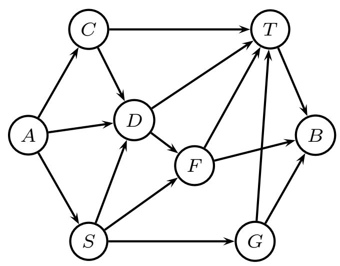
Introduction to Optimization
MS&E 111/ENGR 62, Autumn 2008-2009, Stanford University
Instructor: Ashish Goel
Homework 4. Given 11/5/08. Due 11/12/08 in class.

Collaboration policy: You can solve Problems 1 and 2 with a partner. If you choose to do so, both of you should turn in a copy of your Answer Reports and clearly indicate who you worked with. For all other problems you can discuss general strategies with other students in this class but cannot collaborate on the actual final answer. You cannot discuss the HW with anyone not in the class.

Problem 1



Consider the oil pipeline network in the figure above. The table below gives the capacity of each pipeline (in million gallons per hour), the cost per million gallons of sending oil over a pipeline, and the failure probability.

Edge	Capacity	Cost	Failure Prob.
AC	5	$7000	.4
AD	4	$4000	.9
AS	2	$1000	.1
CD	1	$2000	.2
CT	1	$4000	0
DT	3	$2000	.5
DF	1	$2000	.1
SD	1	$1000	.3
SF	2	$3000	.4
SG	2	$5000	.2
FB	1	$2000	.2
FT	4	$6000	.2
GT	5	$0	1
TB	2	$1000	.6
GB	2	$2000	.8

For each of the following, provide an LP whose solution gives the desired information and solve in Excel:

(a) the cheapest way of sending 1 million gallons per hour from A to B

(b) the cheapest way of sending 5 million gallons per hour from A to B

(c) the most reliable path from A to B

(d) the maximum amount of oil that can be sent from A to B

Problem 2

(a) Suppose that we have N different currencies (e.g., US Dollars, Euros, etc.). For each pair of currencies, (i, j), one unit of currency i can be exchanged into r_{ij} units of currency j.

We wish to find an arbitrage opportunity if it exists, i.e. a sequential series of currency trades that allows us to make a profit with no risk. Formulate this as a min-cost flow problem and provide an LP with a bounded feasible polytope which solves it. Assume no transaction costs.

(b) Can you find an arbitrage opportunity given the currency exchange rates in the spreadsheet currencies.xls? If so, describe the corresponding trades. What is the maximum amount of profit that can be made for each \$1.00 invested in this procedure?

(c) Suppose now that the exchange charges a $p\%$ transaction fee on each unit of currency traded. In other words, if we wish to trade currency i into currency j, $p\%$ of our initial amount of i is "taken away" before doing the exchange.

Modify your LP from (a) to take these fees into account. Assuming $p = 0.50$, how does this change your answer for (b)?

Problem 3

(a) You are given a min-cost flow problem where the total supply is larger than the total demand. Any excess left over supply at any node must be discarded at a cost of f per unit. Briefly describe, in words, how we can model this as a standard min-cost flow problem where the total supply equals the total demand.

(b) Suppose every edge has a lower bound as well as an upper bound on the amount of flow that can go through the edge. For example, a pipeline might rust if a minimum amount of oil is not shipped through that pipeline. What additional conditions (beyond those described in class) should the problem satisfy for you to claim that all basic feasible solutions have integer components? Explain why very briefly.

Problem 4

You are given an $N \times N$ matrix A such that each entry of the matrix is a non-negative number. Further assume that the sum of the entries in any row or column is an integer. You are allowed to round each fractional entry in the matrix, i.e. to change each non-integer entry to either the next higher or next lower integer. Prove that there is a way of rounding each entry such that the row and column sums remain unchanged.

Hint: Consider a min-cost flow problem with $2N$ nodes, one for each row and one for each column. Then, apply the integrality theorem from class along with the results of problem 4 above.

Problem 5 Consider a knapsack problem where a thief has a knapsack with a maximum volume U and weight limit W. The thief is in a warehouse with N goods. Each good i has a weight w_i, a volume u_i, and a value v_i. Additionally, we assume each good to be infinitely divisible, implying not only that we can take fractions of goods, but that we can pack the goods into our knapsack such that the entire space is filled. The thief wants to maximize the value in the knapsack subject to the two capacity constraints.

(a) Set up an LP that will solve this problem. As always, be sure to clearly identify your decision variables.

(b) Assuming there is exactly one optimal solution, what is the maximum number of goods that the thief would take fractionally? Concisely explain why you know this to be true.

(c) Assume now that some subset P of the N goods are precious metals. In addition to the other constraints already mentioned, the thief would like to cap the value of precious metals in the knapsack at M. Again assuming there is exactly one optimal solution, what is the maximum number of goods that the thief would take fractionally now? Why?

(d) Assume now that the thief has decided to enlist some friends, so that there are now k thieves with identical knapsacks, each limited to taking volume up to U, weight up to W, and metals valued up to M. The thieves wish to maximize the aggregate value gathered by the entire set of thieves. Again assuming there is exactly one optimal solution, what is the maximum number of goods that will be taken fractionally by our set of thieves in aggregate (i.e. after you add up the totals taken by all thieves)?