b_1) \right\}$$

The FOC for this problem is the Euler equation

$$\frac{1}{y_0 e_0 - b_0 + Q(b_1)} \frac{\partial Q}{\partial b_1}(b_1) = -\beta V'(b_1) \tag{19}$$

Now, since

$$V(b_1) = \left(\log(e_1) - K_1\right) \int_0^{\overline{e_1}(b_1, K_1)} f(e_1)\, de_1 + \int_{\overline{e_1}(b_1, K_1)}^{\infty} \log(e_1 - b_1) f(e_1)\, de_1$$

it holds that:

$$V'(b_1) = -\int_{\overline{e_1}(b_1,K_1)}^{\infty} \frac{1}{e_1 - b_1} f(e_1)\, de_1 + \frac{\partial \overline{e_1}(b_1,K_1)}{\partial b_1} \underbrace{\left(\log\left(\overline{e_1} - b_1\right) - \log(e_1) - K_1\right)}_{0} f\left(\overline{e_1}\right)$$

Combining, we can rewrite (19) as

$$\frac{1}{y_0 e_0 - b_0 + Q(b_1)} \frac{\partial Q}{\partial b_1}(b_1) = \beta \int_{\overline{e_1}(b_1,K_1)}^{\infty} \frac{1}{e_1 - b_1} f(e_1)\, de_1 \tag{20}$$

Finally, we use (17) and (15) to write with the more familiar form of an Euler equation with bankruptcy,

$$u'(c_0) = \frac{\beta R}{1 - \epsilon\left(\frac{b_1}{1-e^{-K_1}}\right)} \mathbb{E}_{e_1}\left[u'(c_1)\,|e_1 > \frac{b_1}{1 - e^{-K_1}}\right] \tag{21}$$

where $\epsilon(b) \equiv b\lambda(b)$, which satisfies $\epsilon'(b) = \lambda(b) + b\lambda'(b) > 0$. Equation (20) determines the bond policy $b_1(e_0)$ and therefore the consumption policy

$$c_0^r(e_0) = y_0 e_0 - b_0 + Q(b_1(e_0)) \tag{22}$$

CED is increasing, and savings is normal. We prove that amount borrowed b_1 is normal, ie when $e_0' > e_0$, we have $b_1' < b_1$. Since Q is increasing on the left side of the Laffer curve, it follows that $Q' < Q$. Since $ACED = \frac{b_0 - Q}{b_0}$, it then follows that $ACED' > ACED$, so that $ACED$ is increasing.

Suppose not, so there exists a pair $e_0' > e_0$ such that $b_1' > b_1$, and so $Q' > Q$. Then, $c' = e_0' + Q' > c$. But both points are optimal, so from (21) we must have

$$\frac{u'(c)}{u'(c')} = \frac{\mathbb{E}_{e_1}\left[u'(e_1 - b_1)\,|e_1 > \frac{b_1}{1-e^{-K_1}}\right]}{\mathbb{E}_{e_1}\left[u'(e_1 - b_1')\,|e_1 > \frac{b_1'}{1-e^{-K_1}}\right]} \cdot \frac{1 - \epsilon\left(\frac{b_1'}{1-e^{-K_1}}\right)}{1 - \epsilon\left(\frac{b_1}{1-e^{-K_1}}\right)}$$

Now, it is clear from concavity of u and the properties of the conditional expectation that

$$\mathbb{E}_{e_1}\left[u'(e_1 - b_1)\,|e_1 > \frac{b_1}{1 - e^{-K_1}}\right] < \mathbb{E}_{e_1}\left[u'(e_1 - b_1')\,|e_1 > \frac{b_1'}{1 - e^{-K_1}}\right]$$

Moreover, since ϵ is increasing, $1 - \epsilon\left(\frac{b_1'}{1-e^{-K_1}}\right) < 1 - \epsilon\left(\frac{b_1}{1-e^{-K_1}}\right)$. Hence, the ratio on the right is stricly less than 1. At the same time, since u is concave, $u'(c) > u'(c')$, so the ratio on the left is stricly greater than 1. This is a contradiction.

Characterizing the consumption effect of default (*CED*). The default consumption policy is clearly:

$$c_0^d(e_0) = y_0 e_0$$

Hence, the CED is

$$CED\,(e) \equiv \frac{c_0^d\,(e_0) - c_0^r\,(e_0)}{b_0} = \frac{b_0 - Q\,(b_1\,(e_0))}{b_0} \tag{23}$$

For this paper, we assume that $CED\,(0) > 0$. Since CED is increasing by the normality of b_1 and $Q' \geq 0$, we find that $CED > 0$ everywhere.

Default vs repayment decision in period 0, characterization of the repayment region. By the envelope theorem:

$$\frac{\partial V^d}{\partial e_0}\,(e_0) = u'\left(c_0^d\right) = \frac{1}{y_0 e_0}$$

and

$$\frac{\partial V^r}{\partial e_0}\,(e_0) = u'\,(c_0^r) + \frac{\partial b_1}{\partial e_0}\left\{\underbrace{\frac{\partial Q}{\partial b_1} u'\,(c_0^r) + \beta V'\,(b_1)}_{0}\right\} = u'\,(c_0^r)$$

Hence, defining the difference in values as:

$$V^\Delta\,(e_0) \quad = \quad V^r\,(e_0) - V^d\,(e_0)$$

we have:

$$\begin{aligned}
\frac{\partial V^\Delta}{\partial e_0}\,(e_0) &= \frac{1}{y_0 e_0 - b_0 + Q\,(b_1\,(e_0))} - \frac{1}{y_0 e_0} \\
&= \frac{b_0 - Q\,(b_1\,(e_0))}{c_0^r\,(e_0)\,c_0^d\,(e_0)} \\
&= u'\,(c_0^r\,(e_0))\,\frac{c_0^d\,(e_0) - c_0^r\,(e_0)}{c_0^d\,(e_0)} \\
&= u'\,(c_0^r\,(e_0))\,CED\,(e_0)\,\frac{c_0^d\,(e_0)}{b_0}
\end{aligned}$$

which is equation (5) in the text.

Deriving the saver consumption function. Saver problem:

$$\begin{aligned}
&\max u\left(c_0^S\right) + \beta^S u\left(c_1^S\right) \\
\text{s.t.}\quad & c_0^S + \frac{c_1^S}{R} = y_0 + \frac{1}{R} + \Pi
\end{aligned}$$

with constant-elasticity utility function $u\,(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma}$ (σ is the EIS) this gives

$$\left(c_0^S\right)^{-1/\sigma} = \beta^S R\left(c_1^S\right)^{-1/\sigma}$$

so

$$c_1^S = c_0^S \left(\beta^S R\right)^\sigma$$

plugging in,

$$c_0 \left(1 + \left(\beta^S\right)^\sigma R^{\sigma-1}\right) = y_0 + \frac{1}{R} + \Pi$$

so

$$\begin{aligned} c_0^S &= \frac{1}{1+\left(\beta^S\right)^\sigma R^{\sigma-1}} \left\{y_0 + \frac{1}{R} + \Pi\right\} \\ c_1^S &= \frac{\left(\beta^S\right)^\sigma R^\sigma}{1+\left(\beta^S\right)^\sigma R^{\sigma-1}} \left\{y_0 + \frac{1}{R} + \Pi\right\} \end{aligned} \quad (24)$$

this shows that the $MPC^S = \frac{1}{1+\left(\beta^S\right)^\sigma R^{\sigma-1}}$. In the special case of log utility, $\sigma = 1$ and $MPC^S = \frac{1}{1+\beta^S}$.

Proving zero profits for intermediaries. Intermediary just the value of the legacy debts that are repaid:

$$\Pi = (1 - d_0)\, b_0 \quad (25)$$

on all loans going forward, intemediary makes zero profits: at date 0, lends out $Q(b_1)$, at date 1, they get back

$$b_1 \int_{e_1^*(b_1,K_1)}^{\infty} f(e_1)\, de_1 = b_1 \left(1 - F\left(e_1^*\left(b_1, K_1\right)\right)\right)$$

in present value terms, this is

$$-Q(b_1) + \frac{1}{R} b_1 \left(1 - F\left(e_1^*\left(b_1, K_1\right)\right)\right) = 0$$

by (13).

Deriving the current account equation. Summing up budget constraints, we have, for borrowers:

$$c_0^B = \begin{cases} y_0 e_0 - b_0 + Q(b_1) & \text{if repay} \\ y_0 e_0 & \text{if default, } y_0 e_0 \in \left[\underline{e_0}, \overline{e_0}\right] \end{cases}$$

and

$$c_1^B = \begin{cases} e_1 - b_1 & \text{if repay at 1} \\ e_1 & \text{if defaults at 1 or defaulted at 0} \end{cases}$$

This implies that

$$\frac{E\left[c_1^B\right]}{R} = \frac{1}{R} - \frac{b_1}{R}\left(1 - F(b_1)\right)$$

and since

$$Q(b_1) = \frac{b_1}{R}\left(1 - F(b_1)\right)$$

for households of type e_0, we have:

$$c_0^B(e_0) + \frac{\mathbb{E}\left[c_1^B\right]}{R} = \begin{cases} y_0 e_0 + \frac{1}{R} - b_0 & \text{if repay at 0} \\ y_0 e_0 + \frac{1}{R} & \text{if default at 1 or defaulted at 0} \end{cases}$$

Taking the expectation across all agents, we then have

$$\mathbb{E}\left[c_0^B\right] + \frac{\mathbb{E}\left[c_1^B\right]}{R} = y_0 + \frac{1}{R} - b_0\left(1 - d_0\right)$$

For the savers, the intertemporal budget constraint is

$$c_0^S + \frac{1}{R}c_1^S = y_0 + \frac{1}{R} + \Pi \tag{26}$$

then,

$$c_0 + \frac{1}{R}c_1 = y_0 + \frac{1}{R} + \frac{1}{2}\left(\Pi - b_0\left(1 - d_0\right)\right) = y_0 + \frac{1}{R}$$

Hence, if we define aggregate savings (the current account) by

$$s_0 = y_0 - c_0 = nx_0$$

we have that

$$c_1 = 1 + R\left(y_0 - c_0\right) = 1 + Rs_0 = 1 - nx_1$$

with next exports in period 1 being

$$nx_1 = -Rnx_0$$

In other words, if $c_0 > y_0$ in period 0, the country imports and runs a current account deficit, repaying its international debt in period 1.

A.2 Proof of proposition 1

We first derive $\frac{\partial d_0}{\partial K_0}$. An increase in K_0 lowers the value of defaulting, so it shrinks the region of default. For any e_0, the likelihood of repaying is now higher, so it makes any marginal person less likely to default. This says that

$$\frac{\partial V^\Delta}{\partial K_0} = 1$$

so we grow the difference and therefore move the threshold t the left, $\frac{\partial \overline{e_0}}{\partial K_0} < 0$. Then

$$\frac{\partial d_0}{\partial K_0} = f\left(\overline{e_0}\right)\frac{\partial \overline{e_0}}{\partial K_0} < 0 \tag{27}$$

We next derive $\frac{\partial c_0^B}{\partial K_0}$. Since

$$c_0^B = \int_0^{\overline{e_0}} c_0^d(e_0) f(e_0)\, de_0 + \int_{\underline{e_0}}^{\infty} c_0^r(e_0) f(e_0)\, de_0$$

and neither c_0^d nor c_0^r are affected by K_0, we have:

$$\frac{\partial c_0^B}{\partial K_0} = f(\overline{e_0}) \cdot \left(c_0^d(\overline{e_0}) - c_0^r(\overline{e_0})\right) \frac{\partial \overline{e_0}}{\partial K_0} \tag{28}$$

Combining (28) and (27), we have:

$$\frac{\partial c_0^B}{\partial K_0} = \underbrace{\left(\frac{c_0^d(\overline{e_0}) - c_0^r(\overline{e_0})}{b_0}\right)}_{ACED} \cdot b_0 \cdot \frac{\partial d_0}{\partial K_0} \tag{29}$$

Given the consumption function for the saver (24), we have:

$$\frac{\partial c_0^S}{\partial K_0} = MPC^S \cdot \frac{\partial \Pi}{\partial K_0} = MPC^S \cdot b_0 \cdot \frac{\partial(1-d_0)}{\partial K_0} = -MPC^S \cdot b_0 \cdot \frac{\partial d_0}{\partial K_0}$$

while, from (29), we have:

$$\frac{\partial c_0^B}{\partial K_0} = b_0 \cdot \frac{\partial d_0}{\partial K_0} \cdot ACED$$

This implies that

$$\frac{\partial c_0}{\partial K_0} = \frac{1}{2}\frac{\partial c_0^B}{\partial K_0} + \frac{1}{2}\frac{\partial c_0^B}{\partial K_0} = \frac{b_0}{2} \cdot \left(ACED - MPC^S\right) \cdot \frac{\partial d_0}{\partial K_0}$$

which proves Proposition 1.

A.3 Proof of proposition 2

Let

$$W^S = \lambda\left(u\left(c_0^S\right) + \beta^S u\left(c_1^S\right)\right)$$

be the welfare of savers. We have:

$$\frac{dW^S}{dK_0} = \lambda\left(u'\left(c_0^S\right)\frac{dc_0^S}{dK_0} + \beta^S u'\left(c_1^S\right)\frac{dc_1^S}{dK_0}\right)$$

but also:

$$u'\left(c_0^S\right) = \beta^S R u'\left(c_1^S\right)$$

hence,

$$\begin{aligned}\frac{dW^S}{dK_0} &= \lambda u'\left(c_0^S\right)\left(\frac{dc_0^S}{dK_0}+\frac{1}{R}\frac{dc_1^S}{dK_0}\right)\\ &= \lambda u'\left(c_0^S\right)\frac{d\left(c_0^S+\frac{1}{R}c_1^S\right)}{dK_0}\end{aligned}$$

and from the budget constraint (26), we have:

$$\frac{d\left(c_0^S+\frac{1}{R}c_1^S\right)}{dK_0}=\frac{d\Pi}{dK_0}=\frac{d\left(b_0\left(1-d_0\right)\right)}{dK_0}=b_0\left(-\frac{\partial d_0}{\partial K_0}\right)$$

so that

$$\frac{dW^S}{dK_0}=\lambda u'\left(c_0^S\right)b_0\left(-\frac{\partial d_0}{\partial K_0}\right) \tag{30}$$

Next, let

$$\begin{aligned}W^B &= \int W^B\left(e_0\right)f\left(e_0\right)de_0\\ &= \int_0^{\overline{e_0}}\left\{\log\left(y_0e_0\right)+\beta\mathbb{E}\left[\log\left(e_1\right)\right]-K_0\right\}f\left(e_0\right)de_0\\ &\quad+\int_{\overline{e_0}}^{\infty}V^r\left(e_0\right)f\left(e_0\right)de_0\end{aligned}$$

then, the effect of a change in K_0 is

$$\frac{dW^B}{dK_0}=-\underbrace{\int_0^{\overline{e_0}}f\left(e_0\right)de_0}_{d_0}+f\left(\overline{e_0}\right)\left\{\underbrace{V^r\left(\overline{e_0}\right)-V^d\left(\overline{e_0}\right)}_{0}\right\}\frac{d\overline{e_0}}{dK_0} \tag{31}$$

Borrowers at the margin are indifferent, so a rise in K_0 has no effect on welfare from the switchers, and the only effect is inframarginal. Combining (30) and (31), we find

$$\frac{dW}{dK_0}=-d_0+\lambda u'\left(c_0^S\right)b_0\left(-\frac{\partial b_0}{\partial K_0}\right)$$

so

$$\frac{-dW}{dK_0}=d_0-\lambda u'\left(c_0^S\right)b_0\left(-\frac{\partial d_0}{\partial K_0}\right)$$

which proves Proposition 2.

A.4 Proof of proposition 3

In a demand determined economy, we must have

$$c_0\left(y_0;K_0\right)=y_0$$

hence,

$$\frac{\partial c_0}{\partial y_0} dy_0 + \frac{\partial c_0}{\partial K_0} dK_0 = dy_0$$

This implies:

$$dy_0 = \frac{\frac{\partial c_0}{\partial K_0} dK_0}{1 - \frac{\partial c_0}{\partial y_0}}$$

but since $\frac{\partial c_0}{\partial K_0}$ holds y_0 fixed, it is just the small open economy effect on spending discussed in proposition 1. This proves Proposition 3.

A.5 Proof of proposition 4

Let welfare be:

$$W(K_0, y_0)$$

the sensitivity of welfare to its second argument should be understood as not just the effect from raising consumption, but also possibly from raising labor supply (so a negative effect). Then, we have

$$\frac{dW}{dK_0} = \frac{\partial W}{\partial K_0} + \frac{\partial W}{\partial y_0}\frac{\partial y_0}{\partial K_0} = \left(\frac{dW}{dK_0}\right)^{soe} + \tau \cdot \frac{\partial y_0}{\partial K_0} = \left(\frac{dW}{dK_0}\right)^{soe} + \tau \cdot M \cdot \left(\frac{dc_0}{dK_0}\right)^{soe}$$

This proves Proposition 4.

A.6 Proof of proposition 5

First, for savers, we follow the proof of proposition 1. We have:

$$\frac{\partial c_0^S}{\partial K_1} = MPC^S \cdot \frac{\partial \left(y_0 + \frac{1}{R} + \Pi\right)}{\partial K_1} = 0$$

since neither of y_0, R of Π is affected by an announced change at date 0 of a change in K_1.

For borrowers, we first consider a borrower with skill e_0, and then aggregate across borrowers. Since

$$c_0^B = y_0 e_0 - b_0 + Q(b_1, K_1)$$

we have that

$$\frac{\partial c_0^B}{\partial K_1} = \underbrace{\frac{\partial Q}{\partial K_1}(b_1)}_{\text{direct effect}} + \underbrace{\frac{\partial Q}{\partial b_1}(b_1)\frac{db_1}{dK_1}}_{\text{indirect effect through change in borrowing}}$$

where b_1 solves (20). The direct effect is positive from (18). The behavioral response involves an income effect, a substitution effect, and a precautionary saving effect, but we can prove from first principles that the total effect is positive.

Suppose that $K_1 > K_1'$, and that there exists an agent e_0 for whom nevertheless $c < c'$. This agent therefore has $Q < Q'$, so given that he faces a more favorable debt schedule, must have borrowed less, $b_1 < b_1'$. This also implies $\frac{b_1}{1-e^{-K_1}} < \frac{b_1'}{1-e^{-K_1'}}$.But both points are optimal, so from (21) we must have

$$\frac{u'(c)}{u'(c')} = \frac{\mathbb{E}_{e_1}\left[u'(e_1 - b_1)\,|e_1 > \frac{b_1}{1-e^{-K_1}}\right]}{\mathbb{E}_{e_1}\left[u'(e_1 - b_1')\,|e_1 > \frac{b_1'}{1-e^{-K_1'}}\right]} \cdot \frac{1-\epsilon\left(\frac{b_1'}{1-e^{-K_1'}}\right)}{1-\epsilon\left(\frac{b_1}{1-e^{-K_1}}\right)}$$

Now, it is clear from concavity of u and the properties of the conditional expectation that

$$\mathbb{E}_{e_1}\left[u'(e_1 - b_1)\,|e_1 > \frac{b_1}{1-e^{-K_1}}\right] < \mathbb{E}_{e_1}\left[u'(e_1 - b_1')\,|e_1 > \frac{b_1'}{1-e^{-K_1}}\right]$$

Moreover, since ϵ is increasing, $1-\epsilon\left(\frac{b_1'}{1-e^{-K_1}}\right) < 1-\epsilon\left(\frac{b_1}{1-e^{-K_1}}\right)$. Hence, the ratio on the right is stricly less than 1. At the same time, since u is concave, $u'(c) > u'(c')$, so the ratio on the left is stricly greater than 1. This is a contradiction. Hence, total consumption under a more favorable debt schedule must always be positive. This proves Proposition 5.

A.7 Proof of proposition 6

We follow the proof of proposition 2. First,

$$\frac{dW^S}{dK_1} = \lambda u'\left(c_0^S\right)\frac{d\left(c_0^S + \frac{1}{R}c_1^S\right)}{dK_1} = \lambda u'\left(c_0^S\right)\frac{d(\Pi)}{dK_1} = 0$$

since there is no change in the value of profits from a change in K_1. Next,

$$\begin{aligned} W^B &= \int W^B(e_0) f(e_0)\,de_0 \\ &= \int_0^{\overline{e_0}} \{\log(y_0 e_0) + \beta\mathbb{E}[\log(e_1)] - K_0\} f(e_0)\,de_0 \\ &\quad + \int_{\overline{e_0}}^{\infty} \max_{b_1}\left\{\log(y_0 e_0 - b_0 + Q(b_1)) + \beta\int_{\overline{e_1}(b_1)}^{\infty}\log(e_1 - b_1(e_0)) f(e_1)\,de_1 + \beta d_1(b_1)(\log(e_1) - K_1)\right\} f(e_0)\,de_0 \end{aligned}$$

Note again that any effect from the change in $\overline{e_0}$ does not affect welfare since households are indifferent. Then, the envelope theorem implies

$$\begin{aligned} \frac{dW^B}{dK_1} &= \int_{\overline{e_0}}^{\infty}\left\{u'(c_0^r(e_0))\frac{\partial Q}{\partial K_1}(b_1(e_0)) - \beta d_1(b_1(e_0)) f(e_0)\right\} de_0 \\ &= E\left[u'(c_0^r(e_0))\frac{\partial Q}{\partial K_1}(b_1(e_0))\right] - \beta E[d_1(b_1)] \end{aligned}$$

which is Proposition 6.

B Proofs for section 4

B.1 Proof of proposition 7

From $d_0 = F\left(\overline{e_0}\right)$, we have:

$$\frac{\partial d_0}{\partial y_0} = f\left(\overline{e_0}\right)\frac{\partial \overline{e_0}}{\partial y_0}$$

now, since $e_0 y_0$ enter symmetrically into the problem, an increase in y_0 must reduce the $\overline{e_0}$ threshold in proportion, ie:

$$\frac{\partial \overline{e_0}}{\partial y_0} = -\frac{\overline{e_0}}{y_0}$$

the overall effect on the default probability is then

$$\frac{\partial d_0}{\partial y_0} = -f\left(\overline{e_0}\right)\frac{\overline{e_0}}{y_0} < 0$$

This proves Proposition 7.

B.2 Proof of proposition 8

See Auclert and Mitman (2022)