Similarly

$$|(P_1^*\Psi_\alpha, xP_2\Psi_\alpha)| \le C_8 C_7^2(1+\alpha)^2 e^{2\alpha/\delta}\|\psi_\alpha\|^{-2} + \frac{1}{8}(1+\alpha)^2. \tag{B.53}$$

Finally,

$$|(x^{\frac{1}{2}}A\Psi_\alpha, x^{\frac{1}{2}}f_4\Psi_\alpha)| \le C_2' C_8(1+\alpha)^{\frac{1}{2}} \tag{B.54}$$

and analogously for f_5.

We now assume that $\operatorname{supp}\psi \cap \{p : \ x(p) \le \delta/4\}$ is not empty; soon we obtain a contradiction. Under this assumption

$$\|\psi_\alpha\|_{L^2_{sc}(X)} \ge e^{2\alpha/\delta}\|\psi\|_{L^2_{sc}(\{x\le\delta/2\})} \ge C_9 e^{2\alpha/\delta} \tag{B.55}$$

with $C_9 > 0$. Hence, our estimates above and (B.38), together with $|(\Psi_\alpha, V\Psi_\alpha)| \le \sup|V|$ show that

$$(\Psi_\alpha, \Delta\Psi_\alpha) \ge \alpha^2 - C_{10} - \alpha^2(C_{11}e^{-2\alpha/\delta} + \frac{1}{8}). \tag{B.56}$$

Similarly, from (B.39), using (B.40), $[A, V] \in L^\infty(X)$, and that the first term on the right hand side of (B.39) is negative, we have

$$(\Psi_\alpha, 2\Delta\Psi_\alpha) \le C_{12}(1+\alpha)^{3/2} + ((\alpha+\alpha^2) + (1+\alpha)^2)(C_{12}e^{-2\alpha/\delta} + \frac{1}{8}). \tag{B.57}$$

Thus, for sufficiently large α, (B.56) shows that

$$(\Psi_\alpha, \Delta\Psi_\alpha) \ge \frac{3}{4}\alpha^2, \tag{B.58}$$

while (B.57) implies for large α that

$$(\Psi_\alpha, \Delta\Psi_\alpha) \le \frac{1}{2}\alpha^2, \tag{B.59}$$

providing the contradiction. Hence, $\operatorname{supp}\psi$ is a compact subset of the interior of X. Then the standard Carleman-type unique continuation theorem [16, Theorem 17.2.1] implies that ψ vanishes identically as claimed. □

The absence of positive eigenvalues is just a combination of the previous two propositions. Thus, we have proved Theorem 17.6.

References

[1] A. Bommier. Propriétés de la matrice de diffusion, 2-amas 2-amas, pour les problèmes à N corps à longue portée. *Ann. Inst. Henri Poincaré*, 59:237–267, 1993.

[2] T. Christiansen. Scattering theory for perturbed stratified media. *Manuscript*, 1997.

[3] J. Dereziński. Asymptotic completeness of long-range N-body quantum systems. *Ann. Math.*, 138:427–476, 1993.

[4] V. Enss. *Completeness of three-body quantum scattering*, pages 68–83. Lecture notes in mathematics. Springer-Verlag, 1983.

[5] V. Enss. *Quantum scattering theory two- and three-body systems with potentials of short and long range*, pages 39–176. Lecture notes in mathematics. Springer-Verlag, 1985.

[6] L. D. Faddeev. *Mathematical aspects of the three-body problem in the quantum scattering theory*. Steklov Institute, 1963.

[7] R. G. Froese and I. Herbst. Exponential bounds and absence of positive eigenvalues of N-body Schrödinger operators. *Commun. Math. Phys.*, 87:429–447, 1982.

[8] R. G. Froese and I. Herbst. A new proof of the Mourre estimate. *Duke Math. J.*, 49:1075–1085, 1982.

[9] C. Gérard, H. Isozaki, and E. Skibsted. *Commutator algebra and resolvent estimates*, volume 23 of *Advanced studies in pure mathematics*, pages 69–82. 1994.

[10] G. M. Graf. Asymptotic completeness for N-body short range systems: a new proof. *Commun. Math. Phys.*, 132:73–101, 1990.
[11] A. Hassell. Scattering matrices for the quantum 3-body problem. *In preparation.*
[12] A. Hassell. Plane waves for the 3 body Schrödinger operator. *Manuscript*, 1996.
[13] B. Helffer and J. Sjöstrand. *Equation de Schrödinger avec champ magnétique et équation de Harper.*
[14] I. Herbst and E. Skibsted. Free channel Fourier transform in the long range N-body problem. *J. d'Analyse Math.*, 65:297–332, 1995.
[15] L. Hörmander. Fourier integral operators, I. *Acta Mathematica*, 127:79–183, 1971.
[16] L. Hörmander. *The analysis of linear partial differential operators,* vol. 1-4. Springer-Verlag, 1983.
[17] H. Isozaki. Structures of S-matrices for three body Schrödinger operators. *Commun. Math. Phys.*, 146:241–258, 1992.
[18] H. Isozaki. Asymptotic properties of generalized eigenfunctions for three body Schrödinger operators. *Commun. Math. Phys.*, 153:1–21, 1993.
[19] H. Isozaki. A generalization of the radiation condition of Sommerfeld for N-body Schrödinger operators. *Duke Math. J.*, 74:557–584, 1994.
[20] R. Mazzeo and R. B. Melrose. Pseudodifferential operators on manifolds with fibred boundaries. *In preparation.*
[21] R. B. Melrose. Calculus of conormal distributions on manifolds with corners. *International Mathematics Research Notices*, no. 3:51–61, 1992.
[22] R. B. Melrose. *Spectral and scattering theory for the Laplacian on asymptotically Euclidian spaces.* Marcel Dekker, 1994.
[23] R. B. Melrose. *Geometric scattering theory.* Cambridge University Press, 1995.
[24] R. B. Melrose and J. Sjöstrand. Singularities of boundary value problems. I. *Comm. Pure Appl. Math*, 31:593–617, 1978.
[25] R. B. Melrose and J. Sjöstrand. Singularities of boundary value problems. II. *Comm. Pure Appl. Math*, 35:129–168, 1982.
[26] R. B. Melrose and M. Zworski. Scattering metrics and geodesic flow at infinity. *Inventiones Mathematicae*, 124:389–436, 1996.
[27] E. Mourre. Absence of singular continuous spectrum of certain self-adjoint operators. *Commun. Math. Phys.*, 78:391–408, 1981.
[28] E. Mourre. Operateurs conjugés et propriétés de propagation. *Commun. Math. Phys.*, 91:279–300, 1983.
[29] P. Perry, I. M. Sigal, and B. Simon. Spectral analysis of N-body Schrödinger operators. *Ann. Math.*, 114:519–567, 1981.
[30] R. T. Seeley. Complex powers of an elliptic operator. *A.M.S. Symp. Pure Math.*, 10:288–307, 1967.
[31] I. M. Sigal and A. Soffer. N-particle scattering problem: asymptotic completeness for short range systems. *Ann. Math.*, 125:35–108, 1987.
[32] I. M. Sigal and A. Soffer. Asymptotic completeness of N-particle long-range scattering. *J. Amer. Math. Soc.*, 7:307–334, 1994.
[33] J. Sjöstrand. Propagation of analytic singularities for second order Dirichlet problems. *Comm. in PDEs*, 5:41–94, 1980.
[34] E. Skibsted. Smoothness of N-body scattering amplitudes. *Reviews in Math. Phys.*, 4:619–658, 1992.
[35] A. Vasy. Asymptotic behavior of generalized eigenfunctions in N-body scattering. *J. Func. Anal.*, 148:170–184, 1997.
[36] A. Vasy. Structure of the resolvent for three-body potentials. *Duke Math. J.*, to appear.
[37] J. Wunsch. Projections of distributions on manifolds with boundary. *Manuscript.*
[38] D. Yafaev. Radiation conditions and scattering theory for N-particle Hamiltonians. *Commun. Math. Phys.*, 154:523–554, 1993.

Department of Mathematics, MIT, Cambridge, MA 02139
Current address: Department of Mathematics, University of California, Berkeley, CA 94720
E-mail address: andras@math.berkeley.edu