1. Introduction:

A multiple-input multiple-output (MIMO) system consists of multiple antennas at the receiver and transmitter. These multiple antennas can be used to improve the performance of the system through spatial diversity or increase the data rates by spatial multiplexing. One can also use some of the antennas for diversity and some for spatial multiplexing. The number used for diversity and spatial multiplexing depends on the application. Zheng and Tse [1] provide an optimal trade-off curve for multiplexing and diversity in the high SNR regime.

MIMO systems can support higher data rates at the same transmission power and bit-error-rate (BER) requirements i.e. for the same throughput requirement, MIMO systems require less transmission energy. Hence it is tempting to believe that MIMO systems are more energy efficient than single-input single-output (SISO) systems. However the circuit energy consumption of a MIMO system is more than for a SISO system as it has multiple RF chains and requires more signal processing. Several studies on the energy efficiency of MIMO systems have been done.

In this project, we conduct a literature survey on the energy efficiency of MIMO systems. We study the results of the work done in [2],[3] and [4] and break down these various studies in terms of the different systems, their energy consumption for transmission and by the circuit, and the diversity gain and/or multiplexing gain achieved. With this we come up with general system model, which we use to study the impact of increasing the rate on energy consumption at different distances.

In the next section, we give an overview of the energy consumption of MIMO systems by summarizing and analyzing the work done in [2], [3] and [4]. First we introduce the system models used. Then, the energy efficiency of a fixed and variable rate MIMO system is compared to that of a SISO system. In section 3, we give a system model for a MIMO system with multiplexing gain,$r$, and diversity gain,$d_r$. This model is used to analyze the impact of increasing the rate using spatial multiplexing on energy consumption at different transmission distances.

2. Overview of Energy Consumption of MIMO systems:

2.1 System Model:

The following is the system model proposed in [2]. The total average power consumption of a signal path is divided into the power consumption of the power amplifiers and the power consumption of the rest of the circuit components. The power consumed by the power amplifier depends on the transmitted power and is given by
where $E_b$ is the required energy per bit at the receiver for a given BER requirement, $R_b$ is the bit rate, $d$ is the transmission distance, $G_t$ is the transmitter antenna gain, $G_r$ is the receiver antenna gain, $\lambda$ is the carrier wavelength, $M_t$ is the link margin and $N_f$ is the receiver noise figure given by $N_f = N_r / N_0$ where $N_0$ is the single-sided thermal noise power spectral density and $N_r$ is the power spectral density of the total effective noise at the receiver input. It is assumed that the channel experiences a square-law path loss. The power consumption of the power amplifiers is approximated as

$$P_{PA} = (1 + \alpha)P_{out}$$

where $\alpha = \left(\frac{\xi}{\eta}\right) - 1$ with $\eta$ the drain efficiency of the RF power amplifier and $\xi$ the peak-to-average ratio.

For simplicity, the power consumed by the analog RF block only has been modeled. The receiver circuit block is modeled as shown in Fig 1.

$M_t$ is the no. of transmitter antennas and $M_r$ is the no. of receiver antennas. The local oscillator (LO) used by all the RF chains is the same. The power consumption of the circuit components other than the power amplifier is given by

$$P_c = M_t (P_{DAC} + P_{mix} + P_{filt}) + 2P_{syn}$$
$$+ M_r (P_{LNA} + P_{mix} + P_{IFA} + P_{filt} + P_{ADC})$$

[1]"
where $P_{DAC}$, $P_{mix}$, $P_{LNA}$, $P_{IFA}$, $P_{filt}$, $P_{fir}$, $P_{ADC}$, and $P_{syn}$ are the power consumption values for the DAC, the mixer, the low-noise amplifier (LNA), the intermediate frequency amplifier (IFA), the active filters at the transmitter side, the active filters at the receiver side, the ADC and the frequency synthesizer, respectively.

The total energy consumption per bit for a fixed rate system is given by

$$E_{bt} = \frac{P_P + P_c}{R_b} \quad [1]$$

This simplifies to

$$E_{bt} = \left( \frac{\xi}{\eta} \right) \times E_b \times \frac{(4\pi d)^2}{G_t G_r \lambda^2} M_f N_f + P_c / R_b$$

In addition to the path loss, the signal is further attenuated by a scalar fading matrix, $H$ which is modeled such that each entry is a zero-mean circularly symmetric complex Gaussian random variable with unit variance. In [2], the Alamouti scheme is used for the analysis. The instantaneous received SNR is given by

$$\gamma_b = \frac{\|H\|_F^2 \cdot E_b}{M_t \cdot N_o} \quad [2]$$

Thus given a BER requirement, the required $\frac{E_b}{N_o}$ can be found from the corresponding BER equation.

In [3], this model is modified to include the impact of different diversity gains and multiplexing gains and is given as follows.

A MIMO system with $M_t$ transmitting antennas and $M_r$ receiving antennas is considered which is used to transmit and receive a sequence $\{b_t\}$. After applying the space-time code the $M_t$ antennas transmit, $s_t = [s_t(1), ..., s_t(M_t)]^T$. It is assumed that $s_t(i)$ are i.i.d with zero mean and variance $\sigma_s^2$. $H$ is the $M_t \times M_r$ channel gain matrix whose elements are i.i.d complex circular symmetric Gaussian random variables with zero-mean and unit variance. The received signal by the $M_r$ antennas is

$$x_t = [x_t(1), ..., x_t(M_r)]^T$$

which equals

$$x_t = \rho H s_t + v_t \quad [3]$$

where $\rho$ is used to adjust transmission power, $v_t$ is the corresponding AWGN, each of its elements is assumed to have zero mean and variance $\sigma_v^2$. The received SNR at each antenna is

$$SNR = \frac{\rho^2 \sigma_s^2 M_t}{\sigma_v^2} \quad [3]$$

Diversity gain, $d_r$, and the multiplexing gain, $r$, is given by,
where $P_e$ is the bit-error-rate and $R$ is the transmission data rate.

$$d_r = -\lim_{SNR \to \infty} \frac{\log P_e(SNR)}{\log SNR}, \quad r = \lim_{SNR \to \infty} \frac{R(SNR)}{\log SNR} \quad [3]$$

where $[r]$ denotes the maximum integer that is no larger than $r$.

With the multiplexing gain $r$, the achievable data rate is

$$R = r \log SNR$$

From $r$, the diversity gain $d_r$ can be obtained through the equations given above.

From the Chernoff bound of BER,

$$SNR = P_e^{-\frac{1}{d_r}} \quad [3]$$

For a transmission distance $d$, the SNR is related to the average transmission power $P_t$ as

$$P_t = Cd^n SNR \sigma_v^2 = Cd^n \sigma_v^2 P_e^{-\frac{1}{d_r}} \quad [3]$$

where the factor $Cd^n$ denotes the large scale path loss with exponent $n$. From the transmission data rate equation this becomes

$$R = -\frac{r}{d_r} \log P_e$$

If the total data to be transmitted is $N$, the transmission energy is given by

$$J_t = \frac{P_t N}{R} = \frac{Cd^n \sigma_v^2 N d_r P_e^{-\frac{1}{d_r}}}{-\log P_e} \quad [3]$$

The transmission energy is a linear function of $d^n$ and transmission time $N/R$. The circuit energy can be written as $E_c N/R$, where $E_c$ is a constant factor assumed identical for both the transmitter and the receiver. Thus the circuit energy is also a linear function of transmission time.
The SISO equivalent is to have \( d_r = 1 \) and \( r = 0 \). Thus the SISO energy consumption per bit is given by

\[
J_s = \sigma_v^2 SNR_s C d^n N R_s + \frac{2E_c N R_s}{R_s} \triangleq \frac{E_t SNR_s d^n N}{R_s} + \frac{2E_c N R_s}{R_s}
\]

where \( SNR_s \) denotes the SNR required in SISO and \( E_t \triangleq \sigma_v^2 C \).

For MIMO transmissions, the energy consumption per bit is given by,

\[
J_{tc} = K_{tc} E_t \frac{d_r d^n}{r} P_e^{-1} d_r + \frac{K_{tc} E_c (M_t + M_r) d_r}{r}
\]

where \( K_{tc} = \frac{N}{-\log P_e} \).

**2.2 Energy efficiency of fixed rate MIMO systems:**

In [2], fixed rate analysis is done assuming BPSK modulation (\( b=1 \)). It was found that below a certain transmission distance SISO outperforms both MISO 2x1 and MIMO 2x2. This distance is larger for the MIMO case than the MISO case since MIMO adds more circuit energy. If only transmission energy is considered, MISO beats SISO for all distances due to the diversity gain.

![Fig 3. Total energy consumption per bit over d for MISO and SISO [2]](image-url)
2.3 Energy efficiency of variable rate MIMO systems:

The energy efficiency of a MIMO system can be improved by racing to sleep. This was done by exploiting the higher data rates that MIMO systems can support. In [2], this was done by optimizing the constellation size. If $T_{on}$ is the time during which the circuit is active i.e. transmits and receives bits, this parameter is optimized so that the circuit can return to sleep where all the circuits in the signal path are shut down to save energy. The bits per symbol, $b$ is given by

$$b = \frac{L}{BT_{on}}$$

[2]

where B is the modulation bandwidth for the MIMO system and L is the no. of bits in the transmit buffer.

Since MIMO and MISO systems support higher data rates than SISO in Rayleigh fading channels, it is possible to have higher constellation sizes for MISO and MIMO systems with a certain the BER requirement. A larger constellation size implies a higher data rate which decreases the transmission time which in turn reduces the circuit energy consumption. The data rate of the system is given by

$$R_b = \frac{1}{bB} = \frac{L}{T_{on}}$$

An optimal value for $b$ which minimizes the total energy consumption was found by fixing the transmission distance and calculating the total energy consumed per bit for different constellation sizes. It was found that using this optimal constellation size for
transmission, the critical distance over which MISO and MIMO was more energy efficient than SISO was dramatically decreased.

Fig 5. Total energy consumption per bit over b for 2 × 1 MISO [2]

Fig 6. Optimized energy consumption per bit over d for MISO and SISO [2]

<table>
<thead>
<tr>
<th>TABLE II</th>
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<tbody>
<tr>
<td>OPTIMIZED CONSTELLATION SIZE (MISO VERSUS SISO)</td>
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<tr>
<td>d (m)</td>
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<tr>
<td>b_{MISO}</td>
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<tr>
<td>b_{SISO}</td>
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<th>TABLE III</th>
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<tr>
<td>OPTIMIZED CONSTELLATION SIZE (MIMO VERSUS SISO)</td>
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<tr>
<td>d (m)</td>
</tr>
<tr>
<td>b_{MIMO}</td>
</tr>
<tr>
<td>b_{SISO}</td>
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</table>
Also it was seen that MISO is more efficient than MIMO below a certain transmission distance. This is because MIMO has more RF chains than MISO. We see that as the number of transmit or receive antennas increases this critical distance increases i.e. $2 \times 2$ MIMO and $2 \times 1$ MISO has a lower critical distance compared to $2 \times 4$ MIMO and $1 \times 4$ single-input multiple-output (SIMO).

![Fig 7. Optimized energy consumption per bit over d for MIMO and SISO [2]](image)

![Fig 8. Total energy consumption over d for $2 \times 2$ MIMO and $2 \times 1$ MISO [2]](image)
Fig 9. Total energy consumption over $d$ for $1 \times 4$ SIMO and $2 \times 4$ MIMO \[4\]

For this analysis, the higher data rates are obtained by increasing the constellation size i.e. only diversity gain is exploited.

In [3], analysis of the energy efficiency of MIMO transmissions is based on the trade-off between the diversity gain and the multiplexing gain. The system model is as explained in section 2.1.

For a certain $P_e$, the $SNR_x$ and the corresponding SISO energy consumption was found for a fixed transmission distance. For a fixed transmission distance and different multiplexing gain the energy consumption per bit was plotted (fig shows for $d=10m M_t=M_r=2$).

Fig 10. Total energy consumption per bit vs multiplexing gain \[3\]

It was concluded that there is an optimum multiplexing and diversity gain for which the energy consumption is the least. Also, the energy consumption for this diversity and multiplexing gain is lower than the energy consumption for a SISO system. Here the spatial multiplexing is used to obtain higher data rates.
In [4], the system model used is the same as in [2]. Here the energy consumption of different MIMO transmission strategies is compared. Analysis is done using the probability error equations for different transmission strategies in Rayleigh fading, namely, Orthogonal Space-Time Block Coding, Spatial multiplexing with maximum likelihood detection, Spatial multiplexing with decorrelating decision-feedback multiuser detection (can be found in [4]). The data rate $R$ is given by $B \log_2 M$ for SIMO, $rB \log_2 M$ for STBC, $N_T B \log_2 M$ for SM and $B \sum \log_2 M_i$ for adaptive SM, where $B$ is the system bandwidth [4].

From these equations the normalized energy per bit, $\tilde{E}_b/N_o$, was compared to the spectral efficiency for a given BER.

It was concluded that in fading channels the receive diversity improves the energy efficiency by orders of magnitude. STBC adds transmit diversity which furthers lowers the energy consumption. However, for $N_T > 2$, orthogonal designs aren’t effective as full rates can no longer be achieved. SM with ML detection is much more energy efficient especially for high-throughput communications. Given $N_T$, the required SNR increases at a much slower slope as opposed to STBC. Also increasing $N_T$ boosts the throughput at very little extra energy cost.

![Normalized energy per bit vs spectral efficiency](image.png)

**Fig 11.** Normalized energy per bit vs spectral efficiency [4]
However SM with ML may be hindered due to complexity issues. Also, from the plot shown above it was concluded that SM with DDF MUD, satisfactory performance can be got for an over-determined system i.e. $N_T > N_R$, however, when $N_T$ approaches $N_R$ there is a significant degradation. With link adaptation techniques, such as rate adaptation and power control, the performance can be improved. It can even outperform the equal-power equal-rate SM with ML detection.

For a given spectral efficiency and target BER, the comparison of the energy consumption of the various schemes is shown in the figure that follows. From this plot it was concluded that, SM with ML detection outperforms SIMO at a smaller distance. Also 2x4 SM-ML is better than 4x4 SM-ML for $d<140m$ due to the extra circuit energy.

The figure below compares SM-ML with the adaptive spatial multiplexing which is realized through joint antenna selection and link adaptation techniques proposed in [5].

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**Fig 12.** Normalized energy per bit vs spectral efficiency [4]

**Fig 13.** Total energy consumption per bit over $d$ [4]
Both [2] and [3] exploit the higher data rates that MIMO systems can support to reduce transmission time (race to sleep). [3] uses multiple spatial streams to increase the data rate while [2] optimizes the constellation size. [4] compares the energy consumption of different transmission strategies for a given throughput and BER requirement in Rayleigh fading.

3. General system model:

Here we give a system model for a diversity gain, $d_r$, and multiplexing gain, $r$. From [2], the total energy consumption per bit can be expressed as,

$$E_{bt} = \left(\frac{\xi}{\eta}\right) \times \bar{E}_b \times \left(\frac{4\pi d}{G_t G_r \lambda^2}\right)^2 M_t N_f + \frac{P_c}{R_b}$$

where $\bar{E}_b$ is the required energy per bit at the receiver for a given BER requirement, $R_b$ is the bit rate, $d$ is the transmission distance, $G_t$ is the transmitter antenna gain, $G_r$ is the receiver antenna gain, $\lambda$ is the carrier wavelength, $M_t$ is the link margin and $N_f$ is the receiver noise figure given by $N_f = N_r/N_o$ where $N_o$ is the single-sided thermal noise power spectral density, $N_r$ is the power spectral density of the total effective noise at the receiver input, $\eta$ the drain efficiency of the RF power amplifier, $\xi$ the peak-to-average ratio and $P_c$ is the circuit power consumption of the link which is given by,

$$P_c = M_t \left( P_{DAC} + P_{mix} + P_{f\text{ilt}} \right) + 2P_{\text{syn}} + M_r \left( P_{LNA} + P_{mix} + P_{IFA} + P_{f\text{itr}} + P_{ADC} \right)$$

where $P_{DAC}$, $P_{mix}$, $P_{LNA}$, $P_{IFA}$, $P_{f\text{ilt}}$, $P_{f\text{itr}}$, $P_{ADC}$, $P_{\text{syn}}$ are the power consumption values for the DAC, the mixer, the low-noise amplifier (LNA), the intermediate frequency amplifier (IFA), the active filters at the transmitter side, the active filters at the receiver side, the ADC and the frequency synthesizer, respectively, $M_t$ is the number of transmit antennas and $M_r$ is the number of receive antennas.
By the Chernoff bound (in the high SNR regime), we have

$$\bar{P}_b \leq \left( \frac{\bar{E}_b}{M_t N_o} \right)^{-d_r}$$

where $\bar{P}_b$ is the BER.

From this we get an upper bound for the required energy per bit,

$$\bar{E}_b \leq \frac{M_t N_o}{\bar{P}_b \bar{\sigma}_\tau}$$

Using the upper bound, the total energy consumption per bit can be expressed as,

$$E_{bt} = \left( \frac{\xi}{\eta} \right) \times \frac{M_t N_o}{\bar{P}_b \bar{\sigma}_\tau} \times \left( \frac{4 \pi d}{G_t G_r \lambda^2} \right)^2 M_t N_f + \frac{P_c}{R_b}$$

For given constellation size, $M$, and multiplexing gain, $r$, the bit rate, $R_b$ is given by $rB \log_2 M$. Considering that an optimal coding scheme is used, the diversity gain is given by,

$$d_r = (M_t - r)(M_r - r) \quad [1]$$

The energy consumption per bit for SISO system can be found for $M_t = 1$, $M_r = 1$, $d_r = 1$ and $r = 1$.

The energy consumption per bit is calculated at a fixed distance, for different data rates which are achieved by using $r$ number of antennas for multiplexing. Here we consider QPSK modulation i.e. $M = 4$. The values used to calculate the energy consumption per bit is the same as in [2]. They are given in the table below.

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tr>
<td>SYSTEM PARAMETERS</td>
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<table>
<thead>
<tr>
<th>$f_c$ = 2.5 GHz</th>
<th>$\eta = 0.35$</th>
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<tbody>
<tr>
<td>$G_t G_r = 5$ dBi</td>
<td>$\sigma^2 = \frac{N_0}{2} = -174$ dBm/Hz</td>
</tr>
<tr>
<td>$B = 10$ KHz</td>
<td>$\beta = 1$</td>
</tr>
<tr>
<td>$P_{m_{ix}} = 30.3$ mW</td>
<td>$P_{syn} = 50.0$ mW</td>
</tr>
<tr>
<td>$P_b = 10^{-3}$</td>
<td>$T_s = \frac{1}{B}$</td>
</tr>
<tr>
<td>$P_{f_{it}} = P_{f_{itr}} = 2.5$ mW</td>
<td>$P_{LNA} = 20$ mW</td>
</tr>
<tr>
<td>$N_f = 10$ dB</td>
<td>$M_L = 40$ dB</td>
</tr>
</tbody>
</table>

[2]
From the above plots we see that an energy efficient MIMO system, requires all the antennas to be used for increasing the data rates for short transmission distances, i.e. to obtain a high multiplexing gain, and as the transmission distance increases we must use some antennas for diversity and some for multiplexing. For very large transmission distances the antennas must be used for diversity only. This is because at short transmission distance, the total energy consumption is dominated by circuit energy which requires us to reduce the transmission time. As the transmission distance increases, the circuit energy becomes less significant compared to the energy required to transmit over a longer distance. Therefore, we must use some antennas for diversity and others for multiplexing to optimize the overall energy efficiency.
distance increases, the transmission energy starts to dominate the energy consumption. Thus diversity must be used to reduce the required energy per bit.

4. Conclusion:

From the studies we see that, for an energy efficient MIMO system, we must use variable rates. This can be done by optimizing the constellation size or by spatial multiplexing.

Although [3] shows that an optimal diversity and multiplexing point exists, it does not explore how this point changes over transmission distance. From the analysis above we see that for short ranges we must use the antennas for spatial multiplexing and as the transmission distance increases, we must use more antennas for diversity. However, in this analysis we consider the upper bound for the calculations. Further studies can be done by evaluating the required energy per bit using Monte Carlo simulations.

Also the system model proposed in [2] is flexible as it can modified to include more circuit blocks, can be used to analyze different MIMO transmission strategies and can be modified to include the diversity-multiplexing trade-off.

Further research directions would be to compare the energy consumption of a variable-rate MIMO system that increases rates by optimizing constellation size to that of a variable-rate MIMO system that increases rates by spatial multiplexing. Also the impact of the digital circuits on energy consumption must be studied.

5. References:


