Quantum Field Theory → Second Quantization

Quantum Quench Dynamics

In this problem, you will explore the dynamics of a quantum quench, where the parameters of the quantum Hamiltonian and/or action are suddenly changed. In general, this has very interesting effects on the quantum dynamics of a system: you will look at a very simple case in this problem.

We begin by considering the harmonic oscillator given by Hamiltonian

\[ H = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}. \]

Do not yet assume there is a quench: this is the simple harmonic oscillator you are familiar with.

(a) Show that the following is an operator relation for the harmonic oscillator:

\[ x(t) = x(0) \cos(\omega t) + \frac{p(0)}{\omega} \sin(\omega t). \]

Now, let us perform a quantum quench on this system, by replacing \( \omega \) with \( \omega(t) \) as follows:

\[ \omega(t) = \begin{cases} \omega_0 & t < 0 \\ \omega & t \geq 0 \end{cases}. \]

(b) Let |0⟩ correspond to the vacuum state of the harmonic oscillator for \( t < 0 \). Show that if \( t_2 \geq t_1 > 0 \), you can use the result you found above to find with a bit of algebra that:

\[ \langle 0 | x(t_2) x(t_1) | 0 \rangle = \frac{e^{-i\omega(t_2-t_1)}}{2\omega} + \frac{(\omega^2 - \omega_0^2) \cos(\omega(t_2 + t_1)) + (\omega - \omega_0)^2 \cos(\omega(t_2 - t_1))}{4\omega^2\omega_0}. \]

Comment on the result.

Now, let’s consider the following real scalar field theory:

\[ \mathcal{L} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\partial_i \phi)^2 - \frac{1}{2} m(t)^2 \phi^2 \]

with the time dependent mass term

\[ m(t) = \begin{cases} m & t < 0 \\ 0 & t \geq 0 \end{cases}. \]

This corresponds to a scalar field where we suddenly quench the mass and turn it off. Just as in the case of a single oscillator, we expect that the switch of what we mean by the vacuum will cause a spontaneous excitation of fields. However, can we see nontrivial correlation functions of these fields arise after the quench? The answer is almost certainly yes, and you will now show a simple example of this.

(c) Working in 3 spatial dimensions, evaluate the correlator \( \langle 0 | \phi(r,t) \phi(0,t) | 0 \rangle \), where the \( \phi \) fields are evaluated in position space. The two fields are at the same time \( t > 0 \), but separated by a spatial distance \( r \). To do this, expand \( \phi(x,t) \) in terms of creation/annihilation operators for each Fourier mode. Show that in the limit that \( mt \gg 1 \), the correlator becomes approximately

\[ \langle 0 | \phi(r,t) \phi(0,t) | 0 \rangle = \begin{cases} \frac{m}{16\pi r} & 2r < t \\ 0 & 2r > t \end{cases}. \]

(d) Give a physical interpretation of why there is a qualitative change at \( t = 2r \).

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1To do this, determine \( a^\dagger(t) = U(t)a^\dagger U(t) \) and \( a(t) \) defined similarly. To do that, I would consider evolving the state \( a^\dagger |\psi\rangle \) (for any \( |\psi\rangle \)) in time.