Wireless Network Simplification: the Gaussian N-Relay Diamond Network

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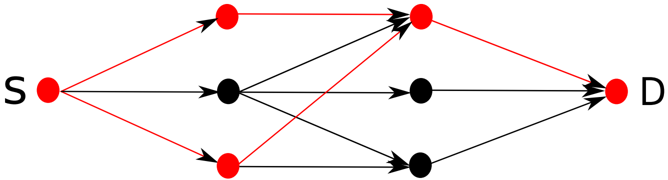
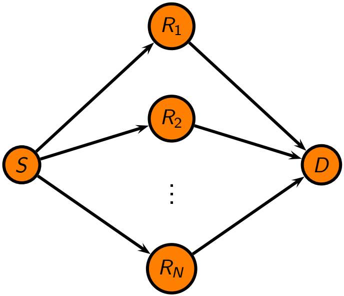
Wireless Network Simplification



Can we maintain (a good part of) the capacity by using a (small) subset of the relays?

Two-Stage Relaying: the N-Relay Diamond Network



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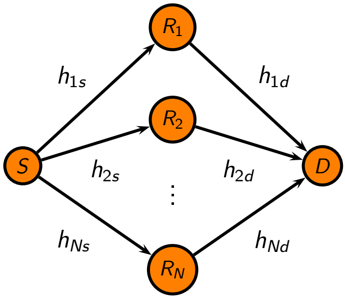

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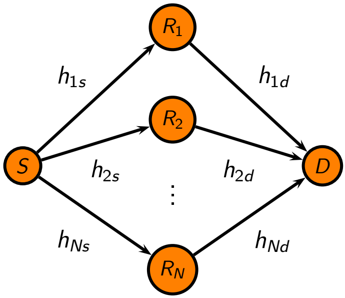

Model: $Y_i = h_{is}X_s + Z_i, \quad i = 1, \ldots, N$ $\qquad Y_d = \sum_{i=1}^{n} h_{id}X_i + Z_d$

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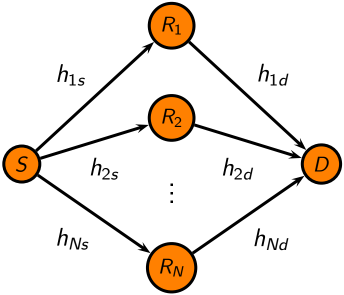

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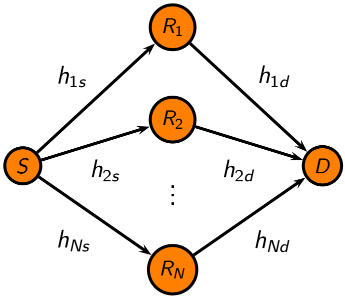

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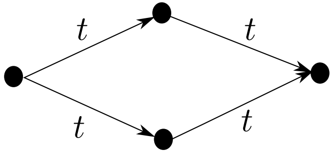
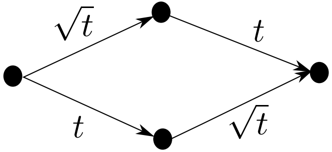
- Relay Selection: Bletsas et al.'2006,...

Diamond Network





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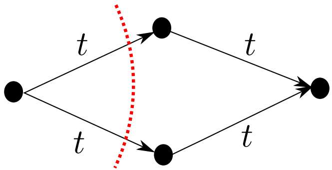

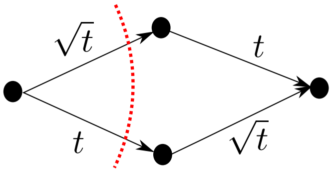

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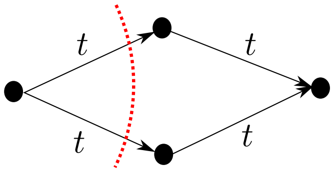

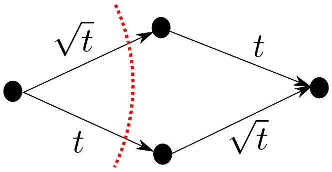

$C \approx \log(1 + 2t^2) \approx 2 \log t$

$C \approx \log(1 + t^2 + t) \approx 2 \log t$

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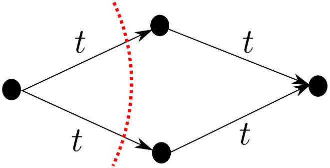

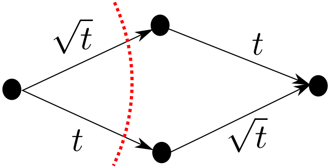

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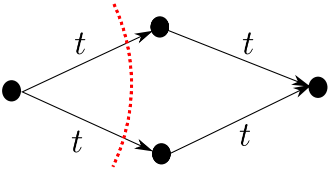

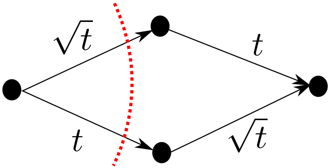

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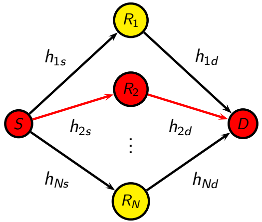
Simple Result



Theorem

In every Gaussian N-relay diamond network, there exists a relay such that routing over this relay achieves a rate

$$C_1 \geq \frac{1}{2}\overline{C} - \frac{1}{2}\max\left(3\log N - 2.75, 2\log N\right).$$

There are channel configurations for an N-relay diamond network such that

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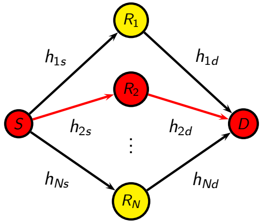

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Main Result

Theorem

In every Gaussian N-relay diamond network, there exists a subset of k relays, such that the capacity of the corresponding Gaussian k-relay diamond sub-network,

$$C_k \geq \frac{k}{k+1}\overline{C} \quad - 3k - \frac{k}{k+1}\max\left(3\log N - 2.75, 2\log N\right)$$

There are configurations of Gaussian N-relay diamond networks such that the capacity of every k-relay sub-network is upper-bounded by,

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As an Approximation Result

The capacity of the Gaussian N-relay diamond network is bounded by

$$\frac{k}{k+1}\,\overline{C} - 3k - \frac{k}{k+1}\max\left(3\log N - 2.75, 2\log N\right) \leq C \leq \overline{C},$$

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The best known capacity approximations by Avestimehr-Diggavi-Tse'08:

- Additive Approximation:

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- Multiplicative Approximation:

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Proof of Main Result

- Cut-set Upper Bound:

$$\overline{C}_k \geq \frac{k}{k+1}\overline{C} - \frac{k}{k+1}\max\left(3\log N - 2.75, 2\log N\right)$$

- Quantize-Map-and-Forward with k relays:

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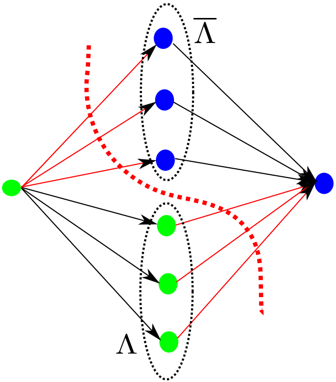


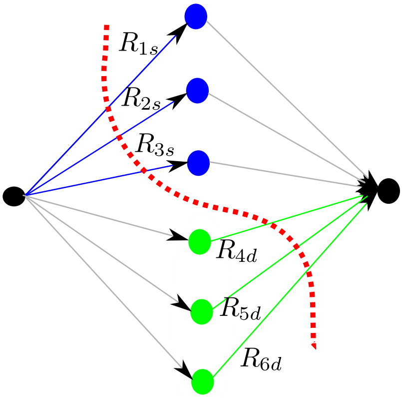
$$\min_{\Lambda} \max_{i \in \overline{\Lambda}} \log(1 + |h_{is}|^2) + \max_{i \in \Lambda} \log(1 + |h_{id}|^2) \leq$$

$$\overline{C} = \sup_{X_s, X_1, \ldots, X_N} \min_{\Lambda} I(X_s, X_\Lambda; Y_d, Y_{\overline{\Lambda}} \,|\, X_{\overline{\Lambda}})$$

$$\leq \min_{\Lambda} \max_{i \in \overline{\Lambda}} \log(1 + |h_{is}|^2) + \max_{i \in \Lambda} \log(1 + |h_{id}|^2) + 3 \log N$$

Proof for $k = 1$

$$\overline{C}^s = \min_{\Lambda} \max_{i \in \overline{\Lambda}} R_{is} + \max_{i \in \Lambda} R_{id}$$



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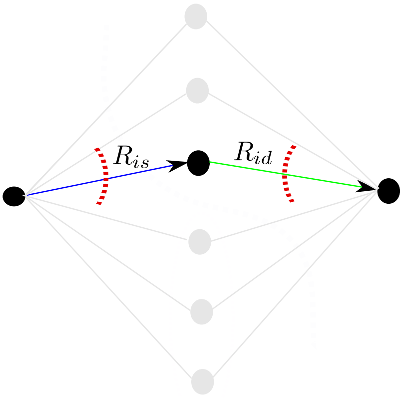

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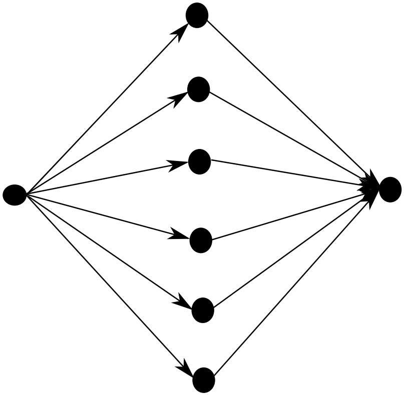
$\exists$ an i s.t. $\overline{C_i}^s \geq \frac{1}{2} \overline{C}^s$.



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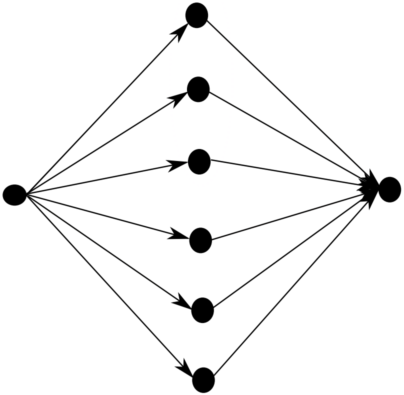

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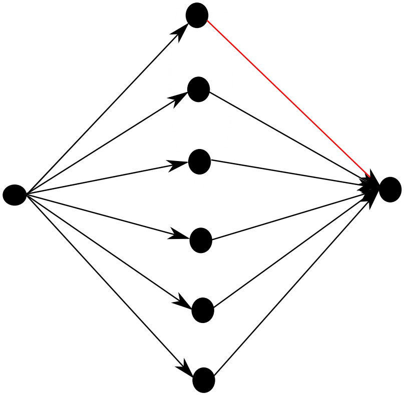

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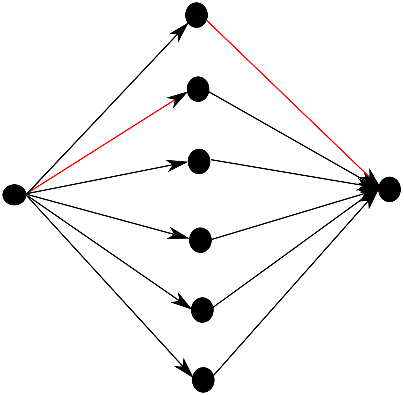

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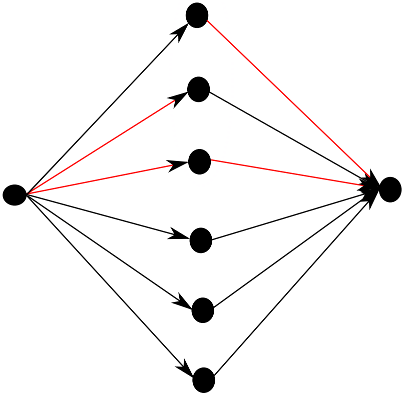

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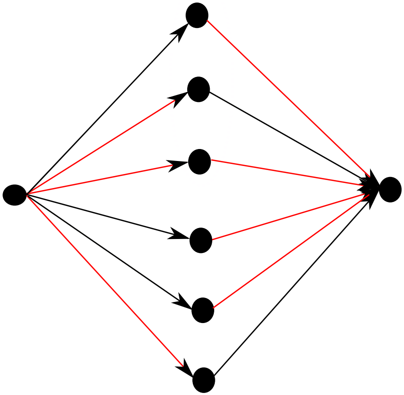

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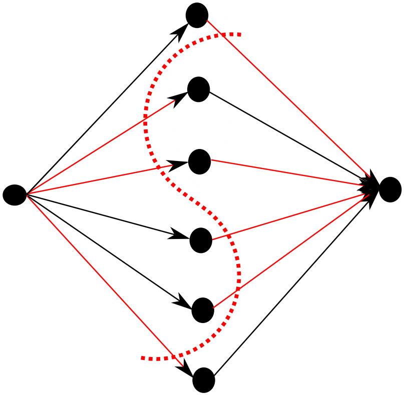

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Then $\overline{C}^s < A$. Contradiction.

Proof for $k = 2$

$$\overline{C}^s = \min_{\Lambda} \max_{k \in \overline{\Lambda}} R_{ks} + \max_{k \in \Lambda} R_{kd}$$

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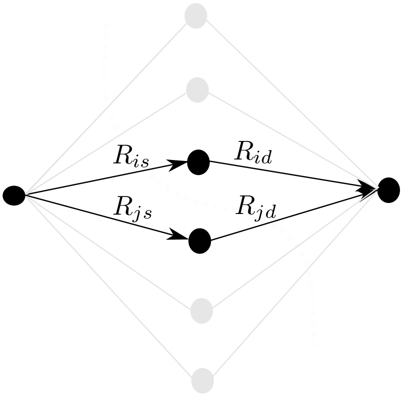

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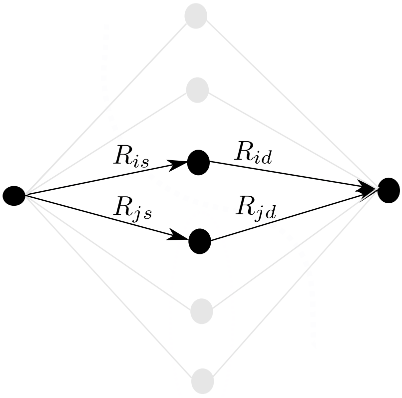

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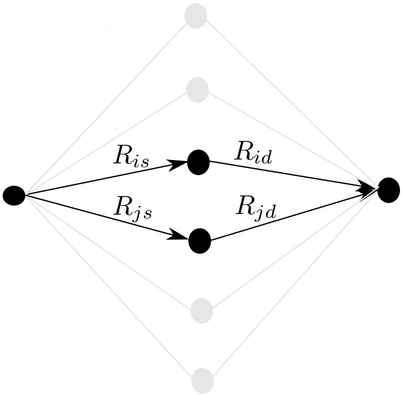

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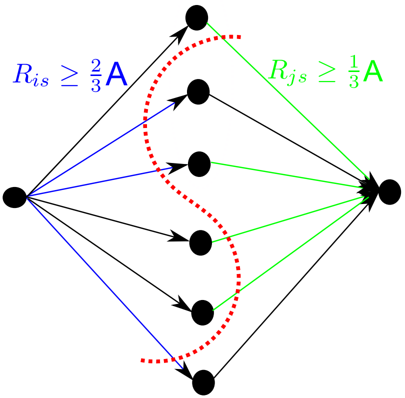

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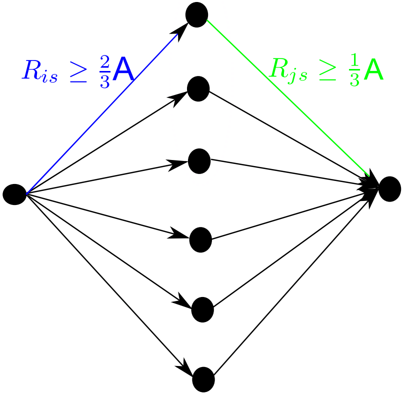

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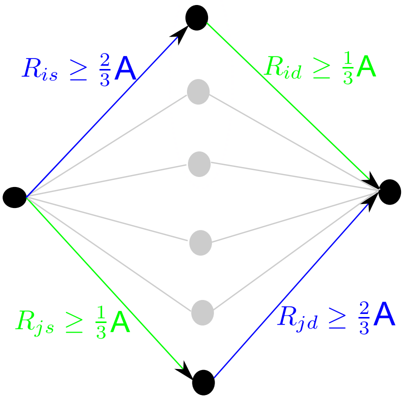
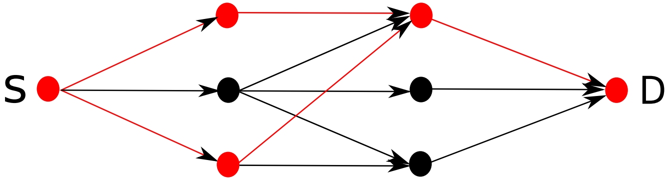


Future Work



- Arbitrary Wireless Networks
- Optimal Resource Utilization
- Optimal Resource Sharing