Inequality and Aggregate Demand

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Can rising income inequality *cause* poor macro performance?

Two major arguments (Stiglitz, etc.):

1. MPCs are negatively correlated with income, so higher income inequality lowers aggregate consumption
2. If inequality comes from more volatile and uncertain incomes, it could raise precautionary savings

Both supported by empirical evidence, both correct in partial eqbm
Inequality and macroeconomic performance

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- Neither survives general eqbm in standard neoclassical models
  - These forces lower real interest rates and raise investment
Inequality and macroeconomic performance

- Can rising income inequality cause poor macro performance?
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- Both supported by empirical evidence, both correct in partial eqbm
- Neither survives general eqbm in standard neoclassical models
  - These forces lower real interest rates and raise investment
- We show that inequality lowers output if monetary policy is slow or unable to react to it (e.g. at the zero lower bound)
  - Quantify the potential effect of 1 & 2 under various mp rules
  - Investment usually falls (‘paradox of thrift’) 
  - Depressed economy even in long run (‘secular stagnation’)


What we do

- Take canonical Huggett-Aiyagari model
  - Add downward nominal wage rigidities (DNWR)
  - Parsimonious, allows focus on household demand
- Calibrate to 2013 U.S.
  - Binding zero lower bound (ZLB): \( r = \pi = i = 0\% \)
  - Mildly depressed employment \( L < 1 \)
- Main qstn: what happens if inequality unexpectedly rises further?
  - Temporarily (income redistribution)
  - Permanently (change in income process)

under various assumptions about fiscal policy

- Key: binding ZLB + DNWR
  - \( \Rightarrow \) most of equilibrium adjustment happens via unemployment
  - In particular steady state \( r \) fixed, \( L \) adjusts to clear markets
Long-run adjustment in asset market

![Graph showing the relationship between real interest rate and total assets](graph.png)
Long-run adjustment in asset market

![Graph showing Asset Demand and Asset Supply]

- **Asset Demand**
- **Asset Supply**

Graph parameters:
- **Real interest rate** \( r \) (bps)
- **Total assets** \( B + K \)
Long-run adjustment in asset market
Long-run adjustment in asset market

![Graph showing the relationship between Real interest rate (bps) and Total assets (B + K).]

- The graph illustrates the long-run adjustment in asset markets.
- The axes represent Real interest rate (bps) and Total assets (B + K).
- The graph includes a point S indicating a specific relationship between the variables.
- The diagram helps visualize how changes in real interest rates affect total assets in the long run.
Contributions 1/2

- Foundation for the transmission mechanism of inequality to output via an aggregate demand channel
- We find that the effects of increasing inequality:
  - are small when inequality = temporary redistribution
  - are small when inequality = permanent redistribution (fixed effects)
  - are potentially large if inequality = increasing risk
  - Always depend on the degree of endogenous inequality and the fiscal response (government spending and public debt)
Our results rely on a new two-step approach to quantifying magnitudes:

\[
\text{Output effect} = (\text{GE multiplier}) \cdot (\text{PE sufficient statistic})
\]

- Sufficient statistics are measurable:
  - Short run: \(\text{Cov}(\text{MPC}, \frac{\partial \nu}{\partial Y})\)
  - Long run: elasticity of savings to idiosyncratic risk

- Multiplier characterizes the response to any aggregate demand shock
  - Depends only on model parameters and policy
Related literature

- Incomplete markets, inequality, and aggregate savings

- Interaction with nominal rigidities
  - Guerrieri-Lorenzoni (2015), Oh-Reis (2013), McKay-Reis (2016)

- Secular stagnation

- Sufficient statistics approaches in macro
Outline

1. Model and equilibrium definition
2. Calibration
3. Income inequality in the short run
4. Income inequality in the long run
Outline

1. Model and equilibrium definition
2. Calibration
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4. Income inequality in the long run
Households

- Mass $\rho(\omega_i)$ of individuals of type $\omega_i$
  - ex-ante identical within type, facing purely idiosyncratic risk
  - idiosyncratic state $\sigma_{it} \in S$, Markov process $\Lambda(\omega)$, at stat. distrib.
  - combined state $s_{it} \equiv (\omega_i, \sigma_{it})$

- Separable preferences, constant EIS $\nu$: $u(c) = \frac{c^{1-\nu-1}}{1-\nu-1}$

- Incomplete markets: bonds and shares + positive nw constraint

\[
\max \quad E \left[ \sum_t \beta^t u(c_{it}) \right]
\]
\[
\text{s.t.} \quad c_{it} + b_{it} + p_t v_{it} = y_t(s_{it}) + (1 + r_{t-1}) b_{it-1} + (p_t + d_t) v_{i,t-1} \\
b_{it} + p_t v_{it} \geq 0
\]
Households

- Mass $\rho(\omega_i)$ of individuals of type $\omega_i$
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- Separable preferences, constant EIS $\nu$: $u(c) = \frac{c^{1-\nu-1}}{1-\nu-1}$

- Incomplete markets: bonds and shares + positive nw constraint

$$\max \mathbb{E} \left[ \sum_t \beta^t u(c_{it}) \right]$$

s.t. $c_{it} + b_{it} + p_t v_{it} = y_t(s_{it}) + (1 + r_{t-1}) b_{it-1} + (p_t + d_t) v_{i,t-1}$

$$b_{it} + p_t v_{it} \geq 0$$

- Perfect foresight: $1 + r_{t-1} = \frac{p_t + d_t}{p_{t-1}}$
Households

- Mass $\rho(\omega_i)$ of individuals of type $\omega_i$
  - ex-ante identical within type, facing purely idiosyncratic risk
  - idiosyncratic state $\sigma_{it} \in S$, Markov process $\Lambda(\omega)$, at stat. distrib.
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- Separable preferences, constant EIS $\nu$: $u(c) = \frac{c^{1-\nu}-1}{1-\nu-1}$

- Incomplete markets: bonds and shares + positive nw constraint

$$\max \mathbb{E} \left[ \sum_t \beta^t u(c_{it}) \right]$$

subject to:

$$c_{it} + b_{it} + p_t v_{it} = y_t(s_{it}) + (1 + r_{t-1}) b_{it-1} + (p_t + d_t) v_{i,t-1}$$

$$b_{it} + p_t v_{it} \geq 0$$

- Perfect foresight: $1 + r_{t-1} = \frac{p_t + d_t}{p_{t-1}}$

- Assets $a_{i,t} = b_{i,t} + p_t v_{i,t}$ summary state, composition indifferent
  - Household with $a_{i,t}$ holds fraction $\theta(a_{i,t})$ in stocks
  - Take $\theta(\cdot)$ directly from data
Household labor income: pre-tax

- Pre-tax labor income:
  \[ z_t(s_{it}) = \begin{cases} \frac{W_t}{P_t} \cdot e_t(s_{it}) & L_t = 1 \\ \frac{W_t}{P_t} \cdot L_t \cdot e_t(s_{it}) \cdot \gamma(s_{it}, L_t) & L_t \leq 1 \end{cases} \]

- \( \frac{W_t}{P_t} \) real wage, \( e_t(s_{it}) \) labor endowment, \( \mathbb{E}_I [e_t(s_{it}) \cdot \gamma(s_{it}, L_t)] = 1 \)
- \( L_t \leq 1 \) fraction of aggregate endowment demanded by firms
- \( \gamma \): incidence of employment \( L < 1 \)
Household labor income: pre-tax

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\[ z_t(s_{it}) = \begin{cases} 
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- \( \gamma \): incidence of employment \( L < 1 \)
  - Equal incidence at \( L \) when \( \gamma(s, L) = 1 \) for all \( s \)
  - Other cases: (in)equality multiplier
Household labor income: pre-tax

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  \[
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  \end{cases}
  \]

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- \( L_t \leq 1 \) fraction of aggregate endowment demanded by firms
- \( \gamma \): incidence of employment \( L < 1 \)
  - Equal incidence at \( L \) when \( \gamma (s, L) = 1 \) for all \( s \)
  - Other cases: (in)equality multiplier
- \( e_t \) main exogenous source of change in labor income inequality
  - Proxy for many plausible underlying causes
  - Can affect separately fixed effect, persistent, or transitory component
Household labor income: post-tax

- Post-tax labor income:
  \[ y_t(s_{it}) = T_t + (1 - \tau_t)(1 - \tau^r)z_t(s_{it}) \]

- Rate \( \tau^r \) earmarked for lump-sum rebate
  - \( T_t = \tau^r (1 - \tau_t) \mathbb{E}_I [z_{it}] \)
  - Govtt adjusts \( \tau_t \) to satisfy its budget constraint
Household labor income: post-tax

- Post-tax labor income:

\[ y_t(s_{it}) = (1 - \tau_t)(\tau' E[I] z_{it}) + (1 - \tau') z_t(s_{it}) \]

- Rate \( \tau' \) earmarked for lump-sum rebate
  - \( T_t = \tau' (1 - \tau_t) E[I] z_{it} \)
  - Govtt adjusts \( \tau_t \) to satisfy its budget constraint

- (\( \tau'_t \) alternative exogenous source of change in income inequality)
Firms and factor markets

- Perfect competition, CRS production $Y_t = F (K_{t-1}, L_t)$
- No adjustment cost to labor: labor demand
  \[ F_L (K_{t-1}, L_t) = \frac{W_t}{P_t} \]
- Convex capital adjustment costs: investment demand
  \[ 1 + \zeta' \left( \frac{I_t}{K_{t-1}} - \delta \right) = q_t \]
  with share price $p_t = q_t K_t$. In steady state $q = 1$, $F_K (K, L) = r + \delta$.
- **Dw wage rigidities**: impose $W_t \geq \kappa W_{t-1}$, when binding $L_t < 1$
  - Assume $0 < \kappa \leq 1$: wages “cannot fall too fast”
  - Why? NK sticky prices have counterfactual implications in HA models
Government policy and equilibrium

- **Fiscal authority** has budget constraint

\[ \tau_t \frac{W_t}{P_t} L_t + B_t = G_t + (1 + r_{t-1}) B_{t-1} \]

follows rules for spending and deficits

\[ \frac{G_t}{Y_{ss}} = \frac{G_{ss}}{Y_{ss}} - \epsilon_{GL} (L_t - L_{ss}) \]

\[ \frac{B_t - B_{t-1}}{Y_{ss}} = -\epsilon_{DL} (L_t - L_{ss}) - \epsilon_{DB} \frac{B_{t-1} - B_{ss}}{Y_{ss}} \]

- Calibrate elasticities \( \epsilon_{DL}, \epsilon_{DB}, \epsilon_{GL} \) to historical U.S. experience
  - cf fiscal rules literature (Leeper, Gali-Perotti, etc.)
Central bank sets nominal interest rate $i_t$, follows either

1. Neoclassical rule: set $i_t$ to achieve $L_t = 1$. Obtain real rate $r^n_t$.
2. ZLB rule:
   
   $$ 1 + i_t = \max \left\{ 1, (1 + r^n_t) \left( \frac{P_t}{P_{t-1}} \right)^\phi \right\} $$

   with $\phi > 1$ (Taylor principle)

3. Constant-$r$ rule
4. Simple Taylor rule in inflation and/or unemployment

Equilibrium definition standard
Outline

1. Model and equilibrium definition

2. Calibration

3. Income inequality in the short run

4. Income inequality in the long run
Benchmark calibration

- Gross income process from Kaplan, Moll, Violante 2016. Has form

\[ \log e_{it} = \xi_{it} + \chi_{it} \]

with \( \xi_{it} \) persistent, \( \chi_{it} \) transitory. Matches:
- Earnings dynamics from W2 data (Guvenen, Ozkhan, Song 2014)
- Cross-sectional earnings distribution without fixed effects
- \( \tau' = 17.5\% \) from Congressional Budget Office (2006)
- Incidence rule \( \gamma(s, L) \) matches W2 ‘worker betas’
  - from Guvenen, Schulhofer-Wohl, Song and Yogo (2017) [GSSY]
  - Alternative: equal incidence (\( \gamma = 1 \))
- Asset allocation \( \theta(a) \) imported parametrically from 2013 SCF
Calibrated $\gamma$ function

![Graph showing elasticity of gross earnings to employment](image-url)
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Main calibration</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>EIS</td>
<td>0.5</td>
<td>Standard calibration</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.962</td>
<td>$r = 0$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$K - L$ elasticity</td>
<td>1</td>
<td>Standard calibration</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor share</td>
<td>87.2%</td>
<td>$\alpha = 1 - (r + \delta) \frac{K}{Y}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>4.0%</td>
<td>NIPA 2013</td>
</tr>
<tr>
<td>$\frac{K}{Y}$</td>
<td>Capital-output ratio</td>
<td>321%</td>
<td>FoF hh. net worth 2013</td>
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<tr>
<td>$\frac{i}{Y}$</td>
<td>Investment rate</td>
<td>12.8%</td>
<td>$\delta \frac{K}{Y}$</td>
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<tr>
<td>$\epsilon_I$</td>
<td>Elasticity of $I$ to $q$</td>
<td>1</td>
<td>Macro invt literature</td>
</tr>
<tr>
<td>$i$</td>
<td>Nominal interest rate</td>
<td>0%</td>
<td>ZLB</td>
</tr>
<tr>
<td>$r$</td>
<td>Eqbm real rate</td>
<td>0%</td>
<td>TIPS yields 2013</td>
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<tr>
<td>$L$</td>
<td>Employment gap</td>
<td>0.975</td>
<td>CBO output gap estimate</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Wage deflation rate</td>
<td>1</td>
<td>$\frac{1+i}{1+r}$</td>
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<tr>
<td>$\frac{B_{ss}}{Y}$</td>
<td>Govtt debt</td>
<td>55.4%</td>
<td>Domestic holdings 2013</td>
</tr>
<tr>
<td>$\frac{G_{ss}}{Y}$</td>
<td>Govtt spending</td>
<td>18.7%</td>
<td>NIPA 2013</td>
</tr>
<tr>
<td>$\epsilon_{GL}$</td>
<td>Response of spending to $L$</td>
<td>0.10</td>
<td>Estimated fiscal rule</td>
</tr>
<tr>
<td>$\epsilon_{DL}$</td>
<td>Response of deficits to $L$</td>
<td>0.75</td>
<td>Estimated fiscal rule</td>
</tr>
<tr>
<td>$\epsilon_{DB}$</td>
<td>Response of deficits to debt</td>
<td>0.07</td>
<td>Estimated fiscal rule</td>
</tr>
</tbody>
</table>
Increasing labor income inequality (1/2)

- Recall income process:
  \[ \log e_{it} = \xi_{it} + \chi_{it} \]

- Our experiments change this to:
  \[ \log e_{it} = \omega_i + A_t \xi_{it} + B_t \chi_{it} - C_t \]
  for various paths \((\omega_i, A_t, B_t)\) that achieve target path for \(sd (\log e_{it})\)
  - \(C_t\) ensures \(\mathbb{E}[e_{it}] = 1\)
  - Main case: \(\omega_i = 0, A_t = B_t \uparrow\)
    - Alternatives: increase entirely due to \(\omega_i, A_t,\) or \(B_t\)
Increasing labor income inequality (2/2)

US labor income inequality

- Log points
- Year


- sd log earnings
- Calibration

Short-run experiment

Source for historical data: Song, Price, Guvenen, Bloom and von Wachter (2016)
Increasing labor income inequality (2/2)

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Increasing labor income inequality (2/2)

US labor income inequality

Log points

Year


sd log earnings Calibration Short-run experiment Long-run experiment

Source for historical data: Song, Price, Guvenen, Bloom and von Wachter (2016)
Outline

1. Model and equilibrium definition
2. Calibration
3. Income inequality in the short run
4. Income inequality in the long run
Partial eqbm effect

- Consider first partial equilibrium effect, holding \((r_t, L_t)\) fixed

Result 1: For any change in after-tax incomes \(dy_{i0}\) st \(\mathbb{E} [dy_{i0}] = 0\), the partial equilibrium change in the path for \(C_t\) is given by

\[
\partial C_t = \text{Cov}_I (MPC_{it}, dy_{i0})
\]

where \(MPC_{it}\) is \(i\)'s spending at date \(t\) of date 0 income. In particular, \(NPV (\partial C) = 0\).
Evaluating $\partial C_0 = \text{Cov}_I (MPC_{i0}, dy_{i0})$

- Quality of approximation within model?

- Fit to data on joint distribution of $(MPC_i, y_i)$? (Use Italian SHIW 2010)

<table>
<thead>
<tr>
<th>Sufficient statistic</th>
<th>Value, Data</th>
<th>Value, Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Cov} \left( MPC_i, \frac{dy_i}{Y} \right) \frac{1}{d\sigma}$</td>
<td>$-0.049$</td>
<td>$-0.045$</td>
</tr>
</tbody>
</table>
General eqbm effect under alternative monetary policies
General eqbm effect under alternative monetary policies

- Output
- Consumption
- Investment
- Government Spending
- Government Bonds
- Capital
- Employment vs s.s.
- Real Interest Rate (bps)

Graphs show the impact of different policies on various economic indicators over time.

- **Output**
  - ZLB (dashed black line)
  - Neoclassical (solid green line)

- **Consumption**
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- **Employment vs s.s.**
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- **Real Interest Rate (bps)**
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General eqbm effect under alternative monetary policies

![Graphs showing the effects of alternative monetary policies on various economic indicators.](image-url)
Relating partial and general equilibrium effect (1/2)

- **Result 2:** the general equilibrium path of aggregate output $Y_t$ is

$$dY = G_Y \cdot \partial C$$

where $G_Y$ is the ‘GE matrix’, independent of the source of shock

- Reflects many equilibrium forces

- In a benchmark case with no investment, no endogenous spending, and constant $-r$ monetary policy:

$$G_Y = \sum_{k=0}^{\infty} M^k$$

where $M_{t,s}$ is incidence-weighted matrix of MPCs

- ‘Intertemporal Keynesian Cross’ (Auclert, Rognlie and Straub 2017)

- Here, for ZLB and constant-$r$, we have $G_Y \simeq I$, so GE $\simeq$ PE.
Relating partial and general equilibrium effect (2/2)

\[ \frac{dY}{Y} = G_Y \cdot \frac{\partial C}{Y} \]
Paradox of Thrift

- Why does investment fall?
- \( Q \)-theory \( \Rightarrow \) given a change in \( \{ r_t, L_t \} \), net investment responds by

\[
d (I_t - \delta K_{t-1}) = \epsilon I \sum_{s=0}^{\infty} \left( \frac{1}{1 + r} \right)^{s+1} \{dMPK_{t+s+1} - dr_{t+s}\}
\]

where \( MPK_t \equiv F_K(K_{t-1}, L_t) \)

- Race between cost of capital effect (↑) and MPK effect (↓)
- Under constant-\( r \) policy, latter always dominates
- Typically also at ZLB, due to limited \( r \) response

- Consistency with market clearing?

\[
\text{Savings} = \text{Investment}
\]

- Redistribution \( \Rightarrow \) rise in \textit{desired} household savings
- But in equilibrium, \( L \downarrow \Rightarrow \) aggregate savings \( \downarrow \), consistent with investment \( \downarrow \)
Outline

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Partial equilibrium effect

Consider again partial equilibrium effect, holding \((r_t, L_t)\) fixed.

- Convergence to higher SS asset level *if inequality implies risk*
Partial equilibrium effect

- Consider again partial equilibrium effect, holding \((r_t, L_t)\) fixed

- Convergence to higher SS asset level \textit{if inequality implies risk}

- Are these magnitudes plausible?

- Compare to micro literature on savings effect of earnings risk

- Large literature, large range of estimates (Browning Lusardi 1996)
Empirical evaluation

- Carroll-Samwick (1997) run in PSID:
  \[
  \log a_i = \alpha_\xi s_{i\xi}^2 + \alpha_\chi s_{i\chi}^2 + \beta Z_i + u_i
  \]
  with \(s_{i\xi}^2\) ≡ variance of innovations to permanent component of earnings, \(s_{i\chi}^2\) ≡ variance of transitory

- Compute equivalent semielasticity in the model

<table>
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<th>Value, Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_\xi)</td>
<td>12.09</td>
<td>12.87</td>
</tr>
<tr>
<td>(\alpha_\chi)</td>
<td>7.11</td>
<td>0.50</td>
</tr>
</tbody>
</table>

- Semielasticities in line with empirical estimates
General eqbm effect under alternative monetary policies
General eqbm effect under alternative monetary policies

- Output
- Consumption
- Investment
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- Government Bonds
- Capital
- Employment vs s.s.
- Real Interest Rate (bps)

Diagram showing the effects of alternative monetary policies on various economic indicators over time.
General eqbm effect under alternative monetary policies
Steady state long-run effect: a special case

- Consider first the following special case:
  - Constant income incidence ($\gamma = 1$)
  - No endogenous fiscal policy ($\epsilon_{GL} = \epsilon_{BL} = \epsilon_{BG} = 0$)
  - Monetary policy: either ZLB or constant-$r$
- Experiment: increase index of inequality $\sigma$
- Asset market clearing:
  \[
  A \left( r, \sigma, \tau, \frac{W}{P}, L \right) = \bar{B} + K
  \]
Consider first the following special case:
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- Monetary policy: either ZLB or constant-$r$

Experiment: increase index of inequality $\sigma$

Asset market clearing (from homotheticity):

$$
(1 - \tau) \frac{\mathcal{W}}{\mathcal{P}} \hat{a}(r, \sigma) = \overline{B} + K
$$

$\hat{a}(r, \sigma)$ partial eqbm savings schedule at $r$ (income=1)
Steady state long-run effect: a special case

Consider first the following special case:

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- No endogenous fiscal policy \((\epsilon_{GL} = \epsilon_{BL} = \epsilon_{BG} = 0)\)
- Monetary policy: either ZLB or constant-\(r\)

Experiment: increase index of inequality \(\sigma\)

Asset market clearing:

\[
\left( \frac{W}{P} L - (G + rB) \right) \hat{a}(r, \sigma) = B + K
\]

\(\hat{a}(r, \sigma)\) partial eqbm savings schedule at \(r\) (income=1)

Government budget constraint:

\[
\tau \frac{W}{P} L = G + rB
\]
Steady state long-run effect: a special case

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- Constant income incidence \((\gamma = 1)\)
- No endogenous fiscal policy \((\epsilon_{GL} = \epsilon_{BL} = \epsilon_{BG} = 0)\)
- Monetary policy: either ZLB or constant-\(r\)

Experiment: increase index of inequality \(\sigma\)

Asset market clearing:

\[
(w(r) L - (G + rB)) \hat{a}(r, \sigma) = B + \kappa(r) L
\]

\(\hat{a}(r, \sigma)\) partial eqbm savings schedule at \(r\) (income=1)

Government budget constraint: \(\tau \frac{W}{P} L = G + rB\)

Factor demand conditions: \(\frac{K}{L} \equiv \kappa(r), \ \frac{W}{P} \equiv w(r)\)
Equilibrium: \((A, L)\) space

\[
(w(r)L - (G + r\overline{B})) \hat{a}(r, \sigma) = \overline{B} + \kappa(r) L
\]
Equilibrium: \((A, L)\) space

\[
(w(r) L - (G + rB)) \hat{a}(r, \sigma) = B + \kappa(r) L
\]
Role of incidence and fiscal response

![Graph showing employment $L$ vs. total assets $B + K$.]
Role of incidence and fiscal response

w. benchmark fiscal policy

Employment $L$

Total assets $B + K$

$S'$
Role of incidence and fiscal response

\[ + \text{GSSY income incidence} \]
Sufficient statistic formula for \( \frac{dY}{Y} \)

- Differentiating SS asset market clearing and using \( \frac{dL}{L} = \frac{dY}{Y} \)

\[
\frac{dY}{Y} = - \frac{1}{\frac{B}{B+K} + \frac{\tau}{1-\tau} + \eta_F + \eta_I} \times \left( \frac{\partial A/A}{\partial \sigma} \right) d\sigma
\]

- \( \frac{\partial A/A}{\partial \sigma} \) average semielasticity of individual savings to \( \sigma \)
- \( \eta_F \) mitigation from fiscal response, \( \eta_I \) effect of income incidence
- In special case: \( \eta_F = \eta_I = 0 \), Multiplier = 2.31
- In main calibration: \( \eta_F, \eta_I > 0 \), Multiplier = 0.3
Extensions

1. Inequality and the $r^*$ decline
2. Change in capital-labor distribution:
   - Decline in labor share due to changing technology or relative price of investment (Piketty/Karabarbounis-Neiman)
   - Monopoly profits (Summers/Krugman)
3. Alternative policy at the zero lower bound
4. Taylor-rule monetary policy
Conclusion

- Canonical macro model of inequality + nominal wage rigidities
  - Allows to study effect of aggregate demand shocks on output, including inequality
  - Very tractable and flexible

- Theory highlights importance of empirical evidence on
  - MPC heterogeneity \( \text{Cov} \left( \text{MPC}, y \right) \) (short-run)
  - Effect of income uncertainty on savings \( \frac{\partial \log A}{\partial \sigma} \) (long-run)
  - Distributional incidence of recessions \( \gamma \) (both)

- Amplification role of private investment

- Stabilizing role of monetary \( (r) \) and fiscal policy \( (G \text{ and } B) \)
Thank you!
Calibrating the retention function

- Model relationship between net income $y$ and gross $z$:

$$\frac{y_{it}}{E[Z_{it}]} = (1 - \tau) \left( \tau_r + (1 - \tau_r) \frac{Z_{it}}{E[Z_{it}]} \right)$$

- CBO data on avg transfers and taxes per nonelderly household by income in 2006: overall AMI $95,000 and

<table>
<thead>
<tr>
<th>Quintiles of market income</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average market income (AMI)</td>
<td>$z_{it}$</td>
<td>12,600</td>
<td>36,100</td>
<td>59,500</td>
<td>89,900</td>
</tr>
<tr>
<td>AMI + Transfers minus taxes</td>
<td>$y_{it}$</td>
<td>25,200</td>
<td>36,300</td>
<td>51,400</td>
<td>72,700</td>
</tr>
</tbody>
</table>

- Yields

$$\frac{y_{it}}{E[Z_{it}]} = 0.143 + 0.666 \frac{Z_{it}}{E[Z_{it}]}$$

with $R^2 = 0.9988$

- Implying

$$\tau_r = \frac{0.143}{0.143 + 0.666}$$
Calibrating household portfolios

- Obtain $\theta (a)$ parametrically from SCF
  - fraction invested in 'shares' as function of total assets $a = b + pv$
  - broad definition: all net worth except deposits and bonds
  - narrow definition: only equity and shares

![Graph showing the relationship between log (net worth/average net worth) and fraction of shares in net worth $\theta (a)$, with fitted curves for SCF 2013: broad and narrow capital.]
Distribution summary statistics

Model

Data (SCF 2013)
Inequality and the $r^*$ decline

Real rate

Basis points

Year


sd log earnings

Level

Year


Real rate and sd log earnings over time.
Policy solutions at the ZLB

Steady state wage inflation $\kappa - 1$ (%)

Steady-state employment

Wage deflation

Effective lower bound

Nominal rate lower bound $\hat{i}$ (%)

Steady-state employment

Inflation target $\pi^*$ (%)

Full employment equilibrium