Translating Electromagnetic Torque into Controlled Motion for use in Medical Implants

Daniel Pivonka, Teresa Meng, and Ada Poon

Abstract—A new propulsion method for sub-millimeter implants is presented that achieves high power to thrust conversion efficiency with a simple implementation. Previous research has shown that electromagnetic forces are a promising micro-scale propulsion mechanism; however, the actual implementation is challenging due to the inherent symmetry of these forces. The presented technique translates torque into controlled motion via asymmetries in resistance forces, such as fluid drag. For a 1-mm sized object using this technique, the initial analysis predicts that speeds of 1 cm/sec can be achieved with approximately 100 μW of power, which is about 10 times more efficient than existing methods. In addition to better performance, this method is easily controllable and has favorable scalability.

I. INTRODUCTION

Implantable devices capable of \textit{in vivo} controlled motion have a wide range of applications, including sensing, imaging, assisting with surgical procedures, and research. Many methods for achieving this goal have been explored, including both mechanical [1], [2], [3] and magnetic [1], [4], [5] approaches. Mechanical methods become increasingly complex and inefficient as they are scaled down, and generally encounter size constraints due to their high power requirements. Passive devices that are controlled with spatially and/or temporally varying magnetic fields tend to require very complex equipment and move increasingly slowly as they are scaled down. Controlling motion with gradient magnetic forces on passive devices has been successful [6], but requires large fields that can only be achieved with MRI. The limitations of MRI constrain the minimum size, and sub-millimeter implants are not possible.

Recent advances in wireless powering for micro-devices can provide significant power to small active devices [7], but locomotion in the sub-millimeter regime still requires highly efficient propulsion. Previous work has demonstrated the advantages of directly converting electrical energy into thrust forces without offering a practical implementation [8]. The method in [8] accomplishes this conversion with the manipulation of forces on current-carrying wires in a static magnetic field. However, the inherent symmetry of these electromagnetic forces becomes a significant problem in attempting to generate a propulsion force. Any current flowing in the device must have a return path; if this were not true, charge would accumulate. However, it is trivial to generate electromagnetic torque, and by taking advantage of asymmetry in the fluid drag characteristics, propulsion can be realized. Alternating the current in a loop of wire will cause it to vibrate, and if there is a preferential direction of fluid drag on the vibrating device, it will experience a net force and move forward. The efficiency of this technique is related to the difference in these drag forces, and this will be discussed in more detail in subsequent sections.

The organization of this paper is as follows. Section 2 outlines a basic theoretical model for the proposed propulsion technique. It first presents the basic rotational kinetics and then illustrates the idea with a device “walking” on land with only static and kinetic friction. The theory is extended to the case of fluid motion, and a formula that approximates the scalability is derived. Section 3 validates the design with numerical Navier-Stokes simulations of the motion in water. There are a variety of shapes that will result in forward motion when rotated in a fluid; for the purposes of this paper, only one simple design is considered. In these simulations, both the torque and net force are calculated. These values are used to determine the power efficiency of the force generation, which is compared against existing methods. Section 4 discusses the results and concludes the paper. In the following sections, boldface letters represent vectors. For a given vector $\mathbf{r}$, $\hat{\mathbf{r}}$ is a unit vector denoting its direction and $r$ denotes its magnitude.

II. THEORETICAL MODEL

The theoretical discussion will include both the rotational kinetics and basic fluid dynamics. Forces due to both buoyancy and gravity will not be considered in this analysis. To create the torque, a simple square loop of wire with side length $L$ is placed in a static magnetic field. For wires perpendicular to the field as shown in Figure 1, the total torque is

\begin{equation}
\tau_{em} = |\mathbf{r} \times \mathbf{F}| = \left| 2 \left( \frac{L^2}{2} \times (I_c \mathbf{L} \times \mathbf{B}) \right) \right| = I_c L^2 B \tag{1}
\end{equation}

where $\tau_{em}$ is the electromagnetic torque, $\mathbf{r}$ is a vector from the pivot point to the applied force, $\mathbf{F}$ is the applied force, $I_c$ is the current flowing through the loop, $\mathbf{L}$ is a vector representing the length and direction of the wire, and $\mathbf{B}$ is the static magnetic field. In the above equation, the center of mass is taken to be the center point of the loop, so each torque arm has a length equal to half the total length. The rotational acceleration of the device can be determined from the torque, and neglecting resistance forces it is simply

\begin{equation}
\alpha = \frac{\tau_{em}}{I_{int}} \tag{2}
\end{equation}

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where $\alpha$ is the angular acceleration and $I_{int}$ is the moment of inertia of the structure. In a “walking” design, one side of the structure experiences static friction (and does not move) and the other side experiences kinetic friction. This basic operation is shown in Figure 1. These additional forces will both modify the torque in the above equation and result in a net propulsion force. The torque about the center of mass becomes

$$\tau_{tot} = \frac{L}{2} [(I_c LB - \mu_s N) + (I_c LB - \mu_k N)]$$

$$= I_c L^2 B - \frac{L}{2} (\mu_s N + \mu_k N)$$

(3)

where $\mu_s$ denotes the coefficient of static friction, $\mu_k$ denotes the coefficient of kinetic friction, and $N$ denotes the normal force. Summing these forces gives the net forward force, which is

$$F = (I_c LB - \mu_k N) - (I_c LB - \mu_s N) = N (\mu_s - \mu_k).$$

(4)

This is valid for small angles of rotation, which corresponds to rapid vibrations of the structure.

![Magnetic Field](image)

**Figure 1:** Current loop experiencing asymmetric resistance forces

The operation of the device in a fluid applies the same principles with fluid drag instead of friction. However, drag will be velocity dependent and therefore coupled to the acceleration. Additionally, drag has fundamentally different behavior as the Reynolds number of the fluid flow changes. To further increase the complexity, the drag is highly shape dependent, especially as the Reynolds number increases. The proposed design takes advantage of this shape dependence to create asymmetries in the drag forces, so these effects cannot be neglected. Due to the complexity, developing an accurate theoretical model is not practical. However, it is possible to derive formulas that provide useful insight into the behavior. If the Reynolds number is high enough, the drag force $D$ is given by

$$D = \frac{1}{2} \rho v^2 C_D A$$

(5)

where $\rho$ is the fluid density, $v$ is the velocity, $A$ is the frontal area, and $C_D$ is a shape factor. Because the device will be rotating, the linear velocity is proportional to the angular velocity and the distance from the pivot point. Once the device begins moving, the velocity will be the sum of the forward component and the component due to the rotation. For simplicity, the forward component of the velocity will not be included in these calculations; there is no effective way of estimating this velocity without simulation. To estimate the total drag force and torque, the formula for drag can be evaluated as an integral along the length of the device. Therefore, the drag force on one half of the device is

$$D = \int_0^{L/2} \frac{1}{2} \rho v^2 (x) C_D h dx = \frac{1}{2} \rho C_D h \int_0^{L/2} (\omega x)^2 dx$$

$$= \frac{1}{48} \rho C_D h L^3 \omega^2$$

(6)

where $\omega$ is the angular velocity and $h$ is the height, implying that $h \cdot dx$ is an area element. For a general shape, $h$ would be a function of $x$ as well, but the above formula assumes height is constant. This formula suggests that any difference in drag is encapsulated in the shape factor, $C_D$. Also, it is clear that the magnitude of the force increases dramatically as size and angular velocity are increased. The total torque on one half of the device can be estimated in a similar way:

$$\tau_d = \int_0^{L/2} \frac{1}{2} \rho (\omega x)^2 C_D x h dx = \frac{1}{128} C_D \rho h L^4 \omega^2.$$ 

(7)

It is important to understand the fluid torque as it will directly impact the rotational acceleration of the device. The actual fluid torque is much more difficult to predict for an asymmetric shape because the asymmetry alters the location of the center of mass. While the above formula does offer a basic understanding of the important parameters, it will not be effective in predicting the torque on an arbitrary shape.

Many assumptions and approximations have been made in this analysis, and as a result it may have significant deviations from the behavior of the actual design. However, the above equations do offer insight into the relative sensitivity of the design parameters and to the effects of scaling to smaller sizes. Assuming that $C_D$ describes the asymmetry, the propulsion force will be proportional to the drag force. Taking $\omega = \alpha t/4$ (the average angular velocity in half of a cycle) and recognizing that $t = \sqrt{2\alpha \theta}/\theta$ if $\alpha$ is constant, an expression can be written for the propulsion force:

$$F_p \propto h L^3 \omega^2 = h L^3 \left( \frac{1}{2} \sqrt{\frac{\alpha \theta}{2}} \right)^2 = h L^3 \frac{\alpha \theta \tau_{rem}}{8 I_{int}}.$$ 

(8)

In the above formula, $t$ represents the time interval of switching the current in the loop, and $\theta$ represents the angle of rotation in this interval. All analyses up to this point make use of the small angle approximation, and so this formula is only valid when $\theta$ is small. The force is directly proportional to $\theta$, which means that it should be as large as possible while still satisfying the small angle criterion. Replacing the torque
with the electromagnetic torque, and realizing that \( I_m \propto L^5 \) and \( h \propto L \), the following expression can be written:

\[
F_p \propto L^4 \frac{\theta I_c L^2 B}{2L^5} \propto L.
\]  

(9)

This result shows that the propulsion force decreases linearly as a representative dimension of the device is scaled down, which is a promising result. This expression assumes that the electromagnetic torque dominates the drag torque, and this is generally true for the sizes and frequencies of interest. However, it also assumes high Reynolds drag, which may not be valid at small sizes. At lower Reynolds numbers, the drag is very difficult to analyze and differences in shape may not have as much of an effect. Additionally, less power will be available as the device becomes smaller, which may limit the available current. Nonetheless, the above result suggests this technique has potential for operating at very small scales.

### III. Simulation

For many reasons, the theoretical model cannot accurately describe the propulsion. Numerical simulations of the fluid mechanics are necessary to effectively evaluate the design. These simulations were performed with the incompressible Navier-Stokes equations in COMSOL. The structure was rotated in water and the forces exerted on it were calculated. This analysis was performed for an asymmetric shape intended to experience asymmetric drag. Two different sizes were compared to evaluate the scalability of the technique. The asymmetric shape and its force profile (denoted with arrows) are shown in the example fluid simulation in Figure 2. Note that it is not necessarily the optimal shape, but it does effectively demonstrate the concepts.

![Figure 2: Simulation of device and its force profile (denoted with arrows).](image)

The net propulsion force is shown in Figure 3 and the fluid torque is shown in Figure 4. To verify the effectiveness of the shape, a symmetric cube was subjected to the same analysis and experienced a net force of zero within the bounds of numerical precision, while the asymmetric shape experienced appreciable forces. As predicted, this propulsion force increases approximately proportional to \( \omega^2 \) for sufficiently high rotation frequencies, which suggests that the high Reynolds model is valid at this scale. Also, at high frequencies, changing the size by a factor of 10 results in a \( 10^5 \) change in force, which agrees with the derived formula for drag. As the frequency drops, there is slight deviation from these behaviors, especially for the smaller device. Additionally, the fluid torque is approximately proportional to \( \omega^2 \), which again agrees with theoretical predictions. The fluid drag torque is plotted in Figure 4. As a reference point, a 1-mA current in a 1-T field results in \( 10^{-9} \) N/m of torque for a 1-mm device and \( 10^{-11} \) N/m for a 0.1-mm device. When the electromagnetic torque is much larger than the fluid drag torque, the angular velocity will be limited by the moment of inertia and the switching time of the current loop rather than the drag. These results
agree with the predicted scaling of the device. Decreasing the size by a factor of 10 results in an increase of ω by a factor of $10^{3/2}$, and comparing the forces at these frequencies in Figure 1 shows roughly a factor of 10 change, as predicted.

With this information, a rough estimate of the thrust efficiency can be determined. Assuming that ±30° rotations are small angles (~15% error) and using the rotational inertia of a 1-mm cube, the required current to achieve a specific angular velocity can be computed. The angular velocity can be related to a thrust force via Figure 3, and from previous work this thrust can be related to velocity [8]. This calculation is shown in Figure 5 in a static magnetic field of .5 T, which can be generated with permanent magnets. With the required current, it is possible to estimate the power for a given velocity. For a mm-sized device, the current to achieve an average angular frequency of 10 Hz is about 10 mA. Assuming the loop of wire has a resistance of roughly 1 Ω, this results in a power consumption of about 100 μW. The thrust force at this frequency is approximately .3 μN, which corresponds to a steady-state velocity on the order of 1 cm/sec. To achieve equivalent results with a mechanical technique requires about 1 mW. This new method performs 10 times better using conservative estimates, highlighting the potential of this design and suggesting that miniature locomotive implants are possible.

IV. CONCLUSION

The purpose of this work is to develop a high efficiency propulsion system for sub-millimeter medical implants. The current methods are not adequate at the scale of interest for several reasons. Mechanical techniques require intricate structures and have low thrust efficiencies, resulting in high power consumption.Magnetically controlling passive implants requires complex equipment, and still suffer size and speed limitations. A consequence of these complications is that most medical implants have no propulsion systems, even in applications that would greatly benefit from one, such as GI tract imaging. In order to achieve sub-millimeter locomotive implants, a new technique is needed.

In past work [8], it was observed that a direct conversion from electrical energy to thrust through electromagnetic forces resulted in very high efficiency. However, there was not a known method for implementing a system that could harness its advantages. Electromagnetic forces are inherently symmetric so there is no net force on an isolated device. It is therefore necessary to introduce some form of asymmetry into the design, and the presented method accomplishes this by inducing asymmetrical drag forces. This method shows promising performance for use in locomotive implants while maintaining simplicity in its design. Additionally, the theoretical analysis predicts favorable scalability, and the simulations verify this claim. With roughly 100 μW, a mm-sized device is predicted to reach a speed of around 1cm/sec. Such a device would be useful in a variety of applications and could revolutionize many medical procedures.

This paper analyzes a simple shape, but it is not necessarily the optimal shape. Further improvement in the asymmetrical forces would directly improve the performance of the device. In addition, analysis can be done to optimize the switching frequency of the oscillations. Higher frequencies result in more frequent but shorter steps, while lower frequencies correspond to longer but slower steps. It is likely that there are optimal operating points that maximize efficiency, speed, or other parameters of interest. Future work will explore these possibilities and begin experimentation with real devices.

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