A CORRECTION TO "PROPAGATION OF SINGULARITIES FOR THE WAVE EQUATION ON MANIFOLDS WITH CORNERS"

ANDRÁS VASY

There is a mistake in the proof of Proposition 7.3[1] of [1], namely a term was omitted in (7.9), so that the displayed equation after (7.15), as well as its analogues after (7.16) do not hold. The term omitted corresponds to the term $|x|^2$ in (7.8) being differentiated by the term $2A\xi \cdot \partial_x$ in the Hamilton vector field appearing in (6.3).

This mistake can be easily remedied as follows. First, after the displayed equation after (7.7) we specify one of the ρ_j slightly more carefully, namely we require

$$\rho_1 = 1 - \tau^{-2}|\zeta|_y^2;$$

note that $d\rho_1 \neq 0$ at q_0 for $\zeta \neq 0$ there. Then

$$|\tau^{-1}W^\flat \omega_0| \leq C_1' \omega_0^{1/2}(\omega_0^{1/2} + |t - t_0|)$$

still holds.

The argument of [1] proceeds with a motivational calculation, followed by the precise version of what is needed. We follow this approach here. So first the correct motivational calculation is presented.

We still have $p|_{x=0} = \tau^2 - |\xi|_y^2 - |\zeta|_y^2$. Thus, the equation after (7.9) can be strengthened to

$$\tau^{-2}|\xi|_y^2 \leq C(\tau^{-2}|p| + |x| + \omega_0^{1/2}),$$

i.e. with $|t - t_0|$ dropped, using that $|\rho_1| = |1 - \tau^{-2}|\zeta|_y^2| \leq \omega_0^{1/2}$. The analogue of (7.9) for ω_0 in place of ω still holds:

$$\begin{aligned}|\tau^{-1}H_p\omega_0| &\leq \tilde{C}_1'' \omega_0^{1/2}(\omega_0^{1/2} + |x| + |t - t_0| + \tau^{-2}|\xi|^2)\\ &\leq C_1'' \omega^{1/2}(\omega^{1/2} + |t - t_0| + \tau^{-2}|p|).\end{aligned}$$

But we also have (and this was the dropped expression)

$$|\tau^{-1}H_p|x|^2| \leq \tilde{C}_1'|x|\,(|x| + |\tau|^{-1}\,|\xi|) \leq C_1'\omega^{1/2}(\omega^{1/2} + (\tau^{-2}|p| + \omega^{1/2})^{1/2}).$$

Thus, the displayed equation after (7.15) becomes (at $p = 0$), with $C_1 = C_1' + C_1''$,

$$\begin{aligned}\tau^{-1}H_p\phi &= H_p(t - t_0) + \frac{1}{\epsilon^2\delta}H_p\omega\\ &\geq c_0/2 - \frac{1}{\epsilon^2\delta}C_1\omega^{1/2}(\omega^{1/2} + |t - t_0| + \omega^{1/4})\\ &\geq c_0/2 - 4C_1(\delta + \frac{\delta}{\epsilon} + (\frac{\delta}{\epsilon})^{1/2}) \geq c_0/4 > 0\end{aligned}$$

Date: December 1, 2008.

1991 *Mathematics Subject Classification.* 58J47, 35L20.

This work is partially supported by NSF grant DMS-0801226.

[1]Below all equation and proposition numbers of the form (7.xx) or 7.xx refer to [1].

provided that $\delta < \frac{c_0}{64C_1}$, $\frac{\epsilon}{\delta} > \max(\frac{64C_1}{c_0}, (\frac{64C_1}{c_0})^2)$, i.e. that δ is small, but ϵ/δ is not too small – roughly, ϵ can go to 0 at most proportionally to δ (with an appropriate constant) as $\delta \to 0$. The rest of the rough argument then goes through.

The precise version is similar. In (7.10) the estimate on the f_i term must be weakened:

$$\tau^{-1} H_p \omega = f_0 + \sum_i f_i \tau^{-1}\xi_i + \sum_{i,j} f_{ij}\tau^{-2}\xi_i\xi_j,$$
$$f_i, f_{ij} \in \mathcal{C}^\infty({}^{\mathrm{b}}T^*X),\ |f_i| \le C_1\omega^{1/2},\ |f_{ij}| \le C_1\omega^{1/2}$$

f_i, f_{ij} homogeneous of degree 0. This affects the estimates on r_i below (7.16):

$$|r_0| \le \frac{C_2}{\epsilon^2\delta}\omega^{1/2}(|t-t_0| + \omega^{1/2}),\ |\tau r_i| \le \frac{C_2}{\epsilon^2\delta}\omega^{1/2},\ |\tau^2 r_{ij}| \le \frac{C_2}{\epsilon^2\delta}\omega^{1/2},$$

and $\operatorname{supp} r_i$ lying in $\omega^{1/2} \le 3\epsilon\delta$, $|t-t_0| < 3\delta$. Thus,

$$|r_0| \le 3C_2(\delta + \frac{\delta}{\epsilon}),\ |\tau r_i| \le 3C_2\epsilon^{-1},\ |\tau^2 r_{ij}| \le 3C_2\epsilon^{-1}.$$

Thus, only the R_i term needs to be treated differently from [1]. We again let $T \in \Psi_{\mathrm{b}}^{-1}(X)$ be elliptic with principal symbol $|\tau|^{-1}$ near $\dot\Sigma$ (more precisely, on a neighborhood of $\operatorname{supp} a$), $T^- \in \Psi_{\mathrm{b}}^1(X)$ a parametrix, so $T^-T = \mathrm{Id} + F$, $F \in \Psi_{\mathrm{b}}^{-\infty}(X)$. Then there exists $R_i' \in \Psi_{\mathrm{b}}^{-1}(X)$ such that for any $\gamma > 0$,

$$\begin{aligned}\|R_i w\| = \|R_i(T^-T - F)w\| &\le \|(R_iT^-)(Tw)\| + \|R_iFw\| \\ &\le 6C_2\epsilon^{-1}\|Tw\| + \|R_i'Tw\| + \|R_iFw\|\end{aligned}$$

for all w with $Tw \in L^2(X)$, hence

$$\begin{aligned}|\langle R_i D_{x_i} v, v\rangle| \le& 6C_2\epsilon^{-1}\|TD_{x_i}v\|\,\|v\| \\ &+ 2\gamma\|v\|^2 + \gamma^{-1}\|R_i'TD_{x_i}v\|^2 + \gamma^{-1}\|F_iD_{x_i}v\|^2,\end{aligned}$$

with $F_i \in \Psi_{\mathrm{b}}^{-\infty}(X)$. Now we use that R_i is microlocalized in an $\epsilon\delta$-neighborhood of $\mathcal{G}$, rather than merely a δ-neighborhood, as in [1], due to the more careful choice of ρ_1: $\mathcal{G}$ is given by $\rho_1 = 0$, $x = 0$, and we are microlocalized to the region where $|\rho_1| \le 3\epsilon\delta$, $|x| \le 3\epsilon\delta$. For $v = \tilde B_r u$, $\tilde B_r = \tilde B \Lambda_r$, Lemma 7.1 thus gives (taking into account that we need to estimate $\|TD_{x_i}v\|$ rather than its square)

$$\begin{aligned}|\langle R_i D_{x_i} v, v\rangle| \le& 6C_2'\epsilon^{-1}(\epsilon\delta)^{1/2}\|\tilde B_r u\|^2 \\ &+ C_0\gamma^{-1}\big(\|G\tilde B_r u\|^2_{H^1(X)} + \|\tilde B_r u\|^2_{H^1_{\mathrm{loc}}(X)} + \|\tilde G P u\|^2_{H^{-1}(X)} + \|Pu\|^2_{H^1_{\mathrm{loc}}(X)}\big) \\ &+ 3\gamma\|\tilde B_r u\|^2 + \gamma^{-1}\|R_i'TD_{x_i}\tilde B_r u\|^2 + \gamma^{-1}\|F_iD_{x_i}\tilde B_r u\|^2,\end{aligned}$$

where the first term is the main change compared to [1]. Its coefficient, $(\delta/\epsilon)^{1/2}$, means that it can then be handled exactly as the R_{ij} term in [1], thus completing the proof.

References

[1] A. Vasy. Propagation of singularities for the wave equation on manifolds with corners. *Annals of Mathematics*, 168:749–812, 2008.

Department of Mathematics, Stanford University, Stanford, CA 94305-2125, U.S.A.
E-mail address: andras@math.stanford.edu