Introducing correlation among fragility functions for multiple components

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ABSTRACT :

Correlations are known to be an important consideration when obtaining accurate probabilistic seismic loss estimation results. Correlations among structural response values are straightforward to measure and incorporate, but it is difficult to incorporate correlations in the element damage predicted by fragility functions. In the most widely used component-based loss estimation procedures (e.g., [1-5]), structural response correlations are incorporated, but typically only the special cases of independent or perfectly-correlated component damage states are allowed. A simple method for considering partially correlated damage is proposed here. Whereas traditional calculations first compute a probability of damage and then consider occurrence of that damage to be random, the method proposed here instead considers the "capacity" of the components to be random, which allows partial correlation of capacities to be easily introduced (because the random component capacity is continuous, unlike the discrete damage states). The approach relies on Monte Carlo simulation, making it compatible with the Monte Carlo analysis approaches used within some existing loss assessment procedures, although purely analytic implementations are also feasible.

KEYWORDS: Fragility function, correlation, loss estimation, uncertainty, performance-based engineering

1 INTRODUCTION

Fragility functions are a well-known method for probabilistically predicting damage to components or structures. These functions provide the probability that a component will equal or exceed some level of damage, as a function of a structural demand parameter. (Fragility functions can also be used to predict damage to entire buildings rather than single components, but the focus here is primarily on component fragility functions.) Damage states for a component are typically not continuous, but rather a discrete set of damage states [6-8]. One fragility function is needed to define occurrence of each potential damage state of each component.

These fragility functions are often used to simulate damage to sets of multiple components in a single structure using Monte Carlo simulation. To predict the damage to a single component, one simulates damage states with relative frequencies specified by the fragility function (or functions, in the case of multiple damage states).These fragility functions are important inputs for component based loss estimation methodologies such as Assembly-Based Vulnerability [2], the PEER framework [1, 4], and the ATC-58 guidelines [3, 5].

Probabilistic dependencies (e.g., correlations) are known to be an important consideration in seismic loss estimation and risk analysis [9-11], but incorporating these correlations is not yet common. Analytically, one can assume independence of damage states, or full dependence of damage states (assuming the same demand level for all components). Partial correlation, however, has not yet been formulated in a tractable manner. Baker and Cornell implemented partial correlation in damage, but only after "collapsing" the discrete damage state prediction and the resulting financial loss distribution into a single continuous distribution of component economic losses for a given level of structural response, so that is not a general solution to the problem [12, 11].

Lee and Kiremidjian proposed a method for specifying correlations among discrete damage states using an optimization approach, but this is cumbersome for rapid implementation and thus likely impractical for general loss estimation approaches [13].

In this paper, a simple and general formulation for correlation in damage states is introduced. By slightly reformulating the traditional fragility function, it is possible to easily introduce correlation among the damage state "capacities" for individual components. The approach is straightforward to implement in Monte Carlo simulations, which are used in Assembly-Based Vulnerability [2] and the under-development ATC-58 guidelines [3, 5]. The approach can also be implemented analytically, but the primary focus here is on Monte Carlo implementation, to maximize compatibility with current loss estimation approaches.

2 DEFINING FRAGILITY FUNCTIONS

2.1 Standard formulation

The typical approach for defining a fragility function is to use a cumulative lognormal distribution function to define the probability that a component will be damaged to a given damage state *i* (or worse) as a function of demand. This function requires two parameters: a median, θ_i, and a log standard deviation (or "dispersion"), β_i. The fragility function can then be written as

$$P(DS \geq ds_i \mid D) = \Phi\left(\frac{\ln(D/\theta_i)}{\beta_i}\right) \tag{1}$$

where D is the level of demand on the component (e.g., an peak interstory drift or peak floor acceleration) and $\Phi(\ )$ is the standard normal cumulative distribution function[1]. An example of this fragility function formulation is shown in Figure 1a.

2.2 An alternative formulation

An alternative, equivalent, formulation of a fragility function is to define a "damage capacity" as the demand level at which the element enters a specified damage state. That damage capacity is defined as a lognormal random variable with median θ_i and dispersion β_i. Monte Carlo simulation is then used to generate individual realizations of damage capacity, and damage is simulated if the specified demand level D is greater than the simulated damage capacity. In this case the fragility is a step function, but the location of the step is random, having the given lognormal distribution. Mathematically, this can be written as

$$P(DS \geq ds_i \mid D) = P(C_i < D) \tag{2}$$

where C_i is the damage capacity for damage state *i*. C_i is lognormally distributed, with median θ and lognormal standard deviation β (i.e., the same parameters as in equation 1. Using random variable notation, we can write this distribution for C_i as

$$C_i \sim LN(\theta_i, \beta_i) \tag{3}$$

where $\sim LN(\cdot)$ is denotes that C_i is a lognormal random variable with the given distribution parameters.

This formulation is shown graphically in Figure 1b. The probability of damage computed using many Monte Carlo simulations will thus be the probability that the damage capacity is less than the specified demand level.

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[1] A normal cumulative distribution function evaluated using ln(D) corresponds to a lognormal distribution for (non-log) D.

This probability, shown as the shaded region of Figure 1b, is exactly equal to the lognormal cumulative distribution function given by equation 1.

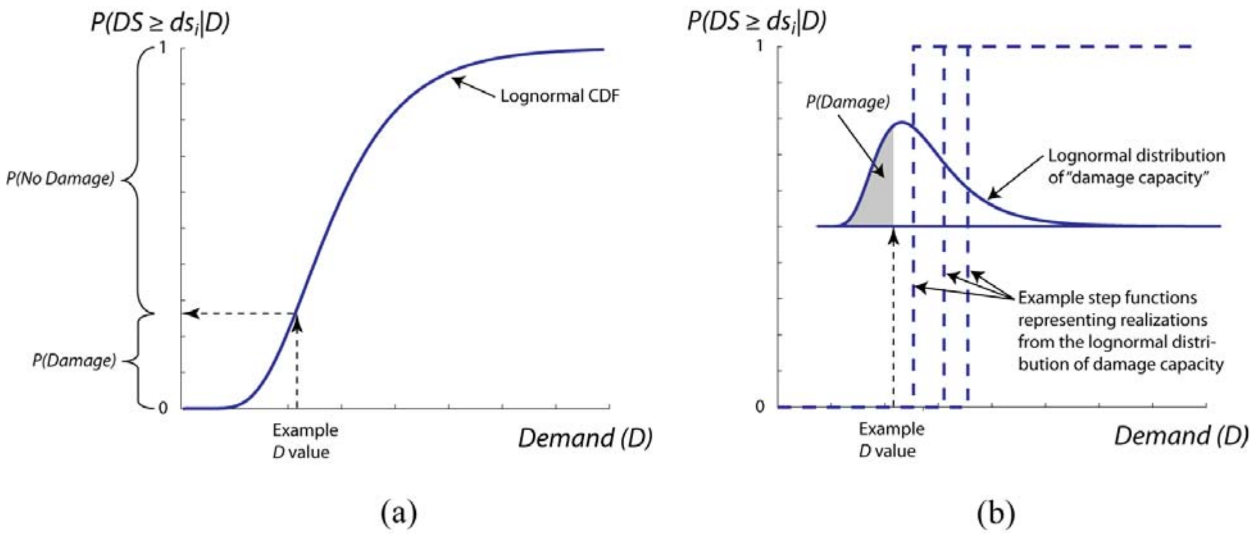

Figure 1: Two equivalent representations of an element fragility. (a) A lognormal CDF specifying the probability of collapse. (b) A step function specifying the probability of collapse, with the location of the step described by a lognormal distribution.

3 IMPLEMENTING CORRELATIONS

When considering only a single element, the two fragility formulations shown in Figure 1 are equivalent. Once multiple elements are considered, however, the advantage of the alternative formulation becomes apparent. Because it is occurrence of the damage state itself that is random in Figure 1a, it is not easy to introduce correlation in damage to multiple elements. Generally one must either assume that the damage state of each element is independent, or assume that the damage states are identical for all elements. Further, the assumption of identical damage states is only possible if the demand level is equal for all elements (so that the probability of damage is equal for all elements). This allows, for example, the assignment of identical damage to all partitions in a given story level with a specified interstory drift, but it does not allow for correlation to be introduced between partitions on different story levels.

The alternative formulation, on the other hand, can easily accept any level of correlation and can incorportate correlations in the absence of equal demands, because the dependence is introduced in the damage capacity rather than the damage state. The damage capacity is a lognormal random variable, so the damage capacity of multiple elements can be easily described by a multivariate lognormal random variable with any admissible correlation structure.

An illustration of simulated results for two elements is shown in Figure 2. The points in the figure represent Monte Carlo simulations of correlated capacity. Damage to individual elements occurs in the shaded regions of the figure. The approach can be used to simulate damage to elements on different stories, by simply letting Demand 1 and Demand 2 differ in Figure 2 (which would correspond, e.g., to having different interstory drift ratios on differing stories).

This approach requires only the ability to simulate correlated lognormal variables. This simulation capacity is already used, for example, in ATC-58 [5, Section F.2.2], so this approach can be adopted within that existing methodology without any major conceptual or programming effort. Alternatively, the probabilities illustrated in Figure 2 could also be evaluated analytically using the multivariate normal cumulative distribution function; while this function is not trivial to compute in high dimensions [14], it is available in commonly used software packages such as Matlab.

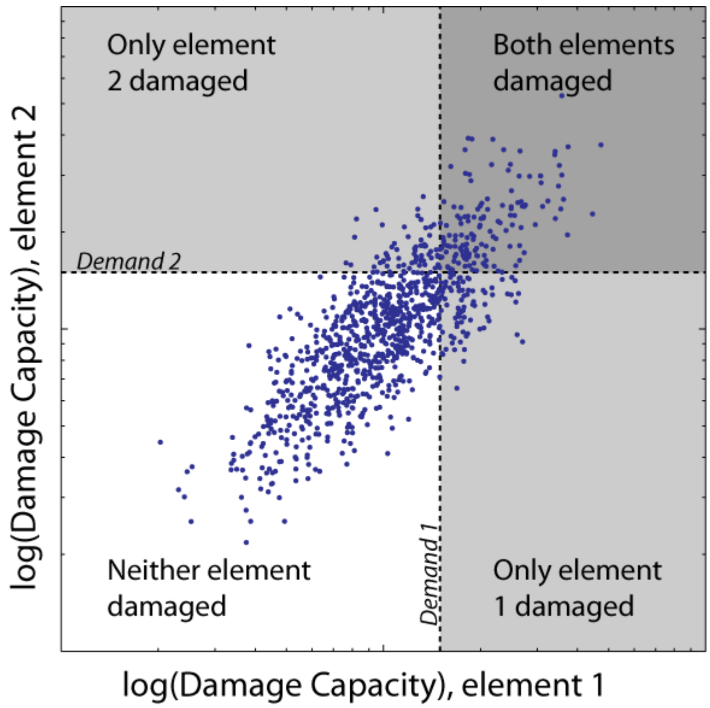

Figure 2: An illustration of the joint distribution of damage capacities for two elements. Points represent Monte Carlo simulations of the damage capacities for the two elements, and shaded regions indicate areas where the structural demand has exceeded the damage capacity for one or both elements.

4 IMPACT OF CORRELATIONS

Figure 3 shows the impact of correlations on the distribution of damage to multiple elements within a structure. This figure shows the distribution of the number of damaged elements out of a total of 10 elements, in the case where each element has a probability of damage of 0.3. Figure 3a shows the case where the damage state of each element is independent (the results in this case correspond to the Binomial distribution). Figure 3d shows the distribution in the case of perfect dependence; all 10 elements fail 30% of the time, and no elements fail the other 70% of the time. Figure 3b and c illustrate two cases of partially correlated fragilities. Given that economic losses are typically a direct function of the number of components in each damage state, it is clear that variations in the damage distributions of the type seen in Figure 3 will directly impact the distribution of overall economic losses in a building.

While Figures 3b and 3c can only be created using the proposed approach, Figures 3a and 3d can also be obtained using traditional approaches. The agreement of these new results with traditional results in those two special cases is a desirable, and shows that this approach merely adds flexibility without altering any of the results in the special cases considered before.

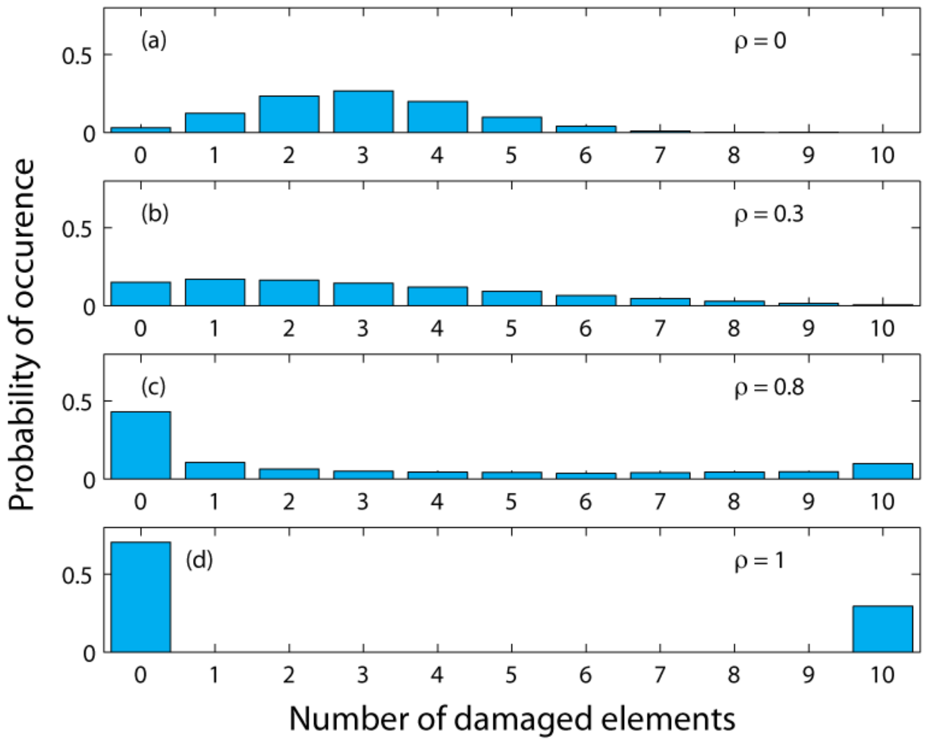

Figure 3: Probability of observing n damaged components out of 10, when the probability of damage of a single component is equal to 0.3, for four correlation coefficient values. (a) ρ=0 (uncorrelated fragilities). (b) ρ=0.3. (c) ρ=0.8. (d) ρ=1 (perfectly correlated fragilities).

5 EXTENTIONS OF THE BASIC FORMULATION

To ready this formulation for practical implementation, two further extensions will be needed. They are discussed briefly here.

5.1 Measuring correlations

To incorporate correlations in loss estimation, it will be necessary to specify a correlation value for the analysis. This is a somewhat challenging task, as correlations can not be measured from test data as easily as median and dispersion values. Two efforts could help with this problem. While it is not possible to measure correlations in capacity directly, it is possible to observe the number of damaged elements on a single floor of a structure, and compare to Figure 3 above. With a reasonable set of observations from earthquake reconnaissance missions, it may be possible to obtain a rough estimate of the correlation level consistent with the observations. Correlation of capacities could also be measured directly from test data if one knew the demand level at which the tested elements first failed (i.e., the damage capacity used in Figure 1b and Figure 2). A reasonable first characterization would be the so-called “equi-correlated” model, where all correlations between similar components are set to a single value [15]. This means that only one correlation coefficient must be estimated, further easing measurement efforts. Methods have also been proposed to calibrate fragility functions while accounting for dependencies among observed damage to multiple components or structures [16]. In addition to providing more accurate estimates of individual component fragilities, these methods also provide information relevant for measuring correlations.

When considering the practical challenges associated with measuring these correlations, one should also keep in mind that even an approximate model will likely improve on state-of-the-practice methods in place today, as any correlation value other than 0 or 1 is likely to be an improved representation of reality.

5.2 Multiple damage states

Building components typically fall into more than one state of damage. Some extra care is needed to adopt the formulation proposed above for multiple damage states, but the challenges are not great. The simplest approach is to simulate capacities for all damage states simultaneously, but without some care, this may lead to Monte Carlo realizations where damage state 2 has a lower capacity than damage state 1[2] (a clearly unrealistic output). A simple solution is to instead simulate damage state 1's capacity as a lognormal random variable, and then simulate damage state 2's capacity as the first capacity plus an additional, incremental, capacity. Following the formulation of equation 3, this approach could be written as

$$C_1 \sim LN(\theta_1, \beta_1) \tag{4}$$

$$C_2 = C_1 + \tilde{C}_2, \quad \text{where } \tilde{C}_2 \sim LN(\theta_2 - \theta_1, \beta_2) \tag{5}$$

$$C_3 = C_2 + \tilde{C}_3, \quad \text{where } \tilde{C}_3 \sim LN(\theta_3 - \theta_2, \beta_3) \tag{6}$$

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where C_i is the damage capacity for damage state i, and is lognormally distributed with median θ_i and lognormal standard deviation β_i. Equations 5 and 6 are written so that the randomness in capacity is composed of the (random) capacity for the preceding damage state, plus some additional randomness associated with the current damage state being considered. Because the incremental capacity is lognormally distributed, it will never take negative values, and thus the damage state capacities will always be properly ordered (with increasing damage levels having increasing capacities and decreasing probabilities of exceedance). That is, $C_1 \leq C_2 \leq C_3 \leq \ldots$, so $P(C_1 < D) \geq P(C_2 < D) \geq P(C_3 < D) \geq \ldots$. Note that equations 5 and 6 define capacities as a sum (rather than product) of lognormal random variables, which is not guaranteed to be lognormal. Example calculations reveal, however, that the resulting capacity distributions are essentially indistinguishable from lognormal distributions.

Figure 4 illustrates the distributions of element damage obtained using this formulation. In each sub-figure there are ten elements that can be damaged, and each element has a 0.3 probability of experiencing damage state 1 or greater, and a 0.1 probability of experiencing damage state 2 or greater. The only difference among the sub-figures is the correlation assumed between damage state capacities of the elements (the same correlation coefficient is assumed for damage state 1 and damage state 2). As with Figure 3, the assumed correlation coefficient is observed to have a great impact on the distribution of damage to multiple elements within a structure.

With this formulation, no additional parameters are required to be estimated for implementation. The θ_i and β_i parameters are already needed for traditional fragility function formulations (e.g., equation 1), so all that is needed are the correlation coefficients for the individual component capacities. If the equi-correlated model is retained and correlation coefficients are assumed constant for all damage states (a reasonable assumption in the absence of detailed observational data), then only the single correlation coefficient is required, as before.

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[2] Where the damage states are ordered such that a higher number indicates a greater level of damage.

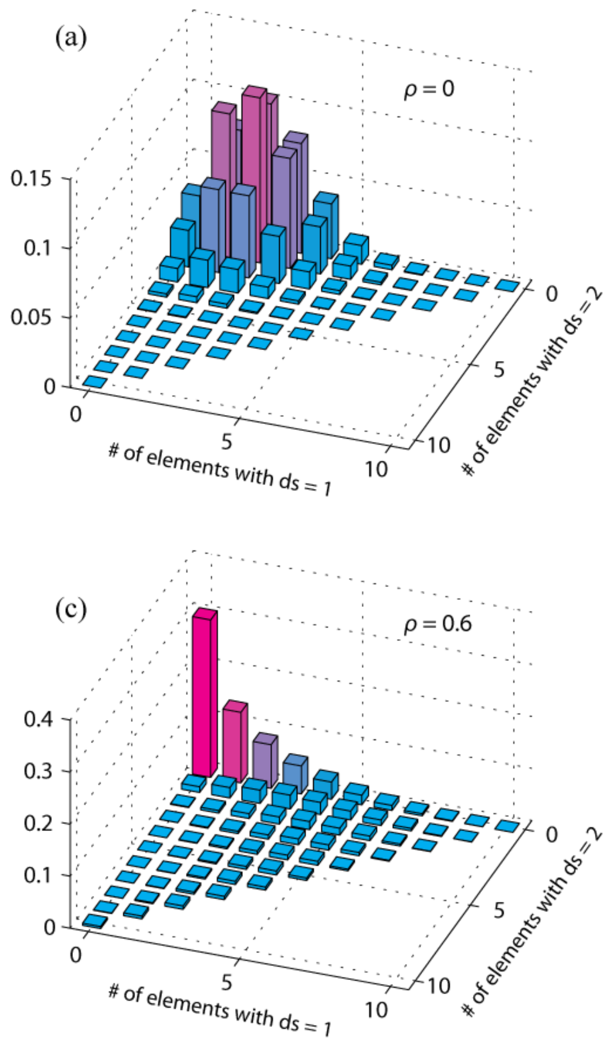

Figure 4: Probability of observing n damaged components out of 10, when there are two potential damage states and $P(DS \geq 1) = 0.3$ and $P(DS \geq 2) = 0.1$, for four correlation coefficient values. (a) ρ=0 (uncorrelated fragilities). (b) ρ=0.3. (c) ρ=0.6. (d) ρ=1 (perfectly correlated fragilities).

6 CONCLUSIONS

A simple method has been presented for modeling correlations among building component damage. Because component damage states are typically discrete, it is difficult to formulate a model with correlation among the damage states which multiple components take.

The methodology utilizes the same parameterization used to define traditional fragility functions (i.e., medians and dispersions), and requires only the additional specification of correlation coefficients between the component "capacities." In the special cases of uncorrelated or perfectly correlated damage, this formulation is equivalent to traditional approaches. This formulation, however, allows for the introduction of partially correlated damage that traditional approaches cannot consider. Given that partial correlation is likely the most realistic description of damage occurrence in a building, this generalization should lead to more accurate estimates of seismic losses.

While this manuscript has focused on damage to multiple components within a single structure, the approach is also applicable to damage to multiple buildings whose damage is believed to be correlated due to shared uncertainty in structural fragility functions. A preliminary application to model damage to a portfolio of buildings demonstrated that this approach is practical and useful for that problem as well [17].

REFERENCES

1 Cornell C.A., Krawinkler H., "Progress and Challenges in Seismic Performance Assessment". *PEER Center News*, 2000. **3**(2).

2 Porter K.A., Kiremidjian A.S., LeGrue J.S., "Assembly-Based Vulnerability of Buildings and Its Use in Performance Evaluation". *Earthquake Spectra*, 2001. **17**(2): 291-313.

3 Bachman R.E., Hamburger R.O., Comartin C.D., Rojahn C., Whittaker A.S. "ATC-58 Framework for Performance-Based Design of Nonstructural Components". *ATC-29-2 Seminar on Seismic Design, Performance, and Retrofit of Nonstructural Components in Critical Facilities*. 2003: Applied Technology Council. 49-61.

4 Moehle J., Deierlein G.G. "A Framework Methodology for Performance-Based Earthquake Engineering". *Proceedings, 13th World Conference on Earthquake Engineering*. 2004. Vancouver, Canada. 13.

5 Applied Technology Council, "ATC-58, Guidelines for Seismic Performance Assessment of Buildings, 35% Draft". 2007, Applied Technology Council: Redwood City, California. 260p.

6 National Institute of Building Sciences (NIBS), *Standardized Earthquake Loss Estimation Methodology (HAZUS 99 Technical Manual)*. 1999, Washington, D.C.: Report prepared for Federal Emergency Management Agency, Federal Emergency Managementp.

7 Porter K.A., Kiremidjian A.S., "Assembly-Based Vulnerability and Its Uses in Seismic Performance Evaluation and Risk-Management Decision-Making". 2001, John A. Blume Earthquake Engineering Center: Stanford. 214p.

8 Aslani H., Miranda E., "Fragility Assessment of Slab-Column Connections in Existing Non-Ductile Reinforced Concrete Buildings". *Journal of Earthquake Engineering*, 2005. **9**(6): 777-804.

9 Miranda E., Aslani H., Taghavi S., "Assessment of Seismic Performance in Terms of Economic Losses", *PEER 2004/05*, P. Fajfar and H. Krawinkler, Editors. 2004, International Workshop on Performance-Based Seismic Design Concepts and Implementation: Bled, Slovenia. 149-160p.

10 Aslani H., Miranda E., "Probabilistic Earthquake Loss Estimation and Loss Deaggregation in Buildings", *Report #154*. 2006, John A. Blume Earthquake Engineering Center: Stanford, CA. 355p.

11 Baker J.W., Cornell C.A., "Uncertainty Propagation in Probabilistic Seismic Loss Estimation". *Structural Safety*, 2008. **30**(3): 236-252.

12 Baker J.W., Cornell C.A., "Uncertainty Specification and Propagation for Loss Estimation Using FOSM Methods", *PEER 2003-07*. 2003, Pacific Earthquake Engineering Research Center, University of California at Berkeley: Berkeley, California. 89p.

13 Lee R., Kiremidjian A.S., "Uncertainty and Correlation for Loss Assessment of Spatially Distributed Systems". *Earthquake Spectra*, 2007. **23**(4): 753-770.

14 Genz A., "Numerical Computation of Multivariate Normal Probabilities". *Journal of Computational and Graphical Statistics*, 1992. **1**(141-149): 27-28.

15 Ditlevsen O., *Uncertainty Modeling: With Applications to Multidimensional Civil Engineering Systems*. 1981, New York; London: McGraw-Hill International Book Co. 412p.

16 Straub D., Der Kiureghian A., "Improved Seismic Fragility Modeling from Empirical Data". *Structural Safety*, 2008. **30**(4): 320-336.

17 Ryu H., Kim J.K., Baker J.W., "A Probabilistic Method for the Magnitude Estimation of a Historical Damaging Earthquake Using Structural Fragility Functions". *Bulletin of the Seismological Society of America*, 2008(in review).