Abstract—A single antenna transmitter is considered with a large number of antennas at the receiver. We propose a simple noncoherent communication scheme based on energy detection that does not require knowledge of instantaneous CSI (channel state information) at either the transmitter or the receiver. We show that the performance of this scheme is the same, in a scaling law sense, as that of the optimal scheme exploiting channel knowledge. We present a constellation design based on the minimum distance criterion and present numerical results to demonstrate the performance of our scheme for representative fading and noise statistics.

Index Terms—Massive MIMO, Non Coherent Communication, Energy Receiver

I. INTRODUCTION

The use of higher and higher frequencies in the design of wireless systems today has made feasible ever increasing numbers of transmitter and receiver antennas. The ever increasing number of antennas at the receiver makes significant performance gains possible [2]. This may be in terms of array (or beamforming) gain or in terms of diversity gain due to large number of independent channel realizations at the receiver. However, as described in [2], a significant challenge in the implementation of such systems is the need for accurate channel state information (CSI) [3]. CSI estimation, even for moderately large multiantenna systems (such as LTE), can take up a significant part of communication resources (≈ 15% in [4]). This problem becomes even more acute as the number of channel parameters that need to be estimated increases. In multiuser systems this commonly manifests itself as the pilot contamination problem which significantly affects TDD operation [2].

In this work, we explore alternate ways of transmission and decoding of information without the use of channel state information at either the transmitter or the receiver. We do assume the statistics to be known and that the channel realizations are independent and identically distributed. A fundamental contribution towards this system model is the notion of unitarily invariant codes. Developed in [6] and [7], this scheme describes a block fading channel model and looks at space-time coding over the Grassman manifold associated with the channel matrix. Fundamental limits on the rate of transmission of information are known in the high SNR regime. Subsequent work shows that Grassman manifold signalling can be effective in multiuser systems also [8], [9]. A fundamental feature of this line of work however is the use of high SNR approximations to derive analytical insights.

In our work we consider the regime of a large number of receiver antennas, with finite transmit power. We consider simple energy based single shot transmission and decoding schemes and show that, for a single user, they can achieve rates which are no different from coherent schemes (i.e. schemes with perfect CSIT and CSIR) in a scaling law sense with an increasing number of antennas.

The rest of the paper is organized as follows. We present the system model in Section II, and then describe and contrast our approach with that of other approaches in the literature in Section III. Sections IV, V contain details of the performance of our achievable scheme. The asymptotic behaviour of our scheme is compared with that of the optimal coherent scheme in Section VII. Finally we conclude with numerical evaluations of our proposed schemes in Section VIII.

II. SYSTEM MODEL

We have one single antenna transmitter and one receiver with $n$ antennas. Mathematically

$$y = hx + \nu$$

with $y \in \mathbb{C}^{n \times 1}$, $\nu \in \mathbb{C}^{n \times 1}$ and each $h_i \sim \mathcal{CN}(\mu, \sigma_h^2)$ and $\nu_i \sim \mathcal{CN}(0, \sigma^2)$. We assume that $|\mu|^2 = \frac{K}{K+1}$ and $\sigma_h^2 = \frac{1}{K+1}$ for some known $K > 0$ (this is the Rician fading model with $E[|h_i|^2] = 1$ [10]). We mention that most of the insights presented in this paper are valid for more general statistics also; we point out the
relevant links in Subsection V-B. We assume that the instantaneous channel realization is unknown to both the transmitter and the receiver and investigate energy based detection schemes for recovering transmitter data, based only on the knowledge of the statistics of the system, i.e., the parameters $K$ and $\sigma^2$. We also assume that every transmission will be associated with an independent channel realization. Thus if $T_c$ denotes the coherence time of the channel, we assume that $T_s = T_c$, where $T_s$ is the symbol time.

We also point out here that in most of this paper we focus on single shot communication, i.e., if $N$ is the size of block length used for communication, most of the discussion in the paper will be for $N = 1$. However, by going to higher block lengths we do see strict benefits. We present an example for $N = 3$ in Subsection VI-C.

We focus on the following encoding and decoding procedure. The transmitter transmits symbols from the constellation $\{\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_L}\}$, subject to an average power constraint

$$\frac{1}{L} \sum_{i=1}^{L} p_i \leq 1. \tag{2}$$

Here $p_i (\in \mathbb{R}^+)$ are the energy levels and $L$ is the constellation size. The above inequalities imply that $p_i \leq L$. Note that the phase of the particular symbols that the transmitter chooses to transmit do not matter, since the receiver uses energy measurements. Keeping this in mind, we define the following equivalent constellation containing the power levels $\mathcal{P} = \{p_1, p_2, \ldots, p_L\}$. The decoder computes

$$\frac{||y||^2}{n} \in \mathbb{R}^+, \quad \text{i.e., it estimates the average received power across all the antennas. Based on its knowledge of the statistics of the channel, it divides the positive real line into non-intersecting intervals or decoding regions, } \mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_L, \quad \text{corresponding to each of the transmitted symbols } \{p_i\}_{i=1}^L \text{ and uses the following decoder:}$$

$$\hat{i} : \left\{ i \in \{1, 2, \ldots, L\} : \frac{||y||^2}{n} \in \mathcal{I}_i \right\}.$$ 

Then the probability of symbol error, when the transmitter transmits at power $p_i$, is defined as

$$P_e(p_i) \triangleq \Pr\{\hat{i} \neq i\}.$$

III. RELATED WORK

An influential work [2] which pointed out the benefits of large numbers of receiver antennas uses the fact that a large number of independent realizations affords many potential benefits to system design. Such benefits include larger beamforming gain, simpler transmission and reception strategies, simpler receiver architecture enabling low cost receivers etc. However, this assumes perfect channel state information in most cases. This is often a bottleneck in many high speed systems [1], [11]. In fact, the channel estimation overhead manifests itself in problems like pilot contamination and performance degradation due to incorrect CSI [1]. Another line of work which examines the efficiency of communication schemes without CSI (i.e., noncoherent communication) often exploit simplifications afforded either by spatial diversity (i.e. Multiple input and multiple output systems) or high SNR (the information theoretic scaling law for the capacity of the noncoherent channel) [7], [6]. While insights from the asymptotic regimes of high SNR and in the limit of large numbers of coherent receiver antennas are critical in helping us characterize the fundamental limits of such systems, the transmission and decoding schemes may involve very high overhead (channel estimation overhead for coherent Massive MIMO and encoding and decoding overhead for Grassman manifold signalling).

In contrast to the work mentioned above, our results hold for finite SNR in the limit of a large number $n$ of receiver antennas. We show that with our energy detection based noncoherent transmission and decoding scheme, large spatial diversity is sufficient to achieve gains very close in a scaling law sense to what would be achieved using a coherent system.

IV. SIMPLIFIED SYSTEM MODEL

We write down an equivalent expression for the statistic computed by the receiver. We have

$$\frac{||y||^2}{n} = \frac{||\mathbf{h} \sqrt{p} + \nu||^2}{n} = \frac{||\mathbf{h}||^2}{n} + \frac{||\nu||^2}{n} + 2 \frac{\text{Re}(\mathbf{h}^* \nu)}{n}. \sqrt{p}.$$ 

We now observe that, in the limit of large $n$, the following results hold from the law of large numbers and the central limit theorem:

$$\frac{||\mathbf{h}||^2}{n} \xrightarrow{n \to \infty} 1, \text{ almost surely,}$$

$$\sqrt{n} \frac{||\nu||^2}{\sigma^2} \xrightarrow{n \to \infty} \sigma^2, \text{ almost surely,}$$

$$\frac{||\mathbf{h}||^2 - n}{\sqrt{n}} \xrightarrow{n \to \infty} \mathcal{N}(0, c_1) \text{ in distribution for some } c_1 > 0,$$

$$\frac{||\nu||^2 - \sigma^2 n}{\sqrt{n}} \xrightarrow{n \to \infty} \mathcal{N}(0, c_2) \text{ in distribution for some } c_2 > 0,$$

$$\frac{2 \text{Re}(\mathbf{h}^* \nu)}{\sqrt{n}} \xrightarrow{n \to \infty} \mathcal{N}(0, c_3) \text{ for some } c_3 > 0.$$
With the above observations, we have the following model
\[
\hat{y} = \left(1 + \frac{\hat{\nu}_1}{\sqrt{n}}\right) p + \sigma^2 + \frac{\hat{\nu}_2}{\sqrt{n}} + \frac{\hat{\nu}_3}{\sqrt{n}} \sqrt{p} \tag{3}
\]
for a large enough \(n\), where \(\hat{\nu}_1, \hat{\nu}_2, \hat{\nu}_3\) approach Gaussian random variables with constant (but possibly different) variances in the limit. We now look into a more precise characterization of the noise appearing into our system. We note that such a “noise”, i.e., factors causing deviations from the value \(p + \sigma^2\), is due to the deviations of the empirical averages like \(\|h\|^2/n, \|\nu\|^2/n\), from their respective ensemble means. We characterize this next by bounding the tail probabilities of the empirical means.

V. Noise characterization and Performance Analysis

We describe in Subsection V-A the bounds for the model described in Section II, and then show how they may be generalized for more general statistics in Subsection V-B. Before that we start with a lemma [12]:

**Lemma 1.** For any \(d > 0\) and zero mean, i.i.d. random variables \(u_1, \ldots, u_n\), we have that \(P \left(\sum_{i=1}^n u_i \leq -d\right) \leq e^{-nI_{2, \tilde{p}}(d)}\), where \(I(d) = \sup_{\theta > 0} (\theta d - \log(E[e^{\theta U}]))\).

We now describe how this lemma may be used to analyze the performance of our system.

A. Rician channel with additive white Gaussian noise

Applying the above lemma in our system model we get that
\[
\frac{1}{n} \sum_{i=1}^n \frac{|y_i|^2}{\bar{p}} - p = z_{re}^2 + z_{im}^2 - \bar{p} - p, \tag{4}
\]
where \(y_i = h_i \sqrt{\bar{p}} + u_i\), \(z_{re} = \text{Re}(\sqrt{\bar{p}} h_i + u_i)\), \(z_{im} = \text{Im}(\sqrt{\bar{p}} h_i + u_i)\), \(z_{re} + i z_{im} \sim \mathcal{CN}(0, p + \sigma^2)\) and \(p\) stands for the power level transmitted. Using the moment generating function of \(|y_i|^2\) we get
\[
E[e^{\theta U}] = E[e^{\theta |y|^2}]e^{-\theta(p + \sigma^2)} = \frac{1 + K}{1 - \bar{p} - K(1 + \theta \sigma^2)}e^{\bar{p}K(1 + \theta \sigma^2)},
\]
where \(\bar{p} = p + \sigma^2\), and \(0 < \theta < \frac{1 + K}{\bar{p} + K \sigma^2}\). This gives us
\[
P \left(\frac{\sum_{i=1}^n u_i}{n} \leq -d\right) \leq e^{-nI_{1, \tilde{p}}(d)},
\]
where
\[
I_{1, \tilde{p}}(d) = \sup_{\theta > 0} (\theta d - \log(E[e^{\theta |y|^2}]))\]
By noting that
\[
\frac{|y|^2}{n} = \frac{\sum_{i=1}^n u_i}{n},
\]
and by using the lemma again for the random variable \(-U\), we get that
\[
P \left(\frac{\sum_{i=1}^n u_i}{n} \leq -d\right) \leq e^{-nI_{2, \tilde{p}}(d)},
\]
where \(I_{2, \tilde{p}}(d) = \sup_{\theta > 0} (\theta d - \log(E[e^{-\theta U}]))\), where
\[
E[e^{-\theta U}] = \frac{1 + K}{1 + K + \theta p + \theta K \sigma^2}e^{-\frac{\theta p + \theta K \sigma^2(1 + \theta p)}{1 + K + \theta p + \theta K \sigma^2}}.\]
Defining
\[
I_{\tilde{p}}(d) = \min(I_{1, \tilde{p}}(d), I_{2, \tilde{p}}(d)),
\]
by an union bound we get that
\[
P \left(\frac{\sum_{i=1}^n u_i}{n} \leq -d\right) \leq e^{-nI_{1, \tilde{p}}(d)} + e^{-nI_{2, \tilde{p}}(d)} \leq 2e^{-nI_{\tilde{p}}(d)}.
\]

We plot the function \(I_{\tilde{p}}(d)\) for some values of the parameter \(d\). We assume \(p = 1, \sigma^2 = 1\) (thus \(\tilde{p} = 2\)) in the following plot.

![Graph](image)

We note that the probability of the absolute value of the noise being larger than a certain threshold \(d\) will vanish exponentially with the number of antennas \(n\) with a positive rate exponent of at least \(I_{\tilde{p}}(d)\) (since \(I_{\tilde{p}}(d) > 0\) for \(d > 0\)). It can also be seen (and rigorously established) that \(I_{\tilde{p}}(d)\) is monotonically increasing with \(d\). Note also how \(I_{\tilde{p}}(d)\) increases with increasing \(K\) (the line of sight component of the Rician fading).

B. Extensions for general statistics

We now indicate how the results change if the channel and noise statistics change such that \(u_i\) (defined analogous to (4)) is any zero mean random variable with finite moment generating function \(E[e^{\theta U}]\) for all \(\theta\) in an open interval around 0. Specifically, we focus on the rate function \(I_{\tilde{p}}(d)\) in Lemma 1. We argue in this section that
for a small $d$, the rate function has a similar form for very general channel and noise statistics. To see this, we note that for small $d$, the supremum of $\theta d - \log(E[e^{\theta U}])$ is also attained at small values of $\theta$. For small $\theta$, we have that

$$\theta d - \log(E[e^{\theta U}]) = \theta d - \log(1 + \theta^2 E[U^2]/2 + o(\theta^2)) \approx \theta d - \theta^2 E[U^2]/2.$$  

This expression is maximized for $\theta = \frac{d}{E[U^2]}$. The corresponding value of $I_p(d)$ is

$$I_p(d) \approx \frac{d^2}{2E[U^2]},$$

which depends on the statistics of $U$ only through $E[U^2]$. $I_p(d)$ depends on the transmitted symbol $p$ and the noise statistics.

The above ideas can be rigorously stated as follows:

**Lemma 2.** The rate function for any arbitrary $U$ satisfies

$$\lim_{d \to 0} \frac{I_p(d)}{d^2} = \frac{1}{2E[U^2]},$$

Note again that $u_i = |h_i \sqrt{p} + \nu_i|^2 - p - \sigma^2$. Due to space constraints, the proof is omitted.

**VI. Constellation Design Problem**

We now indicate how the above insights about the rate function can be used to design transmission and decoding schemes.

**A. Probability of Symbol Error Minimization**

Let’s assume that the user transmits all available power levels $P$ with equal probability. The constellation design problem for minimum average symbol error is

$$\text{minimize}_{P, I_1, \ldots, I_L} P_e \triangleq \frac{1}{L} \sum_{i=1}^L P_i(p_i)$$

subject to $\frac{1}{L} \sum_{i=1}^L p_i = 1, 0 \leq p_i, \forall i.$

The above problem is in general hard. Yet, we can relax the problem by using the insights presented in Subsection V-B.

**B. Constellation Design**

To simplify the constellation design problem, we consider minimum distance decoding. While this may not be optimal in terms of maximizing the error exponent, a high minimum distance does correspond to a good error exponent. This is because of the following:

$$\min_P I_p(d) \approx \min_P \frac{d^2}{2E[U^2]} = \frac{d^2}{2\alpha},$$

where $\alpha \triangleq \max_{\bar{p}} E[U^2]/\bar{p}$.

For the Rician fading model considered in Section II, we have

$$E[U^2] = \sigma^4 + 2p \sigma^2 + \frac{(1 + 2K)p^2}{(1 + K)^2}.$$  

Thus, we see that the minimum distance criterion approaches the optimal error exponent criterion in the limit of large noise variance $\sigma^2$, since for $\sigma^2$ large enough $E[U^2] \approx \sigma^4$. Using this criterion, it follows from the total power constraint (2) that the neighboring points should be separated by

$$d_{\text{min}} \triangleq \frac{2}{L-1}.$$  

Then, the power levels are $p_1 = 0$, $p_2 = \frac{2}{L-1}$, $p_k = \frac{2(k-1)}{L-1}$, $p_L = 2$. At the decoder let us also use the decoding intervals: $I_1 = (0, d_{\text{min}}/2)$, $I_2 = (d_{\text{min}}/2, 3d_{\text{min}}/2)$, $I_L = [2 - d_{\text{min}}/2, \infty)$. Then, defining

$$\tilde{p}_i \triangleq p_i + \sigma^2,$$

we observe that probability of symbol error is upper bounded as follows:

$$\frac{1}{L} \sum_{i=1}^L P_e(p_i) \leq U_L \leq 2e^{-n I_{1,2}\sigma^2(d_{\text{min}}/2)} \approx 2e^{-n d_{\text{min}}^2/(8\alpha)}.$$  

In the above,

$$U_L \triangleq \frac{1}{L} \left( e^{-n I_{1,2}\sigma^2(d_{\text{min}}/2)} + e^{-n I_{2,3}\sigma^2(d_{\text{min}}/2)} + e^{-n I_{L,1}\sigma^2(d_{\text{min}}/2)} + \cdots + e^{-n I_{L-1,L}\sigma^2(d_{\text{min}}/2)} + e^{-n I_{L,L+1}\sigma^2(d_{\text{min}}/2)} \right).$$

**C. Energy-Time Coding**

Until now we have assumed that the transmission is one shot, i.e., the transmitter transmits every coherence time $T_c$. In this subsection we investigate whether transmitting over $N$ coherence times can improve performance. In particular, we present an example where the performance is strictly improved by moving to $N = 3$.

We first describe the details of the system we consider. We keep the same Rician fading model that we introduced in Section II and assume that $N = 3$ and $\sigma^2 \gg 1$, i.e., the noise power is much greater than the average signal power $1$. This also makes $I_p(d)$ (approximately) independent of the constellation point transmitted, and makes the approach of Section VI-B approach the optimal solution, in terms of minimizing $P_e$. Let us also assume equiprobable signalling with power level $1$, and that $L = 2$, i.e., 1 bit is sent every coherence time $T_c = T_s$. If $\sqrt{\tilde{p}_1}, \sqrt{\tilde{p}_2}$, and $\sqrt{\tilde{p}_3}$ represent
the symbols transmitted in each of those time slots then the $P_e$ of the system is dominated by

$$P_e \approx e^{-ncd_{\text{min}}^2}, \quad (8)$$

where $c \approx \frac{1}{2nE[|h|^4]}$ is a constant depending on the channel and noise statistics. For no space time coding,

$$d_{\text{min}} = 2,$$

because the constellation points $(p_1, p_2, p_3)$ can take only $2^2 = 8$ possible values, which can be listed as

$$\mathcal{C} = \{0, 0, 0\}, \{0, 0, 2\}, \{0, 2, 0\}, \{0, 2, 2\}, \{0, 2, 2\}, \{2, 0, 2\}, \{0, 0, 0\}, \{0, 2, 2\}, \{2, 0, 2\}, \{2, 1, 2\}, \{2, 1, 2\}.$$

Thus, for $\sigma \gg p$, energy time coding helps us reduce the probability of symbol error from $e^{-4nc}$ to $e^{-2.18nc}$. Note that the latter may not be the best performance that can be achieved with a block code of length 3 and may be improved upon further by searching more exhaustively over constellation points. The empirical BER performance of the above codes have been presented in Section VIII (Fig. 3).

We now compare the achievable strategies discussed in this section with the optimal throughput with perfect channel state information at the transmitter and the receiver.

VII. ASYMPTOTIC THROUGHPUT CHARACTERIZATION

We first present the scaling behaviour of the ergodic capacity $C \triangleq E[\log(1 + ||h||^2/\sigma^2)]$ of the coherent channel. By the law of large numbers in the limit of large $n$, $||h||^2 \rightarrow 1$ with probability 1 as $n \rightarrow \infty$, thus giving us that $C \approx \log n$. We show that such a scaling behaviour can be achieved by the scheme presented in VI-B. In particular we show the following:

**Theorem 1.** There is a constellation which achieves a rate $K \log n$ per transmission for some $K > 0$ with vanishing probability of error with increasing number of receive antennas $n$.

**Proof:** We saw earlier that the minimum distance achieved with a constellation of size $L$ is given by $d_{\text{min}} = \frac{2}{L-1}$, and the corresponding symbol error probability $P_e$ is upper bounded by $P_e \leq e^{-ncd_{\text{min}}^2}$ (for large $L$, $c$ defined in (8)). By choosing $L = n^K$ for some $K < 0.5$, we see that one can achieve a rate of $\log(n^K) = K \log n$, with a probability of error $P_e$ that satisfies

$$P_e \leq e^{-4n^{(1-2K)c}}.$$

For large enough $n$ this becomes arbitrarily small if $K < 0.5$. This concludes the proof.

VIII. NUMERICAL RESULTS

In Fig. 1 and 2, we present simulation results comparing the performance of our constellations with the upper bounds derived in Section VI-B. We observe that increasing $K$, i.e., the line of sight component has significant benefits (in terms of the number of antennas to achieve a certain probability of error $P_e$) over the case when there is no line of sight.

![Fig. 1. Monte Carlo Estimate of Probability of Symbol Error ($P_e$) and corresponding analytical upper bound ($U_L$) for different number of receive antennas $n$ for Rayleigh Fading ($K = 0$) with $\sigma^2 = 1$.](image)

IX. CONCLUSION AND FUTURE WORK

In this work we presented a simple and novel non-coherent SIMO system with a large number of antennas at the receiver, characterized its performance analytically and via numerical simulations. The proposed system asks for a very simple encoding and decoding procedure, a receiver that only senses the received energy and knows just the statistics of the channel. By explicitly constructing a constellation, we show that the achieved throughput of the proposed system, in the limit of many received antennas, scales as if the receiver had perfect CSI.
In comparison with the coherent communication schemes, the proposed system thrives in the case of very fast time-varying channels since it exploits the huge number of antennas at the receiver by “averaging out” all the fast variations. This scheme could potentially be used in a millimeter wave communication system in which a strong LOS component exists, but the random and difficult to estimate NLOS components make coherent schemes impractical.

Future directions of research include:

1) Identification of the solution of the general optimal constellation design problem as presented in Subsection VI-A.
2) Performance characterization of a similar multiple access channel (MAC) noncoherent SIMO system in which the transmitters encode information in the transmitted energy levels, and the receiver decodes all of them simultaneously by just sensing the received energy levels.
3) Optimal energy time coding schemes.

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