MS&E 235: Internet Commerce

Problem Set 2, Due Tuesday, April 28, 2008

1. In this problem, we will assume that three advertisers have made bids on the same keyword in a search engine. The search engine (which acts as the auctioneer) assigns (α, β) priors to each advertiser, and uses their Gittins' index to compute the winner. The advertisers are:

   - Advertiser (a) has $\alpha = 2$, $\beta = 5$, has bid \$1 per click and no has budget constraint.

   - Advertiser (b) has $\alpha = 1$, $\beta = 4$, pays \$0.2 per impression and additionally \$1 if his ad is clicked. He has no budget constraint.

   - Advertiser (c) has $\alpha = 1$, $\beta = 2$, has bid \$1.5 per click and his ad can only be shown 5 times (including this one).

   There is a single slot, the discount factor is $\theta = 0.95$ and a first price auction is used. Which ad would the auctioneer allocate the slot to? Briefly comment on what a second price auction could charge to the winner.

2. Find a company outside of any company discussed in class whose business model uses a long-tail principle in a critical way (i.e: the long tail is critical for this company). Explain how the long-tail plays a role in this company. Then explain why the long-tail principle is crucial for them: what about their company relies on it?

3. Consider a slight variant of the model for long tail phenomena described in class: Suppose the t-th new user in a system introduces and consumes one new product. This user also consumes k existing products, each with probability proportional to the number of times a product has been consumed already. Let $m_i(t)$ denote the number of times product i has been used up to and including time t. Determine whether the popularity of products follows a long tail. What does the exponent e tend to as k tends to infinity? [20pts]

4. The file movies-hw2.txt is an extract from recently published netflix data. The file contains the number of times the i-th movie was watched.

   (a) Determine whether this data represents a long tail by finding the best fit of the form $a(b + i)^{-e}$. Explain your reasoning. [10pts]

   (b) Compute the median rank, the expected rank, and the standard deviation of a movie watched by a customer. Which of these would you expect to go to infinity as the number of movies inreases but the tail exponent e remains fixed? [10pts]

   (c) Assume that netflix incurs a cost of \$0.90 for shipping and handling of each movie. Assume that a typical consumer rents 8 movies per month. Assume that the top 100 movies have a royalty fee of \$2 that netflix must pay, the next 900 have a royalty of \$1.50, and the rest have a royalty of \$0.50. Compute

Netflix's cost per customer. How sensitive is this number to a $0.25 change in the royalty fee for the first tier? For the third? [10pts]