Non-rigid shape correspondence by matching semi-local spectral features and global geodesic structures

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Introduction

Correspondence detection in context of 3D shape processing

Shape processing: recognition, retrieval, morphing, similarity and self-similarity detection, *etc.*

The Stanford 3D scanning repository
TOSCA high resolution database
Introduction

Rigid vs. non-rigid world

Rigid transformations: rotation, translation, reflections

Non-rigid isometric transformations: Rigid transformations + bending + twisting, without stretching or tearing
Correspondence detection

- $X, Y$ — the shapes we want to match
- $\varphi : X \to Y$ — correspondence function:
  for each $x \in X$ its corresponding point is $\varphi(x)$
Introduction

Non-rigid correspondence detection: previous work

Matching by metric structure comparison

- Memoli’05
- Memoli’07
- Bronstein

Matching shape embeddings in some canonical domain

- Elad’03, Jain’07, Rustamov’07, Mateus’08, Lipman’09

Matching by computing set of rigid transformations

- Zhang’08, Huang’09

Hybrid approaches

- Leordeanu’05, Torresani ‘08, Hu’09, Thorstensen‘09

Similarity and symmetry detection

- Raviv’07, Ovsjanikov’08

Descriptor-based matching

The proposed method

- Sun‘08
- Zaharescu‘09
Problem formulation

Previous work: Close-up

Matching by metric structure comparison
- Anguelov’04
- Memoli’05
- Memoli’07
- Bronstein ‘06
- Tevs‘09
- Bronstein’09

Descriptor-based matching
- Zhang‘08
- Sun‘08
- Zaharescu‘09
Metric structure: definition

- With each \((x, \tilde{x}) \in X\) associate distance measure \(d_x : X \times X \to \mathbb{R}^+\)

- How to compare two metric spaces \((X, d_x)\) and \((Y, d_y)\)?
Problem formulation

Metric structure comparison: previous work

- Compare metrics using Gromov-Hausdorff distance (Gromov’81)

\[ d_{GH}(X, Y) = \inf_{f: X \to Z, g: Y \to Z} d^H_z(f(X), g(Y)) \]

- NP-hard combinatorial problem (MS) or not convex continuous formulation (BBK)

Find initialization using pointwise surface descriptors!
Metric structure comparison: proposed formulation

- Map $X$ onto $Y$

$$d_X(x, \tilde{x})$$

$$d_Y(y, \tilde{y}) = d_Y(\varphi(x), \varphi(\tilde{x}))$$

- The mapping distortion:

$$\text{Distortion}(\varphi) = \sum_{x, \tilde{x} \in X} |d_X(x, \tilde{x}) - d_Y(\varphi(x), \varphi(\tilde{x}))|$$
Problem formulation

Descriptor-based matching

- Zhang’08, Sun’08, Zharescu’09

- Descriptor examples: texture, curvature, Mesh HoG, Heat Kernel Signatures, GPS

- Pointwise distortion:

\[ \text{Distortion}(\varphi) = \sum_{x \in X} \left\| f^X(x) - f^Y(\varphi(x)) \right\| \]
Metric structure vs. descriptor-based matching

• Metric structure-based matching:
  – Preserves global pairwise structure 😊
  – Requires good initialization 😞

• Descriptor-based matching:
  – Fast 😊
  – Local, does not preserve global structure 😞

Combine into one distortion measure:

\[
\text{Distortion}(\varphi) = \sum_{x \in X} \left| f^X(x) - f^Y(\varphi(x)) \right| + \alpha \cdot \sum_{x, \tilde{x} \in X} \left| d_x(x, \tilde{x}) - d_y(\varphi(x), \varphi(\tilde{x})) \right|
\]

Berg’05, Torresani’08, Hu’09, Bronstein’10
The proposed framework: Discrete setting

Problem formulation

- Represent correspondence by a binary matrix $P$

$$P_{ij} = \begin{cases} 
1, & y_j \text{ corresponds to } x_i \\
0, & \text{otherwise}
\end{cases}$$
The proposed framework: Discrete setting

\[ Distortion(\varphi) = \sum_{x \in X} \| f^X(x) - f^Y(\varphi(x)) \| + \alpha \cdot \sum_{x, \tilde{x} \in X} |d_X(x, \tilde{x}) - d_Y(\varphi(x), \varphi(\tilde{x}))| \]

\[ Distortion(P) = \sum_{x_i \in X, y_j \in Y} \| f^X(x_i) - f^Y(y_j) \| P_{ij} + \alpha \cdot \sum_{x_i, x_m \in X, y_j, y_n \in Y} |d_X(x_i, x_m) - d_Y(y_j, y_n)| P_{ij} P_{mn} \]

Denote each correspondence \((i, j)\) by single index \(k\):

\[ P_{ij} \rightarrow p_k \]
\[ \| f^X(x_i) - f^Y(y_j) \| \rightarrow b_k \]
\[ |d_X(x_i, x_m) - d_Y(y_j, y_n)| \rightarrow Q_{kl} \]

Integer Quadratic Programming

\[ Distortion(p) = b^T p + \alpha \cdot p^T Q p \]
The proposed framework: Schematic view

Problem formulation

Descriptors

Distance measure

Application dependent – can be used with any descriptors and metric!

We need **isometry invariant** features and distance measure

Combined distortion measure

\[ \text{Distortion}(p) = b^T p + \alpha \cdot p^T Qp \]

Minimize to obtain the correspondence

\[ P^* \left( \varphi^* : X \rightarrow Y \right) \]

\[ s.t. \quad S p = 1 \]
Problem formulation

**Distance measure**

- Geodesic distance

\[ d_X(x, \tilde{x}) \]

- Other distance measures: diffusion distance (Berard’94, Coifman’06), commute time distance (Qui’07)
Isometry invariant surface descriptors

- Use the Laplace-Beltrami operator $\Delta_M$ (generalization of the Laplacian for surfaces)
- Defined by intrinsic geometry of $M \Rightarrow$ Isometry invariant
- $\Delta_M$ admits orthogonal discrete eigendecomposition $\Delta_M \phi = \lambda \phi$

$X$

$Y$

$0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq ...$
The proposed descriptors

- Similar to the GPS signature (Rustamov’07), we use

\[
\begin{align*}
  f^X(x) &= \left[\phi^X_1(x), \phi^X_2(x), \ldots, \phi^X_K(x)\right], \\
  &\quad \text{with } 0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_K
\end{align*}
\]

\[
f^X(x) \in \mathbb{R}^K
\]

Eigenfunctions are defined up to a sign:
(Jain’07, Mateus’08, Ovsjanikov’08)

\[
\begin{align*}
  f^Y(y) &= \left[s_1 \phi^Y_1(y), s_2 \phi^Y_2(y), \ldots, s_K \phi^Y_K(y)\right], \\
  &\quad \text{with } s \in \{+,-\}^K
\end{align*}
\]
The proposed descriptors: properties

Ovsjanikov et al. ’08: …An intrinsic symmetry of an object induces an extrinsic symmetry (either rotation or reflection) of its GPS embedding. …

Therefore: intrinsic symmetry of $X$ implies extrinsic symmetry of eigenfunctions (corresponding to eigenvalues with unit multiplicity)

$$\phi^X(x) = \phi^X(\tilde{x}) \quad \text{or} \quad \phi^X(x) = -\phi^X(\tilde{x}) \quad \tilde{x} = \text{symm}(x)$$
How many sign sequences are there?

Primary correspondence

Symmetric correspondence

We want to estimate all possible sign sequences
Sign sequence estimation algorithm

I. Find mixed set of correspondences: both primary and symmetrical

II. Cluster correspondences into (# of symmetries + 1) groups

III. Find sign sequence induced by each cluster

\[ s^{(1)} = [-, -, +] \]

\[ s^{(2)} = [-, +, -] \]
The proposed framework

Descriptors based on the Laplace-Beltrami operator

\[ f^X(x), \{ f^{Y,(j)}(y) \} \text{j, } \forall x \in X, y \in Y \]

Geodesic distance (can’t distinguish between different correspondences)

\[ d_x(x, \tilde{x}), d_y(y, \tilde{y}), \]

\[ \forall (x, \tilde{x}) \in X \times X, (y, \tilde{y}) \in Y \times Y \]

Combined distortion measure

\[ \text{Distortion}(p) = b^T p + \alpha \cdot p^T Q p \]

Minimize to obtain the correspondence

\[ P^* \left( \varphi^* : X \rightarrow Y \right) \]

\[ s.t. \quad S p = 1 \]
Integer Quadratic Programming

- NP-hard
- We approximate its solution using MIQP solver by Bemporad’04
- Complexity: exponential in number of correspondences

<table>
<thead>
<tr>
<th>Model</th>
<th>#vertices</th>
<th>Total</th>
<th>IQP</th>
<th>IQP (%)</th>
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<tbody>
<tr>
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<td>256.7</td>
<td>222.3</td>
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<td>hand</td>
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<td>142.3</td>
<td>110.9</td>
<td>78.0</td>
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</tbody>
</table>

Calculation time, in seconds, for 20 correspondences between different models from the TOSCA database, on a laptop with Intel Core 2 Duo T7500 processor and 2GBytes memory.

- Future research: reduce complexity by exploring surface connectivity information
Results

Test cases

- Rigid transformations $\rightarrow$ 100% accuracy
- Approximately isometric transformations
- Intrinsically symmetry shapes
- Different sampling and triangulation
- Scaling
- SHREC database
Different sampling and triangulation

Original (53K) vs. 25K

Original (53K) vs. 5K
Results

Approximately isometric transformations
Results

Scaling
## Results

### SHREC database

<table>
<thead>
<tr>
<th>Transform.</th>
<th>1</th>
<th>≤2</th>
<th>≤3</th>
<th>≤4</th>
<th>≤5</th>
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<tbody>
<tr>
<td>Isometry</td>
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<td>12.55</td>
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<td>11.41</td>
<td>12.78</td>
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<td>Topology</td>
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<td>17.47</td>
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<td>18.21</td>
<td>22.99</td>
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<td>19.01</td>
<td>23.22</td>
<td>23.88</td>
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</table>
Summary

• We proposed a framework for shape matching based on:
  – Metric structures
  – Pointwise surface descriptors

• Formulated matching as Integer Quadratic Programming

• Showed that intrinsic symmetries imply existence of more than one possible correspondence

• Showed how to find all those correspondences

• Future research: efficient optimization, other descriptors and metrics
Thank you!

Questions…