# Budget Aggregation via Knapsack Voting: Welfare-maximization and Strategy-proofness

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## 1. INTRODUCTION

Participatory budgeting [1], which started in South America, is now gaining popularity in the US, with cities like San Francisco, Vallejo, Boston, Chicago and New York adopting this paradigm [6]. With decisions involving millions of dollars across the nation being made this way, a question arises as to how to design voting schemes to aggregate the voters' preferences into a meaningful budget decision. And with many cities adopting digital voting, a key requirement is the amenability of these schemes to implementation via digital tools. There has been some work on addressing this question [4], where a class of schemes called *Knapsack Voting* (inspired by the classical *Knapsack Problem* [5]) was proposed. In particular, two elicitation schemes: one, where the budget constraint is imposed on each vote, and two, where voters compare different items on the ballot according to their *value-for-money*, (i.e., the perceived benefit to society per dollar spent on each item); and appropriate aggregation rules for each, were introduced. We will refer to the former scheme, along with its aggregation rule, as *Knapsack Voting*. In this paper, we show that under a natural model of voter utilities, the Knapsack Voting rule is both *strategy-proof* and *welfare-maximizing*. In addition, we provide an empirical comparison between Knapsack Voting and *K-approval voting*, which is the method currently used in most Participatory Budgeting elections. We see that Knapsack Voting leads to a more economical consideration of projects on the ballot. To do this, we use data collected from the digital voting platform (`pbstanford.org`) that we have deployed in partnership with the local government apparatus in many cities across the nation.

**The Participatory Budgeting Problem** addresses the following scenario: the residents of a city or municipal entity, collectively the set of voters $\mathcal{V}$, vote on a set $\mathcal{P}$ of projects (like roads, lighting, parks, etc.) that they have identified to be worthwhile, where project $p \in \mathcal{P}$ has a cost $c_p$ and there is a fixed total budget of $B$ Dollars (see `pb.cambridgema.gov` for a typical list of projects). Based on these votes, the government has to decide on an allocation that maximizes the total benefit to society.

The voting method currently used by most participatory budgeting elections is **K-approval** voting (see Figure 1), where each voter votes for her top $K$ projects, for some number $K$, say 5. The votes are aggregated by giving each project a score equal to the number of voters that voted for it. The outcome is ascertained by picking projects in descending order of this score till the budget is used up. While this method is simple to implement, it doesn't make voters factor in the cost of the projects.

**Knapsack Voting** was proposed to overcome this drawback [4]. The main idea behind it was imposing the budget constraint on the voter to make her take costs in account while submitting her vote. It was shown to have weak and myopic strategic properties, and its performance with respect to social welfare was not addressed. Its implementation as a digital voting tool was discussed, stopping short, however, of an empirical study of voting data. In this paper, we go a step further and show that Knapsack Voting is strategy-proof and welfare-maximizing for a natural model of voter utilities. We also do a empirical study of Knapsack Voting in contrast to K-approval, using data collected from real Participatory Budgeting elections.

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Fig. 1. Our interface for K-approval

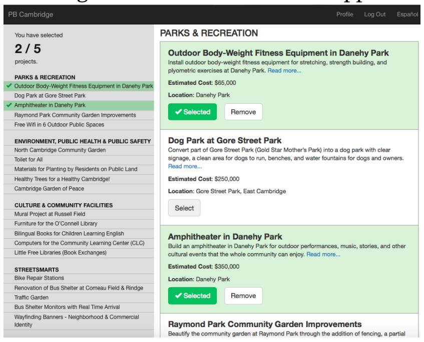
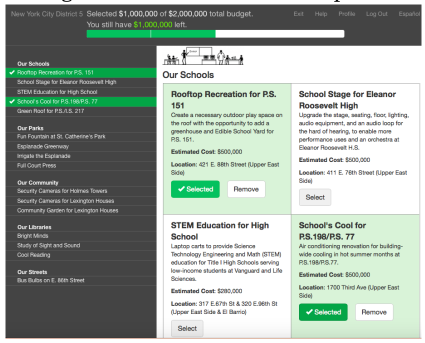



Fig. 2. Our interface for Knapsack vote




## 2. OUR CONTRIBUTIONS

To avoid combinatorial difficulty, we will look at the fractional equivalent of Knapsack Voting, where the voter can choose any valid (integral) allocation of $B$ Dollars among the various projects.

*Definition* 2.1 (*Knapsack Vote*). Each voter $v \in V$ votes for an allocation $\{w_p^v\}_{p\in\mathcal{P}}$, such that $\sum_{p\in\mathcal{P}} w_p^v = B$, where $w_p^v \in \mathbb{Z}_{\geq 0}$

For each $p \in \mathcal{P}$ and any $w_p \in \mathbb{Z}_{\geq 0}$, define $\text{score}(w_p) \triangleq |\{v \in V : w_p^v > w_p\}|$. To define the outcome, we use the following scoring rule:

*Definition* 2.2 (*Knapsack Aggregation*). The outcome is given by $\underset{\sum_{p\in\mathcal{P}} w_p = B}{\arg\max} \sum_{p\in\mathcal{P}} \sum_0^{w_p - 1} \text{score}(w_p)$.

Several impossibility results such as Gibbard-Satterthwaite's [3; 7], and in particular its generalization to voting rules with multiple winners by Duggan-Schwartz [2], rule out the existence of strategy-proof mechanisms for our setting in all generality. Therefore, in order to reason about Knapsack Voting, we will some impose some additional assumptions on voter preferences. We assume a natural model of voter utility, where the "satisfaction" of a voter is determined by the overlap between her preferred budget allocation and the final outcome.

More formally, if $\{w_p^*\}_{p\in\mathcal{P}}$ is the outcome of the elections, then the utility that a voter gets from any project $p$ is equal to $\min\{w_p^v, w_p^*\}$.

*Definition* 2.3 (*Overlap Utility Model*). Each voter $v$ has a preferred allocation $\{w_p^v\}_{p\in\mathcal{P}}$ satisfying the budget constraint, and her utility for an outcome $\{w_p^*\}_{p\in\mathcal{P}}$ is given by $\sum_{p\in\mathcal{P}} \min\{w_p^v, w_p^*\}$..

Under this assumption, the Knapsack Voting rule has two important properties:

THEOREM 2.4 (STRATEGY-PROOFNESS). *For any voter $v$, voting for her true preferred budget allocation $\{w_p^v\}_{p\in\mathcal{P}}$ is a **dominant strategy**.*

THEOREM 2.5 (WELFARE MAXIMIZATION). *The outcome $\{w_p^*\}_{p\in\mathcal{P}}$ of the Knapsack Voting Rule is **welfare-maximizing**, i.e., maximizes the sum of utilities of all voters.*

Together, these amount to substantial theoretical evidence that Knapsack Voting aligns the incentives of the voters with that of the decision maker. Of course, neither property holds for K-approval voting. We omit the proofs of the above theorems, as a full exposition is beyond the scope of this article.

**Deployments and Data Analysis.** While we have deployed our digital voting system (pbstanford.org) in many cities (Boston, Cambridge, Vallejo, New York City, to name a few), we will present data

Table I. Average cost of winning projects, as a fraction of the budget

| | *K-approval* | *knapsack* |
|---|---|---|
| NYC District 8 | 0.20 | 0.12 |
| Cambridge | 0.15 | 0.10 |

collected from New York District 8 (2014) and Cambridge (2015). The trends we demonstrate are similar across all elections. In either election, in addition to the K-approval vote which is the official ballot process (Figure 1), we had an experimental interface for Knapsack Voting (Figure 2).

It is natural to expect that if voters voted in term of the total utility of each project without considering its cost, as in K-approval voting, bigger costlier projects are likely to gain more support; as opposed to Knapsack Voting, where they factor in the costs of the projects. This observation leads us to the following hypothesis:

EMPIRICAL HYPOTHESIS 1. *The K-approval rule biases the outcome towards projects of larger cost compared to the Knapsack rule.*

We illustrate this effect in a couple of ways. First, we look at Figures 3 and 4, where 6-approval and 5-approval were used respectively. Here we lay out the projects in descending order of cost, and plot the cumulative fraction of votes for projects above every cost threshold. We then compare K-approval with Knapsack with respect to the above, and see that this function for K-approval dominates that for Knapsack. This means that as cost of the project increases, it is more likely to be over-represented under K-approval, thereby confirming our hypothesis. Second, we look at the average cost of the winning projects under each method. Table I shows this value as a fraction of the budget in each election. On the average, Knapsack Voting results in a reduction of about 30% in the average cost of the winning projects compared to K-approval.

Fig. 3. Cambridge

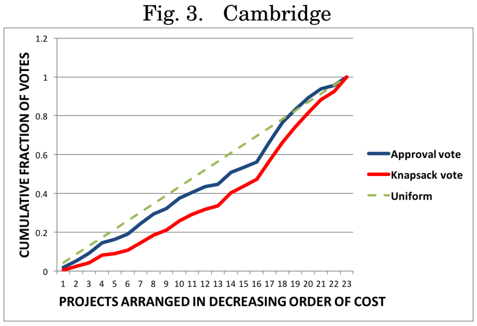
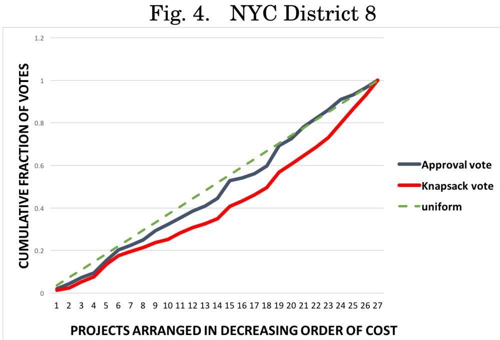



Fig. 4. NYC District 8




## 3. CONCLUSION AND FUTURE WORK

*Knapsack voting* is an intuitive way of eliciting voters' preferences, and aggregating them into a budget outcome. Under a natural assumption on voter utilities, we showed that this rule is *welfare-maximizing* and *strategy-proof*. Similar results can be obtained in the case where there is no hard budget, and there are both expenditure and revenue terms, i.e., voters propose both how to generate revenue from among various avenues, and how to spend it on various projects. We find from an empirical comparison between Knapsack Voting and K-approval, using data from real elections, that the former leads to a more economical consideration of projects on the ballot. We hope that this work will strengthen the case for the adoption of Knapsack Voting in Participatory Budgeting elections.

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