Hierarchical Beamforming
for Large One-Dimensional Wireless Networks

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Abstract—We consider a wireless network with a large number of source-destination pairs distributed over a line. Under line-of-sight propagation, this network has only one degree of freedom for communication. At high SNR, this one degree of freedom can be readily achieved by multi-hop relaying between nodes. At low SNR, however, the performance is determined by the power transfer in the network. We show that none of the existing architectures can achieve optimal capacity scaling. We develop a digital hierarchical beamforming architecture and show that it is scaling optimal. This result reveals a new regime for large wireless networks, where beamforming techniques are needed to achieve capacity.

I. INTRODUCTION

Multi-hop is the traditional communication architecture for wireless adhoc networks. Information is routed from source nodes to destinations via multiple point-to-point transmissions between intermediate nodes acting as relays. Can we design new cooperation architectures for wireless adhoc networks that significantly outperform multi-hop? To shed some light on this question in the large network regime, we follow the *scaling law* approach in [1]-[8]. This approach was initiated in [1] and concentrates on the scaling of the capacity with the number of nodes n when other parameters of the network are coupled with n in two specific ways, leading to the so called *dense* and *extended* scalings. We have shown in [8],[9] that by extending this approach to seeking a multi-parameter family of scaling laws, we can use it to identify the fundamental operating regimes of large wireless networks. Here many different limits (couplings) with respect to the parameters of the network are jointly considered. This allows to uncover a number of qualitatively different operating regimes where the answer to the above raised question is different.

The current understanding of the operating regimes of large wireless networks can be roughly summarized as follows (see [9] for a detailed overview): When SNR is high and there are sufficient spatial degrees of freedom in the network (this is for example the case when the pairwise channels are subject to i.i.d. fading), a hierarchical cooperation architecture based on distributed MIMO transmission can provide significant capacity gain over multi-hop [5]. The situation is more tricky for networks at low SNR. When the SNR is low but there are still sufficient spatial degrees of freedom in the network (as with i.i.d. fading), the optimal architecture depends on the power path loss exponent of the environment α [5], [9]. α determines how fast signal power decays with distance in

an environment and is typically between 2 and 6 in a two-dimensional network. When $2 \leq \alpha < 3$, a bursty version of the hierarchical cooperation architecture significantly outperforms multi-hop and is scaling optimal. When $\alpha \geq 3$, multi-hopping achieves the optimal scaling and no strategy can perform better. Interestingly, none of the two architectures, hierarchical cooperation nor multihop, uses beamforming, which is known to be the right strategy for point-to-point MIMO channels at low SNR [10]. This is because under i.i.d. fading, the distributed MIMO channels available in the network are well-conditioned and beamforming (or waterfilling over the eigenvalues of the channel) provides little gain. Transmitting independent streams from each node during the distributed MIMO transmissions is scaling optimal.

The recent work [6], however, reveals that degrees of freedom in a wireless network can be limited by physical constraints in the spatial channel. This can be thought of as the spatial channel introducing correlation between pairwise gains. In the regime when the available degrees of freedom in the network are very few (determined by the area of the network and the carrier wavelength [8]), they can be readily achieved by multi-hop. This makes multi-hop scaling optimal for such networks in the high SNR regime [6]. However, at low SNR, the performance is determined by the power transfer in the network and it is not clear whether any of the existing architectures achieves the optimal scaling of the capacity.

In this paper, we explore a new regime where the network is *both* limited in power (operating at low SNR) and in spatial degrees of freedom (operating under strongly correlated pairwise channels). We show that a new class of cooperative beamforming architectures outperform classical multi-hopping when α is small. To capture this regime in a simple setup, we focus on a one-dimensional wireless network in a line-of-sight propagation environment. This leads to the extremal case when there is only one degree of freedom for communicating in the network. At high SNR, this single degree of freedom can be readily achieved with multi-hop, or even with simple time-division between the source-destination pairs in the network. At low SNR, we develop a hierarchical beamforming architecture and show that it outperforms existing strategies when $1 \leq \alpha \leq 2$.[1] Here, nodes first broadcast their information to

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[1] The assumption $\alpha \geq 1$ in one-dimensional networks replaces the traditional assumption $\alpha \geq 2$ made in two-dimensional networks.
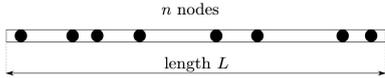
Fig. 1: One-dimensional network

a small cluster around them. This allows to beamform this information and distribute it over a larger cluster in a second step. Continuing in a hierarchical fashion, the information of each source node is broadcasted to the whole network, including the destination node. We show that this broadcasting strategy achieves the optimal scaling when $\alpha = 1$. This is suprising given that the original requirement is to communicate the information of each source node only to its corresponding destination node. A future goal of this research program is to understand the power and spatial degrees of freedom limited regimes of two-dimensional wireless networks.

The multi-stage broadcasting architecture developed in the present paper is reminiscent of other schemes existing in the literature, such as [11], [12], [13], [14]. The setups considered in these references are nevertheless quite different. The problem of broadcast capacity, i.e. the maximum data rate that can be achieved from a single source node to every other node in the network, was considered in [11], [12], [13], while the reverse problem of the data gathering channel was considered in [14]. Our intent here is to primarily study the multiple-unicast scenario. Another difference is that the above references consider primarily high or fixed SNR, while we are interested in the low SNR regime. For one S-D pair, the low SNR regime has been studied in [15], where it was shown that ampify-and-forward based beamforming techniques achieve the optimal scaling in a diamond network. As opposed to this analog strategy, the communication architecture proposed for multiple S-D pairs in the present paper is digital.

II. MODEL

There are n nodes uniformly and independently distributed along a line of length L, as illustrated on Figure 1. Every node is both a source and a destination, and the sources and destinations are randomly paired up one-to-one. All source nodes want to communicate to their destination node at the same rate $R(n)$. The aggregate throughput of the network is defined as $T(n) = nR(n)$.

We assume that communication takes place over a flat channel with bandwidth W and that the received signal $Y_k[m]$ by node k at time m is given by

$$Y_k[m] = \sum_{j \in J} h_{kj} X_j[m] + Z_k[m]$$

where J is the set of transmitting nodes, $X_j[m]$ is the signal sent at time m by node j and $Z_k[m]$ is additive white circularly symmetric Gaussian noise (AWGN) of power spectral density $N_0/2$ Watts/Hz. In a line-of-sight environment, the complex baseband-equivalent channel gain h_{kj} between transmit node

j and receive node k is given by

$$h_{kj} = \sqrt{G} \, \frac{\exp(2\pi i r_{kj}/\lambda)}{r_{kj}^{\alpha/2}} \qquad (1)$$

where G is given by the Friis' formula, λ is the carrier wavelength, r_{kj} is the distance between node k and node j and $\alpha \geq 1$ is the power path loss exponent. In a one-dimensional network, this line-of-sight model clearly departs from the classical i.i.d. phase assumption: it implies in particular that the channel matrix between two distant clusters of nodes is essentially rank 1 when it is full-rank under the i.i.d. model. Finally, we assume full channel state information at both the transmitters and receivers (which is a reasonable assumption in a static line-of-sight environment), as well as a common average power budget per node P.

III. MAIN RESULT

Let us denote by SNR_s the signal-to-noise ratio over the typical nearest neighbor distance in the network. In a one-dimensional network, the typical nearest neighbor distance is $\frac{L}{n}$, therefore, the short-distance SNR is

$$\mathrm{SNR}_s = \frac{GP}{N_0 W} \left(\frac{n}{L}\right)^\alpha.$$

In this paper, we are interested in the low SNR regime, where $\mathrm{SNR}_s = n^{-\gamma}$ for some $\gamma > 0$. We will use the notation $\mathrm{SNR}_s \ll 0$ dB to denote this condition. This models the scenario when the pairwise channels between nearest neighbors are in the low SNR regime. Note that since SNR_s is jointly determined by a number of system parameters P, W and L, this can be the case when the available power per node is small; when the bandwidth is large; or the distances between the nodes are large.

A relatively straightforward analysis reveals that in one-dimensional networks, the multi-hop scheme described in [1] achieves with probability approaching 1 as n increases (with high probability) an aggregate throughput of order

$$T(n) = \Omega \left(\frac{\mathrm{SNR}_s}{\log n}\right)$$

when $\mathrm{SNR}_s \leq 0$ dB and $\alpha \geq 1$. On the other hand, the best known information theoretic upper bound on the throughput scaling of such networks is given in [4]: for $\alpha \geq 1$,

$$T(n) = O\left(\log^3 n\right).$$

This shows that for constant SNR_s, multi-hop cooperation is order optimal and the aggregate throughput is constant, up to logarithmic factors. In the low SNR regime however (that is, when $\mathrm{SNR}_s \ll 0$ dB), the question remains whether a more sophisticated strategy would not allow to achieve higher throughput scaling than multi-hop. We answer this question by the affirmative in the following theorem, in the case where the path loss exponent α lies between 1 and 2.

Theorem 1: Let us assume that $1 \leq \alpha < 2$ and $\mathrm{SNR}_s \ll 0$ dB (i.e. $\mathrm{SNR}_s = n^{-\gamma}$ for some $\gamma > 0$). Then for any $\varepsilon > 0$, there exists a communication scheme (referred to as

"hierarchical beamforming" in the sequel) that achieves the following aggregate throughput with high probability as n gets large:

$$T(n) = \Omega\left(\min\left\{\text{SNR}_s\, n^{2-\alpha-\varepsilon}, n^{-\varepsilon}\right\}\right) \qquad (2)$$

The above aggregate throughput scaling is strictly higher than that achieved by multi-hop. In particular, when $\alpha = 1$ and $\text{SNR}_s \leq 1/n$, $T(n) = \Omega(\text{SNR}_s\, n^{1-\varepsilon})$, which is an order n improvement over multi-hop. The hierarchical beamforming architecture achieving this performance is described in detail in the next section.

Is this strategy optimal or can we do better? Before answering this question, let us introduce the notion of a *broadcasting scheme* below.

Definition 1: A communication scheme achieving a per-node throughput $R(n)$ for n S-D pairs is said to be a broadcasting scheme if at this same rate $R(n)$, all destinations are able to decode the information sent not only by their corresponding source, but also by all the other sources.

As we will see, hierarchical beamforming enters into this category, and so does classical multi-hop in one-dimensional networks (at the price of a small adaptation of the original scheme). The theorem below, together with Theorem 1 above, shows that: a) hierarchical beamforming is scaling optimal when $\alpha = 1$; b) among all *broadcasting schemes*, hierarchical beamforming is scaling optimal when $1 \leq \alpha < 2$, and multi-hopping is when $\alpha \geq 2$.

Theorem 2: Consider a one-dimensional network with $\alpha \geq 1$ and $\text{SNR}_s \ll 0$ dB. Then:

a) The aggregate throughput scaling of any communication scheme is upper bounded with high probability by

$$T(n) = O\left(\min\left\{\text{SNR}_s\, n\log^2 n, \log^3 n\right\}\right)$$

b) The aggregate throughput scaling of any broadcasting scheme is upper bounded with high probability by

$$T(n) = \begin{cases} O\left(\min\left\{\text{SNR}_s\, n^{2-\alpha}\log^2 n, \log^3 n\right\}\right) & \text{if } 1 \leq \alpha < 2 \\ O\left(\text{SNR}_s\, \log^2 n\right) & \text{if } \alpha \geq 2 \end{cases}$$

The proof of this result is omitted due to space limitations, but let us indicate here briefly the main ideas behind the proof. Part a) can be proven using the classical SIMO bound showing that the communication rate from one source to the rest of the network is at most of order $\text{SNR}_s \log^2 n$. For part b), the following observation is useful: in the broadcasting scenario, each destination is waiting for messages from the other $n-1$ nodes in the network. So using the MISO bound around a particular destination allows to upper bound the common rate at which each source should transmit towards this destination.

An interesting open question, that we do not address in the present paper, is whether *any* scaling optimal scheme for the multiple-unicast problem in a one-dimensional network is also a broadcasting scheme or not.

IV. HIERARCHICAL BEAMFORMING

Let us start by considering the situation where the SNR in the network is very low. More precisely, let us assume that

$$\text{SNR}_s \leq n^{\alpha-2} \quad \text{(with } 1 \leq \alpha < 2) \qquad (3)$$

In this regime, many transmissions can take place concurrently in the network (spatial reuse) without creating interference above the noise level. Under this assumption, the lower bound in Eq. (2) reads

$$T(n) = \Omega\left(\text{SNR}_s\, n^{2-\alpha-\varepsilon}\right) \qquad (4)$$

We first sketch the hierarchical beamforming strategy we propose and then proceed to its performance analysis which also provides a more detailed description. Consider one particular source-destination pair $s - d$ in the network. For simplicity, assume that s has one bit to communicate to d. s can communicate this one bit in two steps:

- it can first broadcast this bit to a small cluster of M neighboring nodes around itself.
- the M nodes can then simultaneously transmit this bit to the destination node d by coherently combining their signals.

The beamforming gain due to the coherent combining of the M signals leads to a better performance than simply transmitting the bit from s to d.

From the network point of view, all source-destination pairs have to eventually accomplish these two steps. Step 2 is long-range communication and only one source-destination pair can operate at a time. Step 1 involves local communication and can be parallelized across source-destination pairs. This leads to the following two phases in the operation of the network:

1. The network is divided into clusters of M nodes. Each source node distributes one bit to the M nodes in its cluster. There are M source nodes in a cluster, which can simply take turns to distribute their one bit. When the total interference from the other clusters is below the noise level, this operation can be conducted in parallel among all clusters. At the end of this phase, each node has therefore received (and decoded) one bit from every other node in its cluster.

2. In the second phase, the bits are beamformed to their actual destinations one at a time. Every cluster performs M successive transmissions; in each transmission, the bit of one particular source node in the cluster is beamformed to its destination. There are a total of n succesive beamforming transmissions in this phase, one for each source-destination pair in the network.

A key observation is that this two-phase scheme distributes the bits of every source node to all other nodes in the network, even if this is not what we set for. In the second phase, the beamforming transmissions done one at a time can be decoded not only by the actual destination node but simultaneoulsy by all the nodes in the network. This is a consequence of the fact that the network has only one degree of freedom. The transmitted signals from each cluster can be arranged to coherently combine simultaneously at all the remaining nodes

in the network. Therefore according to Definition 1, this two-phase scheme is a broadcasting scheme.

This brings the idea of recursion. The broadcasting requirement in the first phase can be handled by further dividing each cluster into smaller clusters and use the two-phase broadcast scheme we just described. The two-phase scheme is illustrated in Figure 2. The recursion is summarized in the following lemma.
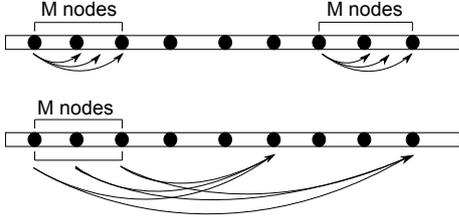

M nodes   M nodes

M nodes

Fig. 2: Two-phase beamforming

Lemma 1: Consider $1 \leq \alpha < 2$ and a one-dimensional network with n nodes and $\mathrm{SNR}_s \ll 0$ dB, subject to an additive external interfering source with bounded average power. If in this network, there exists a broadcasting scheme achieving with high probability an aggregate throughput

$$T(n) = \Omega\left(\mathrm{SNR}_s\, n^{\beta}\right)$$

for some $\beta \leq 2 - \alpha$, then there exists another broadcasting scheme achieving with high probability an aggregate throughput

$$T(n) = \Omega\left(\mathrm{SNR}_s\, n^{f(\beta)}\right)$$

where

$$f(\beta) = 1 - \frac{\alpha(1-\beta)}{2-\beta} \qquad (5)$$

Notice that $f(\beta) > \beta$ for all $1 \leq \alpha < 2$ and $\beta < 2 - \alpha$, so the performance of the new scheme is always strictly better than that of the original one. Figure 3 below illustrates the behavior of $f(\beta)$, for $\alpha = 1$ and $\alpha = 1.5$.
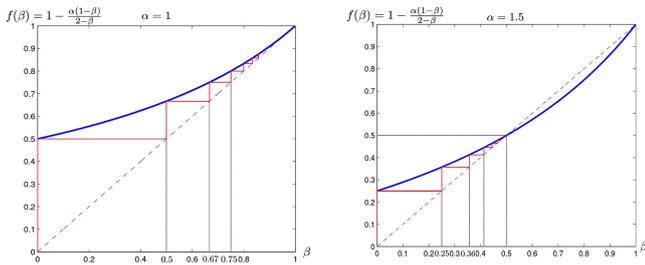

Fig. 3: Growth of the aggregate throughput exponent

Proof of Lemma 1. Consider a network of n nodes and length L, and let us divide it into clusters of length LM/n, with $1 \ll M \ll n$, so that each cluster contains order M nodes with high probability. Based on the assumption made

in the lemma, the following communication scheme will be shown to achieve the desired throughput scaling.

Phase 1. Source nodes broadcast information to every other node inside their cluster, using the original scheme with aggregate throughput $T(M) = \Omega\left(\mathrm{SNR}_s\, M^{\beta}\right)$. This step is parallelized across clusters[2].

Phase 2. For each source node inside a cluster of M nodes, all the nodes inside the cluster simultaneously beamform the received bits to the rest of the network. During this second phase, only one cluster operates at a time.

Performance Analysis. In the first phase, clusters work in parallel. In order to avoid collisions between neighboring clusters, a simple time-division scheme with two rounds is used, where half of the clusters are active at a time: this only affects the throughput by a factor two and allows clear reception of the signals inside each cluster. One can indeed check that because of assumption (3), the average power of the interference caused in one cluster by simultaneous transmissions in the other clusters is bounded.

The broadcasting rate achieved by the scheme inside each cluster is $R(M) = \Omega\left(\mathrm{SNR}_s\, M^{\beta-1}\right)$, so the total time taken by this first phase is upper bounded by

$$t_1 = O\left(\frac{1}{\mathrm{SNR}_s\, M^{\beta-1}}\right)$$

In the second phase, M broadcast transmissions are performed sequentially from each cluster towards the rest of the network. As there are n/M clusters, the total number of transmissions is therefore equal to n (that is, one transmission takes place for each source node). The SNR of each transmission is lower bounded by

$$\mathrm{SNR}_s\, \frac{n}{M}\, M^2\, n^{-\alpha} = \mathrm{SNR}_s\, M\, n^{1-\alpha}$$

where the above factors are explained as follows:

- the factor n/M is due to the fact that each cluster only transmits a fraction M/n of the time, so power can be spared during the rest of the time;

- the factor M^2 is the beamforming gain (notice that because of the line-of-sight channel model (1) and the assumption of a one-dimensional network, it is indeed possible to beamform the signal towards all destinations simultaneously);

- the factor $n^{-\alpha}$ is a lower bound on the power attenuation over distance.

The total time taken by this second phase is therefore upper bounded by

$$t_2 = O\left(\frac{n}{\mathrm{SNR}_s\, M\, n^{1-\alpha}}\right) = O\left(\frac{1}{\mathrm{SNR}_s\, M\, n^{-\alpha}}\right)$$

Optimal cluster size. In order to optimize the throughput of the new scheme, the optimal cluster size M^* should be chosen such that the durations of the two phases are equal, i.e. $t_1 = t_2$, which leads to

$$(M^*)^{\beta-1} = M^*\, n^{-\alpha} \quad \text{i.e.} \quad M^* = n^{\alpha/(2-\beta)} \qquad (6)$$

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[2]Notice that SNR_s, that only depends on the distance between neighboring nodes, remains unchanged for a cluster of M nodes or for the whole network.

(Notice that $\alpha/(2-\beta) \leq 1$, as $\beta \leq 2 - \alpha$ by assumption).

Resulting aggregate throughput. With this cluster size, it is worthwhile noticing that the broadcasting rate of the new scheme is the *same* as the one achieved in each cluster with the original scheme. However, as more nodes participate in the transmission, the aggregate throughput increases as follows:

$$\begin{aligned} T(n) &= n\,R(M^*) = \Omega\left(n\,\mathrm{SNR}_s\,(M^*)^{\beta-1}\right) \\ &= \Omega\left(\mathrm{SNR}_s\,n^{f(\beta)}\right) \end{aligned}$$

where $f(\beta)$ is given in (5). This completes the proof. $\blacksquare$

Let us now explain how applying recursively Lemma 1 allows to obtain the lower bound (4) on the aggregate throughput scaling.

Let us first use multi-hop for broadcasting information at the lowest level of the hierarchy, that is, inside small clusters of M_1 nodes. Note that multi-hop can be easily transformed into a broadcasting scheme in the one-dimensional case without changing its aggregate throughput scaling; since information is routed over a line, each destination already observes the information sent by order n nodes on average. The aggregate throughput achieved inside each cluster is therefore

$$T(M_1) = \Theta\left(\frac{\mathrm{SNR}_s}{\log M_1}\right) = \Omega\left(\mathrm{SNR}_s\,M_1^{\beta}\right) \quad \forall \beta < 0$$

Using then the two-phase scheme described in the proof of Lemma 1, we reach for larger clusters of size M_2 (to be specified below) an aggregate throughput

$$T(M_2) = \Omega\left(\mathrm{SNR}_s\,M_2^{f(\beta)}\right) \quad \forall \beta < 0$$

Iterating this procedure $h-1$ times, until the large cluster of size M_h reaches the network size n, we obtain the following aggregate throughput

$$T(n) = \Omega\left(\mathrm{SNR}_s\,n^{f^{(h-1)}(\beta)}\right) \quad \forall \beta < 0$$

As illustrated on Figure 3, the sequence $f^{(h-1)}(\beta)$ converges to the minimal solution of the equation

$$\beta^* = f(\beta^*)$$

which is given by $\beta^* = 2 - \alpha$ for $1 \leq \alpha < 2$. For a fixed number of hierarchical levels h, the achieved aggregate throughput scaling is therefore $T(n) = \Omega\left(\mathrm{SNR}_s\,n^{2-\alpha-\varepsilon}\right)$, and $\varepsilon > 0$ can be made arbitrarily small by increasing the number h. $\blacksquare$

In addition, let us describe how to compute the optimal cluster sizes $M_1, \ldots, M_h$ in this process. From Eq. (6) in the proof of Lemma 1, we deduce that at level $1 \leq k < h$,

$$M_k = (M_{k+1})^{\alpha/(2-\beta(k))}$$

where $\beta(k)$ is the aggregate throughput exponent achieved at level k. This allows to compute recursively the cluster sizes, starting from $M_h = n$. From this analysis, it turns out that as h gets large, the optimal cluster size M_1 at the lowest level of the hierarchy converges to

$$M_1 = n^{\alpha-1}$$

So when $\alpha = 1$, the hierarchical beamforming scheme starts directly from tiny clusters, whereas when $1 < \alpha < 2$, the optimal communication strategy is first to perform multi-hop inside clusters of size $n^{\alpha-1}$, and then to use hierarchical beamforming. We therefore see that in the latter case, because of the higher value of the path loss exponent α, beamforming only helps when sufficiently many nodes participate to the transmission.

Finally, let us mention what happens at moderately low SNR, i.e. when $n^{\alpha-2} \leq \mathrm{SNR}_s \leq 1$. In this case, the interference felt from the simultaneously transmitting clusters might hurt the transmissions inside a cluster. A simple solution to this problem is to reduce the power used by each node, so as to meet the equality $\mathrm{SNR}_s = n^{\alpha-2}$. In this case, the aggregate throughput of the scheme is arbitrarily close to a constant, which proves the claim made in Theorem 1.

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