Precoding Schemes for MIMO Downlink Transmission

Milind Rao, Abbas Kazerouni, and Omid Aryan
{milind, abbask, aryano}@stanford.edu

I. INTRODUCTION

In multiuser MIMO systems, the Base Station (BS) transmits the message signals to different users in the cell through downlink communication. Since the same frequency and time slot is used in transmission to different users, it is necessary to come up with transmission schemes capable of supressing interference. Interference suppression can be performed using linear precoding and decoding at both the transmitter and the receiver.

In this survey, we will summarize two main precoding design schemes and also provide a comparison of their performance. The first scheme is based on Zero-Forcing Spatial Multiplexing which is proposed in [1]. However there are other variations of zero-forcing beamforming [3]–[5] which place different constraints on the number of users that can be served as the algorithms differ in implementations. In [1], suboptimal methods are introduced to zero out the interference. The second scheme we have taken into account is a Leakage-Based Precoding [2] which maximizes the Signal to Leakage and Noise Ratio (SLNR) of all the user simultaneously instead of zeroing out the interference which serves as a proxy for maximizing the SINR. Here, leakage refers to the amount of the interference caused by a specific user on the other users.

In our system model, we consider a cell containing a BS with $N$ transmit antennas serving $K$ users. Also, $M_i$ is the number of receive antennas for the $i$th user. We will use the notation $\{M_1, M_2, ..., M_K\} \times N$ to represent this scenario, and define $M = \sum_{k=1}^{K} M_k$ as the total number of receive antennas. Now, suppose that the intended message signal for the $i$th is the scalar $s_i$. In the precoding step, this symbol is multiplied by a $N \times 1$ beamforming vector $w_i$, and then the transmitted vector would be

$$x = \sum_{k=1}^{K} w_k s_k = WS$$

where $W = [w_1 \ldots w_K]$ and $S = [s_1 \ldots s_K]^T$. We also assume the following normalizations:

$$\mathbb{E}(|s_k|^2) = 1, \ ||w_k||^2 = 1$$

Consider $H_i$ as the $M_i \times N$ channel matrix whose $(p, q)$th element is the channel coefficient between the $p$th transmit antenna and the $q$th receive antenna. We assume these elements to be i.i.d. zero mean and unit variance complex Gaussian variables. We also assume a slow fading channel with packet based transmission where the
channel does not change over a packet length. Finally, the received signal by the \( i \)th user would be

\[
y_i = H_i x + v_i = H_i WS + v_i = H_i \sum_{k=1}^{K} w_k s_k + v_i
\]

where \( v_i \) is the additive noise with independent Gaussian elements of zero mean and variance \( \sigma_i \).

The linear decoding scheme multiplies the received signal by a matrix \( D_i \) to get an estimate of the transmitted signal, i.e.

\[
\hat{s}_i = D_i (H_i WS + v_i)
\]

and the aim of the precoding and decoding scheme is to design the matrices \( W \) and \( D_i \) so that the effect of the interference is removed in the above estimation.

We assume that channel matrices \( H_i \) are available at the BS, but since there is no cooperation among the different users, complete channel state information is not available to the users. However, the leakage based precoding scheme requires each user to have the information of its own channel state which can be provided either by channel estimation or by feedback from the BS.

II. Zero Forcing Beamforming

In MIMO systems, precoding can be done in multiple ways as described earlier. The optimal method regarding throughput is to maximize SINR for each user but expressions for the precoding matrix cannot be obtained in closed form because of the complex nature of the coupled optimization problem. Another simpler approach which is suboptimal is Zero Forcing (ZF) beamforming where we place the constraint that the interference a user faces is zero. Block Diagonalization (BD) implements ZF beamforming and is a generalization of channel inversion for the multiple antenna scenario. This method may be applied to maximize throughput under this constraint or minimize power such that each user receives a minimum Quality of Service (QoS) guarantee in terms of rate.

The signal the \( j \)th receiver obtains is:

\[
x_j = H_j W_j s_j + H_j \bar{W}_j \bar{s}_j + v_j,
\]

where \( H_j \) is the channel matrix from the base to the \( j \)th user, \( W_j \) is the associated modulation matrix, \( v_j \) is the Gaussian noise received. \( \bar{W}_j \) and \( \bar{s}_j \) are the modulation matrix and transmit data matrix for all other users than user \( j \). Since we place the constraint the the interference is zero, \( H_j \bar{W}_j \bar{s}_j = 0 \) or \( HW \) is a block diagonal matrix.

In the case of single-user communication, the performance gap between precoding knowing the channel matrix \( H \) and without it decreases at high SNR. However, there is a large performance gap at high SNR for multiuser communication as transmissions generate significant co-channel interference and knowing \( H \) is used for mitigating it.

As the users do not communicate with each other, bringing \( H \) to block diagonal form is done by the transmitter. Perfect diagonalization is sub-optimal compared to the block diagonal form as each user is using his antennae separately to receive the signal. This is manifested through lower throughput. To bring \( HW \) to block diagonal form, we
recognize that $\mathbf{W}_j$ must lie in the null space of $\tilde{\mathbf{H}}_j$. From the constraint that the null space of $\tilde{\mathbf{H}}_j$ must have positive dimension for block diagonalization to occur, we have the constraint that $N > \max[\text{rank}(\mathbf{H}_1), \ldots, \text{rank}(\mathbf{H}_K)]$. For the special subcase where receivers have a single antenna, $N > K$. To maximize throughput after constraining the interference to be zero, water-filling can be done on each user’s block of antennae.

The ZF algorithm ensures that interference is zero and this is done at the cost of the received power for each user. Other factors which limit the viability of ZF methods is the fact the partial or complete knowledge of the channel is required at the transmitter and the number of antennas at the transmitter has to be larger than the number of receivers in general.

### III. Leakage-Based Precoding

The second paper of interest is [2], where an alternative approach is proposed for choosing the precoding weights at the transmitter. The technique is called leakage-based precoding. Instead of trying to perfectly cancel out the interference at each user (as is done with zero-force precoding), leakage-based precoding aims at minimizing the interference caused by a signal intended for some user on the remaining users.

In mathematical terms, suppose the power of the desired signal for user $i$ is given by $||H_i w_i||^2$. Furthermore, the power of the interference caused by user $i$ on some other user $k$ is $||H_k w_i||^2$. The leakage is defined to be the total power leaked from user $i$ on all the other users: $\sum_{k=1, k \neq i}^{K} ||H_k w_i||^2$. Clearly, we would like the signal power intended for user $i$ to be large compared to the power leaked to the other users. Thus, we define the signal-to-leakage-noise ratio (SLNR) to be as follows:

$$SLNR_i = \frac{||H_i w_i||^2}{\sum_{k=1, k \neq i}^{K} ||H_k w_i||^2 + N}$$

where $N$ is the noise power at the receiver (user $i$). The goal would thus be to select the vectors $w_i, i = \{1, \ldots, K\}$ such that the above is maximized over $w_i$ subject to $||w_i||^2 = 1$. If we rewrite the above expression as

$$SLNR_i = \frac{||H_i w_i||^2}{\sum_{k=1, k \neq i}^{K} ||H_k w_i||^2 + N}$$

where $\tilde{H}_i = [H_1 \ldots H_{i-1} H_{i+1} \ldots H_K]^T$, then the solution is simply

$$w_i \propto \text{max.eigenvector}( (N I + \tilde{H}_i^* \tilde{H}_i)^{-1} \tilde{H}_i^* H_i)$$

The paper also extends the technique to MIMO systems that employ the Alamouti coding as well as to the multi-stream case. Furthermore, simulations were carried out to compare the BER curve of this technique with that of zero forcing, as is shown in Figure 1. Clearly, the leakage-based method outperforms zero forcing.
IV. COMPARISON

Comparing the zero forcing and leakage-based schemes with one another, the former zeros out the interference whereas leakage based tries to minimize the leaked power. One advantage of the zero forcing method compared to leakage based is the fact that the receivers do not need to have complete channel information. Nevertheless, the number of transmitting antennas in the zero forcing scheme constrains the number of users that can be reliably served. The leakage-based method does not impose such a restriction on the number of antennas. Moreover, leakage-based takes into account the noise of the system when calculating the coefficients while zero forcing does not, which can add to the performance of the scheme.

REFERENCES