MS&E 235: Internet Commerce

Problem Set 3, Due Thursday, May 21, 2009 (IN CLASS for non-SCPD students)

1. Consider a directed 4-cycle, i.e. node A has an edge to node B, B has an edge to C, C has an edge to D, and D has an edge to A.

   (a) Compute the naive PageRank of node A. [5pts]

   (b) Now add an edge from node A to node D. Recompute the naive PageRank of node A. Can you give a plain English explanation of this phenomenon? [8pts]

   (c) Assume that each node is allowed to add additional outgoing edges to the original 4-cycle. More precisely, node A is allowed to have (i) an edge to B alone, or (ii) edges to B and C, or (iii) edges to B and D, or (iv) edges to B, C, and D. The strategy space for B, C, and D are similarly defined. Each node must have an edge pointing to the next node in the cycle, and is allowed to have as many additional outgoing edges as it wants. Edges from a node to itself are not allowed. Also, for any two nodes X and Y, there can be at most one edge from X to Y and at most one edge from Y to X. Each node wants to maximize its own naive PageRank. Prove that the 4-cycle does not represent a Nash equilibrium. [15pts]

   (d) Suppose each node adds an edge to its predecessor in the 4-cycle, i.e. A adds an edge to D, B adds an edge to A, C adds an edge to B, and D adds an edge to C. Prove that this graph (known as a bidirected 4-cycle) represents a Nash equilibrium in the game outlined in part (c). [10pts]

   (e) Find another Nash equilibrium in this game. [7pts]

2. Consider an information network which is trying to predict an event whose intrinsic probability is q. Suppose that the only shares in the system are YES shares. If the event happens, everyone who holds a share gets paid a Dollar. Suppose there exists a risk-neutral insider who knows this probability q.

   (a) (5pts) Let us define the clearing price to be a price such that the insider will neither buy nor sell shares at this price. Find the clearing price of the above market.

   (b) (15pts) Now imagine that this market is used to predict an event whose probability changes with popular perception. If the price of the YES share is p, the probability of the event changes to $(p+q)/2$. Find the clearing price. Use this to comment on the resilience of information markets.

3. (20pts) In class it was shown that by iteratively computing successive values of $\pi^{(i)}$ according to $\pi_k^{(i)} = (1-\epsilon)\sum_{[(j,k)\in E]} \pi_j^{(i-1)}/dj + \epsilon/N$ we converge to the true PageRank, π^*. In particular, it was shown that the error $e^{(i)} \equiv \sum_v |\pi_v^* - \pi_v^{(i)}|$ converges to zero. Compute an upper bound on the number of steps required so that $e^{(i)} \leq \delta$ for some $\delta > 0$.

4. (15pts) Suppose every real individual were to have a unique reviewer id which is used across all the reputation systems on the Internet, i.e. while rating books on Amazon, hotels on tripadvisor, corporations on rapleaf, click-based ranking on search engines, products on epinions, etc. Which major problem associated with online reputation systems will this solve? Is this going to automatically make all these reputation systems robust? Describe one subsidiary business that might emerge that would help merchants/publishers/hotels game the system.