Monetary Policy and the Redistribution Channel*

Adrien Auclert†

January 2016

Abstract

This paper evaluates the role of redistribution in the transmission mechanism of monetary policy to consumption. Three channels affect aggregate spending when winners and losers have different marginal propensities to consume: an earnings heterogeneity channel from unequal income gains, a Fisher channel from unexpected inflation, and an interest rate exposure channel from real interest rate changes. Using a sufficient statistics approach, I show that the latter is plausibly as large as the intertemporal substitution channel in Italian and in U.S. data. A calibrated model reveals that this channel is particularly potent when asset maturities are short, and that the other two redistributive channels can also amplify the effects of monetary policy.

JEL Classification: D31, D52, E21, E52.

---

*This paper is a revised version of Chapter 1 of my PhD dissertation at MIT. I cannot find enough words to thank my advisors Iván Werning, Robert Townsend and Jonathan Parker for their continuous guidance and support. I also thank many seminar participants for their insights. I have particularly benefited from the detailed comments and discussions of Eduardo Dávila, Gauti Eggertsson, Xavier Gabaix, Adam Guren, Gregor Jarosch, Greg Kaplan, Guido Lorenzoni, Ben Moll, Makoto Nakajima, Matthew Rognlie, Alp Simsek, Christian Stoltenberg and Daan Struyven. Thanks to the Macro-Financial Modeling Group for financial support. All errors are my own.

†Princeton University and Stanford University. Email: aauclert@princeton.edu.
1 Introduction

There is a conventional view that redistribution is a side effect of monetary policy changes, separate from the issue of aggregate stabilization which these changes aim to achieve. This view is implicit in most models of the monetary policy transmission mechanism, which feature a representative agent. By contrast, in this paper I argue that redistribution is a channel through which monetary policy affects macroeconomic aggregates, because those who gain from accommodative monetary policy have higher marginal propensities to consume (MPCs) than those who lose. The simple argument goes back to Tobin (1982):

Aggregation would not matter if we could be sure that the marginal propensities to spend from wealth were the same for creditors and for debtors. But [...] the population is not distributed between debtors and creditors randomly. Debtors have borrowed for good reasons, most of which indicate a high marginal propensity to spend from wealth or from current income.

Using consumer theory, I refine Tobin’s intuitions about aggregation and show how they can help us understand the effect of monetary policy on household spending. Monetary expansions tend to increase real incomes, to raise inflation and to lower real interest rates. Not everyone is equally affected by these changes. Suppose that winners have higher MPCs than losers for each change in isolation. Then, I find that three redistributive channels contribute to the increase in aggregate consumer spending.

First, the gains in total earnings from labor and profits that are induced by monetary expansions may be unequal. This is the earnings heterogeneity channel of monetary policy.

Second, unexpected inflation revalues nominal balance sheets, with nominal creditors losing and nominal debtors gaining: this is the Fisher channel, which has a long history in the literature since Fisher (1933). This channel has been explored by Doepke and Schneider (2006), who measure the balance sheet exposures of various sectors and groups of households in the United States to different inflation scenarios. Net nominal positions (NNPs) quantify the exposures to unexpected increases in the price level.

Real interest rate falls create a third, more subtle form of redistribution. These falls increase financial asset prices. But we cannot directly claim that the holders of these assets benefit: instead, we have to consider whether their assets have longer durations than their liabilities. Importantly, these liabilities include their consumption plans, while their assets include their human capital. Unhedged interest rate exposures (UREs)—the difference between all maturing assets and liabilities at a point in time—are the correct measure of households’ balance-sheet exposures to real interest rate changes, just like net nominal positions are for price level changes. For example, agents whose financial wealth is primarily invested in short-term certificates of deposit tend to have positive UREs, while those with large investments in long-term bonds or adjustable-rate mortgage holders tend to have
negative UREs. Real interest rate falls redistribute away from the first group towards the second group: this is what I call the interest rate exposure channel.

In this paper I devote particular attention to this third channel, both because it is novel, and because it is likely to be the most important kind of redistribution created by monetary policy over the business cycle—at least in modern regimes with stable inflation. But my results also provide new insights about the other two channels, and more generally about how an explicit account of heterogeneity affects the transmission mechanism of monetary policy, including via traditional aggregate income and substitution channels.

In the first part of the paper, I uncover all these channels by studying a general aggregation problem. In partial equilibrium, I consider an optimizing agent with a given initial balance sheet, who values nondurable consumption and leisure, and is subject to a transitory change in income, inflation and the real interest rate. I decompose his consumption response into a substitution effect and a wealth effect, and show that the latter is the product of his MPC out of income and a balance-sheet revaluation term in which NNPs and UREs appear. This result is robust to the presence of incomplete markets, idiosyncratic risk, and (certain kinds of) borrowing constraints. In other words, the MPC out of a windfall income transfer is relevant to determine the response of optimizing consumers to inflation— or real interest rate—induced changes in their balance sheets. To the best of my knowledge, this result is new to the incomplete-markets consumption literature.1

I then sum across the individual-level predictions and exploit the fact that financial assets and liabilities net out in general equilibrium to obtain the first-order response of aggregate consumption to simultaneous transitory shocks to output, inflation, and the real interest rate. This response is the sum of five terms, reflecting the contributions from the two aggregate and the three redistributive channels mentioned above. Moreover, the magnitudes of the redistributive channels are given by sufficient statistics: the cross-sectional covariances between MPCs and exposures to each aggregate shock. Since the pioneering work of Harberger (1964), sufficient statistics have been used in public finance to evaluate the welfare effect of hypothetical policy changes in a way that is robust to the specifics of the underlying structural model (see Chetty 2009 for a survey). Mine are useful to evaluate the impact of hypothetical changes in macroeconomic aggregates on aggregate demand in a similarly robust way. All that is required is information on household balance sheets, income and consumption levels, and their MPCs.

In particular, the response of aggregate consumption to the real interest rate is the sum of a redistribution component (which depends on the cross-sectional covariance between MPCs and UREs) and an intertemporal substitution component (which depends on con-

---

1A precursor to this finding is Kimball (1990), who provides equations that characterize MPC, analyzes how it depends on income uncertainty and market structure, and points out that “the marginal propensity to consume out of wealth figures into the interest elasticity of consumption, as a factor in the wealth effect term”.
sumers’ Elasticities of Intertemporal Substitution, or EISs). There is a large literature estimating “the” EIS—a key parameter in dynamic macroeconomic models—and no clear consensus on its value. To assess how large the redistribution part is without taking a stand on the EIS, I define a number \( \sigma_r \) such that the interest rate exposure channel and the substitution channel are equal in magnitude if the EIS is equal to \( \sigma_r \). \( \sigma_r \) is positive when the covariance between MPCs and UREs is negative, in other words when those who gain from falls in real interest rates have higher MPCs. I define a method for measuring UREs, in an exercise similar to Doepke and Schneider’s work on exposures to price changes.

Turning to data, I apply this methodology and estimate \( \sigma_r = 0.12 \) in Italy using a survey containing a self-reported measure of MPC (Jappelli and Pistaferri 2014), and \( \sigma_r = 0.3 \) in the United States using a procedure that exploits the randomized timing of tax rebates as a source of identification for MPC (Johnson, Parker and Souleles 2006). These numbers confirm that there exists an interest rate exposure channel of monetary policy, acting in the same direction as the substitution channel. They also show that this channel is quantitatively significant. Therefore, representative-agent analyses that abstract from redistribution fail to capture an important reason why real interest rates affect consumption.

My reduced-form approach has the virtue of being robust to model misspecification, but it requires precise measures of MPC and URE, which are challenging to obtain jointly. Moreover, it raises important questions about the key theoretical determinants of the sufficient statistics, the robustness of their predictions to large and persistent shocks, and the way in which the interest rate exposure channel interacts with other channels of monetary policy transmission.

In order to answer these questions, I construct a stylized Bewley-Huggett-Aiyagari incomplete markets model with nominal, long-term, circulating private IOUs (as in Huggett 1993) and endogenous labor supply. The model features rich heterogeneity in MPCs and UREs. I calibrate it to gross asset positions and average durations in the U.S. economy and quantitatively evaluate, in its steady-state, the size of my sufficient statistics. I find that the interest rate exposure channel has the same sign, and comparable magnitude, as in my reduced-form analysis. Hence, even if the model is misspecified, its aggregate prediction for the response of consumption to real interest rates through redistribution will be consistent with empirical evidence. I also find that the absolute magnitude of the interest rate exposure channel increases if asset durations are counterfactually shortened. Intuitively, under a shorter maturity structure, debtors—the high-MPC agents in the economy—roll over a larger fraction of their liabilities each period, and their consumption plans are therefore more sensitive to changes in real interest rates.

Turning to the other components of the redistribution channel, the steady-state suffi-

\[ \text{Most studies using aggregate or household-level data point to a number between 0 and 2, with some consensus in macroeconomics for a value below 1 (see for example Hall 2009 or Havránek 2015).} \]
cient statistic for the Fisher channel shows that an unexpected increase in the price level is associated with a substantial increase in aggregate consumption. In line with recent theoretical results in Werning (2015), I also find that the sensitivity of the income distribution to increases in output is an important determinant of the general equilibrium response to monetary policy shocks. In a calibration where, consistent with the empirical findings of Coibion, Gorodnichenko, Kueng and Silvia (2012), increases in GDP reduce income inequality, the earnings heterogeneity channel plays a substantial amplification role. Hence the model suggests that all three redistributive channels can work to amplify the expansionary effects of accommodative monetary policy.

To assess the performance of my sufficient statistics as a predictive tool, I study the economy’s transitional dynamics after an unanticipated monetary policy shock, assuming nominal prices are completely fixed.\(^3\) Despite the endogenous persistence of shocks in the model, the sufficient statistic approach does remarkably well, quantitatively, at capturing the aggregate response of the economy to a transitory monetary policy shock through both direct and general equilibrium channels. Moreover, all of the qualitative predictions from the sufficient statistic exercise carry over to the case of a persistent monetary policy shock.\(^4\)

I illustrate my finding on the role of asset durations by asking the extent to which monetary policy transmission would differ if the U.S. economy only had adjustable rate mortgages instead of its current mix of adjustable and fixed rate mortgages. I find that monetary policy shocks would have more than double their current effect on household nondurable consumption. This is consistent with the results of Calza, Monacelli and Stracca (2013), who find that consumption reacts much more strongly to identified monetary policy shocks in countries where mortgages predominantly have adjustable rates.\(^5\) One interpretation is that the substitution effect is stronger in these countries, since agents effectively participate more in financial markets. My paper offers an alternative interpretation that does not rely on limited participation: in adjustable-rate mortgage (ARM) countries, monetary policy affects household spending predominantly by redistributing wealth.\(^6\)

My final finding is that monetary policy has asymmetric effects in my ARM calibration. While my sufficient statistic correctly predicts the effect of any increase in the policy

\(^3\)This assumption allows me to focus solely on the model’s response to real interest rate changes and its general equilibrium income response. The sufficient statistics shows that nominal price adjustment would further amplify the response through the Fisher channel.

\(^4\)They also hold up well quantitatively, if one uses a representative agent model to scale the response.

\(^5\)It also confirms a widely held-view in policy circles that a country’s monetary policy transmission mechanism is affected by its mortgage structure (Cecchetti 1999; Miles 2004).

\(^6\)Aside from limited participation, I am leaving a number of other redistributive channels out of my analysis. First, since I abstract away from aggregate risk, in my framework monetary policy cannot change risk premia. Second, since I assume that all assets are remunerated at the risk-free rate, my analysis does not address the unequal incidence of inflation due to larger cash holdings by the poor (Erosa and Ventura 2002; Albanesi 2007). Hence my analysis applies most directly to conventional monetary policy actions in modern developed countries with low and stable inflation targets.
rate, it overpredicts the increase in output that results from a sufficiently large fall. This asymmetry comes from the differential response of borrowers at their credit limit to rises and falls in income: while these borrowers save an important fraction of the gains they get from low interest rates, they are forced to cut spending steeply when interest rates rise. The prediction that interest rate hikes lower output more than falls increase it has received support in the empirical literature (Cover 1992; de Long and Summers 1988; Tenreyro and Thwaites 2016). An influential interpretation of this fact, which dates back to Keynes, relies on the presence of downward nominal wage rigidities. My explanation is that MPC differences are smaller for falls than for rises in interest rates, so that the redistribution channel is smaller for the former than for the latter.

My analysis is motivated by an extensive empirical literature documenting that MPCs are large and heterogeneous in the population (see Jappelli and Pistaferri 2010 for a survey), and that they depend on household balance sheet positions. Recently, di Maggio, Kermani and Ramcharan (2014) and Keys, Piskorski, Seru and Yao (2014) have measured the consumption response of households to changes in the interest rates they pay on their mortgages. My theory shows that these papers quantify an important leg of the redistribution channel of monetary policy.

Several papers have focused on the redistributive channels of monetary policy I highlight in isolation. Coibion et al. (2012) propose an empirical evaluation of the earnings heterogeneity channel by measuring how identified monetary policy shocks affect income inequality in the Consumer Expenditure Survey. The Fisher channel has received a great deal of attention in the literature following the work of Doepke and Schneider (2006). For example, on the normative side, Sheedy (2014) asks when the central bank should exploit its influence on the price level to ameliorate market incompleteness over the business cycle. On the positive side, Sterk and Tenreyro (2015) show that the Fisher channel can be a source of effects of monetary policy under flexible prices in a non-Ricardian model. The interest rate exposure channel has, by contrast, not received much attention in the context of monetary policy.

The importance of MPC differences in the determination of aggregate demand is well understood by the theoretical literature on fiscal transfers. MPC differences between borrowers and savers, in particular, have been explored a source of aggregate effects from shocks to asset prices or to borrowing constraints. In Farhi and Werning (2013b), MPCs enter as sufficient statistics for optimal macro-prudential interventions under nominal

---

7See for example Mian, Rao and Sufi 2013; Mian and Sufi 2014; Baker 2014 and Jappelli and Pistaferri 2014.
8Redistribution through real interest rates does play a prominent role, for example, in Bassett (2014)’s study of optimal fiscal policy or in Costinot, Lorenzoni and Werning (2014)’s study of dynamic terms of trade manipulation.
9See Galí, López-Salido and Vallés 2007; Oh and Reis 2012; Farhi and Werning 2013a; McKay and Reis 2015.
10See King 1994; Eggertsson and Krugman 2012; Guerrieri and Lorenzoni 2015; Korinek and Simsek 2014.
rigidities. None of these studies, however, focus on the role of MPC differences in generating aggregate effects of monetary policy.

My dynamic general equilibrium model belongs to a recent literature studying New Keynesian environments with incomplete markets. Guerrieri and Lorenzoni (2015) was the first paper to introduce nominal rigidities in a Bewley-Huggett-Aiyagari model. They did so to study the effect of credit tightening at the zero lower bound. Gornemann, Kuester and Nakajima (2012) were the first to examine the effect of monetary policy itself in such an environment. Calibrating to the earnings heterogeneity evidence of Coibion et al. (2012), they report that a small fraction of wealthy agents gain, and others lose, from contractionary monetary policy shocks. My framework suggests that this can arise from rich households both earning more in profits and gaining from positive unhedged interest rate exposures. New work by Kaplan, Moll and Violante (2016) features a state-of-the-art calibration with both liquid and illiquid assets. They focus on the importance of general equilibrium income effects of monetary policy, which in this paper I further split into an aggregate income channel and an earnings heterogeneity channel.\footnote{Because asset durations are infinitely short in the continuous-time model of Kaplan et al. (2016), their model does not include a Fisher channel—which would otherwise constitute another general equilibrium effect.}

In independent work, McKay, Nakamura and Steinsson (2015) compare, as I do, the aggregate effects of monetary policy shocks when markets are incomplete relative to the representative-agent case, but they focus on shocks announced well in advance. They show that incomplete markets dampen the effect of this forward guidance. This contrasts to the response to contemporaneous monetary policy shocks that I highlight, which tends to be higher under incomplete markets—especially when debt is short-term debt—because of the negative correlation between MPCs and UREs that the model generates.\footnote{This effect is not apparent in the McKay et al. (2015) calibration, in which the government is the only borrower and taxes are only levied on the richest individuals. As a result, when real interest rates fall, the government cuts taxes and this tax-based redistribution channel overwhelms the interest rate exposure channel. I discuss the effects of rebates from negative-URE outside sectors in section 3.3.}

A few other dynamic general equilibrium models examine the impact of mortgage structure on the monetary transmission mechanism. As in my paper, Calza et al. (2013) and Rubio (2011) find, in the calibrations of their models, significantly larger effects from monetary policy shocks under variable- than under fixed-rate mortgages (FRMs). I highlight the role of unhedged interest rate exposures in accounting for these results. Garriga, Kydland and Sustek (2013) study a flexible-price, limited participation model and also find much larger output effects under ARMs than FRMs, mainly acting through investment. Finally, recent contributions by Greenwald (2016) and Wong (2015) have explored the importance of refinancing for monetary policy transmission.

The remainder of this paper is structured as follows. Section 2 presents a partial equilibrium decomposition of consumption responses to shocks into substitution and wealth...
effects. Section 3 provides my aggregation result and discusses the monetary policy transmission mechanism with and without heterogeneity. Section 4 assesses the quantitative magnitude of the interest rate exposure by measuring $\sigma_r$ in survey data. Finally, section 5 builds and calibrates a Huggett model, and uses it to test the accuracy of my sufficient statistics and perform counterfactual experiments. Section 6 concludes.

2 Household balance sheets and wealth effects

In this section, I consider the role of households’ balance sheets in determining their consumption and labor supply adjustments to a transitory macroeconomic shock. I first highlight the forces at play in a life-cycle labor supply model (Modigliani and Brumberg 1954; Heckman 1974) featuring perfect foresight and balance sheets with an arbitrary maturity structure. Balance sheet revaluations and marginal propensities to consume and work play a crucial role in determining both the welfare and the wealth effects from such a shock (theorem 1). Under certain conditions, the positive results from theorem 1 survive the addition of idiosyncratic income uncertainty (theorem 2) and therefore apply to a large class of microfounded models of consumption behavior.

2.1 Perfect-foresight model

Consider a household with separable preferences over nondurable consumption $\{c_t\}$ and hours of work $\{n_t\}$. I assume no uncertainty for simplicity: the same insights obtain when markets are complete. The household is endowed with a stream of real unearned income $\{y_t\}$. He has perfect foresight over the general level of prices $\{P_t\}$ and the path of his nominal wages $\{W_t\}$, and holds long-term nominal and real contracts. Time is discrete, but the horizon may be finite or infinite, so I do not specify it in the summations. The agent solves the following utility maximization problem:

$$\max \sum_t \beta^t \{u(c_t) - v(n_t)\}$$

s.t. $P_t c_t = P_t y_t + W_t n_t + (t-1)B_t + \sum_{s \geq 1} (tQ_{t+s}) (t-1)B_{t+s} - tB_{t+s})$

$$+ P_t (t-1)b_t + \sum_{s \geq 1} (tq_{t+s}) P_{t+s} (t-1)b_{t+s} - tb_{t+s}) \quad (1)$$

In the flow budget constraint (1), $tB_{t+s}$ denotes a nominal payment the household arranges in period $t$ to be paid out to him in period $t + s$, whereas $tb_{t+s}$ denotes a payment in real terms. Correspondingly, $tQ_{t+s}$ is the time-$t$ price of a nominal zero-coupon bond paying

\footnote{Appendix A.1 proves all results under arbitrary non-satiable preferences. Here I present results for separable preferences, since expressions for substitution elasticities take simple and familiar forms in this case. I assume that both $u$ and $v$ are increasing and twice continuously differentiable, with $u$ concave and $v$ convex.}
at \( t + s \), and \( \tau q_{t+s} \) the price of a real zero-coupon bond. This asset structure is the most general one that can be written for this dynamic environment with no uncertainty. The only restriction on the environment is an assumption of no arbitrage, which results in a Fisher equation for the nominal term structure:

\[
\tau Q_{t+s} = (\tau q_{t+s}) \frac{P_t}{P_{t+s}} \quad \forall t, s
\]

I begin the analysis of the consumer problem at \( t = 0 \). The environment allows for a very rich description of the household’s initial holdings of financial assets, denoted by the consolidated claims, nominal \( \{ -1 B_t \}_{t \geq 0} \) and real \( \{ -1 b_t \}_{t \geq 0} \), due in each period. The former could represent deposits, long-term bonds and most typical mortgages. The latter could represent stocks (which here pay a riskless real dividend stream and therefore are priced according to the risk-free discounted value of this stream), inflation-indexed government bonds, and price-level adjusted mortgages. I write real wages at \( t \) as \( w_t \equiv W_t P_t \), the initial real term structure as \( q_t \equiv (0 q_t) \), and impose the present-value normalization \( q_0 = 1 \).

Using either a terminal condition if the economy has finite horizon, or a transversality condition if the economy has infinite horizon, the flow budget constraints consolidate into an intertemporal budget constraint:

\[
\sum_{t \geq 0} q_t c_t = \underbrace{\sum_{t \geq 0} q_t (y_t + w_t n_t)}_{W^H} + \underbrace{\sum_{t \geq 0} q_t \left( (-1 b_t) + \left( \frac{-1 B_t}{P_t} \right) \right)}_{W^F}
\]

(2)

Given \( q_0 = 1 \), the right-hand side of (2) is present-value wealth \( W \), the sum of human wealth \( W^H \) (the present value of all future income) and financial wealth \( W^F \). Since \( \{ -1 B_t \} \) and \( \{ -1 b_t \} \) only enter (2) through \( W^F \), we have:

**Observation 1.** Financial assets with the same initial present value \( W^F \) deliver the same solution to the consumer problem.

For example, this framework predicts that a household holding a mortgage with outstanding nominal principal \( L \) (normalizing the price level at \( P_0 = 1 \), formulates the same plan \( \{ c_t, n_t \}_{t \geq 0} \) for consumption and labor supply irrespective of whether this liability is in the form of an adjustable-rate mortgage (ARM) with \( -1 B_0 = -L \), or a fixed-rate mortgage (FRM) \( -1 B_t = -M \) for \( t = 0 \ldots T \), provided the two mortgages have the same outstanding principal, i.e. \( L = \sum_{t=0}^{T} Q_t M \).

### 2.2 Adjustment after a transitory shock

I now consider an exercise where, keeping balance sheets fixed at \( \{ -1 B_t \}_{t \geq 0} \) and \( \{ -1 b_t \}_{t \geq 0} \), the paths for variables relevant to the consumer choice problem are altered in the following way:
a) all nominal prices rise in proportion, \( \frac{dP_t}{P_t} = \frac{dP}{P} \), for \( t \geq 0 \)

b) all present-value real discount rates rise in proportion, \( \frac{dq_t}{q_t} = -\frac{dR}{R} \), for \( t \geq 1 \)

c) the Fisher equation holds at the new sequence of prices: \( \frac{dQ_t}{Q_t} = -\frac{dR}{R} \) for \( t \geq 1 \)

d) the agent’s unearned income at \( t = 0 \) rises by \( dy \), and his real wage by \( dw \).

This particular variation, depicted in figure 1, captures in a stylized way the major changes in a consumer’s environment that usually follow from a short-lived change in monetary policy: over a period labelled \( t = 0 \), incomes and wages increase, the price level rises due to inflation between \( t = -1 \) and \( t = 0 \), and the real interest rate \( R_0 = \frac{q_0}{q_1} \) falls. While appendix A.1 provides results for any change in the path for \( q_t, P_t, y_t \) and \( w_t \), the particular variation considered here conveys the most important intuitions regarding the role of balance sheets. It also readily extends to setups with incomplete markets, and corresponds to the timing of predictions for macroeconomic aggregates given by the standard New Keynesian model after a one-period change in monetary policy.

I am interested in the first-order change in initial consumption \( dc = dc_0 \), labor supply \( dn \equiv dn_0 \), and welfare \( dU \) that results from this change in the environment. Five important quantities are defined along the initial path: the marginal propensities to consume \( MPC = \frac{\partial c_0}{\partial y_0} \), supply labor \( MPN = \frac{\partial n_0}{\partial y_0} \) and save \( MPS = 1 - MPC + \varphi_0 MPN \), as well as the local elasticities of substitution in consumption \( \sigma \equiv -\frac{\varphi'(c_0)}{\varphi'(c_0)\frac{\partial u'}{\partial y}(c_0)} \) and labor \( \psi \equiv -\frac{\psi'(n_0)}{\psi'(n_0)\frac{\partial v'}{\partial n}(n_0)} \). Separable utility implies that \( MPC \in (0, 1) \), \( MPS \in (0, 1) \) and \( MPN \leq 0 \): in particular, saving and
leisure are ‘normal’. From Slutsky’s equations we then obtain, dropping \( t = 0 \) subscripts whenever unambiguous:

**Theorem 1.** To first order,

\[
dc = \text{MPC} (d\Omega + \psi ndw) - \sigma \text{MPS} \frac{dR}{R} \\
 dn = \text{MPN} (d\Omega + \psi ndw) + \psi n \text{MPS} \frac{dR}{R} + \psi \frac{dw}{w} \\
 du = u'(c) d\Omega
\] (3)

where \( d\Omega = dW - \sum_{t \geq 0} c_t dq_t \), the net-of-consumption wealth change, is given by

\[
d\Omega = dy + ndw + \left( y + wn + \left( -\frac{1}{P_0} B_0 \right) + (-1) b_0 - c \right) \frac{dR}{R} - \sum_{t \geq 0} Q_t \left( -\frac{1}{P_0} B_t \right) \frac{dP}{P} (6)
\]

Unhedged interest Rate Exposure (URE)  
Net Nominal Position (NNP)

These equations separate the wealth and the substitution effects that result from the shock. The relative price changes \( dR \) and \( dw \) generate substitution effects on consumption and labor supply with familiar signs, and magnitudes that are given by a combination of the Frisch elasticities \( \sigma \) and \( \psi \) and marginal propensities. All wealth effects get aggregated into a net term, \( d\Omega \), which affects consumption and labor supply after multiplication by the marginal propensity to consume and work, respectively.

Note that theorem 1 makes no assumption on horizon or the form of \( u \) and \( v \). In appendix A.1 I show that it extends to general utility functions and perturbations, provided one maintains the neoclassical assumption of a linear budget constraint with known prices.

**Net wealth revaluation: determinants and implications.** The net wealth change \( d\Omega \) in (6) is the key expression determining both welfare and wealth effects in theorem 1. This term is a sum of products of balance-sheet exposures by changes in aggregates. The exposure to a one-off rise in the price level is the negative of the present value of the household’s nominal liabilities, also known as their net nominal position (NNP). This term can be computed directly from a survey of the household’s finances. Doepke and Schneider (2006) conduct this exercise for various groups of U.S. households and show that NNP are large and heterogenous in the population: they are very positive for rich, old households and negative for the young middle class with fixed-rate mortgage debt. Theorem 1 shows that these numbers are not only relevant for welfare, but also for the predicted behavioral response to this inflation scenario.

Just as an change in the price level “acts” upon the consumer’s net nominal position, equation (6) shows that a change in the real interest rate acts upon what I call his unhedged

\[^{14}\text{For more general utility functions, the substitution effect is determined by the Hicksian (or compensated) demand elasticities of } c_0 \text{ and } n_0 \text{ with respect to } q_0 \text{ and } w_0. \text{ See appendix A.1.}\]
interest rate exposure, or URE. URE is the difference between all maturing assets (including income) and liabilities (including planned consumption) at time 0. It represents the net saving requirement of the household at time 0, from the point of view of date $-1$. Because it includes the stocks of financial assets that mature at date 0 rather than interest flows, it can significantly diverge from traditional measures of savings, in particular if investment plans have very short durations.

The intuition for why only the difference $y_0 + w_0 n_0 + \left(\frac{-1 B_0}{P_0}\right) + (-1 b_0) - c_0$ enters the wealth effect following a $t = 0$ real interest rate change $dR$ is as follows. To fix ideas, suppose $dR < 0$. The rise in discount factors $\frac{dq_t}{q_t} = -\frac{dR}{R}$ for $t \geq 1$ reflects a rise in the price of future consumption relative to current consumption. Up to a normalization for wealth, this is the same as a fall in the price of current consumption relative to future consumption, $dq_0 = q_0 \frac{dR}{R}$. This fall in the price of $t = 0$ goods benefits those consumers that are supplying more goods than they demand at that date, and conversely hurts the net buyers of current goods. URE is the measure of the net exposure to this price change. Note that URE is also measurable from a survey of household finances that has information on income and consumption.

Observation 2. The composition of a household’s balance sheet is important to understand his consumption, labor supply, and welfare response to changes in interest rates and prices.

Households that invest their wealth in inflation-indexed instruments ($-1 B_t = 0$) have no exposure to price-level changes. Similarly, households who invest their wealth in annuities, nominal or real, that match the difference between their income and their desired consumption ($\left(\frac{-1 B_t}{P_t}\right) + (-1 b_t) = c_t - (y_t + w_t n_t)$) have no exposure to real interest rate changes: their consumption and labor supply responses are purely driven by substitution effects, and their welfare is unaffected to first order. (The second-order term is positive, reflecting the gain from their ability to reoptimize.)

The expression for URE makes clear that the duration of asset plans matters for determining the net wealth change from a change in real interest rates. In this respect, fixed rate mortgage holders are similar to annuitized retirees: if their income covers their overall consumption inclusive of mortgage payments, then their URE is about zero. On the other hand, ARM holders tend to have negative URE, and savers with a large amount of wealth invested at short durations tend to have positive URE. Hence the theory predicts that the former tend to gain and the latter tend to lose from temporary falls in real interest rates. The saver’s consumption may still increase, however, if $\sigma c MPS \geq MPC \cdot URE$—in other words, if the substitution effect dominates the wealth effect.

---

15By contrast, measurement of the exposure to real interest rate changes at any future date requires the knowledge of future income and consumption plans.
Observation 3. Asset value changes give incomplete information to understand the effects of monetary policy on household welfare.

In the model just presented, monetary policy can be thought of as influencing asset values through three channels: a risk-free real discount rate effect \((dR)\), an inflation effect \((dP)\), and an effect on dividends \((dy)\). But these asset value changes do not enter \(d\Omega\) directly, so they are not relevant on their own to understand who gains and who loses from monetary policy, contrary to what popular discussions sometimes imply. For example, it is sometimes argued that accommodative monetary policy benefits bondholders by increasing bond prices. Yet theorem 1 shows that, while increases in dividends do raise welfare, lower real risk-free rates have ambiguous effects on savers. They have no effect on bondholders whose dividend streams initially match the difference between their target consumption and other sources of income. They benefit households who hold long-term bonds to finance short-term consumption, through the capital gains they generate. And they hurt households who finance a long consumption stream with short-term bonds, by lowering the rates at which they reinvest their wealth. Unhedged interest rate exposures, not asset price changes, constitute the welfare-relevant metric for the impact of real interest rate changes on households.

Even though theorem 1 assumes no uncertainty and perfect foresight, it applies directly to environments with uncertainty but where markets are complete, except for the shock that is unexpected (all summations are then over states as well as dates). An important feature of all these environments is that the marginal propensity to consume, \(MPC\), is the same out of all forms of wealth \(\frac{\partial c_0}{\partial y_0} = \frac{\partial c_0}{\partial W}\). The next section relaxes this assumption.

2.3 The consumption response to shocks under incomplete markets

I now consider a dynamic, incomplete-market partial equilibrium consumer choice model. The consumer faces an idiosyncratic process for real wages \(\{w_t\}\) and unearned income \(\{y_t\}\). His utility function has an expected utility form and is separable over time and between consumption and labor supply:

\[
E \left[ \sum_t \beta^t \{u(c_t) - v(n_t)\} \right]
\]

The horizon is still not specified in the summation. As we will see, it will only influence behavior through its impact on the \(MPC\). To model market incompleteness in a general form, I assume that the consumer can trade in \(N\) stocks and as well as in a nominal long-term bond. In period \(t\), stocks pay real dividends \(d_t = (d_{1t} \ldots d_{Nt})\) and can be purchased at real prices \(S_t = (S_{1t} \ldots S_{Nt})\); the consumer’s portfolio of shares is denoted by \(\theta_t\). The long-term bond is modeled as in Hatchondo and Martinez (2009): it can be bought at time \(t\)
at price $Q_t$ and is a promise to pay a geometrically declining nominal coupon with pattern $(1, \delta_N, \delta_N^2, \ldots)$ starting at date $t+1$. The household’s budget constraint at date $t$ is therefore

$$P_t c_t + Q_t (\Lambda_{t+1} - \delta_N \Lambda_t) + \theta_{t+1} \cdot P_t S_t = P_t y_t + P_t w_t n_t + \Lambda_t + \theta_t \cdot (P_t S_t + P_t d_t)$$  \hspace{1cm} (8)

A borrowing constraint limits trading. This constraint specifies that real end-of-period wealth cannot be too negative: specifically,

$$\frac{Q_t \Lambda_{t+1} + \theta_{t+1} \cdot P_t S_t}{P_t} \geq -\frac{\bar{D}}{R_t}$$  \hspace{1cm} (9)

for some $\bar{D} \geq 0$, where $R_t$ is the real interest rate at time $t$. The constraint in (9) is a standard specification for borrowing limits (see for example Eggertsson and Krugman 2012) and we will see that it generates reactions of constrained agents to balance sheet revaluations that are closely related to those of unconstrained agents. Given that the extent to which borrowing constraints react to changes in the macroeconomic environment is an open question, (9) provides an important benchmark.

If the portfolio choice problem just described has a unique solution at date $t-1$, the household’s net nominal position and his unhedged interest rate exposure are both uniquely pinned down in each state at time $t$. This contrasts with the environment in section 2.2, where the consumer was indifferent between all portfolio choices. Here, these quantities are defined as

$$NNP_t \equiv (1 + Q_t \delta_N) \frac{\Lambda_t}{P_t}, \quad URE_t \equiv y_t + w_t n_t + \frac{\Lambda_t}{P_t} + \theta_t \cdot d_t - c_t$$

in other words, as before, $NNP_t$ is the real market value of nominal wealth and $URE_t$ is maturing assets (including income, real coupon payments and dividends) net of maturing liabilities (including consumption).

Consider the predicted effects on consumption resulting from a simultaneous unexpected change in unearned income $dy$, the real wage $dw$, the price level $dP$ and the real interest rate $dR$ that leaves the future unaffected. By no arbitrage, the only effect on asset prices must then come from the change in discounting: $dQ = \frac{dS_j}{S_j} = -\frac{dR}{R}$ for $j = 1 \ldots N$. Writing $MPC = \frac{\partial c}{\partial y}$, $MPN = \frac{\partial n}{\partial y}$ and $MPS = 1 - MPC + wMPN$, the positive results from theorem 1 carry through.

**Theorem 2.** Assume that the consumer is at an interior optimum, or at a binding borrowing constraint with $MPS=0$. Then his first order change in consumption $dc$ and labor supply $dn$ continue to be given by equations (3) and (4). In particular, writing $M\hat{P}C \equiv \frac{MPC}{MPC+MPS}$, the relationship between $dc$ and the total change in income $dY = dy + ndw + wdn$ is

$$dc = M\hat{P}C \left( dY + URE \frac{dR}{R} - NNP \frac{dP}{P} \right) - \sigma c (1 - M\hat{P}C) \frac{dR}{R}$$  \hspace{1cm} (10)

The proof is given in appendix A.2. The intuition for why $MPC$, $MPN$ and $MPS$ are
relevant to understand the response of all agents to changes in the real interest rate and the price level is simple: when the consumer is locally optimizing, these quantities summarizes the way in which he reacts to all balance-sheet revaluations, income being only one such revaluation. When the borrowing limit is binding, consumption and labor supply adjustments depend on the way the borrowing limit changes when the shock hits. Under the specification (9), the changes in \( dR \) and \( dP \) free up borrowing capacity exactly in the amount \( \text{URE} \frac{dR}{R} - \text{NNP} \frac{dP}{P} \). Because preferences are separable, we can obtain a relationship between consumption and the total income change \( dY \) inclusive of the endogenous labor supply response. This relationship involves \( MPC = \frac{MPC}{MPC + MPS} = \frac{MPC}{1 + \psi} \), which is always greater than \( MPC \) and equal to it when labor supply is inelastic (\( \psi = 0 \)).

By showing that the marginal propensity to consume out of transitory income shocks, which has been the focus of a large empirical literature, remains a key sufficient statistic for predicting behavior with respect to other changes in consumer balance sheets, theorem 2 provides important theoretical restrictions, as well as testable implications.

### 3 Aggregation and the redistribution channel

This section shows how the microeconomic demand responses derived in section 2 can aggregate in general equilibrium to explain the economy-wide response to shocks in a large class of heterogenous-agent models (theorem 3).

#### 3.1 Aggregation result

Consider a closed economy populated, at a given time, by \( i = 1 \ldots I \) heterogenous agents with separable preferences (7), where the discount factor \( \beta_i \), the period utility functions \( u_i \) and \( v_i \), and the time horizon can all be \( i \)-specific.

Labor is the only factor of production and firms hire on spot labor markets. Individuals have stochastic endowments of goods \( y_{it} \) and skills, which can affect their real wage \( w_{it} \). They own firm shares delivering profits \( \pi_{it} \). In addition, there exists \( j = 1 \ldots J \) trees, each in unit supply and paying a stream of dividends \( d_{jt} \). Individual \( i \) owns \( \theta_{ijt} \) shares of tree \( j \) at time \( t \). He receives government transfers net of taxes \( t_{it} \). I denote his total endowment, labor, profit and dividend income by \( Y_{it} = y_{it} + w_{it} n_{it} + \pi_{it} + \theta_{ijt} d_{jt} + t_{it} \).

There also exists \( k = 1 \ldots K \) assets in zero net supply, which can be nominal or real. Each agent \( i \) is assumed to be able to trade at least one of these assets, or one tree. All agents have rational expectations and face the same prices on the assets they can trade. They may face a borrowing limit \( D_{it} \), which can also be \( i \)-specific.

There is a government that runs no debt and balances its budget period by period, so that \( \sum_i t_{it} = 0 \). This assumption is made to abstract away from price-induced redistributive
effects between the government and the household sector, and it can be relaxed (see section 3.3). Finally, markets for goods, labor, and assets clear at all times.

This environment nests a large class of one-good general equilibrium models with liquid assets. It can accommodate many assumptions about population structure, asset market structure and participation, heterogeneity in preferences, endowments and skills, as well as the nature of price and wage stickiness.

I am interested in a perturbation of this environment that a) leaves initial balance sheets unchanged and b) upsets individual incomes $dY_i$, nominal prices $dP$ and the real interest rate $dR$ at $t = 0$ only. This exercise is useful to understand the effect of an unexpected shock that has no persistence.\footnote{Note that in general exogenous shocks, even if purely transitory, can generate endogenous persistence in this class of environments (for example through the wealth distribution), in other words affect prices and aggregate quantities at $t \geq 1$. These then affect consumption at $t = 0$, by adding terms to the decompositions in theorems 1 and 2 and deteriorating the approximation. Section 5 examines the size of these effects in a particular application and finds them to be small. More generally, the insights obtained from the analysis with no persistence carry through to environments with persistence, even if simple sufficient statistics can, in general, no longer be derived.}

Aggregation is simplified by several restrictions from market clearing at $t = 0$. Given my assumptions of a closed economy and no government debt, there is no net supply of nominal assets, so asset market clearing requires the cross-sectional average net nominal position $\mathbb{E}_I [NNP_i] \equiv \frac{1}{I} \sum_{i=1}^{I} NNP_i$ to be zero:

$$\mathbb{E}_I [NNP_i] = 0 \quad (11)$$

Moreover, since I assume no capital, agents cannot save or borrow in the aggregate. Goods market clearing then requires that (per capita) aggregate consumption be equal to (per capita) aggregate income from all sources. Omitting $t = 0$ subscripts, this condition is $C \equiv \mathbb{E}_I [c_i] = \mathbb{E}_I [Y_i] \equiv Y$. Combining this with asset market clearing, we obtain

$$\mathbb{E}_I [URE_i] = 0 \quad (12)$$

I assume that my perturbation respects goods market clearing, $dC = \mathbb{E}_I [dY_i] = dY$. Aggregation of consumer responses as described by theorem 2 then shows that the per capita aggregate consumption change can be decomposed as the sum of five channels:

**Theorem 3.** To first order, the changes $dC = dY, dY_i, dP,$ and $dR$ are linked by

$$dC = \mathbb{E}_I \left[ \frac{Y}{Y} M \hat{P} C_i \right] dY + \text{Cov}_I \left( M \hat{P} C_i, dY_i - Y_i \frac{dY}{Y} \right) - \text{Cov}_I \left( M \hat{P} C_i, NNP_i \right) \frac{dP}{P}$$

$$+ \left( \frac{dR}{R} \right) \left( \text{Agg. income channel} \quad \text{Earnings heterogeneity channel} \quad \text{Fisher channel} \quad \text{Interest rate exposure channel} \quad \text{Substitution channel} \right)$$

$$+ \text{Cov}_I \left( M \hat{P} C_i, URE_i \right) - \mathbb{E}_I \left[ \sigma_i (1 - M \hat{P} C_i) c_i \right]$$

The proof is given in appendix A.3. The key step is to aggregate predictions from theo-
rem 2, decomposing i’s individual income change as $dY_i = \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY$ (the sum of an aggregate component and a redistributive component), and using market clearing conditions and $E_1 [dY_i - \frac{Y_i}{Y} dY] = 0$ to transform expectations of products into covariances.

Theorem 3 shows that, in the class of environments I consider, a small set of sufficient statistics is enough to understand and predict the first-order response of aggregate consumption to a macroeconomic shock. Equation (13) holds irrespective of the underlying model generating MPCs and exposures at the micro level, as well as the relationship between $dY$, $dP$ and $dR$ at the macro level. Most of the bracketed terms are cross-sectional moments that are measurable in household level micro-data and are informative about the economy’s macroeconomic response to a shock, independently of the source of this shock. (The two exceptions are the EISs $\sigma_i$, which need to be obtained from other sources, and $dY_i - Y_i dY / Y$, which in general depends on the driving force behind the change in output). I now use this theorem to discuss the channels of monetary policy transmission under heterogeneity. Alternative applications, for example to short-term redistributive fiscal policy or open-economy models, are also possible.

### 3.2 Monetary policy shocks with and without a representative agent

Consider a transitory, accommodative monetary policy shock that, as in figure 1, lowers the real interest rate and raises aggregate income for one period ($dR < 0$, $dY > 0$), and permanently raises the price level ($\frac{dP}{P} > 0$). These are the changes implied by the textbook New Keynesian model with sticky prices and flexible wages after a one-period deviation from the central bank’s zero-inflation targeting policy, so we can apply theorem 3 to understand the consumption response in that model.

The textbook model features a representative agent ($I = 1$) with separable preferences and EIS $\sigma$. Hence all covariance terms in (13) are zero, and we are left with

$$dC = \hat{MPC} dY - \sigma (1 - \hat{MPC}) C \frac{dR}{R}$$

The first term in (14) is a general-equilibrium income effect, and the second term is a substitution effect.\(^{17}\) Solving out for $dC = dY$ gives the textbook response, $\frac{dC}{C} = -\sigma \frac{dR}{R}$. Intuitively, a Keynesian multiplier $\frac{1}{1 - \hat{MPC}}$ works to amplify the initial “first-round” effect from intertemporal substitution. Here this multiplier is entirely microfounded, and in particular takes into account the substitution and wealth effects on labor supply that play out in the background.

Heterogeneity implies a role for redistributive channels in the monetary transmission mechanism. These channels can be signed and quantified by measuring the covariance

\(^{17}\)Since the typical calibration of the representative-agent model implies a low $\hat{MPC}$, the substitution component is typically dominant in this decomposition, as noticed by Kaplan et al. (2016).
terms in equation (13), either within a given model or directly in micro data. I say that they *amplify* the representative-agent response if

\[
\text{Cov}_I(\hat{\text{MPC}}_i, URE_i) < 0 \quad (15)
\]

\[
\text{Cov}_I(\hat{\text{MPC}}_i, NNP_i) < 0 \quad (16)
\]

\[
\text{Cov}_I(\hat{\text{MPC}}_i, d \left( \frac{Y_i}{Y} \right)) > 0 \quad (17)
\]

I now argue that all three of (15), (16) and (17) are reasonable to expect in practice. Inequality (15) says that agents with unhedged borrowing requirements have higher marginal propensities to consume than agents with unhedged savings needs.\(^{18}\) I will show that (15) is true in two micro data cross-sections (section 4) and that it is naturally generated by a model with uninsured idiosyncratic risk (section 5), with a magnitude that depends on asset durations. Because of this interest rate exposure channel, aggregate consumption is more responsive to real interest rates than measures of intertemporal substitution alone would suggest. In other words, the first-round effect of monetary policy is larger than what the representative-agent model predicts.

Inequality (16) says that net nominal borrowers have higher marginal propensities to consume than net nominal asset holders. (16) is also endogenously generated by my model in section 5. It implies that, through its general equilibrium effect on inflation, monetary policy can increase aggregate consumption via a Fisher channel.\(^{19}\)

Inequality (17) says that the increase in aggregate income resulting from monetary accommodation disproportionately benefits high-MPC agents. The empirical evidence suggests that income risk is countercyclical (for example Storesletten, Telmer and Yaron 2004 or Guvenen, Ozkan and Song 2014) and that monetary policy accommodations reduce income inequality (Coibion et al. 2012). Taken together, this suggests that \(d \left( \frac{Y_i}{Y} \right)\) is positive for low-\(Y_i\) agents after monetary accommodations. If, in turn, low-\(Y_i\) agents also have high MPCs, as many studies suggest, then we can expect the earnings heterogeneity channel to further amplify the effects of monetary policy.\(^{20}\)

Away from separable preferences, an additional *complementarity channel* of monetary policy can arise, even in the presence of a representative agent, when preferences are such that increases in hours worked increase the marginal utility of consumption.\(^{21}\)

---

\(^{18}\)Recall that \(\hat{\text{MPC}} = \frac{\text{MPC}}{1 - \frac{\text{MPN}}{\text{MPC}}}\) is equal to the actual MPC when labor supply is inelastic. Since there is very little evidence on MPN heterogeneity, I will throughout assume that the evidence on MPC equally applies to \(\hat{\text{MPC}}\), and that for quantitative purposes we can assume \(\text{MPC} \approx \hat{\text{MPC}}\).

\(^{19}\)Note that this effect from redistribution is conceptually distinct from the effect of future inflation lowering real interest rates, which has nothing to do with nominal redenomination and is present in representative-agent models with persistent shocks to inflation.

\(^{20}\)The role of countercyclical earnings risk in generating amplification from monetary policy shocks has also recently been stressed by Werning (2015).

\(^{21}\)See appendix A.4 for the equivalent of theorem 3 with GHH preferences. This complementarity channel has been argued to be reasonable to explain, for example, the observed hump shapes in the life-cycle profile of earnings.
Independently of the sign of the covariance terms in (13), theorem 3 provides an organizing framework for future research on the role of heterogeneity in the monetary policy transmission mechanism. Additional sources of departures from the representative agent benchmark can come from the interaction between the household sector and other sectors of the economy, as I now discuss.

3.3 Outside assets and trading with other sectors

The market clearing equations (11) and (12) respectively state that the net nominal positions and the unhedged interest rate exposure of the household sector must be zero. Equivalently, unexpected changes in the price level and real interest rate create pure redistribution within the household sector. There are reasons to expect these equalities to fail in the data. It is useful to reflect upon why this might be true and discuss how this may alter the aggregate predictions from the model.

Doepke and Schneider (2006) find that the net nominal position of U.S. households is positive. This means households tend to lose in the aggregate, mostly to the benefit of the government sector, from an unexpected rise in inflation. Similarly, there are reasons to expect to find a positive aggregate URE in the data. The main one is that the household sector tends to be maturity mismatched, holding relatively short-term assets (deposits) and relatively long-term liabilities (fixed-rate mortgages); this is the natural counterpart to the reverse situation in the banking sector. In addition, in periods where the government is increasing its debt and has large flow borrowing requirements, these flows must be financed and households are natural counterparts for them.

Since households are the ultimate claimants on the financial sector and the government, gains to these sectors that occur as a result of lower real interest rates or unexpected inflation must ultimately be rebated back to them. Consider the case of a purely Ricardian model, such as a simple Real Business Cycle model. There, we know that the trading plan between the household and the government is irrelevant. If the government happens to be a flow borrower (have negative URE) when a shock takes place which results in lower real interest rates, this creates a present-value gain to the government, and a lump-sum transfer to the representative household must take place to ensure that both agents’ present-value budget constraints are still satisfied. The same holds true for the maturity-mismatched financial sector, which might rebate gains from lower interest rates through lower fees or higher dividends.22

and consumption (Heckman 1974; Aguiar and Hurst 2005).

22In a world where financial frictions are important, the “stealth recapitalization” of the banking sector (Brunnermeier and Sannikov 2014) from lower real interest rates may have large additional effects on aggregate demand, notably via investment, beyond the ones induced by a rebating of gains to the household sector.
When $\mathbb{E}_t[\text{URE}_i] > 0$, we cannot directly replace the term $\mathbb{E}_t[\hat{MPC}_i\text{URE}_i]$ by a covariance in the expression for theorem 3, and must instead consider the way in which outside sectors rebate gains and losses. Assuming an immediate and uniform rebate, the covariance formula still applies. In practice, rebates might be delayed, and they might target higher or lower MPC agents, so that the precise number may depart from the covariance expression in either direction. In the measurement part that follows, I compute the covariance term as my benchmark, but also consider the value of the ‘partial-equilibrium’ $\mathbb{E}_t[\hat{MPC}_i\text{URE}_i]$ for robustness.  

Although my results in the next sections suggest otherwise, it is interesting to note the theoretical possibility that the interest rate exposure term $\mathbb{E}_t[\hat{MPC}_i\text{URE}_i]$ may not only be positive, but larger than the substitution term in (13). This suggests that in a world in which outside rebates are highly delayed or benefit low-MPC agents, real interest rate cuts could lower aggregate consumption demand, altering significantly the conventional understanding of how monetary policy operates.

4 Measuring the redistribution elasticity of consumption

In a representative-agent model, intertemporal substitution is the only channel of transmission from real interest rates to consumption. The magnitude of this substitution channel depends crucially on the elasticity of intertemporal substitution (EIS). In section 3 I showed that, under heterogeneity, a redistribution component also shapes the aggregate response of consumption to real interest rates. In this section I use my sufficient statistic methodology to quantify this interest rate exposure channel, treating the EIS as a given number $\sigma$. This exercise is not only useful as a measure of the “first-round” response of consumption to monetary policy. It can also serve as an input into any quantitative model in which the EIS matters.

In order to do this, I make several simplifications. I maintain my focus on a purely transitory change in the real interest rate, for which theorem 3 established the existence of a sufficient statistic for the interest rate exposure channel. I assume that all agents have separable preferences and that their $MPN$ is about zero, so that $\hat{MPC} = \frac{MPC}{1+wMPN}$ $\simeq MPC$. Manipulating (13), I find that the partial elasticity of aggregate consumption to the real

---

23 Open-economy considerations can strengthen this rebating logic further. In the international financial accounts of the United States, the Rest of the World has long liabilities (FDI) and shorter assets (Treasury securities)—that is, its aggregate URE is positive—and therefore it tends to lose when interest rates fall, creating additional gains that must ultimately accrue to households.

24 This theoretical possibility is sometimes mentioned in economic discussions of monetary policy. See Raghuram Rajan (“Interestingly [...] low rates could even hurt overall spending”), “Money Magic”, Project Syndicate, November 11, 2013

25 Longer-run changes in real interest rates tend to increase both the intertemporal substitution term and the redistribution term, so the relative magnitudes that I obtain here plausibly extrapolate to these changes as well.
interest rate, \( \frac{\partial C}{\partial R} \), is given by the sum of a redistribution elasticity \( E_r \) and a substitution term \( \sigma S \):

\[
\text{Cov}_I \left( \text{MPC}_i, \frac{\text{URE}_i}{\mathbb{E}_I[c_i]} \right) - \sigma \mathbb{E}_I \left[ (1 - \text{MPC}_i) \frac{c_i}{\mathbb{E}_I[c_i]} \right]
\]

where \( \sigma \equiv \mathbb{E}_I \left[ \sigma_i (1 - \text{MPC}_i) c_i / \mathbb{E}_I[(1 - \text{MPC}_i) c_i] \right] \) is an appropriately-weighted average of \( \sigma_i \)'s, which I will refer to as “the” elasticity of intertemporal substitution, and \( S \) is a scaling factor.

Since all quantities in (18) are measurable at the household or at the group level except for the EIS \( \sigma \), a useful way of organizing the results is to determine the value of the EIS that would make the substitution and the redistribution effects equal in magnitude. I call this value \( \sigma_r \), defined as

\[
\sigma_r \equiv -\frac{E_r}{S} = -\text{Cov}_I \left( \text{MPC}_i, \text{URE}_i \right) / \mathbb{E}_I \left[ (1 - \text{MPC}_i) c_i \right]
\]

Knowing \( \sigma_r \) allows us to say how much a representative-agent model should add to its assumed EIS of \( \sigma \) to correctly predict the magnitude of the economy’s response to one-time shocks to real interest rates.\(^{26}\) I also measure the “no rebate” (NR) version of the redistribution elasticity, \( E_r^{NR} = \mathbb{E}_I \left[ \text{MPC}_i \frac{\text{URE}_i}{\mathbb{E}_I[c_i]} \right] \), which assumes that gains and losses to other domestic agents are not rebated to households and is therefore likely to understate the effects of redistribution on aggregate consumption (see section 3.3).

The literature has used different ways to measure the marginal propensity to consume out of transitory income shocks (see Jappelli and Pistaferri 2010 for a survey). A simple approach has been to ask households to self-report the part of any hypothetical windfall that they would immediately spend. The benefit of this approach is that the windfall can be considered as exogenous for all agents, making this empirical measure of \( \text{MPC} \) close to the \( \frac{\partial c}{\partial y} \) that matters for the theory. The Italian Survey of Household Income and Wealth (SHIW) contains such a question in 2010 (Jappelli and Pistaferri 2014). I use data from this survey for my benchmark measurement exercise.

One concern with self-reported answers to hypothetical situations is that they may not be informative about how households would actually behave in these situations. For this reason, the literature has looked at cleanly identified settings allowing estimation of \( \text{MPC} \) from actual behavior. I also provide a measure of \( \sigma_r \) that uses the variation from the 2001 tax rebates used in Johnson et al. (2006). In the interest of space, these results are presented in appendix B.2.

\(^{26}\)Taking full account of the redistribution channel is more complex than assuming that the representative-agent is \( \sigma_r \)—“more elastic” with respect to real interest rate changes, but the value of \( \sigma_r \) provides a useful rule of thumb.
4.1 Conceptual measurement issues

As defined in section 2.2, $URE_i$ measures the total resource flow that a household $i$ needs to invest over the first period of his consumption plan. From the survey, I construct $URE_i$ as

$$URE_i = Y_i - C_i + B_i - D_i$$

where $Y_i$ is income from all sources including dividends and interest payments, $C_i$ is consumption including expenditures on durable goods but excluding house purchases, as well as mortgage payments and installments on consumer credit, and $B_i$ and $D_i$ represent, respectively, asset and liability stocks that mature over the period.

Even though $E_r$ is a unitless number, the choice of time units is important: $MPC$ needs to be measured over a period consistent with the choice of time units for the numerator and denominator of $E_{[c_i]}$. Ideally, all measurement would be done over a quarter, which is the frequency at which models analyzing monetary policy are calibrated. However consumption, income and MPC in the SHIW are only available at annual frequency. Because it is not obvious how to translate an annual measure of MPC into a quarterly one, I measure $E_r$ once at annual frequency, and once at quarterly frequency where I use $MPC_Q = \frac{MPC_A}{3}$ to reflect the tendency of households with precautionary savings motives to spend more of their income in the first quarter of a one-off transfer receipt.

Given the limited information regarding asset maturities in the SHIW, maturing asset stocks ($B_i$) are difficult to determine precisely. In my quarterly calibrations, I treat time and savings deposits as maturing in the quarter. In annual calibrations, I treat them as maturing within the year. These are likely to be good effective lower and upper bounds for deposit durations. Doepke and Schneider (2006) calculate that, since the beginning of the 2000s, the average duration of U.S. financial assets has been around 4 years. I assume that asset durations in Italy are not too different and count one-fourth of household asset stocks towards $B_i$ in my annual measurement, and one-sixteenth in my quarterly measurement. I treat adjustable-rate mortgages, just as deposits, as maturing liability stocks within the quarter or within the year depending on the calibration. Fixed-rate mortgages are not counted additionally in $B_i$ since mortgage payments—which include amortization—are

\[27 \text{ An extension of the theory presented in section 2 to durable goods reveals that whether or not durable expenditures should be counted towards URE depends on the assumed movements in the relative price of durables as real interest rates change. If price changes offset interest rate changes (as we might expect to be the case for housing), then durable purchases are simple portfolio shifts and should not count. If instead prices are constant, then durable expenditures should be subtracted from URE. (However in this case there is an additional substitution effect between nondurable and durable consumption due to the change in the user cost of durables.) My convention is consistent with the treatment of housing in NIPA data; however my results are essentially unchanged if I do not count any durable expenditure in $C_i$.}

\[28 \text{ This theoretical pattern is also consistent with empirical behavior (see the dynamic specifications of Johnson et al. 2006; Parker, Souleles, Johnson and McClelland 2013 and Broda and Parker 2014).}

21
Table 1: Main summary statistics from the SHIW data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean (€)</th>
<th>Normalized s.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income from all sources ((Y_i, \text{per year}))</td>
<td>36,114</td>
<td>0.90</td>
</tr>
<tr>
<td>Consumption including mortgage payments ((C_i, \text{per year}))</td>
<td>27,976</td>
<td>0.61</td>
</tr>
<tr>
<td>Deposits and maturing assets ((B_i))</td>
<td>14,200</td>
<td>1.45</td>
</tr>
<tr>
<td>ARM mortgage liabilities and consumer credit ((D_i))</td>
<td>6,228</td>
<td>1.03</td>
</tr>
<tr>
<td>Unhedged interest rate exposure ((URE_i, \text{per year}))</td>
<td>16,110</td>
<td>1.92</td>
</tr>
<tr>
<td>Unhedged interest rate exposure ((URE_i, \text{per Q}))</td>
<td>10,007</td>
<td>7.07</td>
</tr>
<tr>
<td>Marginal Propensity to Spend (annual)</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>Count</td>
<td>7,951</td>
<td></td>
</tr>
</tbody>
</table>

"Mean" is the sample mean computed using sample weights. The normalized standard deviation is \(sd_1 \left( \frac{X_i}{E[X_i]} \right)\) for \(X_i = Y_i, C_i, B_i, URE_i\) and \(sd_1 (MPC_i)\) for MPC.

already subtracted from URE as part of the consumption measure.

Measurement error in \(URE\) is a very important issue in this exercise. These errors can stem from many sources, starting from poor data quality (the SHIW measures income, assets and liabilities imperfectly, and consumption very imperfectly) and timing differences in the reporting of consumption and income. My covariance estimates are unbiased provided that the measurement errors in \(URE\) and in \(MPC\) are additive and uncorrelated, which is a strong assumption. One worry, for example, can be that low-MPC households may have systematically lower \(URE\) than my computation suggests, say because they tend to own bank shares, which would bias my estimate of the correlation downward. With these caveats in mind, I proceed to my measurement exercise.

4.2 A quantification of the redistribution elasticity

The 2010 SHIW contains a question which can be used as an empirical measure of \(MPC\).\(^{29}\) Jappelli and Pistaferri (2014) present a detailed analysis of the data and of the empirical determinants of \(MPC\).\(^{29}\) Table 1 reports the main relevant summary statistics from this dataset, with appendix B.1 providing further details. Note that the average \(URE\) is strictly positive in the survey. One reason, in addition to those highlighted in section 3.3, is that consumption is below income at the mean due to underreporting and coverage issues. This makes it all the more important to consider my benchmark estimate \(E_r\) rather than

\(^{29}\)Imagine you unexpectedly receive a reimbursement equal to the amount your household earns in a month. How much of it would you save and how much would you spend? Please give the percentage you would save and the percentage you would spend."

\(^{29}\)Note that the time frame for \(MPC\) is not specified in the question, as issue that is left unresolved in Jappelli and Pistaferri (2014). A follow-up question in the 2012 SHIW separates durable and nondurable consumption, and specifies the time frame as a full year. The equivalent "MPC" out of both durable and nondurable consumption has close to the same distribution as that of \(MPC\) in the 2010 SHIW (respective means are 47 in 2010 and 45 in 2010) which suggests that households tended to assume that the question referred to the full year.
The figure presents the average reported $MPC$ in each percentile of $URE$.

Figure 2: Correlation between $MPC$ and $URE$ in the SHIW population

$E_{NR}$, since the former removes the bias associated with the latter if underreporting in consumption is uncorrelated with MPC.

Figure 2 illustrates that the empirical correlation between $MPC$ and $URE$ is negative in the SHIW. This is reminiscent of the finding from Jappelli and Pistaferri (2014) that MPC covaries with net liquid assets in this survey. A direct implication is that $E_r < 0$: falls in interest rates increase demand via the redistribution channel.

Table 2 computes the key moments $E_{NR}$, $E_r$ and $\sigma_r$ using the household-level information. Sampling uncertainty is taken into account using the survey’s sampling weights. The main quantitative result is that, depending on the frequency at which the estimation is done, $\sigma_r$ is around 0.1.

Combining information from tables 1 and 2, we can decompose the annual $E_r$ measure as

$$E_r = \text{Corr}_l \left( \frac{MPC_i}{E_l[c_i]}, \frac{URE_i}{E_l[c_i]} \right) Sd_l \left( \frac{MPC_i}{E_l[c_i]} \right) Sd_l \left( \frac{URE_i}{E_l[c_i]} \right) \approx -0.09 \times 0.35 \times 1.92$$

The low absolute value for the correlation between $MPC$ and $URE$ suggests that $E_r$ could plausibly be several times larger in a setting with less measurement error in MPCs and UREs. Using a different approach in a U.S. cross-section, appendix B.2 finds, in a main specification, $E_r = -0.24$ and $\sigma_r = 0.30$. Overall, my estimates of $\sigma_r$ fall in a range that many regard as plausible for the EIS itself (for example Hall 1988, or the meta-analysis in Havránek 2015). In other words, the data suggests that the redistribution effect is plausibly as important as the substitution effect in explaining why aggregate consumption
Table 2: Estimates of $\xi_{NR}^r$, $\xi_r$ and $\sigma_r$ using the SHIW

<table>
<thead>
<tr>
<th>Time Horizon</th>
<th>Annual</th>
<th>Quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Estimate</td>
<td>95% C.I.</td>
</tr>
<tr>
<td>No-rebate elasticity</td>
<td>$\xi_{NR}^r$</td>
<td>0.21</td>
</tr>
<tr>
<td>Redistribution elasticity</td>
<td>$\xi_r$</td>
<td>-0.06</td>
</tr>
<tr>
<td>Scaling factor</td>
<td>$\hat{S}$</td>
<td>0.55</td>
</tr>
<tr>
<td>Equivalent EIS</td>
<td>$\sigma_r = -\frac{\hat{\xi}_r}{\hat{S}}$</td>
<td>0.12</td>
</tr>
</tbody>
</table>

All statistics computed using survey weights

responds to changes in real interest rates.

As highlighted above, the main caveat with these calculations is that they are subject to a substantial degree of measurement error. Unhedged interest rate exposures are difficult to measure, both because consumption itself is, and because a precise attribution of financial stocks to flow interest rate exposures is difficult without many more details on the composition of wealth than is usually available. A more involved calculation would also require taking into account indirect interest rate exposures of households through asset ownership and government fiscal rules. As more sources of joint consumption, income and asset data become available, a better empirical understanding of $URE$s will become possible, helping to shape our understanding of the winners and losers from real interest rate changes. Real-time estimates of their covariance with $MPC$s will also provide useful information about the dynamic evolution of the monetary policy transmission mechanism.

5 Monetary policy shocks in a Huggett model

The reduced-form approach developed so far leaves a number of questions unanswered. First, given limitations in the data, the empirical sufficient statistic is bound to be imperfectly measured. Can we rationalize its sign and magnitude in a reasonably calibrated model? What are some key determinants of this number? Second, on the theoretical side, my sufficient statistics give the first-order response of consumption to purely transitory shocks. How robust are its predictions to large and persistent shocks? This is particularly relevant because, in general equilibrium models, even purely transitory exogenous disturbances can be propagated via state variables such as the wealth distribution. Finally, my analysis has highlighted two redistributive channels of monetary policy in addition to the interest rate exposure channel. What does an equilibrium model predict about the Fisher and the earnings heterogeneity channels, and about how all the channels interact?

In order to answer these questions, I build a structural general equilibrium model. Following the work of Guerrieri and Lorenzoni (2015), McKay and Reis (2015) and others,
my model combines the preferences and market structure of the Bewley-Huggett-Aiyagari class with a New Keynesian specification of production. Its consumption side therefore features rich heterogeneity in MPCs and UREs. By contrast, I choose a simple specification for production: I assume that labor is the only factor, that wages are flexible, and that preferences are such that there are no wealth effects on labor supply.\footnote{Since we lack high-frequency microeconomic evidence on the strength of these wealth effects, assuming them away may be a reasonable assumption for the study of cyclical phenomena such as monetary policy. Indeed, other recent heterogenous-agent New Keynesian models (Bayer, Lüticke, Pham-Dao and Tjaden 2015, Kaplan et al. 2016) have made this assumption as well. My model shows that it allows for simple aggregation, but also highlights that it generates complementarities between consumption and labor supply that are important for the aggregate effects of monetary policy, with or without heterogeneity.}

While stylized, my calibration attempts to capture two key features of U.S. data: large gross nominal asset positions and long maturities. I therefore do not seek to match the wealth distribution, and instead follow Huggett (1993) by assuming that all financial claims in the economy are pure circulating private IOUs. This assumption, which counterfactually implies that average wealth is zero, serves two purposes. Theoretically, it allows me to cleanly focus on redistribution within the household sector, sidestepping the issues discussed in section 3.3. Empirically, it captures the fact that total household financial liabilities and interest-paying assets held directly are roughly balanced in U.S. data, as documented in section 5.3. These are arguably the financial claims whose returns are most directly affected by monetary policy. Other key components of household wealth such as equities, pensions and housing, have returns that are heavily influenced by risk and term premia which the model abstracts from.

5.1 Environment

Time is discrete and runs from $t=0$ to infinity. The economy is populated by a continuum of infinitely-lived, ex-ante identical but ex-post heterogenous households indexed by $i \in [0,1]$. Agents face idiosyncratic uncertainty with respect to their productivity $\{e^i_t\}$ and their discount factor $\{\beta^i_t\}$. The process for the exogenous idiosyncratic state $s^i_t = (e^i_t, \beta^i_t)$ is uncorrelated across agents and follows a Markov chain $\Gamma (s^i_t | s^i)$ over time. This Markov chain is assumed to have a stationary distribution $\phi (s)$, which I take to be the cross-sectional distribution of idiosyncratic states at $t=0$. There is no aggregate uncertainty: the path for all macroeconomic variables is perfectly anticipated.

Household $i$ has GHH preferences over the sequence $\{c^i_t, n^i_t\}$:

$$
\mathbb{E} \left[ \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t-1} \beta^i_\tau \right) u \left( c^i_t - e^i_t v \left( n^i_t \right) \right) \right]
$$

(20)

where the felicity function $u (g) = \frac{g^{1-\sigma} - 1}{1-\sigma - 1}$ has constant elasticity of intertemporal substitu-
tion $\sigma$ in net consumption $g$, the disutility function $v(n) = b_n^{\psi - \psi^{-1}}$ has constant elasticity $\psi$ over working hours $n$, and $b$ denotes a constant.

The final good that enters consumers’ utility is produced with a technology

$$Y_t = \left[ \int_0^1 (x^j_t)^{\frac{\epsilon-1}{\epsilon}} dj \right]^\frac{\epsilon}{\epsilon-1}$$

where $x^j_t$ is the quantity of intermediate good $j \in [0, 1]$ used as input and $\epsilon > 1$ is the constant elasticity of substitution across goods. All intermediate goods are produced with a simple, identical linear technology $x^j_t = l^j_t$, where $l^j_t = \int_i e^j_t n^j_t di$ is the number of efficiency units of work entering the production of good $j$, and $n^j_t = \int_j n^j_t dj$.

**Households.** Households only have access one type of nominal, risk-free, long-term bond with rate of decay $\delta_N$ (see section 2.3). They are subject to an affine tax schedule on labor income. Each period, a household with productivity $e^j_t$ maximizes (20) subject to the flow budget constraint

$$P_t c^i_t + Q_t \left( \Lambda_{t+1}^i - \delta_N \Lambda_t^i \right) = (1 - \tau) W_t e^i_t n^i_t + P_t T_t \left( e^j_t \right) + \Lambda_t^i$$

where $P_t$ is the nominal price of the final good, $Q_t$ the nominal price of a bond paying the sequence of coupons $(1, \delta_N, \delta_N^2, \ldots)$ starting in period $t + 1$, $\Lambda_t^i$ the nominal coupon payment owed to household $i$ at time $t$, $\tau$ the marginal tax rate on labor income, $W_t$ the nominal market wage per efficient unit of work, and $T_t \left( e^j_t \right)$ denotes a real lump-sum transfer from the government, which can vary with the household’s current productivity.

A borrowing constraint limits the size of bond issuances so that the market value of real end-of-period liabilities is bounded by a limit $\overline{D_t}$ at time $t$:

$$Q_t \Lambda_{t+1}^i \geq -\overline{D_t} P_t \tag{21}$$

**Firms.** The final good is produced by a perfectly competitive firm, which takes as given the prices $\{P^j_t\}$ of intermediate goods. Profit maximization leads to a final good price of $P_t$, zero profits (so that it is not necessary to be specific about the firm’s ownership), and isoelastic demand for intermediate goods:

$$x^j_t = \left( \frac{P^j_t}{P_t} \right)^{-\epsilon} Y_t \quad \text{where} \quad P_t \equiv \left[ \int_0^1 \left( P^j_t \right)^{1-\epsilon} dj \right]^\frac{1}{1-\epsilon} \tag{22}$$

Labor markets are competitive and wages are fully flexible. Every intermediate-good firm $j \in [0, 1]$ produces under monopolistic competition. The firm’s nominal profits in period $t$, when its current price is $p$, are given by

$$F^j_t(p) = p x^j_t(p) - W_t l^j_t(p) = (p - W_t) \left( \frac{p}{P_t} \right)^{-\epsilon} Y_t \tag{23}$$
I consider two assumptions on price setting. When prices are flexible, producers set them in each period and state to maximize (23). This results in an identical price across all firms in each period \( t \), equal to

\[
P_t = P_t^j = \frac{e}{e - 1} W_t
\]  

(24)

I also consider the opposite assumption of identical, perfectly sticky prices. This is meant to capture the macroeconomic adjustments that take place under nominal rigidities in the simplest possible way; in the equilibria I will consider, incentives to change prices vanish in the long run. Under identical sticky prices, every firm \( j \) has a price \( P_t^j = P_t \) at time \( t \) and cannot change it. It accommodates the demand \( x_t^j \) that is forthcoming at that price by hiring workers at the going real wage, and its profits are determined by \( F_t^j(P_t) = F_t(P) \).

**Fiscal policy.** I assume that the government owns all the firms. Each period, it collects their nominal profits and runs the personal income tax system. Moreover, it maintains a strict balanced budget every period, and therefore sets the lump-sum transfer equal to total collections:

\[
P_t \int T_t(e_t^i) \, di = \int F_t^j(P_t^j) + \tau \int W_t e_t^i n_t^i di
\]  

(25)

This stylized representation allows fiscal policy to control the flow of non-labor income in the economy, determining its incidence on the income distribution with the function \( T_t(\cdot) \).\(^{32}\) I assume that this function is characterized by a constant \( t^* = 1 - (1 - \tau) \frac{e}{e - 1} \) and a free parameter \( \gamma \) as follows. Define \( t_t \equiv \frac{\int F_t^j(P_t^j) + \tau \int W_t e_t^i n_t^i di}{P_t Y_t} \) as the real tax intake in period \( t \) as a share of GDP. I specify that \( T_t(e) = t^* + (t_t - t^*) \left( \gamma + (1 - \gamma) \frac{e}{E[e]} \right) \) \( (26) \)

The rule in (26) clearly satisfies the government budget constraint (25). All households obtain the same tax rebate if \( \gamma = 1 \), or when \( t_t = t^* \). If \( \gamma < 1 \) the tax system favors high-\( e \) agents when \( t_t > t^* \) and low-\( e \) agents when \( t_t < t^* \). As the next section shows, \( \gamma \) turns out to determine the sign and the magnitude of the model’s earnings heterogeneity channel.

5.2 Equilibrium analysis

Appendix C.2 defines perfect-foresight equilibria with flexible prices and with fully sticky prices. Under flexible prices, the real interest rate \( R_t \equiv \frac{1 + \delta N Q_t + 1}{Q_t + 1} \) is determined in equilibrium, while under fully sticky prices it is controlled by the central bank. Both classes of equilibria have the following key properties.

\(^{32}\)A strictly equivalent assumption would be to let households own equal nontradable shares in firms instead of taxing profits, and to change the \( T_t \) function by a constant. The benefit of my formulation is that it isolates directly what matters for the earnings heterogeneity channel.
Because of my specification of labor supply, all agents work the same amount of hours \( n_t \), a function of the real wage \( \frac{W_t}{P_t} \). In turn, this implies that GDP is a function of the real wage \( \frac{W_t}{P_t} \), and therefore GDP is a constant \( Y^* \) irrespective of initial conditions. Under sticky prices, I call a boom a situation where \( Y_t > Y^* \) (and real wages are high) and a recession the opposite situation where \( Y_t < Y^* \) (and real wages are low).

The total tax rebate as a share of GDP is always equal to \( t_t = 1 - (1 - \tau) \frac{W_t}{P_t} \), and therefore \( t_t = t^* \) under flexible prices. Under sticky prices, \( t_t \) falls in booms and rises in recessions—a consequence of countercyclical markups. The parameter \( \gamma \) in (26) controls the incidence of this effect on the income distribution. Indeed, defining \( Y_i^t = \frac{W_i^t n_i^t + T_i(e_i^t)}{P_t} \) as \( i \)'s total real earnings (inclusive of the lump-sum rebate), appendix C shows that

\[
\frac{Y_i^t}{Y_t^*} = (1 - t^*) \frac{e_i^t}{E[e]} + t^* + \gamma (t_t - t^*) \left( 1 - \frac{e_i^t}{E[e]} \right)
\]

Hence the sign of \( \gamma \) determines the direction of the earnings heterogeneity channel in the model. When \( \gamma \) is positive, recessions (\( t_t > t^* \)) favor agents with below-average productivity (\( e_i^t < E[e] \)). This could capture the effects of automatic stabilizers. When \( \gamma = 0 \), cyclical conditions leave the income distribution unchanged. Finally, when \( \gamma < 0 \) the post-tax income distribution expands in recessions, capturing countercyclical earnings risk.

In the steady state we can compute the income-MPC covariance \( \mathcal{M} = \mathbb{E} \left[ \frac{Y^t}{Y} \text{MPC}^t \right] \), the income-MPC covariance \( C_Y = \mathbb{Cov} \left( \text{MPC}^i, \frac{Y_i}{Y} \right) \), the redistribution elasticities with respect to the price level \( P \) and the real interest rate \( r \)—respectively, \( \mathcal{E}_P = -\mathbb{Cov} \left( \text{MPC}^i, \frac{\text{NNP}^i}{C} \right) \) and \( \mathcal{E}_r = \mathbb{Cov} \left( \text{MPC}^i, \frac{\text{URE}^i}{C} \right) \)—as well as the Hicksian scaling factor \( S \) and a statistic \( T \) which captures consumption-labor complementarities. Appendix C.3 shows:

**Proposition 1.** Assume that the steady-state is perturbed by a shock that changes \( dY \) and \( dR \) for one period only, and revises all future prices by \( dP \). Then the following equation gives the first order response of aggregate consumption \( dC \):

\[
\frac{dC}{C} = \mathcal{M} \frac{dY}{Y} + \frac{\gamma}{\psi} C_Y \frac{dY}{Y} + \mathcal{E}_P \frac{dP}{P} + (\mathcal{E}_r - \sigma S) \frac{dR}{R} + T \frac{dY}{Y}
\]

In particular, if \( dP = 0 \), the response of consumption \( dC = dY \) is

\[
\frac{dC}{C} = \frac{1}{1 - \mathcal{M} - T - \frac{\gamma}{\psi} C_Y} (\sigma_r + \sigma) \frac{dR}{R}
\]
heterogeneity, Fisher channel, interest rate exposure and substitution channels. The last term corresponds to a complemetarity channel, which stems from GHH preferences. Consider a transitory monetary policy shock under fully sticky prices: in this case \( dP = 0 \) and the nominal interest rate change is \( dR \). If we assume negligible effects of the shock in future periods, we obtain (29), which gives the consumption response in impulse-multiplier form. The sum \( \sigma + \sigma_r \) gives the effect on consumption from the substitution and aggregate wealth effects resulting from \( dR \), and these effects then amplified in general equilibrium by the product \( \mu \times \varrho \): \( \mu \) is the representative-agent multiplier that arises from the presence of complementarities between consumption and labor supply, and \( \varrho \) is an additional multiplier summarizing the role of heterogeneity in the general equilibrium adjustment of incomes, with \( \varrho = 1 \) for a representative agent, and \( \varrho \) decreasing in \( \gamma \) provided that \( C_Y < 0 \).

I now calibrate the model, compute the value of its steady-state sufficient statistics, consider how well the formulas in proposition 1 perform to explain the model’s response to monetary policy shocks.

5.3 Steady-state calibration and solution method

I perform my calibration at quarterly frequency. I target an annual equilibrium real interest rate of 3% and a household debt/PCE ratio 113%—the U.S. level for 2013, which in that year is virtually equal to the stock of interest-paying assets held by the household sector.\(^{33}\) I also target an average asset duration of 4.5 years. This is an average of the durations of U.S. household assets and liabilities reported by Doepke and Schneider (2006) at the end of their data sample (see their figure 3). I consider a flexible-price steady-state with no inflation: \( \Pi = 1 \). These considerations imply a choice of \( \delta_N = 0.95.\(^{34}\)

Since the moments of the redistribution channel all feature a prominent role for MPCs, a minimal requirement on the calibration is that it generates average marginal propensities to consume that are in line with the empirical evidence. I target an average quarterly marginal propensity to consume of 0.24—which is close to a consensus number from the empirical literature. I achieve the joint objectives of matching this number and my target debt level through a combination of relatively tight borrowing limits (\( D = 7.8 \), or 195% of per capita annual consumption, when the natural borrowing limit is over 1300%) and a preference process where agents slowly alternate slowly between patience (discount factor \( \beta^P \)) and impatience (discount factor \( \beta^I \)).\(^{35}\)

---

\(^{33}\)According to the the U.S. Financial Accounts (Board of Governors Z.1 release), in 2013 households held interest-paying liabilities worth $13trn and interest-paying assets worth $12.8trn. I define the former as the sum of mortgages (table L.217, line 7) and consumer credit (table L.222, line 1), the latter as time and savings deposits (L.205[13]) and credit market instruments (the sum of L.208[18], L.209[6], L.210[6], L.211[8], L.212[13], L.216[36], L.217[14] and L.222[3]).

\(^{34}\)The duration of the nominal bond is \( \frac{R^E}{R^E - \delta_N} = D \), so \( \delta_N = R \left( 1 - \frac{1}{D} \right) \). \( \delta_N = 0 \) and \( D = 1 \) for short-term debt.

\(^{35}\)Specifically, I specify that the stationary population distribution must contain patient and impatient agents in
Table 3: Calibration parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impatient discount factor</td>
<td>β^I 0.959</td>
</tr>
<tr>
<td>Patient discount factor</td>
<td>β^P 0.994</td>
</tr>
<tr>
<td>Elasticity of labor supply</td>
<td>ψ 1</td>
</tr>
<tr>
<td>Elasticity of substitution in net consumption</td>
<td>σ 0.5</td>
</tr>
<tr>
<td>Steady-state labor wedge</td>
<td>τ^* 0.40</td>
</tr>
<tr>
<td>Degree of cyclicality in income risk (benchmark)</td>
<td>γ 0</td>
</tr>
<tr>
<td>Asset/liability coupon decay rate</td>
<td>δN 0.95</td>
</tr>
<tr>
<td>Borrowing limit (% of annual per capita consumption)</td>
<td>D 185%</td>
</tr>
</tbody>
</table>

Table 3 summarizes my benchmark parameters. The elasticity of labor supply ψ plays no special role in the steady-state equilibrium of the model. However, it does play an important role in determining the response of real wages to monetary policy shocks. Since the structural vector autoregression evidence (for example Christiano, Eichenbaum and Evans 2005) is for a muted response, I calibrate ψ on the high end of what standard estimates from analyses of panel data imply, and set ψ = 1. I set the elasticity of intertemporal substitution in net consumption to σ = 0.5, which is well within the range of typical calibrations. Since I am ultimately interested in comparing the substitution and the redistribution channel, this allows me not to stack the cards a priori against the substitution channel.

The mix of labor income tax τ and the elasticity of substitution between goods ε is irrelevant, conditional on the “labor wedge” τ^* = 1 – (1 – τ) ε / (ε – 1). I calibrate this wedge jointly with the earnings process. The degree of cyclicality in income risk γ does not affect the steady-state of the model. In a benchmark I set it equal to γ = 0, and later relax this assumption.

Finally, I normalize Y^* = n^* = 1 per quarter. Further details on the calibration and the numerical solution technique—a version of Carroll (2006)’s endogenous gridpoints—are provided in appendix C.7.

5.4 Calibration outcomes

Table 4 displays summary outcomes of the model calibration, focusing on key sufficient statistics. 22% of borrowers are at a binding borrowing limit, and these agents account for 13% of aggregate consumption, so that their response to shocks is an important determinant of the aggregate effect. The model generates a large cross-sectional dispersion equal numbers, and that consumers stay in their patience state for 50 years on average. This process is meant to capture slow-moving preference heterogeneity. It is similar to the one which Krusell and Smith (1998) found useful to match the wealth distribution and which Carroll, Slacalek and Tokuoka (2014) used to generate high marginal propensities to consume on average in the population.
of MPCs around their average level of 0.24, with some agents with high cash-on-hand only slightly above the permanent-income level (below 0.01), and many constrained agents with much larger MPCs. Hence the model delivers a distribution of MPCs that is empirically reasonable.\footnote{More details on calibration outcomes are available in appendix C.9.}

Given the long maturities, the dispersion in steady-state UREs is moderate. The redistribution elasticity $\mathcal{E}_r = -0.09$ and the equivalent EIS $\sigma_r = 0.14$ have magnitudes that are comparable to the ones I obtained in the empirical evidence from section 4. This is a success of the calibration, since I did not target these moments. An alternative would have been, for example, to target $\mathcal{E}_r = -0.24$ as implied by the U.S. data, and to find the asset duration parameter $\delta_N$ that rationalizes it: this exercise delivers $\delta_N = 0.87$, corresponding to assets with 1.8 years of duration, which is not unreasonable. Whether as cross-checks or as direct targets for calibration, sufficient statistics play a promising role in disciplining heterogeneous-agent general equilibrium models going forward.

With this success, I now compute a counterfactual steady-state with shorter maturities. I imagine that all U.S. mortgages have adjustable rates, and that all asset maturities are correspondingly shortened. Keeping in mind this model-implied symmetry and the fact the average duration of U.S. ARMs is above a quarter (e.g. Stanton and Wallace 1999), I consider a plausible reduction of durations to three quarters, corresponding to $\delta_N = 0.67$. In this exercise, steady-state interest rate exposures become much larger in both directions.\footnote{Appendix C.4 proves that changes in $\delta_N$ do not alter the steady-state of the model, only the interest rate exposures that sustain it.} The equivalent EIS rises to $\sigma_r = 1.11$, which is now above my calibrated level of $\sigma$. This finding implies a very important role for the maturity structure in determining the aggregate effects of monetary policy changes, as explored in section 5.5.

One surprising feature of the model comes from the elasticity of aggregate consumption with respect to increases in the price level, $\mathcal{E}_P = 1.77$. This number implies very pow-
erful redistribution through the Fisher channel. There is a strong intuition for this result. Inflation redistributes along the asset dimension, which in this class of models is highly correlated with MPC (a consequence of the concavity of the consumption function). This implies that, allowing for even a moderate degree of price adjustment, the Fisher channel would generate very large amplification of monetary policy shocks. Pursuing this effect further would at least require comparing $\mathcal{E}_P$ to its empirical counterpart and modeling more carefully the adjustment of borrowing constraints to unexpected inflation. Here, I only conclude that my model is suggestive of an amplification role for the Fisher channel in the monetary policy transmission mechanism, and leave a more detailed investigation to future research.

5.5 The effects of monetary policy shocks

Starting from the steady-state, I now assume that the central bank unexpectedly lowers the nominal interest rate at $t = 0$, and then lets it gradually return to its steady-state level at a rate $0 \leq \rho_R < 1$. Because prices are fully sticky, this translates into a path for the real interest rate of

$$R_t - R^* = \rho_R (R_{t-1} - R^*), \quad t \geq 1$$

This corresponds to the typical monetary policy shock that is analyzed in the New Keynesian literature, for the special case with no nominal price adjustment.

Figure 3 displays the impulse responses of the real interest rate $R_t$ and output $Y_t$ to a transitory and a persistent monetary policy shock, corresponding respectively to $\rho_R = 0$ (left column) and $\rho_R = 0.5$ (right column). The initial impulse is a 100 annualized basis points fall in the nominal interest rate, and therefore the real interest rate depicted in panel A. Consider first the output effect from a transitory shock. The left graph of panel B shows that in the benchmark U.S. calibration (the solid red line), the impulse on impact is 40% above that of an equivalent representative-agent economy (the grey line). As reported in table 5, the sufficient statistic formula in (29) delivers this prediction with less than 1% error. The formula sheds light on the source of the 40% amplification under heterogeneity. First, because $\sigma_r = 0.14$, the interest rate exposure channel amplifies by $\frac{\sigma_r}{\sigma_r + \psi} = 28\%$. Second, because $\varrho = 1.10$, general equilibrium income adjustments amplify by a further 10%.

The graph also visually confirms—and table 5 verifies—the hypothesis that effects on future aggregates are negligible, which was necessary to use equation (29) in prediction. This vindicates the use of the statistic approach to deriving model solutions, despite the

---

38I specify that the borrowing limit $\{\mathcal{D}_t\}$ adjusts in response to such a shock as to hold the real coupon payment in the next period fixed: $\mathcal{D}_t = Q_t \bar{a}$. Appendix C.7 provides details on this choice and how to interpret it.
Figure 3: Monetary policy shock (left: $\rho_R = 0$, right: $\rho_R = 0.5$)
presence of the wealth distribution as an endogenous state variable. Turning to the effect of a persistent shock in the right graph of panel B, the benchmark output response is now 43% above its representative-agent counterpart. In other words, the insights derived from sufficient statistics that are obtainable for transitory shocks carry over to more persistent changes, both qualitatively and quantitatively.

**Importance of the maturity structure.** For my counterfactual ARM-only economy, the green line in panel C shows that the response is more than twice as large as the benchmark U.S. response (123% above when the shock is transitory, 102% above when it is persistent). Through the lens of the sufficient statistics, this is unsurprising: because $\sigma_{\text{ARM}}^r = 1.11$, we would predict a response that is $\frac{\sigma_{\text{ARM}}^r - \sigma_{\text{FRM}}^r}{\sigma_{\text{ARM}}^r + \sigma_{\text{FRM}}^r} = 150\%$ higher. The actual response is somewhat below this number, and there is also a small amount of persistence for a few quarters. Section 5.6 will shed further light on the source of this effect.

There are two equivalent ways of interpreting the more muted response of the economy to monetary policy shocks under longer asset durations. The first is that long durations reduce the endogenous amount of unhedged interest rate exposures—making everyone’s consumption less sensitive to changes in real interest rates. A second and more subtle interpretation is that under longer asset maturities, expansionary monetary policy creates more capital gains for asset holders and additional upward revaluation of liabilities for borrowers. These capital gains and losses redistribute against the economy’s MPC gradient, and therefore make monetary policy less potent in affecting output.

This role of the maturity structure in monetary policy transmission is consistent with the cross-country structural VAR evidence presented in Calza et al. (2013). It suggests that wealth redistribution is the primary reason why monetary policy affects consumption in a country like the United Kingdom, where mortgages have adjustable rates.

**Role of the cyclicity of earnings risk.** My benchmark calibration assumed that $\gamma = 0$, implying that monetary policy leaves the income distribution unchanged, with each agent’s income varying in proportion to aggregate GDP. Panel C considers relaxing this assumption. Under a calibration with $\gamma = 1$, a monetary expansion disproportionately benefits the highest-income individuals—in the model, this is the effect of a less progressive tax system, also interpretable as a reduced role for automatic stabilizers in booms. Table 3 and the blue line of Panel D show that, under this calibration, the impact effect from a monetary policy shock is reduced by around 40% relative to benchmark, placing it just above the representative-agent response, both for the transitory and the persistent monetary policy shock. The sufficient statistic approach sheds light on this: the $\varrho$ multiplier drops from $\varrho = 1.10$ to $\varrho = 0.83$ under this adverse effect from earnings heterogeneity.
Table 5: Outcomes vs predictions from sufficient statistic

<table>
<thead>
<tr>
<th></th>
<th>Rep. agent</th>
<th>Benchmark</th>
<th>ARM only</th>
<th>Auto. stab.</th>
<th>Cc. risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual $\frac{dC}{C}, \rho_R = 0$</td>
<td>0.22</td>
<td>0.31</td>
<td>0.69</td>
<td>0.24</td>
<td>0.44</td>
</tr>
<tr>
<td>Rep. agent multiplier ($\mu$)</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>Heterogeneity multiplier ($\varrho$)</td>
<td>1.00</td>
<td>1.10</td>
<td>1.11</td>
<td>0.86</td>
<td>1.63</td>
</tr>
<tr>
<td>Total elasticity ($\sigma + \sigma_r$)</td>
<td>0.50</td>
<td>0.64</td>
<td>1.61</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>Prediction: $-\mu \varrho (\sigma + \sigma_r) \frac{dR}{R}$</td>
<td>0.22</td>
<td>0.31</td>
<td>0.78</td>
<td>0.24</td>
<td>0.46</td>
</tr>
<tr>
<td>Error</td>
<td>0.3%</td>
<td>-0.5%</td>
<td>-12.8%</td>
<td>-0.2%</td>
<td>-4.7%</td>
</tr>
</tbody>
</table>

Contribution to total effect from:
- Aggregate income channel: 1% 15% 17% 15% 15%
- Earnings heterogeneity channel: 0% 0% 0% -9% 10%
- Interest rate exposure channel: 0% 7% 23% 9% 5%
- Substitution channel: 40% 24% 10% 31% 17%
- Complementarity channel: 60% 55% 53% 55% 55%

Actual $\frac{dC}{C}, \rho_R = 0.5$ | 0.44 | 0.63 | 1.28 | 0.48 | 0.90 |

Notes: This table presents the impact ($t = 0$) effect on consumption and output of a monetary policy shock. The top part gives the computed effect $\frac{dC}{C}$ from a transitory shock, and compares it to the prediction in (29). The middle part presents the decomposition in (28). Rows may not sum to 100% due to errors in the approximation formula as well as to rounding errors. The bottom line presents the computed response from a persistent shock.

In contrast to the implication from a positive $\gamma$, Coibion et al. (2012) have recently used CEX data to show that the standard deviation of after-tax income tends to fall by about 1% in response to a 100 basis point identified monetary policy expansion. This is also consistent with a large body of empirical evidence on countercyclical earnings risk. I search for the value of $\gamma$ that captures the Coibion et al. (2012) elasticity exactly, and find $\gamma = -1.125$. The orange lines of Panel D show that, under this new calibration, the earnings heterogeneity channel results in a further amplification of 42% relative to the benchmark, which results in total amplification of 110% of monetary policy shocks relative to the representative agent. This is because the heterogeneity multiplier is now $\varrho = 1.63$.

Conclusion on the role of heterogeneity in the transmission mechanism. Table 5 summarizes my key quantitative findings. When using the sufficient statistics approach in prediction using equation (29), the resulting formula often gets very close to the actual response, and the relative conclusions about the strength of effects tend to carry over to more persistent shocks. Relative to the representative-agent response, heterogeneity amplifies because of an interest rate exposure channel—consistent with my empirical evidence from section 4—and because of the general equilibrium adjustment of incomes.
When using the sufficient statistics approach as as a *quantitative decomposition*, a few points emerge. First, heterogeneity reduces the role of the substitution channel and increases the role of the aggregate income channel. This is consistent with recent results by Kaplan et al. (2016), who perform a similar decomposition in a richer model, using model-implied policy functions instead of sufficient statistics. Second, there is a substantial role for the interest rate exposure channel, especially with short asset durations. Third, when calibrated to match the empirical evidence in Coibion et al. (2012), the earnings heterogeneity channel plays an additional amplification role, consistent with the results in Gornemann et al. (2012). Factoring in a role for the Fisher channel as suggested by the discussion in section 5.4, it therefore appears that *all* of the channels highlighted in the analysis of this paper—in particular, all of the redistributive channels—can play an amplifying role in the transmission of monetary policy shocks.

### 5.6 Asymmetric effects of increases and cuts in interest rates

Motivated by the 13% discrepancy between the sufficient statistic prediction and the full impulse response for my ARM calibration, this section documents the effects as I vary the sign and the magnitude of the initial monetary impulse. Figure 4 shows the result of this exercise. The 13% discrepancy is the consequence of a general pattern of departures from
the sufficient statistic prediction that arises only in my ARM calibration for large enough cuts in interest rates. This asymmetric effect can be traced back to the asymmetric behavior of the 22% of agents who are at their borrowing limit in response to increases and falls in income. While these agents have to cut consumption one for one in response to income falls, their MPC out of moderate increases is below 0.3. Because their debt is short term in the ARM calibration, falls in interest rates effectively act as reductions in payments on their credit limit, and therefore as increases in income. In the aggregate, this generates an effective reduction in MPC differences that is strong enough to affect the quantitative magnitude of the redistribution channel. Increases in interest rates do not have the same feature, since the MPC of borrowers out of increases in interest payments is exactly one, as captured by the sufficient statistics.

This type of asymmetric effects of monetary policy changes receives support from the empirical evidence (see for example Cover 1992; de Long and Summers 1988 and recently Tenreyro and Thwaites 2016). My explanation, which has to do with asymmetric MPC differences in response to policy rate changes, provides an alternative to the traditional Keynesian interpretation of this fact, which relies on downward nominal wage rigidities.39

6 Conclusion

This paper contributes to our understanding of the role of heterogeneity in the transmission mechanism of monetary policy. I identified three important dimensions along which monetary policy redistributes income and wealth, arguing that each of them was likely to be a source of aggregate effects on consumption. My classification holds in many environments and provides a simple, reduced-form approach to computing aggregate magnitudes. Hence it can guide future work on the topic, both theoretical and empirical.

An important finding of my paper is that the monetary policy transmission mechanism operates differently in economies with short and long asset maturities, due to the different nature of the redistribution caused by changes in interest rates. This finding expands upon the popular view that lower interest rates benefit holders of long-term assets and are prejudicial to holders of short-term assets. When assets have relatively long maturities, lower real interest rates indeed tend to benefit asset holders with lower MPCs, making interest rate cuts less effective at increasing aggregate demand. An implication is that monetary policy is intertwined with policies such as government debt maturity management and mortgage design.

39While my U.S. benchmark calibration does not feature asymmetric effects of interest rates, in practice, the refinancing option embedded in fixed rate mortgages in the United States is likely to create an asymmetric effect in the opposite direction from the one I stress here. See recent work by Wong (2015) for theory and empirical evidence along these lines.
My results capture some of the general equilibrium, macroeconomic consequences of the presence of large and heterogeneous marginal propensities to consume, which are a robust feature of household micro data. Beyond the role of wealth redistribution for the macroeconomic effects of fiscal or monetary policy, this raises other questions—such as the role of inequality in the distribution of income in determining aggregate demand—which I leave for future research.

References


Appendix for Online Publication

A Proofs for sections 2 and 3

A.1 Proof of theorem 1

Theorem 1 in the main text is a special case of a general decomposition that holds for arbitrary nonsatiable preferences $U$ over $\{c_t\}$ and $\{n_t\}$ and for any change in the price level $\{P_0, P_1, \ldots\}$, the real term structure $\{q_0 = 1, q_1, q_2, \ldots\}$, the agent’s unearned income sequence $\{y_0, y_1, \ldots\}$ and the stream of real wages $\{w_0, w_1, \ldots\}$, with the nominal term structure adjusting instantaneously to make the Fisher equation hold at the post-shock sequences of interest rates and prices. The utility maximization problem is then

$$\max U (\{c_t, n_t\})$$

$$\text{s.t. } P_tC_t = P_t y_t + W_t n_t + (t-1)B_t + \sum_{s \geq 1} (tQ_{t+s}) \left((t-1)B_{t+s} - tB_{t+s}\right) + P_t (t-1)b_t + \sum_{s \geq 1} (tq_{t+s}) P_{t+s} (t-1)b_{t+s} - tB_{t+s}$$

and the first order date-0 responses of consumption, labor supply and welfare to the considered change are, in this case, given by

$$dc_0 = MPCd\Omega + c_0 \left(\sum_{t \geq 0} e^h_{c_0,q_t} \frac{dq_t}{q_t} + \sum_{t \geq 0} e^h_{c_0,w_t} \frac{dw_t}{w_t}\right)$$

$$dn_0 = MPNd\Omega + n_0 \left(\sum_{t \geq 0} e^h_{n_0,q_t} \frac{dq_t}{q_t} + \sum_{t \geq 0} e^h_{n_0,w_t} \frac{dw_t}{w_t}\right)$$

$$dU = U_{c_0} d\Omega$$

where $e^h_{x_0,y_t} = \frac{\partial x_0}{\partial y_t} \frac{y_t}{x_0}$ for $x \in \{c,n\}$ and $y \in \{q,w\}$ are Hicksian elasticities and $d\Omega = dW - \sum_{t \geq 0} c_t dq_t$, the net-of-consumption wealth change, is given by

$$d\Omega = \sum_{t \geq 0} (q_t y_t) \frac{dy_t}{y_t} + \sum_{t \geq 0} (q_t w_t n_t) \frac{dw_t}{w_t}$$

Real unearned income change

Real earned income change

$$+ \sum_{t \geq 0} q_t \left(y_t + w_t n_t + \left(-1\frac{B_t}{P_t}\right) + (-1b_t) - c_t\right) \frac{dq_t}{q_t} - \sum_{t \geq 0} Q_t \left(-1\frac{B_t}{P_0}\right) \frac{dP_t}{P_t}$$

Revaluation of net savings flows

Revaluation of net nominal position
Applying this general formula to variation considered in the main text yields:

\[ dc_0 = \text{MPC} d\Omega + c_0 \left( \partial_{c_0,q_0} \frac{dR_0}{R_0} + \partial_{c_0,w_0} \frac{dw_0}{w_0} \right) \]  
(A.2)

\[ dn_0 = \text{MPN} d\Omega + n_0 \left( \partial_{n_0,q_0} \frac{dR_0}{R_0} + \partial_{n_0,w_0} \frac{dw_0}{w_0} \right) \]  
(A.3)

\[ dU = U_{c_0} d\Omega \]

Moreover, when preferences are separable, I show below that

\[ \epsilon_{h,c_0,q_0} = -\sigma \text{MPC} \]  
(A.4)

\[ \epsilon_{c_0,w_0} = \psi \frac{w_0 n_0}{c_0} \text{MPC} \]  
(A.5)

\[ \epsilon_{n_0,q_0} = \psi \text{MPS} \]  
(A.6)

\[ \epsilon_{n_0,w_0} = \psi (1 + w_0 \text{MPN}) \]  
(A.7)

hence theorem 1 follows.

Proof of the general statement. It is convenient to define the expenditure function over the sequences \{\text{q}_t\} and \{\text{w}_t\}:

\[ e(\{\text{q}_t\}, \{\text{w}_t\}, U) = \min \left\{ \sum_{t} q_t (c_t - w_t n_t) \text{ s.t. } U(\{c_t, n_t, h_t\}) \geq U \right\} \]

and let \text{c}_{h,t}, \text{n}_{h,t} be the resulting Hicksian demands. The envelope theorem implies a version of Shephard’s lemma:

\[ e_{\text{q}_t} = c_t - w_t n_t \quad \forall t \]

\[ e_{\text{w}_t} = -q_t n_t \quad \forall t \]

Define the indirect utility function to attain unearned wealth \( \tilde{W} = \sum_{t \geq 0} q_t \left( y_t + (-b_t) + \left( -\frac{b_t}{P_t} \right) \right) \) (wealth exclusive of earned income whose price is changing) as

\[ V(\{q_t\}, \{w_t\}, \tilde{W}) = \max \left\{ U(\{c_t, n_t\}) \text{ s.t. } \sum_{t} q_t (c_t - w_t n_t) = \tilde{W} \right\} \]

and let \text{c}_{t}, \text{n}_{t} be the resulting Marshallian demands. Differentiating along the identities

\[ c_{0,t}(\{q_t\}, \{w_t\}, U) = c_0(\{q_t\}, \{w_t\}, e(\{q_t\}, \{w_t\}, U)) \]

\[ n_{0,t}(\{q_t\}, \{w_t\}, U) = n_0(\{q_t\}, \{w_t\}, e(\{q_t\}, \{w_t\}, U)) \]
we find that Marshallian derivatives are

\[ \frac{\partial c_h}{\partial q_t} = \frac{\partial c_0}{\partial q_t} + \frac{\partial c_0}{\partial w} e_{q_t}, \quad \frac{\partial c_h}{\partial w_t} = \frac{\partial c_0}{\partial w_t} + \frac{\partial c_0}{\partial w} e_{w_t} \]

\[ \frac{\partial n_h}{\partial q_t} = \frac{\partial n_0}{\partial q_t} + \frac{\partial n_0}{\partial w} e_{q_t}, \quad \frac{\partial n_h}{\partial w_t} = \frac{\partial n_0}{\partial w_t} + \frac{\partial n_0}{\partial w} e_{w_t} \]

denoting \( MPC \equiv \frac{\partial c_0}{\partial w} / \frac{\partial c_0}{\partial q} \) and \( MPN \equiv \frac{\partial n_0}{\partial w} / \frac{\partial n_0}{\partial q} \) (under the present value normalization \( q_0 = 1 \)) and using Shephard’s lemma we obtain

\[ \frac{\partial c_h}{\partial q_t} = \frac{\partial c_0}{\partial q_t} + MPC \cdot (c_t - w_t n_t) \]
\[ \frac{\partial c_h}{\partial w_t} = \frac{\partial c_0}{\partial w_t} - MPC \cdot q_t n_t \]
\[ \frac{\partial n_h}{\partial q_t} = \frac{\partial n_0}{\partial q_t} + MPN \cdot (c_t - w_t n_t) \]
\[ \frac{\partial n_h}{\partial w_t} = \frac{\partial n_0}{\partial w_t} - MPN \cdot q_t n_t \]

Applying a Taylor expansion to the consumption function \( c_0(q_t, w_t, P_t, y_t) \) and using the above values for derivatives evaluated at the initial sequence \( q_t, w_t, P_t, y_t \), we have that the first-order differential is

\[ dc_0 = \sum_{t \geq 0} \frac{\partial c_0}{\partial q_t} dq_t + \sum_{t \geq 0} \frac{\partial c_0}{\partial w_t} dw_t + \frac{\partial c_0}{\partial W} d\bar{W} \]

\[ = \sum_{t \geq 0} \left( \frac{\partial c_0}{\partial q_t} - MPC \cdot (c_t - w_t n_t) \right) dq_t + \sum_{t \geq 0} \left( \frac{\partial c_0}{\partial w_t} + MPC \cdot q_t n_t \right) dw_t \]

\[ + MPC \left( \sum_{t \geq 0} \left( y_t + (-1) b_t \right) + \left( \frac{-1}{P_t} B_t \right) \right) dq_t - \sum_{t \geq 0} q_t \left( \frac{-1}{P_t} B_t \right) \frac{dP_t}{P_t} + \sum_{t \geq 0} q_t dy_t \]

\[ = c_0 \sum_{t \geq 0} q_t \frac{\partial c_0}{\partial q_t} dq_t + c_0 \sum_{t \geq 0} w_t \frac{\partial c_0}{\partial w_t} dw_t + MPCd\Omega \]

where

\[ d\Omega = \sum_{t \geq 0} q_t dy_t + \sum_{t \geq 0} q_t n_t dw_t \]

\[ + \sum_{t \geq 0} \left( y_t + w_t n_t + (-1) b_t \right) + \left( \frac{-1}{P_t} B_t \right) \left( c_t \right) dq_t - \sum_{t \geq 0} q_t \left( \frac{-1}{P_t} B_t \right) \frac{dP_t}{P_t} \]

\[ = \sum_{t \geq 0} q_t y_t \frac{dy_t}{y_t} + \sum_{t \geq 0} q_t w_t n_t \frac{dw_t}{w_t} \]

\[ + \sum_{t \geq 0} q_t \left( y_t + w_t n_t + (-1) b_t \right) + \left( \frac{-1}{P_t} B_t \right) \left( c_t \right) \frac{dq_t}{q_t} - \sum_{t \geq 0} Q_t \left( \frac{-1}{P_t} B_t \right) \frac{dP_t}{P_t} \]

where the last line uses the Fisher equation, \( \frac{q_t}{P_t} = \frac{Q_t}{P_t} \) to rewrite future real wealth in date-0.
terms. Using the same calculation for $n_0$, we obtain the labor supply response formula in theorem 1. The welfare response follows from application of the envelope theorem to the indirect utility function:

$$
\frac{\partial V}{\partial q_t} = -U_c^0 \{c_t, n_t, h_t\} \cdot (c_t - w_t n_t)
$$

$$
\frac{\partial V}{\partial w_t} = U_c^0 \{c_t, n_t, h_t\} \cdot (q_t n_t)
$$

$$
\frac{\partial V}{\partial W} = U_c^0 \{c_t, n_t, h_t\}
$$

therefore a Taylor expansion yields the first-order differential

$$
dU = \sum_{t \geq 0} \frac{\partial V}{\partial q_t} dq_t + \sum_{t \geq 0} \frac{\partial V}{\partial w_t} dw_t + \frac{\partial V}{\partial W} d\tilde{W}
$$

$$
= U_c^0 \{c_t, n_t, h_t\} \cdot \left( \sum_{t \geq 0} (w_t n_t - c_t) dq_t + \sum_{t \geq 0} q_t n_t dw_t + d\tilde{W} \right)
$$

$$
= U_c^0 \{c_t, n_t, h_t\} \cdot d\Omega
$$

as was to be shown. \qed

Suppose that utility has the separable form

$$
U (\{c_t, n_t\}) = \sum_t \beta^t \{ u (c_t) - v (n_t) \}
$$

(A.8)

where $u$ is any continuous, strictly increasing and concave function and $v$ is any continuous, increasing and convex function. I now derive the values for all elasticities $\varepsilon_{x_0,y_t}^h = \frac{\partial^2 U^x}{\partial x_0 \partial y_t}$, in particular (A.4)–(A.7).

**Marginal propensity to consume.** Inverting the first-order conditions

$$
u' (c_t) = \beta^{-t} \left( \frac{q_t}{q_0} \right) u' (c_0) \tag{A.9}
$$

$$
\nu' (n_t) = \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v' (n_0) \tag{A.10}
$$

and inserting the resulting values for $c_t$ and $n_t$ into the budget constraint

$$
\sum_{t \geq 0} \frac{q_t}{q_0} (c_t - w_t n_t) = W
$$
we obtain
\[ c_0 + \sum_{t \geq 1} \frac{q_t}{q_0} (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u' (c_0) \right] - w_0 \left( n_0 + \sum_{t \geq 1} \frac{q_t w_t}{q_0 w_0} (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v' (n_0) \right] \right) = W \]
(A.11)

Writing \( MPC = \frac{\partial c_0}{\partial W} \), \( MPN = \frac{\partial n_0}{\partial W} \) and differentiating (A.11) with respect to \( W \) we obtain
\[ MPC \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u'' (c_0)}{u'' (c_t)} \right) - w_0 MPN \left( 1 + \sum_{t \geq 1} \frac{q_t w_t}{q_0 w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{w_t}{w_0} \frac{v'' (n_0)}{v'' (n_t)} \right) = 1 \]
(A.12)

moreover, the intratemporal first order condition
\[ v' (n_0) = w_0 u' (c_0) \]
(A.13)

implies
\[ \frac{v'' (n_0)}{v' (n_0)} MPN = w_0 u'' (c_0) MPC \]
\[ \frac{v'' (n_0)}{v' (n_0)} MPN = \frac{u'' (c_0)}{u' (c_0)} MPC \]

so, using the definition of the local elasticities of substitution,
\[ -\sigma (c_t) c_t u'' (c_t) = u' (c_t) \]
(A.14)
\[ \psi (n_t) n_t v'' (n_t) = v' (n_t) \]
(A.15)

we see that \( MPC \) and \( MPN \) are related through
\[ MPN = -\frac{\psi (n_0) n_0}{\sigma (c_0) c_0} MPC \]

Inserting into (A.12), this gives
\[ MPC = \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u'' (c_0)}{u'' (c_t)} + \frac{\psi (n_0) w_0 n_0}{\sigma (c_0) c_0} \sum_{t \geq 1} \frac{q_t w_t}{q_0 w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v'' (n_0)}{v'' (n_t)} \right)^{-1} \]
(A.16)

as well as
\[ MPS = 1 - MPC + w_0 MPN \]
\[ = MPC \left( \sum_{t \geq 1} \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u'' (c_0)}{u'' (c_t)} \right) + \frac{\psi (n_0) w_0 n_0}{\sigma (c_0) c_0} \sum_{t \geq 1} \frac{q_t w_t}{q_0 w_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v'' (n_0)}{v'' (n_t)} \right) \]
(A.17)
Expressions (A.16) and (A.17) can also be rewritten using the fact that (A.9)-(A.10) together with (A.14)-(A.15) yield

\[
\beta^{-t} \left( \frac{q_t}{q_0} \right) u''(c_t) = \sigma (c_t) c_t \sigma (c_0) c_0 \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v''(n_t) = \psi (n_t) n_t \psi (n_0) n_0
\]

So, for example

\[
\text{MPC} = \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \sigma (c_t) c_t \sigma (c_0) c_0 \frac{\psi (n_0) n_0}{\sigma (c_0) c_0} \left( 1 + \sum_{t \geq 1} \frac{q_t}{q_0} \left( \frac{w_t}{w_0} \right) \psi (n_t) n_t \right) \right)^{-1}
\]

**Hicksian elasticities.** The solution to

\[
e (\{q_t\}, U) = \min \left\{ \sum_t q_t c_t \quad \text{s.t.} \quad U (\{c_t\}) \geq U \right\}
\]

also involves the first-order conditions (A.9)-(A.10), from which we obtain

\[
u (c_t) = u \left( (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u' (c_0) \right] \right) \quad \text{and} \quad v (n_t) = v \left( (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v' (n_0) \right] \right)
\]

attaining utility \( U \) requires that the initial values \( c_0, n_0 \) satisfy

\[
u (c_0) + \sum_{t \geq 1} \beta^t u \left( (u')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) u' (c_0) \right] \right) - v (n_0) - \sum_{t \geq 1} \beta^t v \left( (v')^{-1} \left[ \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) v' (n_0) \right] \right) = U
\]

(A.18)

For \( s \geq 1 \), differentiating with respect to \( q_s \) along the indifference curve (A.18) results in

\[
\frac{\partial c_0}{\partial q_s} \left( u' (c_0) + \sum_t \beta^t \frac{u' (c_t)}{u'' (c_t)} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u'' (c_0)}{u'' (c_t)} \right) - \frac{\partial n_0}{\partial q_s} \left( v' (n_0) + \sum_t \beta^t \frac{v' (n_t)}{v'' (n_t)} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{v'' (n_0)}{v'' (n_t)} \right) + \beta^s \frac{u' (c_s)}{u'' (c_s)} \left( \beta^{-s} \frac{1}{q_0} u' (c_0) \right) - \beta^s \frac{v' (n_s)}{v'' (n_s)} \left( \beta^{-s} \frac{1}{q_0} \frac{w_s}{w_0} v' (n_0) \right) = 0
\]

dividing by \( u' (c_0) \) and using (A.9), (A.13), (A.14) and (A.15) we find

\[
\frac{\partial c_0}{\partial q_s} \left( 1 + \sum_t \frac{q_t}{q_0} \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{u'' (c_0)}{u'' (c_t)} \right) - \frac{\partial n_0}{\partial q_s} \frac{w_0}{w_0} \left( 1 + \sum_t \frac{q_t}{q_0} w_t \beta^{-t} \left( \frac{q_t}{q_0} \right) \frac{v'' (n_0)}{v'' (n_t)} \right)
\]

\[
= \frac{1}{q_0} \sigma (c_s) c_s + \frac{1}{q_0} \psi (n_s) w_s n_s
\]
moreover, differentiating (A.13) we also find
\[
\frac{\partial n_0}{\partial q_s} = -\frac{\psi(n_0)}{\sigma(c_0)} \frac{n_0 \partial c_0}{c_0 \partial q_s}
\]
so we conclude that
\[
\frac{\partial c_0}{\partial q_s} \bigg|_{U} = \frac{1}{q_0} \sigma(c_s) \frac{c_s}{c_0} \text{MPC} \left( 1 + \frac{\psi(n_s)}{\sigma(c_s)} \frac{w_s n_s}{c_s} \right)
\]
and that the associated Hicksian elasticity is
\[
\epsilon^h_{c_0 q_s} = \frac{q_s}{q_0} \sigma(c_s) \frac{c_s}{c_0} \text{MPC} \left( 1 + \frac{\psi(n_s)}{\sigma(c_s)} \frac{w_s n_s}{c_s} \right)
\]
Similarly, differentiating (A.18) with respect to \( q_0 \) yields
\[
\frac{\partial c_0}{\partial q_0} \frac{u'(c_0)}{\text{MPC}}^{-1} = \sum_{t \geq 1} \beta^t \frac{u'(c_t)}{u''(c_t)} \left( \beta^{-t} \frac{q_t}{q_0} \frac{q_t}{q_0} \right) - \sum_{t \geq 1} \beta^t \frac{v'(n_t)}{v''(n_t)} \left( \beta^{-t} \frac{q_t}{q_0} \frac{w_t}{w_0} \right)
\]
so we recognize the expression in (A.17)
\[
\frac{\partial c_0}{\partial q_0} \frac{u'(c_0)}{\text{MPC}}^{-1} = -\frac{1}{q_0} \sigma(c_0) c_0 \left( \sum_{t \geq 1} \beta^{-t} \left( \frac{q_t}{q_0} \frac{q_t}{q_0} \right) \frac{u''(c_t)}{c_0} \frac{q_t}{q_0} + \frac{w_0 n_0 \psi(n_0)}{c_0} \sum_{t \geq 1} \beta^{-t} \left( \frac{q_t}{q_0} \frac{w_t}{w_0} \right) \frac{v''(n_t)}{c_0} \frac{q_t}{q_0} \frac{w_t}{w_0} \right)
\]
and therefore finally
\[
\frac{\partial c_0}{\partial q_0} \bigg|_{U} = -\frac{c_0}{q_0} \sigma(c_0) \frac{\text{MPS}}{\text{MPC}}
\]
with a Hicksian elasticity of
\[
\epsilon^h_{c_0 q_0} = -\sigma(c_0) \frac{\text{MPS}}{\text{MPC}}
\]
which is (A.4). For \( s \geq 1 \), differentiating with respect to \( w_s \) along the indifference curve (A.18) similarly results in
\[
\frac{\partial c_0}{\partial w_s} \frac{u'(c_0)}{\text{MPC}}^{-1} = \beta^s \frac{v'(n_s)}{v''(n_s)} \beta^{-s} \frac{q_s}{q_0} \left( \frac{1}{w_0} \right) v'(n_0)
\]
\[
\frac{\partial c_0}{\partial w_s} \frac{\text{MPC}}{\text{MPC}}^{-1} = \frac{q_s}{q_0} \psi(n_s) n_s
\]
hence
\[
\frac{\partial c_0}{\partial w_s} \bigg|_U = MPC \frac{q_s}{q_0} \psi (n_s) n_s
\]
with associated elasticity
\[
e_{c_0,w_s}^h = MPC \frac{q_s}{q_0} \left( \frac{\psi (n_s) w_s n_s}{c_0} \right)
\]
A similar procedure can be used to differentiate with respect to \(w_0\): from (A.13) we obtain
\[
\frac{\partial n_0}{\partial w_0} = -\frac{\psi (n_0)}{\sigma (c_0)} \frac{n_0}{c_0} \frac{\partial q_s}{\partial q_s} + \psi (n_0) \frac{n_0}{w_0}
\]
and differentiating along (A.18) we therefore obtain
\[
\frac{\partial c_0}{\partial w_0} u' (c_0) MPC^{-1} + \psi (n_0) \frac{n_0}{w_0} \left( v' (n_0) + \sum_{t \geq 1} \beta^t v' (n_t) \beta^{-t} \left( \frac{q_t}{q_0} \right) \left( \frac{w_t}{w_0} \right) \frac{v'' (n_0)}{v'' (n_t)} \right)
\]
\[
= \sum_{t \geq 1} \beta^t v' (n_t) \beta^{-t} \frac{q_t}{q_0} \left( \frac{w_t}{w_0} \right)^2 \frac{v'' (n_0)}{v'' (n_t)} v' (n_0)
\]
We conclude by noticing that \(v' (n_0) = \psi (n_0) n_0 v'' (n_0)\), so
\[
\frac{\partial c_0}{\partial w_0} \bigg|_U = MPC \psi (n_0) n_0
\]
and
\[
e_{c_0,w_0}^h = MPC \left( \psi (n_0) \frac{w_0 n_0}{c_0} \right)
\]
which is (A.5). Finally, elasticities for \(n_0\) result from a final differentiation of (A.13):
\[
\begin{align*}
\epsilon_{n_0,q_s}^h &= -\frac{\psi (n_0)}{\sigma (c_0)} \epsilon_{c_0,q_s}^h \quad s \geq 0 \\
\epsilon_{n_0,w_s}^h &= -\frac{\psi (n_0)}{\sigma (c_0)} \epsilon_{c_0,w_s}^h \quad s \geq 1 \\
\epsilon_{n_0,w_0}^h &= \psi (n_0) \left( 1 - \frac{1}{\sigma (c_0)} \epsilon_{c_0,w_0}^h \right) \\
&= \psi (n_0) \left( 1 - \frac{\psi (n_0) w_0 n_0}{\sigma (c_0) c_0} \right) MPC \\
&= \psi (n_0) (1 + w_0 \text{MPN})
\end{align*}
\]
which in particular give (A.6) and (A.7).

**Values of all elasticities in a steady-state with no growth.** It is insightful to look at the value of elasticities just derived in an infinite horizon model where \(\frac{q_s}{q_0} = \beta^s\) and \(w_s = w^*, \forall s\). These prices correspond to those prevailing in a steady-state with no growth of an infinite
horizon model, and the resulting elasticities are relevant, for example, to determine the impulse responses in many RBC models. The first order conditions imply that consumption and labor supply are constant. Let us call the solutions \( c^* \) and \( n^* \), respectively. Writing \( \vartheta \equiv \frac{w^* n^*}{c^*} \) for the share of earned income in consumption and \( \kappa \equiv \frac{\psi \sigma}{1 + \psi \sigma} \in (0, 1) \), the results above imply

\[
\begin{array}{c|cccc|c}
\hline
\epsilon_h & q_0 & q_{s,s} & w_0 & w_{s,s} & \text{Marg. propensity} \\
\hline
c_0 & -\sigma \beta & \sigma (1 - \beta) \beta^s & \sigma \kappa (1 - \beta) & \sigma \kappa (1 - \beta) \beta^s & \text{MPC} (1 - \kappa) (1 - \beta) \\
n_0 & \psi \beta & -\psi (1 - \beta) \beta^s & \psi (1 - \kappa (1 - \beta)) & -\psi \kappa (1 - \beta) \beta^s & \text{MPN} - \frac{1}{\psi} \kappa (1 - \beta) \\
\hline
\end{array}
\]

A.2 Proof of theorem 2

After dividing through by \( P_t \), defining real bond positions as \( \lambda_t \equiv \frac{\Lambda_t}{P_{t-1}} \) and writing \( \Pi_t \equiv \frac{P_t}{P_{t-1}} - 1 \) for the inflation rate between \( t - 1 \) and \( t \), the budget constraint (8) becomes

\[
c_t + Q_t \left( \frac{\lambda_{t+1} - \delta_N \frac{\lambda_t}{\Pi_t}}{} \right) + (\theta_{t+1} - \theta_t) \cdot S_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot d_t
\]

In this notation, the consumer’s date-\( t \) net nominal position is

\[
NNP_t = (1 + Q_t \delta_N) \frac{\lambda_t}{\Pi_t}
\]

while his unhedged interest rate exposure is:

\[
URE_t = y_t + w_t n_t + \frac{\lambda_t}{\Pi_t} + \theta_t \cdot d_t - c_t = Q_t \left( \frac{\lambda_{t+1} - \delta_N \frac{\lambda_t}{\Pi_t}}{} \right) + (\theta_{t+1} - \theta_t) \cdot S_t
\]

His optimization problem can be represented using the recursive formulation

\[
\begin{align*}
\max_{c_t,\lambda',\beta'} u(c) - v(n) + \beta \mathbb{E} & \left[ V(\lambda',\theta';y',w',Q',\Pi',d',S') \right] \\
\text{s.t.} & \quad c + Q(\lambda' - \delta_N \frac{\lambda}{\Pi}) + (\theta' - \theta) S = y + wn + \frac{\lambda}{\Pi} + \theta d \\
& \quad Q\lambda' + \theta' S \geq \frac{D}{R}
\end{align*}
\]

(A.19)

The function \( V \) corresponds to the value from optimizing given a starting real level of bonds \( \lambda' \) and shares \( \theta' \), and includes the possibility of hitting future borrowing constraints.
I consider the predicted effects on \( c \) and \( n \) resulting from a simultaneous unexpected change in unearned income \( d\bar{y} \), the real wage \( dw \), the price level \( \frac{d\bar{P}}{\bar{P}} = \frac{d\bar{P}}{\bar{P}} \) and the real interest rate \( d\bar{R} \), which result in a change in asset prices \( \frac{dQ}{Q} = \frac{dS_j}{S_j} = -\frac{d\bar{R}}{\bar{R}} \) for \( j = 1 \ldots N \). By leaving the future unaffected, this purely transitory change does not alter the value from future optimization starting at \( (\lambda', \theta') \)— that is, the function \( W \) is unchanged. I claim that, provided the consumption and labor supply functions are differentiable, their first order differentials are

\[
 dc = MPC \left( dy + n (1 + \psi) dw + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) - \sigma c MPS \frac{d\bar{R}}{\bar{R}} \quad (A.20)
\]

\[
 dn = MPN \left( dy + n (1 + \psi) dw + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) + \psi n MPS \frac{d\bar{R}}{\bar{R}} + \psi n \frac{dw}{w} \quad (A.21)
\]

where \( \sigma \equiv -\frac{\omega'(c)}{\omega'(c)} \) and \( \psi = \frac{\omega'(n)}{w'(n)} \) are the local elasticities of intertemporal substitution and labor supply, respectively, \( MPC = \frac{dc}{dy} \), \( MPN = \frac{dc}{dn} \) and \( MPS = 1 - MPC + wMPN \).

Once (A.20) and (A.21) are established, we can transform these relationships into one between \( dc \) and \( dY = dy + ndw + wdn \) as follows. Rewrite the equations as

\[
 dc = MPC \left( dY + \psi dw - wdn + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) - \sigma c MPS \frac{d\bar{R}}{\bar{R}}
\]

\[
 wdn - \psi ndw = wMPN \left( dY + \psi dw - wdn + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) + \psi wn MPS \frac{d\bar{R}}{\bar{R}}
\]

Hence

\[
 wdn - \psi ndw = \frac{1}{1+wMPN} \left\{ wMPN \left( dY + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) + \psi wn MPS \frac{d\bar{R}}{\bar{R}} \right\}
\]

which, inserted into the expression for \( dc \) yields

\[
 dc = MPC \left( 1 - \frac{wMPN}{1+wMPN} \right) \left( dY + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) - \sigma c MPS \left( 1 + MPC \frac{\psi wn}{\sigma c} \frac{1}{1+wMPN} \right) \frac{d\bar{R}}{\bar{R}}
\]

But \( MPC \frac{\psi n}{\sigma c} = -MPN \) so this is

\[
 dc = \left( \frac{MPC}{1+wMPN} \right) \left( dY + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) - \sigma c \frac{MPS}{1+wMPN} \frac{d\bar{R}}{\bar{R}}
\]

and noting that \( 1 + wMPN = MPC + MPS \) we can finally rewrite this in terms of \( \hat{MPC} = \frac{MPC}{MPC+MPS} \) as

\[
 dc = \hat{MPC} \left( dY + URE \frac{d\bar{R}}{\bar{R}} - NNP \frac{d\bar{P}}{\bar{P}} \right) - \sigma c \left( 1 - \hat{MPC} \right) \frac{d\bar{R}}{\bar{R}}
\]

In order to prove (A.20) and (A.21), there are two cases to consider. In the first case, the consumer is at a binding borrowing limit. The problem is then a static choice between \( c \) and
In the second case, the consumer is at an interior optimum. The result then follows from application of the implicit function theorem to the set of $N + 2$ first-order conditions which, together with the budget constraint, characterize the solution to the problem in (A.19). Here, to simplify the notation and the proof, I first prove the statement in the case where all variables are changing but $N = 0$, and then consider the case with stocks ($N > 0$) but without bonds and assuming only $R$ is changing.

**Case 1. Binding borrowing limit.**

**Proof.** The consumption of an agent at the borrowing limit is given by

$$c = wn + Z$$  \hspace{1cm} (A.22)

where

$$Z = z + (1 + Q\delta_N) \frac{\lambda}{\Pi} + \theta \cdot (d + S) + \frac{D}{R}$$

Given that $dS = -\frac{S}{R} dR$, $dQ = -\frac{Q}{R} dR$ and $d \left( \frac{1}{\Pi} \right) = -\frac{1}{\Pi^2} d\Pi = -\frac{1}{\Pi} \frac{dP}{P}$, we have

$$dZ = dy - (1 + Q\delta_N) \frac{\lambda}{\Pi} \frac{dP}{P} + \left( Q\delta_N \frac{\lambda}{\Pi} + \theta \cdot S + \frac{D}{R} \right) \left( -\frac{dR}{R} \right)$$

Next, given that the borrowing limit is binding, the consumer is making a static choice between $c$ and $n$ given the budget constraint (A.22). We can then apply the results of section A.1 to find

$$dc = MPC \left( dZ + w \left( 1 + \psi \right) \right)$$

$$dn = MPN \left( dZ + w \left( 1 + \psi \right) \right) + \psi ndw$$

which yields the desired result. \hfill \square

**Case 2a.** $N = 0$, all variables changing  
I first prove the following lemma.

**Lemma A.1.** Let $c(z, w, q, b)$ and $n(z, w, q, b)$ be the solution to the following separable consumer choice problem under concave preferences over current consumption $u(c)$ and assets $V(a)$, and convex preferences over hours worked $v(n)$:

$$\max \quad u(c) - v(n) + V(a)$$

$$\text{s.t.} \quad c + q (a - b) = wn + z$$

A11
Assume \( c() \) and \( n() \) are differentiable. Then the first order differentials are

\[
\begin{align*}
dc &= MPC (dz + n(1 + \psi)dw - (a - b)dq + qdb) - \sigma cMPS \frac{dq}{q} \\
\frac{dn}{dn} &= MPN (dz + n(1 + \psi)dw - (a - b)dq + qdb) + \psi nMPS \frac{dq}{q} + \psi n \frac{dw}{w}
\end{align*}
\]

where \( MPC = \frac{\partial c}{\partial z} \), \( MPN = \frac{\partial n}{\partial z} \) and \( MPS = 1 - MPC + wMPN = 1 - MPC \left(1 + \frac{wn \psi}{c \sigma}\right)\).

Proof. The following first-order conditions are necessary and sufficient for optimality:

\[
\begin{align*}
\frac{u'(c)}{c} = \frac{1}{w} v'(n) = \frac{1}{q} V'(a)
\end{align*}
\]

(A.23)

I first obtain the expression for \( MPC \) by considering an increase in income \( dz \) alone. Consider how that increase is divided between current consumption, leisure and assets. (A.23) implies

\[
\begin{align*}
\frac{u''(c)}{c} dc &= \frac{1}{w} \frac{v''(n)}{n} dn = \frac{1}{q} V''(a) da
\end{align*}
\]

(A.24)

where the changes \( dc, dn \) and \( da \) are related to \( dz \) through the budget constraint

\[
\begin{align*}
dc + qda &= wdn + dz
\end{align*}
\]

(A.25)

Define \( MPC = \frac{\partial c}{\partial z} \), \( MPN = \frac{\partial n}{\partial z} \) and \( MPS = q \frac{\partial a}{\partial z} \). Then (A.24) implies

\[
\begin{align*}
\frac{MPN}{MPC} &= w \frac{u''(c)}{v''(n)} = \frac{u''(c)}{u'(c)} \frac{v'(n)}{v''(n)} = -\frac{n \psi}{c \sigma} \\
\frac{MPS}{MPC} &= \frac{q^2 u''(c)}{V''(a)} = \frac{q}{c} \frac{V'(a)}{V''(a)}
\end{align*}
\]

where \( \sigma \equiv -\frac{u'(c)}{v''(c)} \) and \( \psi \equiv \frac{v'(n)}{wn'(n)} \). Hence the total marginal propensity to spend is

\[
1 - MPS = \frac{\partial c}{\partial z} - w \frac{\partial n}{\partial z} = MPC \left(1 + \frac{wn \psi(n)}{c \sigma(c)}\right) = 1 - \frac{q^2 u''(c)}{V''(a)} MPC
\]

(A.26)

and the marginal propensity to consume is

\[
MPC = \frac{1}{1 + \frac{q^2 u''(c)}{V''(a)} - w^2 \frac{u''(c)}{v''(n)}} = \frac{V''(a) v''(n)}{V''(a) v''(n) + q^2 u''(c) \frac{v''(n)}{V''(a)}}
\]

Consider now the overall effect on \( c, n \) and \( a \) of a change in \( q, w, z \) and \( b \). Applying the implicit
function theorem to the system of equations

\[
\begin{align*}
\nu' (n) - \omega u' (c) &= 0 \\
\nu' (a) - q u' (c) &= 0 \\
c + q (a - b) - \omega n - z &= 0
\end{align*}
\]

results in the following expression for partial derivatives:

\[
\begin{bmatrix}
\frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial n} & \frac{\partial c}{\partial w} \\
\frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b} \\
\frac{\partial w}{\partial q} & \frac{\partial w}{\partial z} & \frac{\partial w}{\partial n} & \frac{\partial w}{\partial b}
\end{bmatrix}
= -\begin{bmatrix}
-wu''(c) & 0 \\
-q u''(c) & 0 \\
1 & -w & q
\end{bmatrix}^{-1}
\begin{bmatrix}
0 & 0 & -u'(c) & 0 \\
-u'(c) & 0 & 0 & 0 \\
(a - b) & -1 & -n & -q
\end{bmatrix}
\]

(A.27)

now

\[
\det (A) = \nu'' (n) V'' (a) - \omega^2 u'' (c) V'' (a) + q^2 u'' (c) \nu'' (n) = \frac{V'' (a) \nu'' (n)}{MPC}
\]

and so

\[
A^{-1} = \frac{MPC}{V'' (a) \nu'' (n)} \begin{bmatrix}
\omega V'' (a) & -\nu'' (n) q & \nu'' (n) V'' (a) \\
q^2 u'' (c) + V'' (a) & -wqu'' (c) & wu'' (c) V'' (a) \\
qwu'' (c) & w^2 u'' (c) - \nu'' (n) & qu'' (c) \nu'' (n)
\end{bmatrix}
\]

therefore, the first row of (A.27)

\[
\begin{bmatrix}
\frac{\partial c}{\partial q} & \frac{\partial c}{\partial z} & \frac{\partial c}{\partial n} & \frac{\partial c}{\partial w}
\end{bmatrix} = MPC \begin{bmatrix}
-w & q \\
-\frac{\nu''(n)}{V''(a)} & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & -u'(c) & 0 \\
-u'(c) & 0 & 0 & 0 \\
(a - b) & -1 & -n & -q
\end{bmatrix}
\]

(A.28)

Using (A.26) we find

\[
-q \frac{u'(c)}{V'' (a)} MPS = \frac{\sigma c}{q} \frac{q^2 u'' (c) M}{MPC} = \frac{\sigma c}{q} MPS
\]

so that the first column of the matrix equation (A.28) reads

\[
\frac{\partial c}{\partial q} = \frac{\sigma c}{q} MPS - (a - b) \frac{\sigma c}{q} MPS
\]
The second and fourth column of \((A.28)\) yield directly

\[
\frac{\partial c}{\partial z} = \text{MPC} \\
\frac{\partial c}{\partial b} = q\text{MPC}
\]

Finally, using \((A.23)\) we have

\[
\frac{w}{v''(n)} u'(c) = \frac{v'(n)}{v''(n)} = \psi n
\]

so that the third column of \((A.28)\) reads

\[
\frac{\partial c}{\partial w} = \text{MPC}\psi n + \text{MPC}n \\
= \text{MPC} (1 + \psi) n
\]

The first-order total differential \(dc\) is then

\[
dc = \frac{\partial c}{\partial z} dz + \frac{\partial c}{\partial b} db + \frac{\partial c}{\partial q} dq + \frac{\partial c}{\partial w} dw
\]

\[
= \text{MPC} \left( dz + qdb - (a - b) dq + (1 + \psi) ndw \right) + \sigma c\text{MPS} \frac{dq}{q}
\]

as claimed. Similarly, after using \(\text{MPN} = \text{MPC} w \frac{u''(c)}{v''(n)}\), the second row of \((A.27)\) is

\[
\begin{bmatrix}
\frac{\partial n}{\partial q} & \frac{\partial n}{\partial z} & \frac{\partial n}{\partial w} & \frac{\partial n}{\partial b}
\end{bmatrix}
= \text{MPN} \begin{bmatrix}
-\frac{q^2 + V''(a) u''(c)}{w V''(a)} & \frac{q}{V''(a)} & 0 & 0 & -u'(c) & 0 \\
b' & 0 & 0 & 0 & -n & -q
\end{bmatrix}
\]

Using \((A.26)\) we find

\[
-q \frac{u'(c)}{V''(a)} \text{MPN} = \frac{\sigma c}{q} \frac{q^2 u''(c)}{V''(a)} \text{MPC} \left( \frac{-n\psi}{\sigma c} \right) = -\frac{n\psi}{q} \text{MPS}
\]

Again the first column yields

\[
\frac{\partial c}{\partial q} = -\frac{n\psi}{q} \text{MPS} - (a - b) \text{MPN}
\]

The second and fourth column of \((A.28)\) yield directly

\[
\frac{\partial c}{\partial z} = \text{MPN} \\
\frac{\partial c}{\partial b} = q\text{MPN}
\]
Finally, since

\[
\left( q^2 + \frac{V''(a)}{u''(c)} \right) \frac{u'(c)}{V''(a)} = -\sigma c \left( q^2 \frac{u''(c)}{V''(a)} + 1 \right)
\]

the third column yields

\[
\frac{\partial n}{\partial w} = \frac{1}{w} \psi n (MPS + MPC) + MPN n = \frac{1}{w} \psi n (1 + wMPN) + MPN n = \psi n \frac{1}{w} + MPN (n + \psi n)
\]

The first-order total differential \( dn \) is then

\[
dn = \frac{\partial n}{\partial z} dz + \frac{\partial n}{\partial b} db + \frac{\partial n}{\partial q} dq + \frac{\partial n}{\partial w} dw
\]

\[
= MPN \left( dz + qdb - (a - b) dq + (1 + \psi) ndw \right) - \psi n MPS \frac{dq}{q} + \psi n \frac{dw}{w}
\]

\[\text{(A.31)}\]

**Proof of theorem 2 in case 2a).** If the policy functions are differentiable and the consumer is at an interior optimum, then the conditions of lemma A.1 are satisfied: the borrowing constraint is not binding so can be ignored, and the value function is concave per standard dynamic programming arguments. The notation of theorem 2 can be cast using that of the lemma by using the mapping

\[
q \equiv Q \quad z \equiv y + \frac{\lambda}{\Pi} \quad a \equiv \lambda' \quad b \equiv \delta_N \frac{\lambda}{\Pi}
\]

with \( \frac{dP}{\Pi} = \frac{dQ}{\Pi} = -\frac{dR}{\Pi} \). Hence \( dz = dy - \frac{\lambda}{\Pi} \frac{dP}{\Pi}, db = -\delta_N \frac{\lambda}{\Pi} \frac{dP}{\Pi} \) and \( \frac{dq}{q} = -\frac{dR}{\Pi}; \) so

\[
dz + qdb - (a - b) dq = dy - (1 + Q\delta_N) \frac{\lambda}{\Pi} \frac{dP}{\Pi} + \left( \lambda' - \delta_N \frac{\lambda}{\Pi} \right) \frac{dR}{\Pi}
\]

Inserting this equation into (A.29) and (A.31) yields the desired result. \(\square\)

**Case 2b) \(N > 0\), no bonds, only \(R\) changing.** Since we are not considering changes in wages, it is sufficient to restrict the analysis to a choice between consumption and labor supply. The following lemma then proves the result for \( dc \). The result for \( dn \) follows as a straightforward extension.

**Lemma A.2.** Let \( c(\theta, Y, R) \) be the solution to the following consumer choice problem under concave

A15
preferences over current consumption \( u(c) \) and assets \( W(\theta') \)

\[
\max_{c,\theta'} \quad u(c) + W(\theta') \\
\text{s.t.} \quad c + (\theta' - \theta) S = Y + \theta d
\]

where \( \frac{dS}{dR} = -\frac{S}{R} \). Then, to first order

\[
dc = MPC \left( dY + URE \frac{dR}{R} \right) - \sigma(c) c (1 - MPC) \frac{dR}{R}
\]

where \( \sigma(c) \equiv -\frac{u'(c)}{cu''(c)} \) is the local elasticity of intertemporal substitution, \( MPC = \frac{\partial c}{\partial Y} \), and \( URE = Y + \theta d - c \)

Proof. The following first-order conditions characterize the solution

\[
S_i^t u' \left( Y + \theta d - (\theta' - \theta) S \right) = W_{\theta_i} (\theta') \quad \forall i = 1 \ldots N \tag{A.32}
\]

Consider first an increase in income \( dY \) alone. Differentiating along (A.32) we find

\[
S_i^t u''(c) \left( 1 - \sum_j S_j^t d\theta'_j \right) = \sum_j W_{\theta_i \theta_j} (\theta') \frac{d\theta'_j}{dY} \quad \forall i \tag{A.33}
\]

Define \( \eta^i \equiv S_i^t \frac{d\theta'_i}{dY} \). Then (A.33) rewrites

\[
\sum_j \left( \frac{1}{S_j^t S_j^t} W_{\theta_i \theta_j} (\theta') + u''(c) \right) \eta^j = u''(c) \quad \forall i
\]

Defining the matrix \( M \) with elements

\[
m_{ij} \equiv \frac{1}{S_i^t S_j^t} W_{\theta_i \theta_j} (\theta') + u''(c)
\]

this system can also be written in matrix form as

\[
M \eta = u''(c) \mathbf{1}
\]

or

\[
\eta = u''(c) M^{-1} \mathbf{1}
\]

The budget constraint then implies that

\[
MPC = \frac{dc}{dY} = 1 - \sum_j \eta^j = 1 - u''(c) m \tag{A.34}
\]
where \( m \) is defined as
\[
   m \equiv 1M^{-1}1 \quad (A.35)
\]

Next, consider an increase in the real interest rate \( dR \). Differentiating along (A.32) we now have
\[
   \frac{dS_i}{dR} u'(c) + S_i u''(c) \left( - \sum_j S_j \frac{d\theta^j}{dR} - \sum_j S_j \left( \theta^j - \theta^i \right) \right) = \sum_j W_{\theta^{j|i}} \left( \theta^i \right) \frac{d\theta^j}{dR} \quad \forall i
\]

Using \( \frac{dS_i}{dR} = -\frac{dR}{R} \), this rewrites
\[
   -\frac{S_i}{R} u'(c) + S_i u''(c) \left( - \sum_j S_j \frac{d\theta^j}{dR} + \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \right) = \sum_j W_{\theta^{j|i}} \left( \theta^i \right) \frac{d\theta^j}{dR} \quad \forall i \quad (A.36)
\]

Defining now \( \gamma^j \equiv \frac{S_j}{R} d\theta^j \), (A.36) shows that \( \gamma^j \) solves
\[
   \sum_j m_{ij} \gamma^j = -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \quad \forall i
\]

which rewrites in matrix form
\[
   M \gamma = \left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \right) 1
\]

or
\[
   \gamma = \left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \right) M^{-1}1 \quad (A.37)
\]

Differentiating with respect to \( R \) along the budget constraint \( c = Y + \theta d - \left( \theta' - \theta \right) S \), we next see that
\[
   \frac{dc}{dR} = - \sum_j S_j \frac{d\theta^j}{dR} + \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) = - \sum_j \gamma^j + \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right)
\]

inserting (A.37) and using the definition of \( m \),
\[
   \frac{dc}{dR} = - \left( -\frac{1}{R} u'(c) + u''(c) \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \right) m + \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \quad (A.38)
\]

rearranging terms and using \( u'(c) \equiv -c\sigma(c) u''(c) \) we find
\[
   \frac{dc}{dR} = -c\sigma(c) \frac{c}{R} u''(c) m + \sum_j \frac{S_j}{R} \left( \theta^j - \theta^i \right) \left( 1 - u''(c) m \right)
\]
But using the expression for $MPC$ in (A.34), this is simply

\[
\frac{dc}{dR} = -\sigma(c) \frac{c}{R} (1 - MPC) + \sum_j \frac{S_j^i}{R} \left( \theta'^i - \theta^i \right) \text{MPC}
\]

and using the budget constraint $\sum_j S_j^i \left( \theta'^i - \theta^i \right) = (\theta'^i - \theta^i) \cdot S_i = URE$ we obtain

\[
\frac{dc}{dR} = -\sigma(c) \frac{c}{R} (1 - MPC) + \frac{1}{R} URE \cdot \text{MPC}
\]

(A.39)

Finally, considering a simultaneous change in income and the real interest rate, combining (A.34) and (A.39) we obtain the first order differential

\[
dc = MPC \left( dY + URE \frac{dR}{R} \right) - \sigma(c) c (1 - MPC) \frac{dR}{R}
\]

as was to be shown. \hfill \Box

A.3 Proof of theorem 3

Given the assumption of fixed balance sheets and purely transitory shocks, Theorem 2 shows that

\[
dc_i = \hat{MPC}_i \left( dY_i + URE_i \frac{dR}{R} - NNP_i \frac{dP}{P} \right) - \sigma_i c_i (1 - \hat{MPC}_i) \frac{dR}{R}
\]

where, in the notation of the section, the instantaneous change in income inclusive of the labor supply response, given no insurance and a fixed aggregate dividend, is

\[
dY_i = dy_i + n_i dw_i + w_i dn_i + d\pi_i + dt_i
\]

which we can decompose as

\[
dY_i = \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY
\]

and note that, since $E_I[Y_i] = Y$,

\[
E_I \left[ dY_i - \frac{Y_i}{Y} dY \right] = dY - \frac{E_I[Y_i]}{Y} dY = 0
\]

(A.40)

Hence,

\[
dc_i = \hat{MPC}_i \left( \frac{Y_i}{Y} dY + dY_i - \frac{Y_i}{Y} dY + URE_i \frac{dR}{R} - NNP_i \frac{dP}{P} \right) - \sigma_i c_i (1 - \hat{MPC}_i) \frac{dR}{R}
\]
and taking a cross-sectional average
\[
dC = \mathbb{E}_I \left[ \frac{Y_i}{Y} \hat{MPC}_i \right] dY + \mathbb{E}_I \left[ \hat{MPC}_i \left( dY_i - \frac{Y_i}{Y} dY \right) \right] - \mathbb{E}_I \left[ \hat{MPC}_i NNP_i \right] \frac{dP}{P} \\
+ \left( \mathbb{E}_I \left[ \hat{MPC}_i \hat{URE}_i \right] - \mathbb{E}_I \left[ \sigma_i (1 - \hat{MPC}_i) c_i \right] \right) \frac{dR}{R}
\]

and using the market clearing conditions (11), (12), together with (A.40) we find
\[
dC = \mathbb{E}_I \left[ \frac{Y_i}{Y} \hat{MPC}_i \right] dY + \text{Cov}_I \left( \hat{MPC}_i, dY_i - \frac{Y_i}{Y} dY \right) - \text{Cov}_I \left( \hat{MPC}_i, NNP_i \right) \frac{dP}{P} \\
+ \left( \text{Cov}_I \left( \hat{MPC}_i, \hat{URE}_i \right) - \mathbb{E}_I \left[ \sigma_i (1 - \hat{MPC}_i) c_i \right] \right) \frac{dR}{R}
\]
as claimed. Note that we can also rewrite this expression in elasticity terms by dividing by per-capita consumption \( C = Y = \mathbb{E}_I [c_i] \), and noting that \( \frac{1}{Y} \left( dY_i - \frac{Y_i}{Y} dY \right) = \frac{dY_i}{Y} - Y_i \frac{dY}{Y} = d \left( \frac{Y_i}{Y} \right) \).

We then find
\[
\frac{dC}{C} = \mathbb{E}_I \left[ \frac{Y_i}{Y} \hat{MPC}_i \right] \frac{dY}{Y} + \text{Cov}_I \left( \hat{MPC}_i, d \left( \frac{Y_i}{Y} \right) \right) - \text{Cov}_I \left( \hat{MPC}_i, \frac{NNP_i}{\mathbb{E}_I [c_i]} \right) \frac{dP}{P} \\
+ \left( \text{Cov}_I \left( \hat{MPC}_i, \hat{URE}_i \right) - \mathbb{E}_I \left[ \sigma_i (1 - \hat{MPC}_i) \frac{c_i}{\mathbb{E}_I [c_i]} \right] \right) \frac{dR}{R}
\]

**A.4 Extension to GHH preferences**

I first derive the consumer theory results for GHH preferences. This gives an equivalent to theorem 1 for this class of preferences. I then derive the equivalent of theorem 2 for such preferences. I adopt the parametric utility specification:

\[
U \left( \{c_t, n_t\} \right) = \sum_t \beta^t u \left( c_t - v \left( n_t \right) \right) \\
u \left( c \right) = c^{1 - 1 \over 1 - \sigma} \\
v \left( n \right) = b n^{1 + \psi \over 1 + \psi}
\]

**Theorem A.1.** To first order, under GHH preferences

\[
dc = \text{MPC} d\Omega + \phi n dw - \sigma \left( 1 - \left( \frac{\psi}{\psi + 1} \right) \frac{wn}{c} \right) c\text{MPS} \frac{dR}{R} \tag{A.41}
\]
\[
dn = \phi n \frac{dw}{w} \tag{A.42}
\]
\[
dU = u' \left( c - v \left( n \right) \right) d\Omega \tag{A.43}
\]
where $d\Omega = dW - \sum_{t\geq 0} c_t dq_t$, the net-of-consumption wealth change, is given by

$$d\Omega = dy + ndw + URE \frac{dR}{R} - NNP \frac{dP}{P}$$  \hspace{1cm} (A.44)

The following sections establish that

$$MPN = 0$$  \hspace{1cm} (A.45)

$$\epsilon^h_{c_0,q_0} = -\sigma \left(1 - MPC\right) \left(1 - \left(\frac{\psi}{\psi + 1}\right) \frac{w_0 n_0}{c_0}\right)$$  \hspace{1cm} (A.46)

$$\epsilon^h_{c_0,w_0} = \psi \frac{w_0 n_0}{c_0}$$  \hspace{1cm} (A.47)

$$\epsilon^h_{n_0,q_0} = 0$$  \hspace{1cm} (A.48)

$$\epsilon^h_{n_0,w_0} = \psi$$  \hspace{1cm} (A.49)

The proof then proceeds exactly as in section A.1.

**Marginal propensities.** The problem’s first-order conditions are

$$c_t - v(n_t) = (c_0 - v(n_0)) \left[ \beta t \left(\frac{q_t}{q_0}\right)\right]^\sigma \quad n_t = \left(\frac{w_t}{b}\right)^\psi$$  \hspace{1cm} (A.50)

so that

$$v(n_t) = \frac{1}{b\psi} t^{\psi + 1} \quad w_t n_t - v(n_t) = \frac{1}{b\psi} t^{\psi + 1}$$

Inserting into the budget constraint

$$\sum_{t\geq 0} q_t \left(c_t - w_t n_t\right) = W$$

we obtain

$$\left(c_0 - \frac{1}{b\psi} \frac{\psi}{1 + \psi} w_0^{\psi + 1}\right) \sum_{t} \beta^t \left(\frac{q_t}{q_0}\right)^{1-\sigma} - \frac{1}{b\psi} \frac{1}{1 + \psi} \sum_{t\geq 0} q_t w_t^{\psi + 1} = W$$  \hspace{1cm} (A.51)

Leading to the explicit consumption function

$$c_0 = \frac{1}{b\psi} \frac{\psi}{1 + \psi} w_0^{\psi + 1} + \left(\sum_{t} \beta^t \left(\frac{q_t}{q_0}\right)^{1-\sigma}\right)^{-1} \left(W + \frac{1}{b\psi} \frac{1}{1 + \psi} \sum_{t\geq 0} q_t w_t^{\psi + 1}\right)$$
Marginal propensities follow:

\[
MPC = \left( \sum_t \beta_t^{ct} \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right)^{-1} \\
MPN = 0
\]

implying in particular (A.45).

Hicksian elasticities. The solution to

\[
e (\{q_t\}, \{w_t\}, U) = \min \left\{ \sum_t q_t (c_t - w_t n_t) \text{ s.t. } U (\{c_t, n_t\}) \geq u \right\}
\]

involves the same first-order conditions (A.50). So \(c_0\) is the solution to

\[
\left( \sum_t \beta_t^{ct} \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right) u \left( c_0 - \frac{1}{b\psi} \frac{\psi}{1 + \psi} w_0^{\psi+1} \right) = U \tag{A.52}
\]

From \(n_0 = \left( \frac{w_0}{b} \right)^\psi\) we immediately have

\[
\epsilon_{n_0,w_0}^h = \psi \\
\epsilon_{n_0,w_s}^h = 0 \quad s \geq 1 \\
\epsilon_{n_0,q_s}^h = 0 \quad s \geq 0
\]

hence (A.48) and (A.49). Meanwhile, implicit differentiation of (A.52) leads to

\[
\epsilon_{c_0,w_0}^h = \frac{w_0}{c_0} \frac{dc_0}{dw_0} = \frac{1}{b\psi} \frac{\psi}{w_0^{\psi+1}} = (\psi + 1) \frac{v (n_0)}{c_0} \\
\epsilon_{c_0,w_s}^h = 0 \quad s \geq 1 
\]

(A.47) follows by noticing that, from the first order condition \(v' (n_0) = w_0\), we have

\[
\frac{v (n_0)}{c_0} = \left( \frac{\psi}{\psi + 1} \right) \frac{w_0 n_0}{c_0} \tag{A.53}
\]

Moreover, for \(s \geq 1\)

\[
\left( \sum_t \beta_t^{ct} \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right) u' (c_0 - v (n_0)) \frac{dc_0}{dq_s} + u (c_0 - v (n_0)) \frac{1-\sigma}{q_s} \left( \beta^{qs} \left( \frac{q_s}{q_0} \right)^{1-\sigma} \right) = 0
\]
so that
\[
\frac{dc_0}{dq_s} = \left( \sum_t \beta^t \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right)^{-1} \frac{u(c_0 - v(n_0))}{u'(c_0 - v(n_0))} \left( \frac{\beta^s}{q_s} \left( \frac{q_s}{q_0} \right)^{1-\sigma} \right)
\]

using
\[
u (c_0 - v(n_0)) = \frac{\sigma}{\sigma - 1} u'(c_0 - v(n_0)) (c_0 - v(n_0))
\]
we obtain
\[
\epsilon^h_{c_0,q_s} = \frac{q_s}{c_0} \frac{dc_0}{dq_s} = \sigma \text{MPC} \left( \beta^s \left( \frac{q_s}{q_0} \right)^{1-\sigma} \right) \left( 1 - \frac{v(n_0)}{c_0} \right)
\]

While for \(s = 0\),
\[
\left( \sum_{t \geq 0} \beta^t \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right) u'(c_0 - v(n_0)) \frac{dc_0}{dq_0} + u(c_0 - v(n_0)) \frac{(\sigma - 1)}{q_0} \left( \sum_{t \geq 1} \beta^t \left( \frac{q_t}{q_0} \right)^{1-\sigma} \right) = 0
\]
therefore
\[
\epsilon^h_{c_0,q_0} = -\sigma \text{MPC} \left( \frac{1}{\text{MPC}} - 1 \right) \left( 1 - \frac{v(n_0)}{c_0} \right)
\]
\[
= -\sigma (1 - \text{MPC}) \left( 1 - \frac{v(n_0)}{c_0} \right)
\]

which, combined with (A.53), proves (A.46). Evaluating these elasticities in the case of an infinite horizon model where \(\frac{q_s}{q_0} = \beta^s\) and \(\frac{w_s}{w_0} = 1\), and defining \(\xi = \left( \frac{1 - \frac{v(n^*)}{c^*}}{c^*} \right) = \left( 1 - \frac{m^* n^*}{c^*} \right) \left( \frac{\psi}{1 + \psi} \right) \in (0,1)\), we obtain:

<table>
<thead>
<tr>
<th>Table A.2: Steady-state moments, GHH preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon^h)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>(c_0)</td>
</tr>
<tr>
<td>(n_0)</td>
</tr>
</tbody>
</table>

### Incomplete markets.

Consider the same setup as in section 2.3, except that the utility function is replaced by
\[
\mathbb{E} \left[ \sum_t \beta^t u(c_t - v(n_t)) \right] = \frac{c^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \quad u(c) = \frac{c^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \quad v(n) = b \frac{n^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \quad (A.54)
\]
Consider the same change in initial conditions as in section 2.3. Denote by $\xi$ the share of net consumption in gross consumption at date 0:

$$\xi = 1 - \frac{v(n)}{c} = 1 - \left( \frac{\psi}{\psi + 1} \right) \frac{wn}{c}$$

**Theorem A.2.** Assume that the consumer is at an interior optimum, or is at a binding borrowing constraint with MPS=0. Then his first order changes in consumption $dc$ and labor supply $dn$ are given by

$$dc = MPC \left( dy + n (1 + \psi) dw + URE dr - NNP \frac{dP}{P} \right) + (1 - MPC) \psi ndw - \sigma c\xi (1 - MPC) dr$$

$$dn = \psi n dw$$

Instead of contributing equally to the increase in consumption $dc$ as in theorem 2, the two components of the increase in earnings induced by higher wages no longer have a symmetric role. The non-behavioral part is treated by the consumer as unearned income and contributes to an increase in consumption through the MPC. The behavioral part ($wdn = \psi ndw$), however, now has a one-for-one effect on consumption, due to the particular form of complementarity between hours and consumption assumed here.

Again, the proof of theorem A.2 follows from a lemma.

**Lemma A.3.** Let $c(z,w,q,b)$ and $n(z,w,q,b)$ be the solution to the following separable consumer choice problem under concave preferences over current consumption $u(c)$ and assets $V(a)$, and convex preferences over hours worked $v(n)$:

$$\max \quad u(c - v(n)) + V(a)$$
$$\text{s.t.} \quad c + q (a - b) = wn + z$$

then

$$dc = MPC (dz + ndw - (a - b) dq + qdb) - \sigma c\xi (1 - MPC) \frac{dq}{q} + n\psi dw$$

(A.55)

$$dn = \psi n dw$$

where $\sigma \equiv -\frac{u'(g)}{g u'(g)}$ and $\psi = \frac{v'(n)}{nv'(n)}$ are the local elasticities of intertemporal substitution in net consumption and labor supply, respectively, $MPC = \frac{dc}{dz}$ and $\xi = 1 - \frac{v(n)}{c}$.

**Proof.** Define net consumption as $g = c - v(n)$. The following first-order conditions are neces-
sary and sufficient for optimality:

\[ u'(g) = \frac{1}{q} V'(a) \quad \text{and} \quad v'(n) = w \]  

(A.56)

As in the proof of lemma A.1, first obtain the expression for MPC by considering an increase in income \( dz \) alone. Since the real wage is unchanged, \( dn = 0, dc = dg \) and that increase is divided between current consumption and assets alone. (A.23) and the budget constraint implies

\[ u''(g) dc = \frac{1}{q} V''(a) da \]  

(A.57)

\[ dc + qda = dz \]  

(A.58)

so

\[ MPC = \frac{1}{1 + q^2 \frac{u''(g)}{V''(a)}} \]

Consider now the overall effect on \( c, n \) and \( a \) of a change in \( q, w, z \) and \( b \). Differentiation of \( v'(n) = w \) yields

\[ dn = \frac{1}{v''(n)} dw = \frac{v'(n)}{v''(n)} \frac{dw}{w} = \psi_n \frac{dw}{w} \]

net income, \( y^{net} = wn - v(n) + z \) changes by

\[ dy^{net} = ndw + (w - v'(n)) dn + dz = ndw + dz \]

Implicit differentiation of

\[
\begin{cases}
V'(a) - qu'(g) = 0 \\
g + q(a - b) - y^{net} = 0
\end{cases}
\]

results in the following expression for partial derivatives:

\[
\begin{bmatrix}
\frac{\partial \xi}{\partial q} & \frac{\partial \xi}{\partial y^{net}} & \frac{\partial \xi}{\partial b} \\
\frac{\partial \xi}{\partial q} & \frac{\partial \xi}{\partial y^{net}} & \frac{\partial \xi}{\partial b}
\end{bmatrix}
= - \begin{bmatrix}
qu''(g) & V''(a) \\
1 & q
\end{bmatrix}^{-1} \begin{bmatrix}
-u'(g) & 0 & 0 \\
a - b & -1 & -q
\end{bmatrix}
= \frac{MPC}{V''(a)} \begin{bmatrix}
q & -V''(a) \\
-1 & -qu''(g)
\end{bmatrix} \begin{bmatrix}
-u'(g) & 0 & 0 \\
a - b & -1 & -q
\end{bmatrix}
\]

so we obtain

\[
\begin{bmatrix}
\frac{\partial \xi}{\partial q} & \frac{\partial \xi}{\partial y^{net}} & \frac{\partial \xi}{\partial b}
\end{bmatrix}
= \frac{MPC}{V''(a)} \begin{bmatrix}
qu'(g) - V''(a)(a - b) & V''(a) & qV''(a)
\end{bmatrix}
\]
using

\[-q \frac{u'(g)}{V''(a)}\text{MPC} = \frac{g\sigma}{q} \frac{q^2 u''(g)}{V''(a)}\text{MPC} = \frac{g\sigma}{q} \left( \frac{q^2 u''(g)}{1 + q^2 u''(g)} \right) = \frac{g\sigma}{q} (1 - \text{MPC})\]

we obtain

\[
\begin{bmatrix}
\frac{\partial g}{\partial q} & \frac{\partial g}{\partial y_{net}} & \frac{\partial g}{\partial b}
\end{bmatrix} = \begin{bmatrix}
\frac{g\sigma}{q} (1 - \text{MPC}) - \text{MPC} (a - b) & \text{MPC} & q\text{MPC}
\end{bmatrix}
\]

Hence

\[
dg \simeq \frac{\partial g}{\partial y_{net}} dy_{net} + \frac{\partial g}{\partial b} db + \frac{\partial g}{\partial q} dq
\]

\[
= \text{MPC} \left( dy_{net} + dq \right) - (a - b) dq + g\sigma (g) \left(1 - \text{MPC}\right) \frac{dq}{q}
\]

Moreover,

\[
dc = dg + v'(n) dn = dg + \frac{v'(n)}{v''(n)} dw = dg + \psi ndw
\]

So

\[
dc \simeq \text{MPC} \left( dz + ndw + dq \right) - (a - b) dq + g\sigma (g) \left(1 - \text{MPC}\right) \frac{dq}{q} + n\psi dw
\]

using \(g = c \left(1 - \frac{v(n)}{c}\right) = c\xi\) we finally obtain

\[
dc \simeq \text{MPC} \left( dz + ndw + dq \right) - (a - b) dq + c\sigma\xi \left(1 - \text{MPC}\right) \frac{dq}{q} + n\psi dw
\]

\[
\begin{array}{c}
\text{Proof of theorem A.2.} \\
\text{The notation of theorem A.2 can be cast in that of lemma A.3 by using the}
\end{array}
\]

\[
\begin{array}{c}
\text{mapping} \\
q \equiv Q \\
z \equiv y + \frac{\lambda}{\Pi} \\
a \equiv \lambda' \\
b \equiv \delta_N \frac{\lambda}{\Pi}
\end{array}
\]

A25
with \( \frac{dP}{P} = \frac{d\Pi}{\Pi} \) and \( \frac{dQ}{Q} = -\frac{dR}{R} \). Hence \( dz = dy - \frac{\lambda}{\Pi} \frac{dP}{P}, db = -\delta_N \frac{\lambda}{\Pi} \frac{dP}{P} \) and \( \frac{dq}{q} = -\frac{dR}{R} \), so

\[
dz + n \left(1 + \psi(n)\right) dw + qdb - (a - b) dq = dy + n \left(1 + \psi(n)\right) dw - \left(1 + Q\delta_N\right) \frac{\lambda}{\Pi} \frac{dP}{P} + \left(\lambda' - \delta_N \frac{\lambda}{\Pi}\right) Q \frac{dR}{R}
\]

Inserting this equation into (A.55) yields the desired result. \(\square\)

**Aggregate consumption response with GHH preferences.**

The following version of theorem 3 obtains with GHH preferences.

**Theorem A.3.** Under GHH preferences, to first order, the changes \( dC = dY, dY_i, dw_i, dP \) and \( dR \) are linked by

\[
dC = \underbrace{\mathbb{E}_I \left[ \frac{Y_i}{Y} \hat{MC}_i \right]}_{\text{Aggregate income channel}} dY + \underbrace{\text{Cov}_I \left( \hat{MC}_i, dY_i - \frac{dY}{Y} \right)}_{\text{Earnings heterogeneity channel}} - \underbrace{\text{Cov}_I \left( \hat{MC}_i, NNP_i \right)}_{\text{Fisher channel}} \frac{dP}{P} \\
+ \underbrace{\text{Cov}_I \left( \hat{MC}_i, URE_i \right)}_{\text{Interest rate exposure channel}} - \underbrace{\mathbb{E}_I \left[ \sigma_i \xi_i \left(1 - \hat{MC}_i\right) c_i \right]}_{\text{Substitution channel}} \frac{dR}{R} + \underbrace{\mathbb{E}_I \left[ \psi_i \left(1 - MP_i\right) n_i dw_i \right]}_{\text{Complementarity channel}}
\]

(A.59)

**Proof.** Same as that of theorem 3, aggregating up from theorem A.2 instead of theorem 2. \(\square\)

### B  Data appendix

#### B.1 Additional details from the 2010 Survey of Household Income and Wealth

Tables B.1 presents summary statistics with more information on the distributions than those available in table 1. A striking feature of the Italian data is that fewer than 10% of households report to own mortgages. The share of adjustable mortgages is around 50%, which is consistent with official sources (Eurosystem 2009).
Table B.1: Summary statistics from the Italian SHIW 2010

<table>
<thead>
<tr>
<th>Variable</th>
<th>count</th>
<th>mean</th>
<th>p5</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>p95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income from all sources ($Y_i$, €/year)</td>
<td>7,951</td>
<td>36,114</td>
<td>9,565</td>
<td>19,857</td>
<td>30,719</td>
<td>45,340</td>
<td>81,320</td>
</tr>
<tr>
<td>Consumption incl. mortgage payments ($C_i$, €/year)</td>
<td>7,951</td>
<td>27,976</td>
<td>10,700</td>
<td>17,060</td>
<td>24,000</td>
<td>33,600</td>
<td>57,600</td>
</tr>
<tr>
<td>Deposits and maturing assets ($B_i$, €)</td>
<td>7,951</td>
<td>14,200</td>
<td>0</td>
<td>1,000</td>
<td>5,156</td>
<td>15,054</td>
<td>50,000</td>
</tr>
<tr>
<td>ARM mortgage liabilities and consumer credit ($D_i$, €)</td>
<td>7,951</td>
<td>6,228</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>26,800</td>
<td></td>
</tr>
<tr>
<td>Unhedged interest rate exposure (€/yr)</td>
<td>7,951</td>
<td>16,110</td>
<td>-21,862</td>
<td>1,093</td>
<td>10,974</td>
<td>26,646</td>
<td>71,610</td>
</tr>
<tr>
<td>Unhedged interest rate exposure (€/Q)</td>
<td>7,951</td>
<td>10,007</td>
<td>-17,328</td>
<td>594</td>
<td>6,407</td>
<td>16,871</td>
<td>52,054</td>
</tr>
<tr>
<td>Total fixed-income financial assets (€)</td>
<td>7,951</td>
<td>15,133</td>
<td>0</td>
<td>0</td>
<td>4,285</td>
<td>99,000</td>
<td></td>
</tr>
<tr>
<td>Total financial liabilities (€)</td>
<td>7,951</td>
<td>27,481</td>
<td>0</td>
<td>1,359</td>
<td>7,000</td>
<td>24,064</td>
<td>91,104</td>
</tr>
<tr>
<td>Marginal Propensity to Spend</td>
<td>7,951</td>
<td>47</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

All statistics are computed using survey weights

B.2 Estimation using the CEX

Perhaps the best identified setting to measure MPCs are the large-scale 2001 and 2008 U.S. tax rebates, whose timing of receipt was randomized (Johnson et al. 2006; Parker et al. 2013). Since these studies exploit variation in timing for a policy announced ahead of time, they identify the MPC out of an expected increase in income. This is, in general, different from the theoretically-consistent MPC out of an unexpected increase. In a benchmark incomplete market model, unless borrowing constraints are binding and are not adjusting in response to the expected tax rebate, two consumers that differ only in their timing of receipt should adjust their consumption profile by similar amounts when they receive the news (reflecting the net gain from the present value of the transfer as well as Ricardian offsets) and not react differentially when they receive the transfer. However, to the extent that borrowing constraints are rigid and binding, or if households are surprised by the receipt despite its announcement, the estimation gets closer to the MPC that is important for the theory; and in general provides a lower bound for it.40

I use the data from the 2001 rebates collected in the U.S. Consumer Expenditure Survey (CEX) and analyzed in Johnson et al. (2006) to conduct a measurement of UREs and estimation of redistribution elasticities that provides an alternative to the Italian data used in section 4.2.

B.2.1 Data: Consumer Expenditure Survey, 2001-2002 (JPS sample)

My data for the Consumer Expenditure Survey comes from the Johnson et al. (2006) (JPS) dataset, which I merged with the main survey data to add information on total consumption expenditures, as well as assets and liabilities separately. The dataset covers households with interviews between February 2001 and March 2002. I restrict my sample to households who have income information. This leaves me with 9,443 interviews of 4,583 households.

40See Kaplan and Violante (2014) for another discussion of the interpretation of the coefficients estimated by JPS, and a model in which even households with positive total assets can be at a binding limit on their liquid transactions account, and modify their consumption upon receipt but not upon news of the transfer.
I define maturing assets are the sum of total assets in checking, brokerage and other accounts, savings, S&L, credit unions and other accounts as well as one fourth of the amount in U.S. savings bonds. Because the CEX has poor data on mortgages, I count all of “total amount owed to creditors” from the fifth interview towards \( D_i \) is order to calculate my unhedged interest rate exposure measure. Table B.2 presents summary statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>count</th>
<th>mean</th>
<th>p5</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>p95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income from all sources ($/year)</td>
<td>9,443</td>
<td>45,617</td>
<td>6,700</td>
<td>18,612</td>
<td>36,000</td>
<td>62,828</td>
<td>115,000</td>
</tr>
<tr>
<td>Consumption incl. mortgage payments ($/year)</td>
<td>9,443</td>
<td>36,253</td>
<td>9,544</td>
<td>18,724</td>
<td>28,464</td>
<td>44,114</td>
<td>90,296</td>
</tr>
<tr>
<td>Deposits and maturing assets ($/year)</td>
<td>9,443</td>
<td>7,147</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,100</td>
<td>30,100</td>
</tr>
<tr>
<td>Consumer credit ($)</td>
<td>9,443</td>
<td>2,872</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,000</td>
<td>14,600</td>
</tr>
<tr>
<td>Unhedged interest rate exposure ($/yr)</td>
<td>9,443</td>
<td>13,639</td>
<td>-41,948</td>
<td>-6,000</td>
<td>6,192</td>
<td>27,616</td>
<td>84,126</td>
</tr>
<tr>
<td>Unhedged interest rate exposure ($/Q)</td>
<td>9,443</td>
<td>6,616</td>
<td>-16,514</td>
<td>-2,288</td>
<td>1,377</td>
<td>7,813</td>
<td>35,520</td>
</tr>
<tr>
<td>Net liquid assets ($)</td>
<td>7,202</td>
<td>7,340</td>
<td>0</td>
<td>5</td>
<td>1,329</td>
<td>6,400</td>
<td>38,000</td>
</tr>
</tbody>
</table>

All statistics are computed using survey weights

**B.2.2 The redistribution elasticity from the 2001 tax rebates in the CEX**

In this section I compute the MPC out of the 2001 tax rebate using the Johnson et al. (2006) (JPS) procedure, stratifying by URE. I then use the estimates by bin to form a measure of the redistribution elasticity. To be specific, I split the sample into \( J \) groups ranked by their URE. I then run the main JPS estimating equation

\[
C_{i,m,t+1} - C_{i,m,t} = \alpha_m + \beta X_{i,t} + \sum_{j=1}^{J} MPC_j R_{i,t+1} QURE_{ij} + u_{i,t+1} \tag{B.1}
\]

where \( C_{i,m,t} \) is the level of household \( i \)'s consumption expenditures in month \( m \) and at date \( t \), \( \alpha_m \) are month fixed effects absorbing seasonal variation in expenditures, \( X_{i,t} \) are the controls used by JPS in their main specification (age and changes in family composition), \( R_{i,t+1} \) is the dollar amount of the rebate at \( t + 1 \), and \( QURE_{ij} \) is a dummy indicating that household \( i \)'s URE is in group \( j = 1 \ldots J \). This procedures exploits variation in timing of the rebate across households in the same exposure group to identify the propensity to consume out of the expected one-time transfer that the stimulus payment provides.

In each URE bin, I next calculate the average normalized URE, \( NURE_{jr} \), as the average over households in group \( j \) of \( \frac{URE_j \bar{c}}{c} \), where \( \bar{c} \) is average consumption expenditure in the sample. I finally compute my estimators as

A28
The figure presents the estimated \( MPC \), together with 95% confidence intervals, in each \( URE \) bin.

**Figure B.1:** \( MPC \) estimated in \( URE \) bins (JPS procedure, food consumption)

\[
\hat{\varepsilon}^{NR}_r = \frac{1}{J} \sum_{j=1}^{J} MPC_j NURE_j
\]

\[
\hat{\varepsilon}_r = \hat{\varepsilon}^{NR}_r - \left( \frac{1}{J} \sum_{j=1}^{J} MPC_j \right) \left( \frac{1}{J} \sum_{j=1}^{J} NURE_j \right)
\]

\[
\hat{S} = 1 - \left( \frac{1}{J} \sum_{j=1}^{J} MPC_j \right)
\]

where \( MPC_j \) is the point estimate in group \( j \) from (B.1). In order to take into account sampling uncertainty, I compute the distribution of these estimators using a Monte-Carlo procedure, resampling the panel at the household level with replacement.

Figure B.1 illustrates the procedure for \( J = 3 \), using expenditures on food as the headline consumption estimate. There is a clear gradient in \( MPC \), with households with lower (and on average negative) \( URE \) displaying a much higher marginal propensity to consume, confirming my claim that \( \varepsilon_r < 0 \).

Table B.3 repeats the exercise of table 2, where this time the moment estimation is done at the group and not the individual level. The quantitative results are large. Using food consumption as the source of \( MPC \) estimation in (B.1), \( \sigma_r \) is estimated to be 0.3, which is well within the range of typical values for the EIS. Using all nondurable consumption instead, \( \sigma_r \) becomes as

---

Note that I simply take \( \hat{S} \) to be the sample counterpart to \( 1 - E_I [MPC] \). The procedure cannot simultaneously give an estimate of the covariance between \( MPC \) and consumption. In the SHIW data, the difference between average \( MPC \) and consumption-weighted \( MPC \) is small, so this is unlikely to significantly affect the value of \( \sigma_r \).
Table B.3: Moments of the redistribution channel computed using the JPS procedure

<table>
<thead>
<tr>
<th>Consumption measure</th>
<th>Food</th>
<th>All nondurable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Estimate</td>
<td>95% C.I.</td>
</tr>
<tr>
<td>No-rebate elasticity ( \hat{E}_{NR} )</td>
<td>-0.12 [-0.27; 0.02]</td>
<td>-0.33 [-0.65; -0.02]</td>
</tr>
<tr>
<td>Redistribution elasticity ( \hat{E}_r )</td>
<td>-0.24 [-0.42; -0.07]</td>
<td>-0.64 [-0.97; -0.32]</td>
</tr>
<tr>
<td>Scaling factor ( \hat{S} )</td>
<td>0.82 [0.69; 0.95]</td>
<td>0.56 [0.33; 0.78]</td>
</tr>
<tr>
<td>Equivalent EIS ( \hat{\sigma}_r = -\frac{\hat{E}_r \hat{S}}{\hat{S}} )</td>
<td>0.30 [0.05; 0.54]</td>
<td>1.15 [0.24; 2.07]</td>
</tr>
</tbody>
</table>

Confidence intervals are bootstrapped by resampling households 100 times with replacement.

Confidence intervals are bootstrapped by resampling households 100 times with replacement.

high as 1.15, suggesting that redistribution may even play a dominant role in the transmission of shocks to real interest rates. Even the no-rebate elasticity \( \hat{E}_{NR} \) is negative, so that the negative correlation \( \hat{E}_r \) is strong enough to overwhelm the effect of a positive aggregate URE.

Tables B.4 and B.5 also shows these results to be robust to using medians instead of means within URE bins, and to using any different number of bins \( J \) for 2 to 10. In every case, the correlation is very negative and the standard deviation of MPC is substantial. Even though the standard deviation of URE is only 1.27 in this dataset (table 1), these numbers combine with those of section 4.2 to suggest that \( \hat{\sigma}_r \) is plausibly large in magnitude, and give the redistribution channel of monetary policy quantitative support.

Table B.4: Estimates from table B.3 using median instead of mean URE per bin

<table>
<thead>
<tr>
<th>Consumption measure</th>
<th>Food</th>
<th>All nondurable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{E}^{PE}_r )</td>
<td>-0.065 [-0.14; 0.01]</td>
<td>-0.17 [-0.33 -0.02]</td>
</tr>
<tr>
<td>( \hat{E}_r )</td>
<td>-0.123 [-0.21; -0.04]</td>
<td>-0.32 [-0.48; -0.16]</td>
</tr>
<tr>
<td>( \hat{S} )</td>
<td>0.82 [0.69; 0.95]</td>
<td>0.56 [0.33; 0.78]</td>
</tr>
<tr>
<td>( \hat{\sigma}_r = -\frac{\hat{E}_r \hat{S}}{\hat{S}} )</td>
<td>0.149 [0.03; 0.27]</td>
<td>0.58 [0.12; 1.04]</td>
</tr>
</tbody>
</table>

Confidence intervals are bootstrapped by resampling households 100 times with replacement.

Table B.5: Estimated \( \hat{\sigma}^* \), using \( J \) bins of URE (consumption measure: food expenditures)

<table>
<thead>
<tr>
<th>( J )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\sigma}_r )</td>
<td>0.25</td>
<td>0.30</td>
<td>0.29</td>
<td>0.28</td>
<td>0.35</td>
<td>0.31</td>
</tr>
<tr>
<td>95% C.I.</td>
<td>[0.03; 0.47]</td>
<td>[0.05; 0.55]</td>
<td>[0.05; 0.52]</td>
<td>[0.04; 0.53]</td>
<td>[0.08; 0.62]</td>
<td>[0.00; 0.60]</td>
</tr>
</tbody>
</table>

Confidence intervals are bootstrapped by resampling households 100 times with replacement.
C Details on the structural model of section 5

C.1 Analysis of policy functions

The consumer’s idiosyncratic state is given by the combination of \( s^i_t \) and his real bond position \( \lambda^i_t \equiv \frac{\Lambda^i_t}{\Pi^i_{t-1}} \). From his point of view, the relevant components of the aggregate state are \((w_t, T_t(e), Q_t, \Pi_t, \bar{D}_t)\), where \( \Pi_t \equiv \frac{P_t}{P_{t-1}} \) denotes the inflation rate at \( t \) and \( w_t \equiv (1 - \tau) \frac{W_t}{P_t} \) the post-tax real market wage. His optimization problem is characterized by the Bellman equation:

\[
V_t(\lambda, s) = \max_{c, n, \lambda'} u(c - e(s)v(n)) + \beta(s) \mathbb{E}[V_{t+1}(\lambda', s') | s]
\]

subject to

\[
c + Q_t\left(\lambda' - \delta_N \frac{\lambda}{\Pi_t}\right) = w_t e(s) n + T_t(e(s)) + \frac{\lambda}{\Pi_t}
\]

(C.1)

**Proposition C.1.** The policy function for hours worked \( n_t(\lambda, s) \) is independent of \( \lambda \) and \( s \) and is given by \( n_t(\lambda, s) = b^{-\psi}w^{\psi}_t \). The consumption policy function \( c_t(\lambda, s) \) is concave in bond holdings \( \lambda \).

**Proof.** Maximization of (C.1) with respect to \( n \) immediately leads to the first order condition:

\[
e(s)v'(n) = e(s)bn^{\psi - 1} = e(s)w_t \quad \Rightarrow \quad n_t(\lambda, s) = b^{-\psi}w^{\psi}_t
\]

(C.2)

Define \( i \)'s net consumption \( g^i_t \) as \( g^i_t \equiv c^i_t - v(n_t) \) and \( i \)'s net income function as

\[
z(w) \equiv \max_{\hat{n}} \{w \cdot \hat{n} - v(\hat{n})\} = \frac{1}{1+\psi}wn = \frac{1}{1+\psi}b^{-\psi}w^{1+\psi}
\]

The problem in (C.1) can be rewritten as a maximization over \( g \) and \( \lambda \)

\[
V_t(\lambda, s) = \max_{g, \lambda'} u(g) + \beta(s) \mathbb{E}[V_{t+1}(\lambda', s') | s]
\]

subject to

\[
g + Q_t\left(\lambda' - \delta_N \frac{\lambda}{\Pi_t}\right) = e(s)z(w_t) + T_t(e(s)) + \frac{\lambda}{\Pi_t}
\]

(C.3)

When the borrowing constraint binds, the consumption function is linear in \( \lambda \). Otherwise, an Euler equation characterizes the consumer’s optimal consumption plan. An adaptation of the Carroll and Kimball (1996) results then shows that the net consumption function \( g \) is concave in \( \lambda \), and strictly concave provided that the consumer is exposed to residual income risk, i.e. \( z(w_t) + \frac{\partial T_t(e)}{\partial e} = z(w_t) + \gamma (\tau_t - \tau^*) Y_t > 0 \). It follows that \( c \) is concave in \( \lambda \), since \( c = g + ev(n) \) where \( n \) is independent of \( \lambda \).
C.2 Equilibrium definitions

Define the real interest rate between \( t \) and \( t + 1 \) as \( R_t \equiv \frac{1 + \delta_N Q_{t+1}}{Q_{t-1}} \). When prices are fully flexible, the path for real interest rates \( R_t^* \) is determined in equilibrium. Through its influence on the nominal bond price \( Q_t \), the monetary authority controls the path of the inflation rate \( \Pi_t \) directly (for example, pinning it down through a Taylor rule). All price level changes are perfectly anticipated by households and firms, and have no effect on real variables.

**Definition C.1.** Given an initial distribution \( \Psi_0 (s, \lambda) \) over idiosyncratic states and bond positions, an initial price level \( P_0 \), and a path for inflation \( \{ \Pi_t \} \) and borrowing limits \( \{ D_t \} \), a flexible-price equilibrium is a sequence of consumption rules \( \{ c_t (s, \lambda) \} \), next-period bond choices \( \{ \lambda_{t+1} (s, \lambda) \} \), distributions \( \{ \Psi_t (s, \lambda) \} \) and aggregate prices \( \{ R_t, Q_t, \Pi_t, W_t, \lambda_t \} \) and quantities \( \{ n_t, C_t, Y_t, T_t \} \) such that: consumers solve the optimization problem in (C.1), final and intermediate-goods firms maximize profits, leading to (24), the price level evolves according to \( P_t = \Pi_t P_{t-1} \) and bond prices according to \( R_t = \frac{1 + \delta_N Q_{t+1}}{Q_{t-1}} \), the government’s budget constraint (25) is satisfied with \( T_t (e) = T_t \), markets for labor, intermediate goods, final output and bonds clear,

\[
\int_l^l 1_i dl = \int_l^l e_i n_i dl \tag{C.4}
\]

\[
C_t \equiv \int c_t (s, \lambda) d\Psi_t (s, \lambda) = Y_t \tag{C.5}
\]

\[
Q_t \int \lambda_{t+1} (s, \lambda) d\Psi_t (s, \lambda) = 0 \tag{C.6}
\]

and the evolution of the bond distribution is consistent with \( \{ \lambda_{t+1} (s, \lambda) \} \).

When prices are fully sticky, the inflation rate is constant at \( \Pi_t = 1 \), and the monetary authority, by changing the nominal interest rate, controls the real interest rate \( R_t \) directly. It is natural to think of initial conditions, including the identical price \( \bar{P} \) for all firms, as being inherited from a steady-state with flexible prices.

**Definition C.2.** Given an initial distribution \( \Psi_0 (s, \lambda) \) over idiosyncratic states and bond positions, an initial common price for all firms \( \bar{P} \), and paths for the real interest rate \( \{ R_t \} \) and borrowing limits \( \{ D_t \} \), a sticky-price equilibrium is a sequence of consumption rules \( \{ c_t (s, \lambda) \} \), next-period bond choices \( \{ \lambda_{t+1} (s, \lambda) \} \), distributions \( \{ \Psi_t (s, \lambda) \} \) and aggregate prices \( \{ \Pi_t, Q_t, P_t, W_t, \lambda_t \} \) and quantities \( \{ n_t, C_t, Y_t, T_t (e), \bar{T}_t \} \) such that: consumers solve the optimization problem in (C.1), the final goods firm maximizes profits, intermediate goods firms satisfy demand at price \( \bar{P} \), the price level is constant (\( \Pi_t = 1 \)), bond prices evolve according to \( Q_t = \frac{1 + \delta_N Q_{t+1}}{R_t} \), the government’s budget constraint (25) is satisfied with \( T_t (e) \) satisfying (26), markets for intermediate goods and labor clear (C.4), markets for final goods (C.5) and bonds (C.6) clear, and the evolution of the bond distribution is consistent with \( \{ \lambda_{t+1} (s, \lambda) \} \).
C.3 Aggregation and the nature of earnings heterogeneity

The following propositions are aggregation results that obtain under both flexible and sticky prices.

**Proposition C.2.** *(Per capita) GDP is equal to $Y_t = \kappa \psi_t$ where $\kappa \equiv b^{-\psi} \mathbb{E}[e]$ is a constant.*

Proposition C.2 illustrates the simplicity of aggregation in this model. GDP is a function of aggregate labor supply, which depends only on the net-of-tax real wage $w_t$.\[^{42}\]

**Proof.** Integrating (C.4) using households’ first-order condition for labor supply (C.2) as well as the existence of a stationary distribution for idiosyncratic states, I obtain

$$\int \epsilon^i n^i di = b^{-\psi} \mathbb{E}[e] \psi^i \quad (C.7)$$

where $\kappa$ is constant. Next, combining $x^j_t = l^j_t$ and (22) using intermediate-good market clearing, integrating across firms and making use of the fact that $P^j_t = P_t$ for all $j$, and finally using labor market clearing (C.4), GDP is equal to

$$Y_t = \int_{j=0}^{1} x^j_t dj = \int_{j=0}^{1} l^j_t dj = \int \epsilon^i n^i di = \kappa w^i \quad (C.8)$$

\[\square\]

**Proposition C.3.** The total tax rebate as a share of GDP is:

$$t_t = \frac{\int T_t (\epsilon^i) di}{Y_t} = \tilde{\tau}_t \quad (C.9)$$

where $\tilde{\tau}_t \equiv 1 - w_t$ is the labor wedge, a summary measure of the economy’s distortions.

**Proof.** A high $\tilde{\tau}_t$ indicates a distorted economy, which may result from a high tax rate $\tau$, high monopoly power $\epsilon$ or a negative output gap (a recession) under sticky prices. In the model, there is therefore a direct link between the labor wedge $\tilde{\tau}_t$ and the government tax take as a share of GDP, $t_t$. Moreover $\tilde{\tau}_t$, as well as output $Y_t$, are constant under flexible prices. \[\square\]

Using the government budget constraint constraint (25), the definition of firm profits in (23),

\[^{42}\]If I had allowed intermediate-goods prices to differ, a summary measure of heterogeneity in production outcomes $\Delta_t \equiv \int_{j=0}^{1} \left( \frac{P^j_t}{P_t} \right)^{-\epsilon} dj$ would also enter the expression for $Y_t$. 

A33
labor market clearing (C.4), and the relationship between aggregate hours and output in (C.8)

\[
\begin{align*}
P_t \int T_t (e_i) \, di &= \int \int F_j (P_j) \, dj + \tau \int \int W_t e_i n_i \, di \\
&= \int \int P_j e_i \, dj - W_t \int \int t_i \, dj + \tau \int \int W_t e_i n_i \, di \\
&= P_t Y_t - (1 - \tau) W_t Y_t
\end{align*}
\]

hence

\[
t_t = \int T_t (e_i) = Y_t \left( 1 - (1 - \tau) \frac{W_t}{P_t} \right) = Y_t \left( 1 - w_t \right)
\]

**Proposition C.4.** Under flexible prices, \( t_t = \tau_t = 1 - (1 - \tau) \frac{e-1}{e}, \) and \( Y_t = Y^* = \kappa (1 - \tau^*) \).\)

Proposition C.4 follows from the observation that, under flexible prices, equation (24) determines a constant post-tax real wage \( w^* = (1 - \tau) \frac{e-1}{e} \). Under sticky prices, I call a *boom* a situation where \( Y_t > Y^* \) (equivalently, \( w_t > w^* \) and \( \tau_t < \tau^* \)) and a *recession* the opposite situation where \( Y_t < Y^* \) and \( \tau_t > \tau^* \).

**Proof.** Under flexible prices, equation (24) implies that

\[
w_t = (1 - \tau) \frac{W_t}{P_t} = (1 - \tau) \frac{e-1}{e} \equiv w^*
\]
a constant, and therefore \( \tau_t = 1 - w^* \equiv \tau^* \) is constant, and \( Y_t = \kappa (w^*) \equiv \kappa (1 - \tau^*) \equiv Y^* \) is constant as well. This is true in every period, irrespective of the distribution of wealth. \(\square\)

Together, propositions C.3 and C.4 show that the flexible-price labor wedge \( \tau^* \) is a natural benchmark for the tax rebate function in (26). Note that, from the definition of \( t^* \), we have \( t^* = \tau^* \). Under flexible prices, this implies that all agents obtain the same rebate \( T^* = \tau^* Y^* \). Under sticky prices, Proposition C.3 shows that the total tax rebate as a share of GDP falls in booms and rises in recessions—a consequence of countercyclical markups. The parameter \( \gamma \) in (26) controls the incidence of this effect on the income distribution. Indeed, define total nonfinancial income \( Y_i \) as \( i \)'s real earnings inclusive of the lump-sum rebate,

\[
Y_i \equiv w_i e_i n_i + T_t \left( e_i \right)
\]

**Proposition C.5.** Under the tax system in (26), the function \( \overline{\tau} (\overline{\tau}) = \tau^* + \gamma (\overline{\tau} - \tau^*) \) summarizes the effect of cyclical conditions, as given by the labor wedge \( \overline{\tau}_t \), on the income distribution:

\[
\frac{Y_i}{Y_t} = (1 - \overline{\tau} (\overline{\tau}_t)) \frac{e_i}{E [e]} + \overline{\tau} (\overline{\tau}_t) \quad (C.10)
\]

Plugging in the expression for \( \overline{\tau} \), we obtain (26) in the main text. Expression (C.10) is also quite informative: note that, as \( \overline{\tau} (\overline{\tau}_t) \) varies from zero to one, agents' relative incomes alternate...
between their fundamental level—which, given that all agents work the same number of hours, is simply given by productivity $e$—and perfect equality. Proposition C.5 makes clear that the sign of $\gamma$ determines the direction of the earnings heterogeneity channel in the model. When $\gamma$ is positive, the post-tax income distribution compresses in recessions. This could capture the effects of automatic stabilizers. When $\gamma = 0$, $\tau$ is constant at $\tau^*$, and cyclical conditions leave the income distribution unchanged. Finally, when $\gamma < 0$ the post-tax income distribution expands in recessions. This latter assumption captures countercyclical earnings risk (for example Storesletten et al. 2004 or Guvenen et al. 2014).

**Proof.** Note from (C.2) and (C.7) that

$$w_i e_i n_i^t = w_i \frac{1}{b^\psi} e_i^\psi w_i^\psi = w_i \frac{e_i^t}{E[e]} Y_t = (1 - \tau_t) \frac{e_i^t}{E[e]} Y_t$$

(C.11)

Adding the tax rebate as defined by the rule in (26) and using the equilibrium relation $\tau_t = t$, the total income of an individual with productivity $e_i^t$ relative to average income is

$$\frac{Y_i^t}{Y_t} \equiv (1 - \tau_t) \frac{e_i^t}{E[e]} + \tau_t (\tau_t) + (\tau_t - \tau_t (\tau_t)) \frac{e_i^t}{E[e]}$$

$$= (1 - \tau_t (\tau_t)) \frac{e_i^t}{E[e]} + \tau_t (\tau_t)$$

C.4 Equilibrium and steady-state with flexible prices

As can be seen from proposition C.4, under flexible prices output—and therefore aggregate consumption—is a constant determined by the degree of monopoly power and the tax system. It follows that (unexpected) redistributive policies—such as targeted lump-sum transfers from one group of agents to another, or inflationary shocks that erode the real value of debts and assets—do not affect aggregate output.\footnote{These shocks do, however, change relative consumption and welfare levels, as well as the market-clearing real interest rate, in a way that depends on the strength of the redistribution channel. See the working paper version of this paper, Auclert (2015), for an analysis of these effects.} This result can be viewed as a useful benchmark, highlighting the importance of general equilibrium when thinking through the aggregate effects of redistributive policy.

I define a steady-state as a flexible-price equilibrium with constant debt limit $D$ and inflation $\Pi$, attaining a constant real interest rate $R^*$ and a stationary distribution for bonds $\Psi (s, \lambda)$. The steady-state has the following important property:

**Proposition C.6.** Two economies that differ only in their maturity structure of financial assets and liabilities $\delta_N$ attain the same flexible-price steady-state interest rate $R^*$, with identical joint distribution over bond market values $b = Q \lambda$ and idiosyncratic states $s$.\footnote{These shocks do, however, change relative consumption and welfare levels, as well as the market-clearing real interest rate, in a way that depends on the strength of the redistribution channel. See the working paper version of this paper, Auclert (2015), for an analysis of these effects.}
The logic behind this proposition is simple. When agents face a constant term structure of interest rates $R^*$, short and long-term assets span the same set of contingencies, and the specification of a constant borrowing limit $\overline{D}$ in (21) is also neutral with respect to maturity. Crucially, unhedged interest rate exposures do vary with $\delta_N$. Changing $\delta_N$ therefore allows me to change asset durations and the strength of the interest rate exposure channel without changing any of the other steady-state properties of the model.

Proof. In a steady-state flexible price equilibrium with constant inflation rate $\Pi$ and debt limit $\overline{D}$, real wages are the constant $w^* = 1 - \tau^*$, the tax intercept is the constant $T^* = \tau^* Y^*$, and the bond price is the constant $Q = \frac{1}{\Pi R - \delta_N}$. Hence, the budget constraint and borrowing constraint in (C.3) rewrite

$$g + \frac{1}{\Pi R - \delta_N} \lambda' = e(s) z(w^*) + T^* + \frac{R}{\Pi R - \delta_N} \lambda$$

$$\frac{1}{\Pi R - \delta_N} \lambda' \geq -\overline{D}$$

Define gross assets as $a = \frac{1}{\Pi R - \delta_N} \lambda + \overline{D}$ and cash-on-hand as

$$\chi = e(s) z(w^*) + T^* + \frac{R}{\Pi R - \delta_N} \lambda + \overline{D}$$

with next-period value

$$\hat{\chi} = e(s') z(w^*) + T^* + R(\hat{a} - D) + \overline{D}$$

The consumer’s net consumption policy is then the solution to the stationary Bellman equation

$$W(\chi; s) = \max_{\hat{a} \geq 0} u(\chi - \hat{a}) + \beta \mathbb{E} \left[ W(e(s') z(w^*) + T^* + R\hat{a} - \overline{D} (R - 1)) | s \right] \quad (C.12)$$

The maturity structure parameter $\delta_N$ does not enter equation (C.12). Hence in steady state, aggregate consumption $g = \chi - \hat{a} + ev(n)$ is unaffected by this parameter conditional on a level of $b = Q\lambda$ and $s$. Since the same is true of aggregate labor supply, all policy functions are unaffected by $\delta_N$ and hence equilibrium is as well.

C.5 Equilibrium with fully sticky prices

In a fully sticky-price equilibrium, when the central bank temporarily and unexpectedly sets a real interest rate $R_t$ below its “natural” level that prevails under flexible prices $R^*_t$, aggregate consumption increases due to both a substitution and a redistribution effect, and firms accommodate by producing more. As proposition C.2 shows, an increase in production $Y_t$ requires an increase in the real wage $w_t$. This in turn leads to a fall in the labor wedge $\tilde{\tau}_t = 1 - w_t$, which can result in a change in the income distribution following (C.10). I now show how the analysis

A36
of section 3 can be used to understand the aggregate consumption response that results from all of these effects.

C.6 Using redistribution channel moments to predict impulse responses

**Definition C.3.** The moments of the redistribution channel in the steady-state of the model are defined as follows. Gross-of-tax income-weighted MPC is \( M^g \equiv \mathbb{E}_I \left[ \frac{\epsilon}{\psi} \text{MPC}^i \right] \), (net-of tax) income-weighted MPC is \( M \equiv \mathbb{E}_I \left[ \frac{\epsilon}{\psi} \text{MPC}^i \right] \), consumption-weighted MPC is \( M^c \equiv \mathbb{E}_I \left[ \frac{\epsilon}{\psi} \text{MPC}^i \right] \), and the income-MPC covariance is \( C_Y \equiv \text{Cov}_I \left( \text{MPC}^i, Y^i \right) \). The redistribution elasticities with respect to the price level \( P \) and the real interest rate \( r \) are, respectively, \( E_P \equiv -\frac{\text{Cov}_I \left( \text{MPC}^i, \text{NNP}^i \right)}{\mathbb{E}_I \left[ \text{NNP}^i \right]} \) and \( E_r \equiv \frac{\text{Cov}_I \left( \text{MPC}^i, \text{URE}^i \right)}{\mathbb{E}_I \left[ \text{URE}^i \right]} \).

From these moments we can form the Hicksian scaling factor: defining \( \xi \equiv 1 - \frac{\psi}{1 + \psi} \left( 1 - \tau^* \right) \), it is here equal to \( S = \xi \left( 1 - \left[ \frac{1}{\xi} M^c + \left( 1 - \frac{1}{\xi} \right) M^g \right] \right) \). Using \( S \) we can then define the equivalent EIS \( \sigma_r = -\frac{\xi}{S} \). We also define \( T \equiv \left( 1 - M^g \right) \left( 1 - \tau^* \right) \) as the consumption-labor complementarity factor. All of the cross-sectional moments in definition C.3 can be directly computed from the policy functions in steady-state, and can be compared to their empirical counterparts. The concavity of the consumption function implies that \( E_P > 0 \). It also implies that \( E_r < 0 \) provided that all assets and liabilities are short term \( (\delta_N = 0) \), since in this case \( URE^i \) is simply \( Q \lambda_i \). In the calibration we will see that \( E_r < 0 \) also holds for longer maturities, providing a rationale for the empirical evidence in section 4. A key result, however, will be that \( E_r \) becomes less negative when \( \delta_N \) increases.

The cross-sectional moments from definition C.3 are useful to predict the response of aggregate consumption to macroeconomic shocks that last for one period, as the following application of theorem A.3 (which is proposition 1 in the main text) indicates.

**Proposition C.7.** Assume that the steady-state is perturbed by a shock that changes \( dY \) and \( dR \) for one period only, and revises all future prices by \( dP \). Assume that labor markets clear so that \( \frac{d\lambda}{\psi} = \frac{1}{\psi} \frac{dY}{Y} \) and \( \frac{d\tau}{1 - \tau} = -\frac{d\lambda}{\psi} \). Then the following equation gives the first order response of aggregate consumption \( dC \):

\[
\frac{dC}{C} = M \frac{dY}{Y} + \frac{T}{\psi} C_Y \frac{dY}{Y} + E_P \frac{dP}{R} + (E_r - \sigma S) \frac{dR}{R} + T \frac{dY}{Y} \tag{28}
\]

If, in particular \( dP = 0 \), the response of consumption \( dC = dY \) is

\[
\frac{dC}{C} = -\mu \rho \left( \sigma_r + \sigma \right) \frac{dR}{R} \tag{29}
\]

where \( \mu = \frac{\xi}{T} \) and \( \rho = \frac{1 - \left( \frac{1}{\xi} M^c + \left( 1 - \frac{1}{\xi} \right) M^g \right)}{1 - \left( \frac{1}{\xi} M + \left( 1 - \frac{1}{\xi} \right) M^g + \frac{1}{\psi} C_Y \right)} \).
Proof. First, rewrite theorem A.3 in terms of elasticities, using the fact that all individuals have a common net EIS and Frisch elasticity ($\sigma^i = \sigma, \psi^i = \psi$). Dropping time subscripts for simplicity given that the initial conditions are a steady-state, and using market clearing

\[ C = E_f \left[ c^i \right] = Y \]

we obtain

\[
\frac{dC}{C} \simeq E_f \left[ \frac{Y^i}{Y} MPC^i \right] \frac{dY}{Y} + \frac{1}{Y} \text{Cov}_I \left( MPC^i, dY^i \frac{dY}{Y} - Y^i \frac{dY}{Y} \right) - \text{Cov}_I \left( MPC^i, \frac{NNP^i}{E_f \left[ c^i \right]} \right) \frac{dP}{P}
\]

\[ + \left( \text{Cov}_I \left( MPC^i, \frac{URE^i}{E_f \left[ c^i \right]} \right) - \sigma \cdot E_f \left[ \left( 1 - \frac{e^i \psi(n^i)}{c^i} \right) \left( 1 - MPC^i \right) \frac{c^i}{E_f \left[ c^i \right]} \right] \right) \frac{dR}{R}
\]

\[ + \psi E_f \left[ \left( 1 - MPC^i \right) n^i E^i \right] dw_i \]

Note first that $n^i = n$ for every agent by proposition C.2, and therefore

\[ S = E_f \left[ \left( 1 - MPC^i \right) \frac{c^i}{C} \right] - E_f \left[ \frac{e^i \psi(n^i)}{c^i} \left( 1 - MPC^i \right) \frac{c^i}{C} \right] \]

\[ = 1 - E_f \left[ MPC^i \frac{c^i}{C} \right] - \frac{\psi(n)}{C} E_f \left[ e^i \left( 1 - MPC^i \right) \right] \]

\[ = 1 - M^c - \frac{\psi(n)}{C} E_f \left[ e^i \right] (1 - M^s) \]

But $C = Y = E_f \left[ e^i \right] n$, and

\[ \psi'(n) = w = 1 - \tau^* = bn^i \psi^{-1} \Rightarrow \frac{\psi(n)}{n} = \frac{\psi}{1 + \psi} (1 - \tau^*) \quad (C.13) \]

so that in steady state, $\frac{\psi(n)E_f[e]}{C} = \frac{\psi}{1 + \psi} (1 - \tau^*)$. Define $\xi \equiv \left( 1 - \frac{\psi}{1 + \psi} (1 - \tau^*) \right)$, we obtain

\[ S = \xi \left( 1 - \left[ \frac{M^c + (\xi - 1) M^s}{\xi} \right] \right) \]

\[ = \xi \left( 1 - \left[ \frac{1}{\xi} M^c + \left( 1 - \frac{1}{\xi} \right) M^s \right] \right) \quad (C.14) \]

Similarly, from (C.11) we have

\[ \frac{e^i n}{Y} = \frac{e^i}{E_f \left[ e^i \right]} = \frac{e^i}{E_f \left[ e^i \right]} \frac{1 - \tau^*}{w} \]
so that

$$\psi \frac{1}{Y} \mathbb{E}_t \left[ \left( 1 - \text{MPC}^i \right) n^i e^i \right] dw = \psi (1 - \tau^*) \mathbb{E}_t \left[ \left( 1 - \text{MPC}^i \right) \frac{e^i}{\mathbb{E}_t[e]} \right] \frac{dw}{w}$$

$$= \psi (1 - \tau^*) (1 - \mathcal{M}^i) \frac{dw}{w}$$

Next, using (C.10),

$$\frac{Y^i}{Y} = \frac{e^i}{\mathbb{E}[e]} + \bar{\tau} \left( 1 - \frac{e^i}{\mathbb{E}[e]} \right)$$

and since \( \bar{\tau}'(\tau^*) = \gamma \),

$$dY^i - \frac{Y^i}{Y} dY = Y \left( 1 - \frac{e^i}{\mathbb{E}[e]} \right) \gamma d\bar{\tau}$$

$$= - \frac{Y^i - Y}{1 - \bar{\tau}} \gamma d\bar{\tau}$$

With the last line following from \( \bar{\tau}(\bar{\tau}) = \bar{\tau} = \tau^* \) at steady state. Hence

$$\frac{1}{Y} \text{Cov}_t \left( \text{MPC}^i, dY^i - \frac{Y^i}{Y} dY \right) = - \text{Cov}_t \left( \text{MPC}^i, \frac{Y^i}{Y} \right) \gamma \frac{d\bar{\tau}}{1 - \bar{\tau}}$$

The link between \( dY, d\bar{\tau} \) and \( dw \) is then provided by differentiating the market clearing conditions. From \( 1 - \bar{\tau} = w \) we obtain

$$- \frac{d\bar{\tau}}{1 - \bar{\tau}} = \frac{dw}{w}$$

as from \( Y = \kappa w^\psi \), we obtain

$$\frac{dY}{Y} = \psi \frac{dw}{w}$$

Combining all results yields the formula in (28).

**Response to a monetary policy shock.** In response to a shock to \( dR \), prices are fixed \( dP = 0 \), and solving out (28) for \( dC = dY \) we obtain
\[
\frac{dC}{C} = \frac{\varepsilon_r - \sigma S}{1 - \mathcal{M} - \frac{1}{c} \mathcal{C}_Y - (1 - \mathcal{M}^{\delta}) (1 - \tau^*)} \frac{dR}{R}
\]

\[
= -\frac{S}{\tau^* - \mathcal{M} + \frac{1}{c} \mathcal{C}_Y + \mathcal{M}^{\delta} (1 - \tau^*)} (\sigma_r + \sigma) \frac{dR}{R}
\]

\[
= -\frac{S}{\tau^* \left( 1 - \left[ \frac{1}{c} \mathcal{M} + (1 - \frac{1}{c}) \mathcal{M}^{\delta} + \frac{c}{\mathcal{C}_Y \tau^*} \right] \right)} (\sigma_r + \sigma) \frac{dR}{R}
\]

where as usual \( \sigma_r \equiv -\xi \varepsilon_r \). Using the expression for \( S \) in (C.14), we therefore obtain

\[
\frac{dC}{C} = -\kappa \frac{1 - \left( \frac{1}{c} \mathcal{M}^c + (1 - \frac{1}{c}) \mathcal{M}^{\delta} \right)}{1 - \left( \frac{1}{c} \mathcal{M} + (1 - \frac{1}{c}) \mathcal{M}^{\delta} + \frac{c}{\mathcal{C}_Y \tau^*} \right)} (\sigma_r + \sigma) \frac{dR}{R}
\]

which is equation (29).

**Representative agent model.** Since a representative agent has \( \mathcal{M} = \mathcal{M}^c = \mathcal{M}^{\delta} \) as well as \( \mathcal{C}_Y = 0 \) and \( \sigma_r = 0 \), (29) becomes

\[
\frac{dC}{C} = -\kappa \frac{\sigma_r}{\tau^*} \frac{dR}{R} = -\mu \sigma \frac{dR}{R}
\]

This equation can be understood by observing that a representative agent with discount rate \( \beta \) satisfies an Euler Equation

\[
\frac{C_{t+1} - v (n_{t+1})}{C_t - v (n_t)} = (\beta R_t)^{\sigma}
\]

which, letting \( r_t = \log \frac{R_t}{R^*} \), \( \hat{c}_t = \log \frac{C_t}{C^*} \) and \( \hat{n}_t = \log \frac{N_t}{N^*} \), can be approximated as

\[
\frac{C}{C - v (n)} \left( \hat{c}_{t+1} - \hat{c}_t \right) - \frac{v' (n)}{C - v (n)} \left( \hat{n}_{t+1} - \hat{n}_t \right) = \sigma r_t
\]

with \( \frac{C}{C - v (n)} = \frac{1}{1 - \frac{v (n)}{C}} = \frac{1}{\xi} \), and \( \frac{v' (n) n}{C - v (n)} = v' (n) = 1 - \tau^* \). Hence we obtain the loglinear “dynamic IS curve”

\[
\hat{c}_{t+1} - \hat{c}_t = \sigma \xi r_t + (1 - \tau^*) \left( \hat{n}_{t+1} - \hat{n}_t \right)
\] (C.15)

Equation (C.15) illustrates the amplification mechanism inherent in the complementarities between hours and consumption. Following a shock to the real interest rate, consumption rises, and this change must be matched by a change in hours worked, \( \hat{c}_t = \hat{n}_t \). In turn, this raises the marginal utility of consumption. In equilibrium, \( \hat{c}_t \) follows the following dynamic equation in response to interest rate shocks:

\[
\hat{c}_t = \hat{c}_{t+1} - \frac{\tau^*}{\tau^*} \sigma r_t
\]
Equation (29) since the shock to $r_t$ lasts for one period only and the central bank is assumed to stabilize income in the long-run, $\hat{c}_\infty = 0$.

C.7 Details on the calibration and solution technique

C.7.1 Calibration of the earnings process

The steady-state wedge $\tau^*$ plays a crucial role in the analysis. As proposition C.5 makes clear, $\tau^* = \tau(\tau^*)$ determines the degree of inequality in earnings, and hence the strength of the precautionary savings motive, given a process for idiosyncratic uncertainty. From proposition C.7 it is also clear that $\tau^*$ is crucially related to the output multiplier when prices are sticky. I therefore calibrate it jointly with the productivity process as follows.

Since Lillard and Weiss (1979) and MaCurdy (1982), a large literature has fitted earnings processes to panel data on labor earnings, in particular to PSID data on male earnings. A consensus from the literature is that the earnings process features an important degree of persistence: the data on annual, log pre-tax labor earnings is reasonably described by an $AR(1)$ process with a large autoregressive root, possibly a unit root.

Since my model with infinitely-lived agents exploits the existence of a stationary distribution to define steady-state aggregates, I postulate that individual-level productivity follows an $AR(1)$ process in logs at quarterly frequency

$$\log e^i_t = \rho \log e^i_{t-1} + \sigma_e \sqrt{1 - \rho^2} e^i_t \quad e^i_t \sim N(0, 1)$$

(C.16)

with $\rho < 1$. (C.16) admits the stationary distribution $\log e^{SS} \sim N(0, \sigma_e^2)$. In the steady-state of my model, since every agent works the same number of hours, pre-tax earnings $e^i_t n^i_t$ are proportional to $e^i_t$. This implies that the steady-state variance of log earnings is also $\sigma_e^2$. Moreover the process, sampled at annual frequency, is an $AR(1)$ with root $\rho^4$.

In the Panel Study of Income Dynamics (PSID), if one considers the entire sample, the cross-sectional standard deviation of log head pre-tax earnings in 2009 is 1.04. This number been relatively stable over time since 1968. I therefore set $\sigma_e = 1.04$. I then discretize the idiosyncratic productivity process by using a 10-point Markov Chain using the procedure described in Tauchen (1986). Figure C.1 plots the PSID Lorenz curve for post-tax earnings against that obtained in the model, illustrating that a lognormal distribution fits the majority of the earnings distribution very well.

Typical calibrations of the earnings process in this class of models (for example, Aiyagari 1995; Floden and Lindé 2001 or Guerrieri and Lorenzoni 2015) assume smaller standard deviations for log earnings than 1.04, because they calibrate residual earnings uncertainty and therefore do not seek to match the earnings distribution as I do here. In my model, the driver of consumption-smoothing behavior is post-tax-and-transfer earnings, whose standard deviation is controlled by $\tau^*$. I therefore set $\tau^* = 0.4$ to match the post-tax labor earnings distribution.
that these studies typically take (with a standard deviation of logs of 0.6).

The value of $\rho^4$ is more controversial. Two papers that estimate the process in the PSID and use it to calibrate an incomplete-market model are Heaton and Lucas (1996) and Floden and Lindé (2001). The former use a value of 0.53, while the latter use 0.91. I settle for $\rho^4 = 0.8$, implying a quarterly degree of persistence $\rho = 0.9457$.

C.7.2 The behavior of constrained agents after monetary policy shocks

As explained briefly in footnote 38, I specify that the borrowing limit $\{\mathcal{D}_t\}$ adjusts in response to such a shock so as to hold the real coupon payment in the next period fixed: $\mathcal{D}_t = Q_t \bar{d}$, or equivalently

$$\frac{\Lambda_{t+1}}{P_t} \geq -\bar{d} \quad \text{(C.17)}$$

In addition to being a natural one, the specification of the adjustment process for borrowing limits in (C.17) implies that proposition C.7 holds exactly, including for agents at a binding borrowing limit. It is crucial to understand how these agents are affected depending on the maturity of the debt in the economy, $\delta_N$. In the experiments I consider inflation is $\Pi_t = 1$, so that nominal and real interest rates are equal. Consider an agent with income $Y^i_t$ who maintains himself at the borrowing limit in an initial steady-state where the real interest rate is $R$ and the bond price is constant at $Q_t = \frac{1}{R - \delta_N}$. His consumption is equal to his income, minus the interest payment on the value of the borrowing limit $\mathcal{D} = Q_t \bar{d}$:

$$c^i_t = Y^i_t - (R - 1) \mathcal{D}$$
Across economies with different debt maturities \( \delta_N \), \( \bar{D} \) is a constant, so that the steady-state payments are the same, but the exposure of these payments to real interest rate changes differ. Indeed we can decompose:

\[
(R - 1) \bar{D} = (R - \delta_N) \bar{D} - \bar{D} (1 - \delta_N) = \bar{d} + \text{URE}
\]

where \( \bar{d} \equiv (R - \delta_N) \bar{D} \) is the part that is precontracted and \( \text{URE} \equiv -\bar{D} (1 - \delta_N) \) the part that is subject to interest changes. Hence, economies with different \( \delta_N \) involve very different levels of unhedged interest rate exposures for borrowing-constrained agents, ranging from the full principal \(-\bar{D}\) when \( \delta_N = 0 \) to none when \( \delta_N = 1 \). In the benchmark calibration with \( \delta_N = 0.95 \), highly-indebted low-income agents use their full income for interest payments and amortization \( \bar{d} \), and then borrow, as on a home equity line of credit, to maintain their consumption level. Hence they are only mildly affected by changes in interest rates. On the other hand, my ARM calibration is closer to one in which all debt is short-term, in which case \( c_i = Y_i - (R_i - 1) \bar{D} \) for constrained agents: these are agents that have to refinance the entire principal \( \bar{D} \) every period, leading to large swings in their consumption as interest rates change.

C.8 Computational method

C.8.1 Method of endogenous gridpoints

I use the method of endogenous gridpoints (Carroll 2006) to solve for consumer policy functions. This is a computationally efficient solution method based on policy function iteration, which avoids costly root-solving operations and is applicable to any standard incomplete market problem with CRRA utility functions (see for example Guerrieri and Lorenzoni 2015).

The computation involves finding the policy function for net consumption \( g_t(\lambda, s) \) on a fine grid for \( \lambda \) (2000 points) and a discrete grid for \( s \) (20 points: 2 states for \( \beta \) and 10 states for \( z \)). From \( g_t \), I recover the policy function for gross consumption \( c_t(\lambda, s) = g_t(\lambda, s) + e(s) \nu(n_t) \).

My GHH preference assumptions imply that the policy function for hours worked is simply \( n_t = b - \psi w_t^\theta \).

When the borrowing constraint binds, which happens for \( \lambda \leq \lambda^*_t \) for some \( \lambda^*_t \), the policy function is given by

\[
g_t(\lambda, s) = Z_t(e(s)) + \lambda \left( 1 + Q_t \frac{\delta_N}{\Pi_t} \right) + \bar{D}_t \tag{C.18}
\]

where \( Z_t(e) = \frac{1}{1+\psi} \xi \psi w_t^{1+\psi} + T(e) \). For \( \lambda > \lambda_t \) the borrowing constraint is not binding, and defining the real interest rate by

\[
R_t = \frac{1 + \delta_N Q_{t+1}}{Q_t \Pi_{t+1}}
\]
the solution is characterized by the Euler equation

$$g_t^{-\sigma^{-1}} = \beta_t R_t \mathbb{E}_t \left[ (g_{t+1})^{-\sigma^{-1}} \right]$$  \hspace{1cm} (C.19)

The idea behind endogenous gridpoints is to start from a given state today $s$ and a target bond level in the next period $\lambda'$. The budget constraint

$$\lambda' = \frac{1}{Q_t} \left( Z_t (e(s)) + \lambda \left( 1 + Q_t \delta_t \Pi_{t+1} - g \right) \right)$$  \hspace{1cm} (C.20)

implies that the pairs $(\lambda, g)$ that are consistent with $\lambda'$ are on a straight line. Moreover, given a guess for the policy function $g_{t+1}(\cdot, \cdot)$, there is a unique value of $g$ consistent with an optimal choice of $\lambda'$ tomorrow, given by

$$g = \left( \beta_t R_t \mathbb{E}_t \left[ g_{t+1} (\lambda', s')^{-\sigma^{-1}} \big| s \right] \right)^{-\sigma}$$  \hspace{1cm} (C.21)

Hence by varying the target bond level $\lambda'$, one traces out the policy function $g_t(\lambda, s)$ in the region $\lambda > \overline{\lambda}$. This is very efficient computationally since it can be performed on the grid for $\lambda'$ which is used to store $g_{t+1}(\lambda', s')$. The calculation only involves:

a) Finding $g$ using (C.21), which only involves power operations and linear combinations using the Markov transition matrix for $s$

b) Finding $\lambda$ by solving one linear equation in one unknown in (C.20)

c) Defining $\lambda^*_t$ as the bond value today that corresponds to $\lambda' = \frac{D_t}{Q_t}$, since this is the highest level of bonds for which the consumer chooses to be at the borrowing limit tomorrow with his Euler equation holding with equality

d) If $\lambda^*_t > \frac{D_t - 1}{Q_t - 1}$, completing the policy function on an arbitrary grid for $\left[ \frac{D_t - 1}{Q_t - 1}, \lambda^*_t \right]$ using (C.18)

e) Interpolating the resulting policy function back to the grid for $\lambda$

Figure C.2 illustrates the construction of the policy function for state $s = 1$ in the model calibration. Consider targeting a bond level $\lambda' = 0$. This yields a value for consumption through the Euler Equation (C.19) indicated by the dashed yellow line. It also yields a set of pairs $(g, \lambda)$ consistent with $\lambda' = 0$ through the budget constraint (C.20), as indicated by the solid purple line. The intersection of these two lines yields a new point of the policy function over $\lambda$. Varying $\lambda'$ in this way we trace out this policy function (solid blue line) over the range where the Euler equation holds. The policy function is completed by the set of points consistent with borrowing at the limit (solid red line).
C.8.2 Flexible price steady-state

In a flexible price steady-state with constant productivity $A$, inflation $\Pi$ and borrowing limit $D$, the consumer faces a constant sequence $(w_t, T_t, Q_t, \Pi_t, \bar{D}_t) = \left(1 - \tau^*, \tau^*Y^*, \frac{1}{\Pi R - \delta N}, \Pi, \bar{D}\right)$ where $R$ is the interest rate that prevails in steady-state.

I find the steady-state interest rate $R$ using the following classic bisection procedure:

a) Start with a guess for $R$ and for the consumption policy function $g^0(\lambda, s)$

b) Iterate on $g_t(\lambda, s)$ using the procedure described in C.8.1 until $g_{t+1} - g_t$ is sufficiently small. By construction, $g = g^{SS}(b, y)$ then satisfies the functional equation

$$g(\lambda, s)^{-\sigma^{-1}} = \beta(s) \text{RE} \left[ g \left( \frac{1}{Q} \left( Z^* (e(s)) + \lambda \left( 1 + \frac{\delta N}{\Pi} \right) - g(\lambda, s) \right), s' \right)^{-\sigma^{-1}} | s \right]$$

(C.22)

c) Use the inverse policy function for next period bonds $\lambda(\lambda', s) = [\lambda']^{-1}(\lambda', s)$, which is computed as part of the endogenous gridpoints method, to find the stationary conditional distribution for bonds $\Psi(\lambda|s)$, as the fixed point of the operator mapping $\Psi_t$ to $\Psi_{t+1}$,

$$\Psi_{t+1}(\lambda'|s') = \sum_s \Psi_t \left( [\lambda']^{-1}(\lambda', s) | s \right) \frac{\text{Pr}(s_t = s)}{\text{Pr}(s_{t+1} = s')} \Pi(s'|s)$$

d) Check that goods market clear, $\int c(\lambda; s) d\Psi(\lambda, s) = Y^*$. If they do not, adjust $R$ in the
direction of market clearing and repeat (one must first determine whether steady-state consumption is locally increasing or decreasing in $R$)

C.8.3 Transitional dynamics following a monetary policy shock

Here I describe how to compute transitional dynamics for monetary policy shocks in the fully sticky price version of the model. An unexpected shock to monetary policy perturbs an equilibrium with no inflation and prices all initially equal. This implies a path $R_t$ for the real interest rate. Assume that the economy returns to steady-state by time $T$ (in my computations, $T = 200$ when the shock has persistence $\rho = 0.5$)

a) Start by assuming that real wages and output stay constant at their steady-state level: $w_t = 1 - \tau^*, Y_t = Y^*, T_t = \tau^*Y^*$. Using the path of $R_t$, compute the bond price path

$$Q_t = \frac{1 + \delta_N Q_{t+1}}{R_t}$$

backwards starting from $Q_T = \frac{1}{1-\delta_N}$.

b) Given the path for $(w_t, T_t, Q_t, \Pi_t = 1, D_t)$, compute policy functions backwards, starting from $g_T = g^{SS}$, using the method of endogenous gridpoints described in C.8.1.

c) Starting from the conditional bond distribution that prevails in the initial steady-state, and using the transitional inverse policy function for next period bonds computed as part of step b), compute the conditional bond distributions along the transition using

$$\Psi_{t+1}(\lambda'|s') = \sum_s \Psi_t \left( [\lambda']^{-1}(\lambda', s) | s \right) \frac{Pr(s_t = s)}{Pr(s_{t+1} = s')} \Pi(s'|s)$$

d) See if goods market clear, $\int c_t(\lambda; s) d\Psi_t(\lambda, s) = Y(w_t) = \kappa w^*_t$. If they do not, adjust the path of real wages in the direction of goods market clearing, correspondingly adjust individual-level taxes $\frac{T_{t}(e)}{T_t} = \tau^* + (1 - w_t - \tau^*) \left( \gamma + (1 - \gamma) \frac{\epsilon}{E_{t+1}} \right)$ and repeat until convergence.

C.9 Calibration outcomes

As figure C.3 shows, my model generates a large dispersion in MPCs, with some agents with high cash-on-hand only slightly above the permanent-income level (below 0.01), and many constrained agents with much larger MPCs. Figure C.4 shows that these agents are disproportionately impatient agents, especially low-income impatient agents.

Note that while the MPC of agents exactly at the borrowing limit is equal to 1 by definition, as is clear from figure C.3, the Lagrange multiplier on their borrowing constraint is small.
enough that even a small positive transfer (lower than the amount it takes to move them to the next point on the asset grid) leads them to start smoothing substantially, as their MPC falls discontinuously to a number below 0.5. In other words, the large persistence in earnings creates an incentive for households to smooth non-infinitesimal positive shocks to income. This fact is behind the quantitative asymmetry of the reaction to positive and negative shocks to interest rates discussed in section 5.6.

Finally, figure C.5 shows the wealth distribution in the model. As stressed in the main text, this is a purposefully stylized Huggett model with average household wealth equal to zero. Hence its calibration cannot replicate the wealth distribution. While we have at our disposal models that do a good job at matching the distribution of net positions (see for example Kaplan et al. 2016), there is currently no good model generating the gross positions (such as the size of mortgage balances) that we observe in the data. A model such as mine which places emphasis
on these gross positions must therefore give up on matching the wealth distribution. For the purpose of evaluating the aggregate effect of a short-term change in monetary policy, this is not an overwhelming concern, since my results show that it is much more important to match the empirical covariance between MPCs and UREs, which my model does achieve.

References


