Multi-region active contours with a single level set function

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Contributions
- A method for multi-region image segmentation using active contours.
- Novel active contour evolution based on the Voronoi Implicit Interface Method (VIIM) [1].
- Level set formulation with a single level set function.
- Number of regions or their intensity statistics unknown a priori.
- Applicable with various region and boundary appearance models [2].

Active Contours
Goal: segment image I(x) into multiple regions \( \{ \Omega_i \} \).
Approach: region boundaries are modelled by a curve \( C \), minimizing an energy functional \( E(C) \).

\[
E(C) = E_{	ext{int}}(C, I) + \mu E_{	ext{ext}}(C, I)
\]

\( C_t = -\frac{\partial E}{\partial C} = F_n \)

\( C(t) = \{ x \mid \phi(x) = 0 \} \)

\( \phi \) implicitly as the zero level set of a 2D level set function \( \phi(x) \).

Implicit treatment of multi-point junctions: No gaps or overlaps!

Level Set Formulation
Define \( C \) implicitly as the zero level set of a 2D level set function \( \phi(x) \):

\[
C = \{ x \mid \phi(x) = 0 \}
\]

\( C_t = \nabla \phi \)

2-region segmentation:
- Object = \( \{ x \mid \phi(x) > 0 \} \)
- Background = \( \{ x \mid \phi(x) < 0 \} \)

Caveat: Extension to > 2 regions is non-trivial!

Voronoi Implicit Interface Method (VIIM)
A method for tracking multiple evolving regions using a single level set function

\[
\phi(x) > 0 \implies \text{unsigned distance from } C
\]

Observation: Motion of zero-level set corresponding to the interface \( C \) is bracketed by the motion of its surrounding \( \epsilon \)-level sets.

The algorithm
1. Define \( C(0) \) and \( C_t = -\frac{\partial E}{\partial C} = F_n \).
2. Extend \( F \) to \( \epsilon \)-level sets of \( C \), \( F_{\epsilon,n} \).
3. Evolve \( \epsilon \)-level sets of \( \phi(x) \) using the VIIM.
4. Reconstruct \( C \) from \( \epsilon \)-level sets. Return to Step (2).

Image appearance models
- Region competition model with geodesic active contours regularization

\[
E(C, \{ \Omega_i \}) = \sum_{i=1}^{M} \int_{\Omega_i} -\log P(I(x) | \Omega_i) + \frac{\mu}{2} \int_{\partial \Omega_i} g(C(x)) dx
\]

Special case: Piecewise constant model

\[
E_{\text{disc}}(C, \{ \epsilon_i \}) = \sum_{i=1}^{M} \int_{\Omega_i} (-\epsilon_i)^2 dx
\]

Pairwise dissimilarity model

\[
E_{\text{disc}}(C) = \int_{\Omega} \sum_{i,j, \Omega_i \cap \Omega_j \neq \emptyset} w(x,y) dxdy, \text{ where } w(x,y) = \text{Dissimilarity}(x,y).
\]

Multi-region Segmentation
The algorithm
1. Define \( C(0) \) and \( C_t = -\frac{\partial E}{\partial C} = F_n \).
2. Extend \( F \) to \( \epsilon \)-level sets of \( C \), \( F_{\epsilon,n} \).
3. Evolve \( \epsilon \)-level sets of \( \phi(x) \) using the VIIM.
4. Reconstruct \( C \) from \( \epsilon \)-level sets. Return to Step (2).

References

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