MS&E 235: Internet Commerce

Problem Session 2 Solutions

1. Multi-armed bandits.

   Consider a telephone sales company (a company that calls you on your phone and offers to sell you goods) that sells only one good. This company does not distribute the good but simply takes a one dollar commission from another company that does. This company has (endless lists of) telephone numbers ready to be called for the first time. The team has called phone numbers with area code "201" fifty times and got one sale, and numbers with area code "918" one hundred forty times and got three sales.

   - The CEO and owner are fighting over which area code the team should focus on next, and they ask you to offer your opinion. What is your opinion? (Hint: you may need to try several discount factors).

     The problem can be modeled as a multi-armed bandit problem, with two slot machines: (1,49) and (3,147).

     With theta = 0.90. The gittins values are 0.23 and 0.21 respectively. With theta = 0.95, 0.51 and 0.44. We can argue that for theta near one, we would expect this trend (of the first machine being better) to hold because the first machine has higher value of information. Choose 201.

   - How would your answer change if "201" area code phone calls each cost one cent more per call in long distance charges?

     However, when there is a fee of one cent per call we must modify the equation in each cell in the gittins spreadsheat by adding a -0.01 to the second term. The new gittins index is 0.16; Choose 918.

     Actually the best option is to call an new area code; which option the executives have yet to consider. This is because a (1,1) machine has higher gittins index then them all.

2. Individualistic and Social Choices.

   Consider the scenario similar to the system described in class. At time t, consumer number t comes into the system, first buys 1.0 units of a unique good – called good number t, and then buys units of goods 1, 2, ..., t-1 and some additional units of good t. In this second move he buys 1.0 goods in total, and buys the goods proportional to their percentage market share.

   - Determine the number of goods 1, 2, ..., 10 purchased at the end of time t = 10.

     t = 1: 2

     t = 2: 2.67 1.33

     t = 3: 3.2 1.6 1.2

     t = 4: 3.66 1.83 1.37 1.14

     ...

     t = 10: 5.68 2.84 1.77 1.55 1.40 1.28 1.19 1.11 1.05

   - Fit a curve of the form $m(i) = a * i^{-e}$ to this data, by finding the constants a and e that give the best fit.

     Plot t = 10 vs 1, 2, 3, ..., 10. Select "insert chart". Right click on colored line, and choose "add trendline". Choose "power". Choose to "display equation" and "display R^2".

     $$y = 5x^{-.7}$$
     $$R^2 = 0.9832$$

   - Based on the constant e that best fit the data, would you call this a long or short tail?

     This is a long tail because the error is small. In fact this is a tail with infinite median because $e < 1$.

3. Consider the market for widgets. The most popular brand sells 2.7 times as many items as the second most popular brand. The second most popular brand sells 2.7 times as many items as the third, and so on with the third and fourth, etc.

   - Would you consider this a short or long tail? This is perhaps a short tail. Fitting a trendline to fake data shows 10 percent error. (Also look at $a * x^k * e^{-x}$)
   - What percentage of the market share is in the third position onward (down the tail)?
     Integrate from 1 to t of e^{-x} gives $e^{-1} - e^{-t}$. Thus the first three goods compose of $(e^{-1} - e^{-3})/(e^{-1}) = 1 - e^{-2}$ of the market share. Thus the rest of the goods compose of $e^{-2} = 14$ percent of the market share.