



Incorporating modeling uncertainties in the assessment of seismic collapse risk of buildings

Abbie B. Liel^{a,*}, Curt B. Haselton^b, Gregory G. Deierlein^c, Jack W. Baker^c

^a Department of Civil, Environmental and Architectural Engineering, University of Colorado, Boulder, 80309, United States

^b Department of Civil Engineering, California State University, Chico, CA 95929, United States

^c Blume Earthquake Engineering Center (MC: 3037), Department of Civil and Environmental Engineering, Stanford University, Stanford, CA 94305, United States

ARTICLE INFO

Article history:

Available online 26 July 2008

Keywords:

Performance-based earthquake engineering
Collapse
Modeling uncertainties
Response surface
Monte Carlo
Seismic reliability

ABSTRACT

The primary goal of seismic provisions in building codes is to protect life safety through the prevention of structural collapse. To evaluate the extent to which current and past building code provisions meet this objective, the authors have conducted detailed assessments of collapse risk of reinforced-concrete moment frame buildings, including both 'ductile' frames that conform to current building code requirements, and 'non-ductile' frames that are designed according to out-dated (pre-1975) building codes. Many aspects of the assessment process can have a significant impact on the evaluated collapse performance; this study focuses on methods of representing modeling parameter uncertainties in the collapse assessment process. Uncertainties in structural component strength, stiffness, deformation capacity, and cyclic deterioration are considered for non-ductile and ductile frame structures of varying heights. To practically incorporate these uncertainties in the face of the computationally intensive nonlinear response analyses needed to simulate collapse, the modeling uncertainties are assessed through a response surface, which describes the median collapse capacity as a function of the model random variables. The response surface is then used in conjunction with Monte Carlo methods to quantify the effect of these modeling uncertainties on the calculated collapse fragilities. Comparisons of the response surface based approach and a simpler approach, namely the first-order second-moment (FOSM) method, indicate that FOSM can lead to inaccurate results in some cases, particularly when the modeling uncertainties cause a shift in the prediction of the median collapse point. An alternate simplified procedure is proposed that combines aspects of the response surface and FOSM methods, providing an efficient yet accurate technique to characterize model uncertainties, accounting for the shift in median response. The methodology for incorporating uncertainties is presented here with emphasis on the collapse limit state, but is also appropriate for examining the effects of modeling uncertainties on other structural limit states.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

Comprehensive assessment of the risk of earthquake-induced structural collapse requires a robust analytical model that captures nonlinear behavior and, also, explicit consideration of the many important sources of uncertainty. The largest uncertainty lies in characterizing the earthquake ground motion. Uncertainties in ground motion intensity are commonly represented by a site-specific hazard curve, which relates spectral intensity to the frequency of exceedance; the additional uncertainties associated with frequency content and other attributes of the ground motion records are termed 'record-to-record' variabilities. Apart from ground motions, there are uncertainties in simulating the structural response, which relate to the analysis method and the extent to which the idealized model accurately represents real behavior. Where de-

tailed nonlinear response history analysis is used to simulate structural response, a primary source of modeling uncertainty lies in definition of the analysis model parameters – specifically the strength, stiffness, deformation capacity, and energy dissipation characteristics of the building components – as compared to the components' actual behavior.

This study involves probabilistic assessment of structural collapse risk through nonlinear response history simulation, which incorporates the uncertainties associated with ground motions and structural modeling. However, the primary focus of this study is on modeling parameter uncertainties and how to realistically and expediently quantify their effects in nonlinear response history analysis. Past research (e.g. [1,2]) has indicated that modeling uncertainties associated with damping, mass, and material strengths have a relatively small effect on the overall uncertainty in seismic performance predictions, but these studies have focused primarily on pre-collapse performance of structures. In contrast, Ibarra and Krawinkler [3] have shown that the uncertainty

* Corresponding author.

E-mail address: abliel@stanford.edu (A.B. Liel).

associated with modeling deformation capacity and post-peak softening response of component element models can have a significant influence on the predicted collapse performance. This study builds on the work by Ibarra and Krawinkler to quantify the significance of modeling uncertainties associated with component deformation capacity and other parameters critical to collapse prediction of reinforced concrete moment frame buildings. Though we use reinforced concrete frame structures for illustration purposes, the procedure developed for incorporating modeling uncertainties is systematic and applicable to other structural systems.

To begin, we provide an overview of the collapse assessment procedure and results for reinforced concrete (RC) moment frames in high seismic regions. We then review methods for quantifying the effects of uncertainty in element and system level modeling, and propose a procedure that combines response surface analysis and Monte Carlo methods. This procedure is applied to six case study frame structures of varying heights and ductility capacity. The results of these case studies indicate that modeling uncertainties tend both to increase the dispersion (σ_{ln}) and also to shift the median (\bar{m}) of the probability distribution for structural response. We compare the response surface results with first-order second-moment reliability methods, which are easier to implement but rely on simplifying assumptions that do not necessarily apply. Finally, we propose a new simplified method (“ASOSM”), which captures the critical effects of modeling uncertainties, but requires less computational time than the response surface based method. Throughout this study, we focus primarily on the effects of modeling uncertainties on the assessment of collapse risk, but also demonstrate the applicability of response surface based method to structural response limit states other than collapse.

2. Overview of collapse assessment procedure and results

The procedure for collapse assessment utilizes the performance-based earthquake engineering methodology developed by the Pacific Earthquake Engineering Research Center, which provides a probabilistic framework for relating ground motion intensity to the structural response and building performance through nonlinear time-history simulation [4]. Assessment of global side-sway collapse capacity is based on the incremental dynamic analysis (IDA) technique [5]. In IDA, the structural model, which captures both material and geometric nonlinearities, is analyzed for a specific ground motion record. This time-history analysis is repeated, each time increasing the scale factor on the input ground motion, until that record causes structural collapse, as identified by runaway interstory drift displacements. This process is repeated for an entire suite of ground motion records.¹ In our study, the ground motion intensity measure is the spectral acceleration at the first mode period of the building [$S_a(T_1)$]. The outcome of the IDA procedure is a collapse fragility function, a cumulative probability distribution that defines the probability of structural collapse as a function of the ground motion intensity. The median value of the fragility (or median “collapse capacity”) corresponds to the ground motion intensity that causes collapse in half of the records of the ground motion suite. For a given structural model, the uncertainty in the collapse fragility from IDA represents the so-called record-to-record variability.

Several different metrics can be used to quantify collapse performance, either in absolute terms or relative to the earthquake

intensity used for design. In the United States, design levels for seismic effects in building codes are based on the definition of a “maximum considered earthquake” or MCE. Accordingly, the collapse capacity can be described through the following metrics: (a) the collapse capacity margin, equal to the ratio of median collapse capacity obtained from IDA to the MCE intensity, (b) the probability of collapse conditioned on the MCE (or other hazard level of interest), and (c) the mean annual frequency of collapse, obtained by integrating the collapse probability distribution with the hazard curve for a particular site.

This procedure has been applied to assess the performance of both ductile and non-ductile RC frame buildings. The ductile frames represent designs that conform to current building code requirements, whereas the non-ductile frames represent older buildings that do not meet current building code design and detailing requirements, and typically exhibit worse seismic performance. The nonlinear analysis models consist of the two-dimensional three-bay frame, as shown in Fig. 1a. Modeled in OpenSees [6], the simulation model captures both material nonlinearities in beams, columns, and beam-to-column joints and large deformation ($P-\Delta$) effects. Inelasticity in the beams, columns, and joints are modeled with concentrated springs idealized by the backbone response curve shown in Fig. 1b and the associated hysteretic rules developed by Ibarra et al. [7]. An important attribute of the inelastic model is that it captures both in-cycle and between-cycle strength degradation, the former being particularly important for realistic simulation of collapse behavior [3,8]. Properties of these inelastic springs are obtained from calibration to experimental tests of reinforced concrete beam-columns and joints, as described by Haselton [8]. These spring properties are calibrated to mean or expected values of the structural components. When used in combination with nonlinear geometric transformations and robust convergence algorithms, these structural models are capable of simulating structural response into the collapse limit state. These collapse models have been used in several applications, including the validation of seismic performance factors for building codes in the ATC-63 project [9,10].

Haselton [8] evaluated the collapse capacity of 30 ductile reinforced concrete moment frames of varying height (1–20 stories) which were designed according to current building code provisions (ASCE 7-02, ACI 318-02 and IBC 2003 requirements for ‘special’ moment frames). The buildings are assumed to be located at a site in Los Angeles, for which the hazard curve has been defined through probabilistic seismic hazard analysis [11]. The calculated collapse margins (relative to the MCE) range from 1.1 to 2.1 and the conditional probabilities of collapse at the MCE vary from 0.12 to 0.47. When the collapse fragility is combined with the site hazard curve, the mean annual frequency of collapse ($\lambda_{collapse}$) varies between 2.2×10^{-4} and 25.5×10^{-4} collapses/year, corresponding to collapse return periods from 400 to 4500 years. These collapse assessments are conservative (i.e. overstating the collapse risk), because they do not include an adjustment for spectral shape [8]. The authors have also conducted a similar study of non-ductile reinforced concrete frame structures [12]. The collapse assessment results for the six case-study structures are reported in Table 1. (Note that the collapse rates are obtained by assuming all random variables to be ergodic. The approximation is not strictly true, as uncertain model parameters take a single value fixed over a structure’s lifetime, while ground motion intensities will take a unique value for each earthquake. The error introduced by this approximation is trivial, however, for the small rates of interest here [13].)

The collapse metrics reported here include the effects of both record-to-record and modeling uncertainties. The record-to-record uncertainties are calculated from the IDA results, where the logarithmic standard deviation ($\sigma_{ln, RTR}$) ranges between 0.35 and 0.45

¹ For this study, the ground motions were selected to represent large earthquakes with moderate fault-rupture distances (i.e., non-near-field conditions). This is the basic Far-Field ground motion set assembled by Haselton and Kircher as part of an Applied Technology Council project, ATC-63 [9]. These records were selected without consideration of epsilon, a measure of spectral shape which has been shown to have a significant impact on collapse capacity [8].

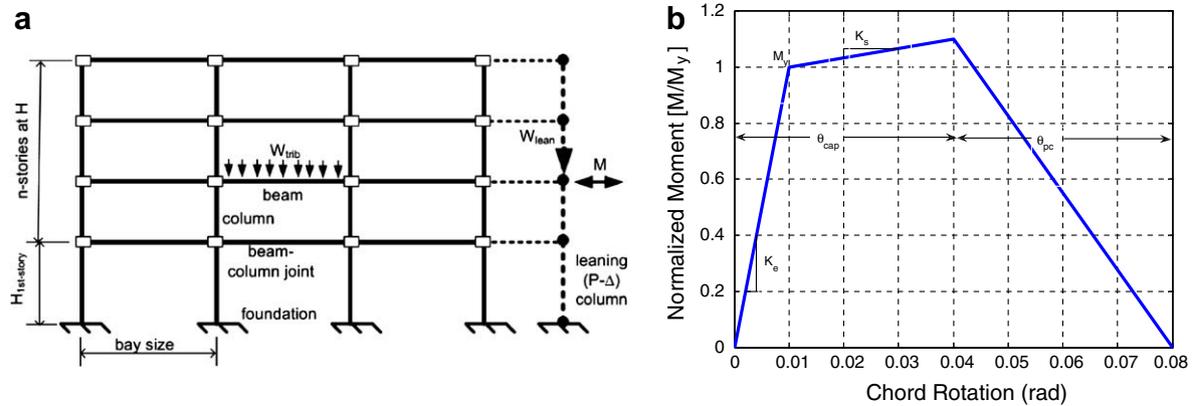


Fig. 1. Schematic diagram of analytical model for frame structures, showing: (a) generalized two-dimensional model configuration and (b) nonlinear material features of beam–column hinges.

Table 1
Collapse metrics for case study reinforced concrete frame structures

Number of stories	Frame ductility	Framing system	T_1 (s) ^a	Mean $S_{a_{g,m}}$, collapse (g)	Margin compared to MCE	λ_{col} ($\times 10^{-4}$)
1	Ductile	Space frame	0.42	2.95	2.11	1.2
4	Ductile	Perimeter frame	1.12	1.3	1.71	1.7
12	Ductile	Perimeter frame	2.01	0.61	1.32	6.7
2	Non-ductile	Perimeter frame	1.04	0.71	0.89	15
4	Non-ductile	Space frame	1.98	0.3	0.65	62
12	Non-ductile	Space frame	2.26	0.35	0.83	22

^a All metrics refer to the model with mean parameters, and are obtained from the subset of 20 earthquake records.

depending on the structure of interest. Haselton [8] determined the modeling uncertainty through detailed study of a 4-story reinforced concrete frame using the FOSM procedure and the mean estimates approach (discussed later). He calculated the logarithmic standard deviation associated with modeling uncertainties ($\sigma_{ln, modeling}$) of 0.45. When combined with the record-to-record uncertainties using SRSS, the modeling uncertainty increased the aggregate uncertainty by about 40–60%, depending on the structure. Although Haselton does not predict a shift in the median or margin associated with collapse, this increase of dispersion (σ_{ln}) has a significant impact on both the computed conditional collapse probabilities and mean annual frequencies of collapse, indicating the importance of accurately incorporating modeling uncertainties in the assessment. When structural modeling uncertainties are excluded from the analyses, the computed $\lambda_{collapse}$ is only 0.3×10^{-4} to 8.3×10^{-4} collapses/year.

3. Treatment of modeling uncertainties

3.1. Techniques for incorporating modeling uncertainties

A variety of approaches have been used to study the effects of these modeling uncertainties on the fragilities for structural response. These approaches range from methods that simplify the calculations by discretely interrogating the effects of one or more model random variables to specialized structural reliability methods and more general Monte Carlo-type methods.

Sensitivity analyses provide a straightforward method for interrogating the effects of modeling uncertainties on response quantities of interest. The effect of each random variable on structural response is determined by varying a single modeling parameter and re-evaluating the structure's performance. These studies, such as those conducted by Esteva and Ruiz [14], Porter et al. [1], Ibarra and Krawinkler [3], and Aslani [15], are used to select those modeling parameters that have the most significant impact on the response. While useful for identifying trends in the behavior, sensitivity analyses alone are not sufficient to quantify the effect of modeling uncertainties in the collapse risk assessment.

First-order-second-moment (FOSM) reliability methods can be used to propagate modeling uncertainties to quantify their effect on the collapse fragility [16]. Here, we use FOSM to predict the parameters of the response distribution directly rather than a probability of failure or reliability index (β). Where X represents the set of model random variables with mean values M , in FOSM, the limit state $g(x)$ is linearized using a Taylor series expansion about the mean ($x = M$), such that the mean of the fragility is unchanged ($\mu_g = g(M)$) and the variance of the response due to sources of modeling uncertainty is computed from the gradients of $g(x)$. Where the limit state function does not have a defined functional form, the needed gradients of the linearized limit state function can be obtained through perturbation of individual random variables in a series of sensitivity analyses. However, the linear approximation may be problematic when the limit state functions are highly nonlinear. FOSM will not predict a shift in the mean value of the fragility resulting from the effects of modeling uncertainties.

Several researchers have explored the effects of modeling uncertainties with FOSM, including Ibarra and Krawinkler [3], and Lee and Mosalam [2]. In the study described previously, Haselton [8] investigated the effects of modeling uncertainties on the collapse capacity of a code-conforming 4-story reinforced concrete frame. Haselton used sensitivity analysis results to compute the relationship between model random variables and structural response for the gradients needed in FOSM calculations. For the most realistic modeling case, the logarithmic standard deviation contribution from modeling and design uncertainties on collapse capacity is 0.45, which is roughly equivalent in magnitude to the record-to-record variability.² This work by Haselton et al. provides the basis for comparison with the present study.

² Both Haselton [8] and Ibarra and Krawinkler [3] use a one-sided gradient for the FOSM computations, calculating the slope separately in the two directions away from the mean and using the higher value (higher rate of change).

Another class of reliability-based methods that might be considered for this problem are the first-order reliability method (FORM) and related second-order reliability method (SORM). These methods use linear or quadratic approximations, respectively, of the failure surface, and the approximations are centered around a design point (the point on the failure surface associated with the highest probability of failure). These methods are very effective at handling large numbers of random variables, and the approximation is very good at low failure probabilities. Further, probabilities are computed directly, unlike FOSM where only means and variances are computed. The challenge with using FORM/SORM is that the fragility function is a complete probability distribution for collapse capacity, so its specification requires calculations of the body of the distribution, where the FORM/SORM approximations are not as good as in the tails of the distribution. Further, specification of the complete fragility function requires repeated FORM/SORM calculations at many limit-state thresholds (i.e. each ground motion intensity level), resulting in much greater computational expense than the FOSM approach. For these reasons, the older FOSM approach is generally preferred to FORM/SORM for incorporating modeling uncertainties into fragility functions.

An alternative approach uses Monte Carlo methods to determine the effect of modeling uncertainties on the structural response predictions [17,18]. Using Monte Carlo, one can generate realizations of each modeling random variable, which are inputted into a simulation model, and the model is then analyzed to determine the collapse capacity. When the process is repeated for hundreds or thousands of sets of realizations, a distribution of collapse capacity results associated with the input random variables is obtained. The simplest sampling technique to generate the realizations of model random variables is based on random sampling using the distributions defined for the input modeling random variables, though other techniques, known as variance reduction, can decrease the number of simulations needed. Porter et al. [19] used Monte Carlo methods to predict structural damage in an existing 6-story non-ductile reinforced concrete frame building located in Van Nuys, California based on a set of uncertain model random variables. Their study employed a two-dimensional non-deteriorating structural model. In another study, Zhang and Ellingwood [20] investigated the effects of uncertain material properties on structural stability problems using a Monte Carlo approach (and compared it with a perturbation approach). While conceptually straightforward, these Monte Carlo procedures can become computationally very intensive if the time required to evaluate the limit state for each set of realizations of model random variables is non-negligible. For this reason, past seismic reliability studies using Monte Carlo analysis have tended to use less computationally intensive structural models (e.g. [19]) than the degrading, highly nonlinear models in this study.

The computational effort associated with Monte Carlo methods can be reduced when combined with response surface analysis [17,21]. A response surface is a simplified functional relationship or mapping between the input random variables and the limit state criterion, such as collapse capacity of a structure. The price of this efficiency is a loss of accuracy in the estimate of the limit state, which depends on the degree to which the highly nonlinear predictions of structural response can be accurately represented by the simplified response surface. Ibarra and Krawinkler [3] analyzed the collapse capacity of a single degree-of-freedom oscillator and used a response surface to represent the collapse capacity as a function of one of the model random variables, post-capping stiffness. In that particular case, Ibarra and Krawinkler's study found that the simplified FOSM procedure, the full Monte Carlo procedure, and the combined response surface/Monte Carlo approach all produced comparable results. However, this observation is based on a single degree-of-freedom model and only one random

variable, and may not be easily generalized to multiple random variables and degrees of freedom.

Whichever procedure is utilized, correlations between the input random variables may significantly affect the extent to which modeling uncertainties impact the performance assessment [8,22]. For the nonlinear structural analyses considered here, questions about correlation involve both correlations between the multiple model parameters associated with a single structural component, and correlations between parameters for multiple components in a building. There is insufficient data to quantify these correlations, so values are typically based on expert judgment. In general, increased correlation tends to increase the dispersion (σ_{ln}) in the response quantity of interest (e.g. [8,15]) and, hence, the fully correlated case is often considered to be conservative.

3.2. Combination of sources of uncertainty

Once the effects of modeling uncertainties have been predicted there remains significant debate related to interpretation of these results, centering on how the effects of modeling uncertainties should be combined with the effects of other sources of uncertainty, such as record-to-record uncertainties. For this purpose, different sources of uncertainty are sometimes characterized as either 'aleatory' (randomness) or 'epistemic' (lack of knowledge) [23].

One approach for combining the effects of different sources of uncertainty is the confidence interval approach, e.g. [24,25]. The confidence interval method is illustrated by the collapse fragilities shown in Fig. 2a. Record-to-record variability (treated as aleatory) is shown by the cumulative distribution function obtained directly from IDA analyses, and the epistemic uncertainty (related to modeling variability) creates the distribution on the median of that cumulative distribution. The distribution associated with epistemic uncertainty in this case may be obtained from FOSM, Monte Carlo methods, or expert judgment. In order to make predictions at a specified confidence level, the aleatory distribution is shifted to the appropriate percentile on the epistemic distribution. For example, if the median of the aleatory distribution is shifted to the 10% probability of exceedance of the epistemic distribution, then the probabilities associated with the shifted aleatory distribution in Fig. 2a are consistent with a 90% prediction of confidence. In other words, the 90% confidence measure implies a 90% probability that the true collapse capacity is higher than the collapse capacity predicted by the shifted fragility function. Although this approach is conceptually appealing, the resulting structural performance predictions become highly dependent on the level of confidence chosen, as shown in [8]. For example, at a spectral demand of $S_a(T_1) = 1$ g, the probability of collapse is close to zero for the median (50% confidence) estimate and over 0.4 for the 90% confidence estimate. In addition to the high sensitivity in results, this method requires distinguishing between aleatory and epistemic uncertainties, which is a subjective and debatable distinction.

A second approach, referred to as the mean estimates approach, can be used to combine the contributions of record-to-record and modeling uncertainties in structural response fragilities, provided that certain assumptions are made. When aleatory (record-to-record) uncertainties only are considered, the structural response is well-described by a lognormal distribution [24], with logarithmic mean (λ) and standard deviation (σ_{ln}). In the mean estimates approach, it is assumed that the epistemic (modeling) uncertainty describes uncertainty in the logarithmic mean (λ), and that this random variable is also lognormally distributed with log mean μ_λ and log standard deviation σ_λ . The random variables associated with epistemic and aleatory uncertainty are assumed to be independent. It can be shown that when these two distributions are combined the resulting distribution is also lognormal with the log-

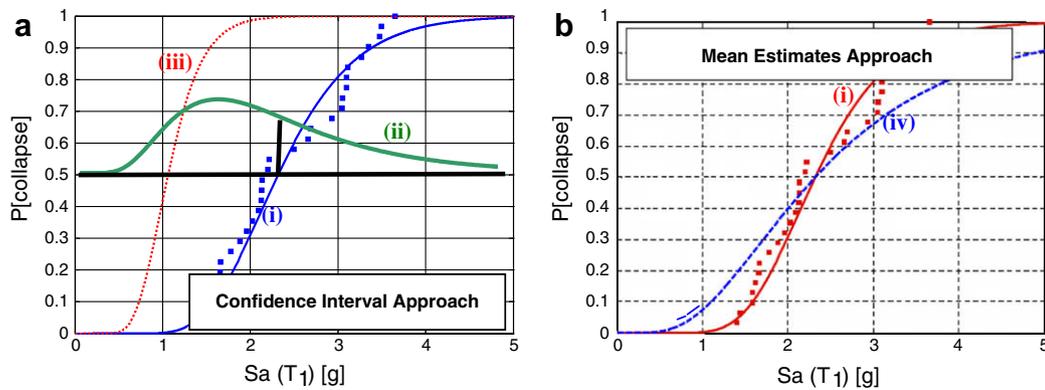


Fig. 2. Collapse fragilities for a 4-story reinforced concrete ductile frame structure, illustrating: (a) the confidence interval approach and (b) the mean estimates approach. Legend: (i) distribution of collapse capacity due to aleatory (record-to-record) uncertainties only; (ii) distribution of the median of the collapse capacity distribution due to epistemic (modeling) uncertainties; (iii) aleatory distribution shifted to the 10th percentile of the epistemic distribution, i.e. “90% confidence level”; (iv) distribution with expanded variance (SRSS) to account for epistemic and aleatory uncertainties.

arithmetic mean of μ_i (and median = $\exp[\mu_i]$), and a logarithmic variance that is the sum of the two logarithmic variances [26]. Thus, when the mean estimates approach is used, the median is unchanged when modeling uncertainties are incorporated, but the variance increases, as shown in Fig. 2b. The results derived from this approach are not sensitive to whether individual uncertainties are classified as aleatory or epistemic, which is helpful when the classification of a particular uncertainty is not obvious.

The third approach, adopted here, uses Monte Carlo simulation over the uncertain model parameters to generate a set of alternate potential descriptors of the real structural response fragility. The record to record uncertainty is captured in the individual fragility curve variances. A combined fragility, incorporating both aleatory and epistemic uncertainties, is then computed as the expected value of the probability over the Monte Carlo realizations at each spectral acceleration level.³ This approach is explained in more detail below. Unlike the mean estimates approach, the more general formulation here allows for modeling uncertainty to both shift the median and the variance of the fragility function. It also does not assume independence between the aleatory and epistemic random variables, as was required above.

3.3. Proposed procedure for evaluating effects of modeling uncertainties

Owing to the relative advantages and disadvantages of the various methods and the significance of various sources of uncertainty in the assessment process, the response surface methodology in combination with a Monte Carlo approach is proposed as the preferred method to quantify the effects of modeling uncertainties on structural response. The most complete method, the full Monte Carlo procedure, is infeasible because of the computationally intensive nature of the time history analysis in this study (it takes approximately 300 min to compute the median collapse capacity for one set of realizations of the input random variables, and hundreds of realizations would be needed for each structure considered). FORM/SORM methods would also require many evaluations in order to obtain a continuous prediction of the probability of collapse as a function of spectral acceleration. The simplest method, FOSM with mean estimates approach, is unable to capture the potential shift in the median of the distribution associated with the effects of modeling uncertainties, and, as a result, it

provides an insufficient representation of the effects of model uncertainties.

In the response surface based method, sensitivity analyses are first used to probe the effects of modeling variables on the median collapse capacity of the system. The results of the sensitivity analysis provide the inputs to regression analysis used to create the response surface, which represents the median collapse capacity as a function of model random variables. The response surface is idealized by a second-order polynomial functional form, which is capable of representing nonlinear limit states and interactive effects between the model random variables. Engineering judgment is used to confirm that the functional form is a realistic representation of the limit state, particularly where it is extrapolated beyond the region where sensitivity analysis data is available. Following creation of the response surface, a Monte Carlo procedure is used to obtain a suite of sample realizations for the set of random variables under consideration. For each set of realizations, the median collapse capacity of the structure is computed from the response surface. The outcome is a set of simulated collapse fragilities for the structure. At a given spectral acceleration level, each individual fragility curve will provide a probability representing record-to-record uncertainties, and the variation of these probabilities among the simulations represents the effect of model uncertainty. We compute the expected value of these probabilities at each spectral acceleration level to obtain the structural collapse fragility. These studies focus largely on the collapse limit state, but the same methodology is equally applicable to other limit states for which a response surface can be defined.

4. Evaluation of the effects of modeling uncertainties on case study structures

4.1. Overview and discussion of 4-story ductile frame structure

The proposed method to assess modeling uncertainties is illustrated by applying it to the set of case study reinforced concrete buildings that include both ductile and non-ductile design details. All frames have 6.1 m (20 ft) or 7.6 m (25 ft) bay spacings and 4.0 m (13 ft) story heights, except for the first story which has a 4.6 m (15 ft) height. Three different building heights are considered (1, 4, and 12-story ductile structures; 2, 4, and 12-story non-ductile structures). The frames are modeled as shown in Fig. 1. The details of the design and the collapse assessment for these structures are available in [8] and [12].

The assessment of modeling uncertainties focuses on uncertainties in the modeling parameters that define the lumped plasticity

³ This approach does not preclude computation of confidence intervals, if needed. For a 90% confidence level, for example, compute the 90th percentile of the probabilities at each spectral acceleration level, rather than the expected (mean) value.

Table 2
Uncertainties in modeling parameters for RC beams, columns and joints

Random variable ^a	σ_{\ln}	Source
Beam or column strength	0.19	Panagiatakos and Fardis [29]
Beam or column stiffness	0.33	Haselton [9]
Beam or column rotation capacity ($\theta_{\text{cap,pl}}$)	0.59	Haselton [9]
Beam or column post-capping rotation capacity (θ_{pc})	0.72	Haselton [9]
Beam or column cyclic deterioration (λ)	0.50	Haselton [9]
Joint strength	0.39	Mitra and Lowes [27]
Joint stiffness	0.39	Same as joint strength
Joint rotation capacity ($\theta_{\text{cap,pl}}$)	0.59	Limited data; same as beam-columns
Joint post-capping rotation capacity (θ_{pc})	0.72	limited data; same as beam-columns
Joint cyclic deterioration (λ)	0.50	Limited data; same as beam-columns

^a Normalized variables, with $\mu_{\ln} = 0$.

plastic hinges for beams, columns and joints. The beam–column hinges are modeled using an inelastic spring model developed by Ibarra, Medina and Krawinkler [7]. The element backbone (Fig. 1b) and hysteretic rules are defined by six parameters: flexural strength (M_y), initial stiffness (K_e), post-yield (hardening) stiffness (K_s), capping point (θ_{cap}), post-capping deformation capacity (θ_{pc}) and cyclic deterioration (λ).⁴ These parameters are assumed to be lognormally distributed, where the mean and standard deviation are obtained from previous research; Table 2 summarizes the logarithmic standard deviation for the parameters of each type of component. The joint modeling parameters, also shown in Table 2, are based on representative data from Mitra and Lowes [27] and on engineering judgment where insufficient data is available. Modeling uncertainties associated with the beam–column joints are neglected for the ductile moment frame structures, because capacity design provisions and transverse reinforcement requirements for joints have been shown to be sufficient to ensure that failure occurs outside the joints. For simplicity, other parameters related to element level modeling (e.g. pinching and residual strength) and system level behavior (e.g. damping, mass, live and dead loading) are not considered; earlier sensitivity studies found that modeling variables related to component strength and deformation capacity are the dominant model parameters affecting collapse assessment [8].

Independent assessment of each of the random variables described in the preceding discussion is computationally prohibitive, given the analysis time that would be required to assess combinations of the five random variables for each plastic hinge location and beam–column joint in the frame. To further reduce the number of variables under consideration, correlations are assumed between parameters within each component and between components in the building. At the element level, two meta random variables are created. The *strength* meta variable represents the strength and stiffness model parameters (M_y, K_e) in an element, implying that the strength and stiffness are perfectly correlated within each element. The *ductility* meta variable does the same for ductility parameters ($\theta_{\text{cap}}, \theta_{\text{pc}}$, and λ), such that plastic rotation capacity, cyclic deterioration, and post-capping rotation capacity are assumed to be perfectly correlated. At the structural level, the *strength* and *ductility* meta variables are assumed to be perfectly correlated with like variables among all like components in the entire structure. These correlation assumptions leave six meta variables: beam strength, beam ductility, column strength, column ductility, joint strength and joint ductility. Each meta variable is a standard lognormal random variable (with $\mu_{\ln} = 0$ and $\sigma_{\ln} = 1$),

⁴ In this study, hardening stiffness (K_s) is neglected because of its very small influence on collapse capacity.

which can be mapped to the model parameters of interest. While these assumptions are loosely supported by observations from the model calibration study of reinforced concrete columns [8], there is insufficient empirical evidence to quantify correlations and the assumed correlations are made primarily for tractability.

Based on the definition of these meta random variables, sensitivity analyses are conducted to quantify the effects of each meta modeling variable on the structural response. The realizations of random variables used in the sensitivity analysis are based on central composite design, including star points (in which only one random variable is changed at a time) and factorial points (capturing interactions between the random variables) [21]. In total, 33 sensitivity analyses were conducted for each ductile structure based on the four meta random variables of interest. For the non-ductile frames, 93 sensitivity analyses were necessary to account for the joint strength and ductility meta variables in addition to the column and beam strength and ductility variables. Each random variable was perturbed ± 1.7 standard deviations⁵ away from the mean individually, and in combinations with other random variables at $\pm 1\sigma$. For each sensitivity analysis, a nonlinear model is created with modified element material properties, and the incremental dynamic analysis is run with a subset of 20 earthquake records.⁶ The nonlinear IDA collapse assessment procedure is conducted as described in Section 2 and in more detail in [8,12,28].

To examine the effects of modeling uncertainty on structural behavior, two distinct limit states are considered for the 4-story ductile moment frame building, corresponding to: (a) exceedance of 1% interstory drift and (b) collapse. The fragility functions are defined in terms of the spectral acceleration at the structure's fundamental period.

A summary of sensitivity analysis results for the two limit states of interest for the 4-story ductile frame are shown in Fig. 3, where Figs. 3a and c provide a histogram of the 33 analyses for each limit state and Figs. 3b and d provide a tornado diagram of sensitivity results. As shown in Fig. 3b, of the four random variables, column strength has the largest effect on the median collapse capacity, followed by column ductility, beam strength and beam ductility. Beam strength has an inverse effect on collapse since the weaker beams tend to delay the formation of unfavorable story mechanisms. Comparing the two different limit states (Figs. 3a and c), it is apparent that modeling uncertainties are more significant for the collapse limit state than the 1% interstory drift limit state. In particular, the beam and column ductility meta variables have virtually no effect on the 1% drift fragility, as these uncertainties are related to highly nonlinear structural behavior that does typically not occur before 1% interstory drift.

The sensitivity analyses also demonstrate that the random variables have an asymmetric effect on the response, e.g., decreases in ductility tend to have proportionally more significant effects on collapse capacity than increases. This behavior is further illustrated in Fig. 4, where we observe the effects of saturation in collapse capacity. These nonlinearities are particularly acute in the collapse limit state, but are also apparent at the 1% drift limit state. This characteristic cannot be captured by the linearized limit state functions in FOSM analysis.

The sensitivity analysis results are used to create a response surface that describes each limit state as a function of the input random variables. The response surface is idealized by a second-or-

⁵ $1.7 \approx \sqrt{3}$; chosen for practicality (and with reference to values typically used in the experimental design literature).

⁶ This subset of earthquake records was chosen to reduce the computational time needed to conduct the study. The response spectra of this subset were verified to be characteristic of the response spectra of the whole suite of records, but the collapse capacities reported may be somewhat different than those reported elsewhere because of the smaller number of records used.

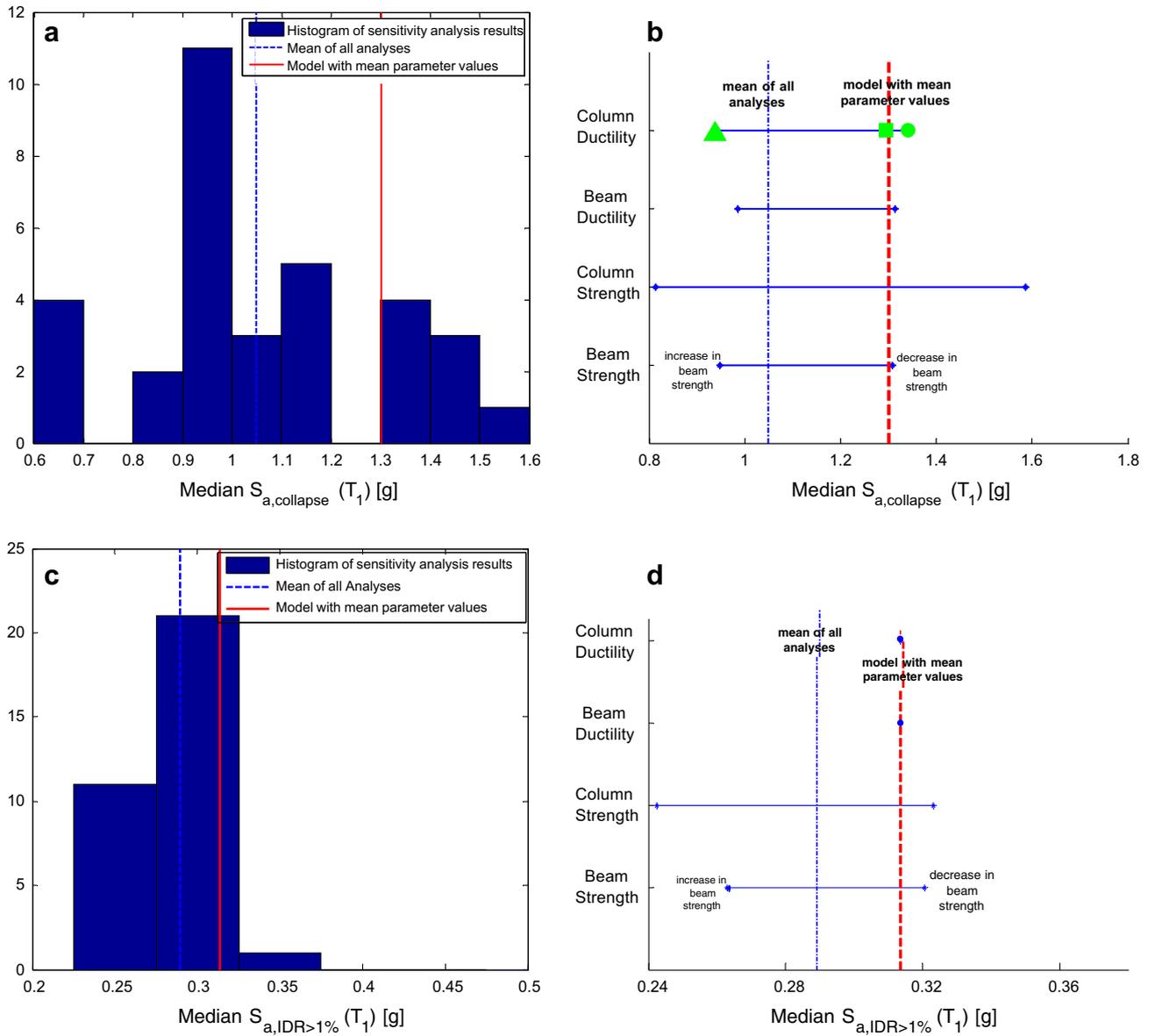


Fig. 3. Histogram showing the results of 33 sensitivity analyses for the median spectral acceleration corresponding to (a) the collapse capacity and (c) the 1% interstory drift limit state. Tornado diagram from sensitivity analysis results, demonstrating the effect of varying each meta variable individually ($\pm 1.7\sigma$) for: (b) median collapse capacity and (d) 1% interstory drift limit state. The markers on column ductility in Fig. 3b are shown for easy comparison to Fig. 4.

der polynomial that is fitted to the data through standard regression analysis (e.g., the ‘regress’ function in Matlab). As opposed to linearized limit state for standard FOSM methods, the quadratic response surface enables representation of the nonlinearities and asymmetries in the relationship between the model random variables and the structural response.

For the 4-story ductile moment frame example, the quadratic response surfaces for the two limit states, median collapse capacity [$\hat{m}_{collapse} = \exp(\mu_{ln,Sa,col})$] and 1% interstory drift [$\hat{m}_{IDR>1\%} = \exp(\mu_{ln,Sa,IDR>1\%})$] are given by the following equations,

$$\begin{aligned} \mu_{ln,Sa,col} = & 0.26 - 0.077(BS) + 0.20(CS) + 0.073(BD) \\ & + 0.098(CD) - 0.052(BS^2) + 0.064(BS)(CS) \\ & - 0.019(BS)(BD) + 0.078(BS)(CD) \\ & - 0.045(CS^2) + 0.052(CS)(BD) - 0.043(CS)(CD) \\ & - 0.044(BD^2) + 0.019(BD)(CD) - 0.047(CD^2) \end{aligned} \quad (1)$$

$$\begin{aligned} \mu_{ln,Sa,IDR>1\%} = & -1.15 - 0.063(BS) + 0.085(CS) - 0.025(BS^2) \\ & + 0.067(BS)(CS) - 0.038(CS^2) \end{aligned} \quad (2)$$

where BS refers to beam strength, BD to beam ductility, CS to column strength and CD to column ductility meta variables. The response surface of Eq. (1) is evaluated according to statistical measures of goodness of fit with R^2 equal to 0.99 and a p -value of 1.11×10^{-16} . In addition, the variance inflation factors are computed to be $\ll 10$, indicating that collinearity is not a problem. The fit of Eq. (2) yields similarly robust values. A graphical representation of the response surface for median collapse capacity (Eq. (1)) is shown in Fig. 5. As expected, column strength, column ductility and beam ductility all have a positive effect on the median collapse capacity, while beam strength has an inverse effect.

The response surface provides a good representation of the sensitivity analysis results in the region $\pm 1.7\sigma$ for each of the normalized random variables. However, when extrapolated outside this

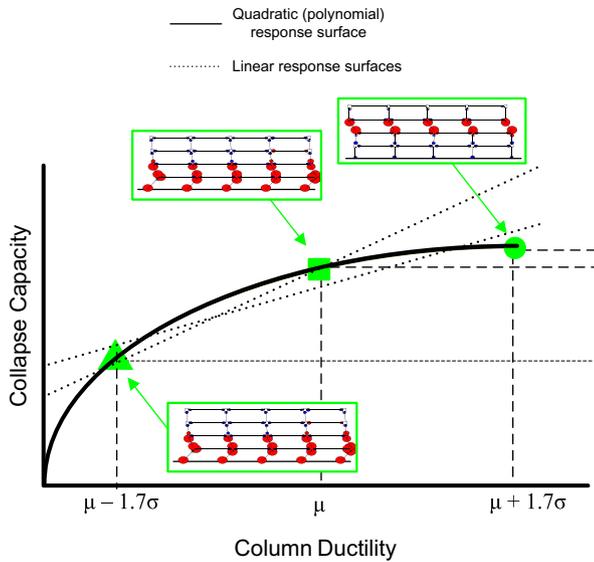


Fig. 4. Illustration of nonlinear relationship between model random variables (e.g. column ductility) and structural response (e.g. collapse capacity). The quadratic response surface provides a good fit to the data, while the linear response surface(s) are only able to capture average trends. The nonlinearities are largely due to the structure’s many possible collapse modes, illustrated by the superimposed 4-story frame structures.

region, it is possible that the fitted second-order response surface may not increase monotonically as we would expect. In these cases, we modified the fitted response surface to have monotonic behavior. These changes were implemented for completeness, but they do not have a large influence on the final results because these discrepancies only exist in the region rarely sampled in the Monte Carlo procedure.

Using the calculated response surfaces as a surrogate for time-history analyses, the Monte Carlo method is used to incorporate the effects of the uncertain model random variables on the predicted limit state fragilities. The model random variables are sampled 10,000 times, each time generating a set of realizations that are consistent with the assumed lognormal distribution of the meta model random variables. For each set of realizations, the re-

sponse surface is used to calculate the median capacity of the structure. Therefore, from each of the ten thousand sets of realizations, predictions of the median collapse capacity and 1% interstory drift limit state are obtained. As noted previously, since a reduced set of ground motion records is used to assess the response surface, only the median collapse point is extracted from the surface. The logarithmic standard deviation due to record-to-record uncertainties is assumed to be constant over the response surface and set equal to the value obtained for the collapse analyses of the median structural model using the full record set (44 records). This assumption is made for practical convenience, since conceptually it is possible to re-estimate σ_{in} for each realization, provided that the analyses used to generate the response surface are based on a sufficiently large number of ground motion records.

The final step is to recreate the limit state fragility function, based on the Monte Carlo results, to include both the effects of record-to-record and modeling uncertainties. Each Monte Carlo realization is associated with a different fragility describing the probability of failure as a function of spectral acceleration. As an example, shown in Fig. 6a are results for the probability of collapse at $Sa(T_1) = 1.91$ g for all the 10,000 Monte Carlo realizations. The final fragility probability is the expected value of the collapse probability at each spectral acceleration level. Fig. 6b illustrates the effects of modeling uncertainties showing both the collapse fragility curve for the mean structural model considering only record-to-record uncertainties (the lower curve), and the collapse fragility including both record-to-record and modeling uncertainties (the upper curve). The superposition of histograms of the probabilities at selected spectral acceleration levels in Fig. 6b are included to demonstrate the method through which the upper curve is obtained.

Using the response surface procedure we observe that modeling uncertainties tend to both increase the dispersion (σ_{in}) in the structural response fragility and shift the prediction of the median. Fig. 7 illustrates the fragility curves for the two limit states (collapse and 1% drift), where the proposed response surface based method for including modeling uncertainties (Figs. 7a and c) is contrasted with a FOSM approach (Figs. 7b and d). Several observations can be drawn from these figures. First, comparing the plots in Figs. 7a versus b and c versus d, the response surface based method captures the shift in the median point, which is not captured by the FOSM-type approaches. This inability to predict the shift in the

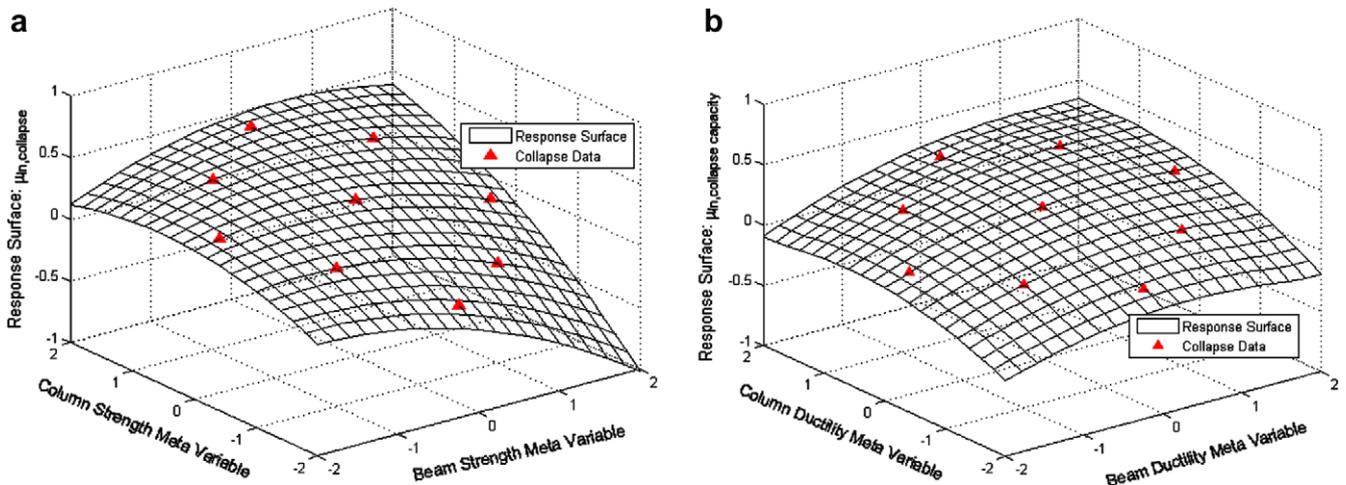


Fig. 5. Graphical representation of the polynomial response surface for collapse capacity of the 4-story ductile moment frame. Each of these represents a slice of a multi-dimensional surface. In (a) the effects of column strength and beam strength are shown, while beam ductility and column ductility meta variables are held constant (at 0, their mean values); likewise, (b) illustrates the effects of varying beam and column ductility.

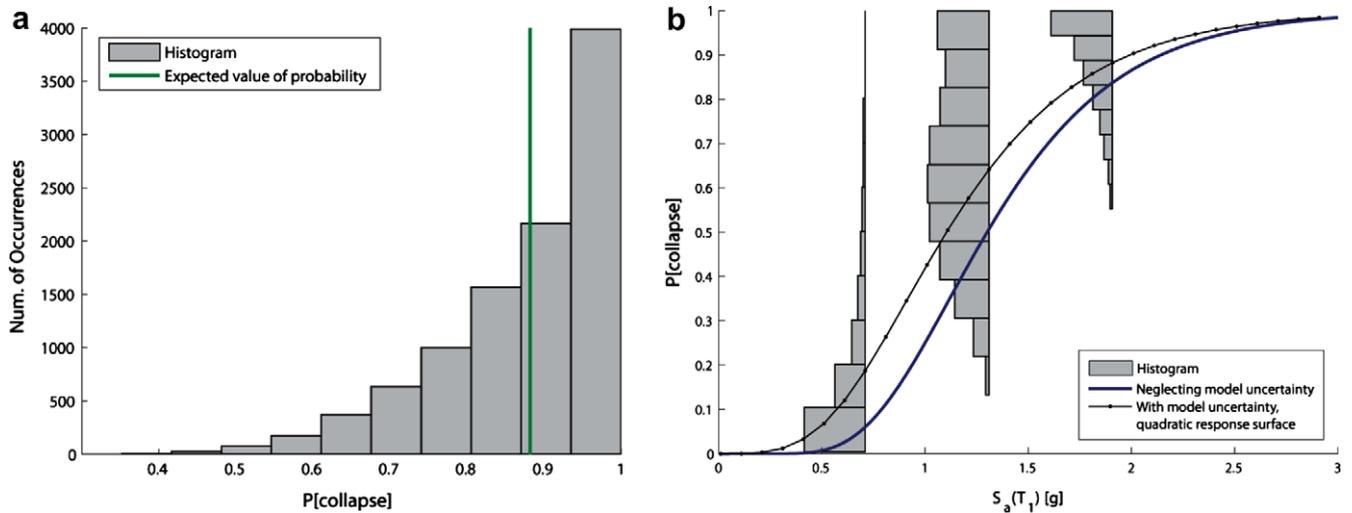


Fig. 6. (a) Histogram of collapse probabilities obtained from Monte Carlo realizations at $S_a(T_1) = 1.91$ g and (b) computed collapse fragilities with histograms superimposed at selected S_a levels.

median is a significant limitation of FOSM, especially at the collapse limit state, for which we observe a 19% decrease in the median collapse capacity of the 4-story ductile moment frame. Overall, the comparison of Figs. 7a and c demonstrates that the model random variables have a less significant impact on pre-collapse limit states, and we observe both a smaller shift in the median ($\sim 3\%$) and a smaller increase in the logarithmic standard deviation. The lesser effect of modeling uncertainties on the one-percent drift limit state is unsurprising, because much of the model uncertainty relates to element deformation capacity and cyclic degradation properties that do not have a significant effect when nonlinear deformations in the structure are much smaller.

The results illustrated in Figs. 7a and c are somewhat contrary to the conventional expectation that the effect of modeling uncertainties is to flatten the response fragility, but not to shift the median. For example, suppose we use the FOSM with mean estimates approach to quantify the impacts of model uncertainties on the fragility representing exceedance of one-percent interstorey drift, obtaining the results shown in Fig. 7d; we observe the characteristic flattening from incorporating additional sources of uncertainty. The same results are obtained if we use the response surface based method, provided that the response surface is linear (as illustrated in Fig. 7b). Therefore, it is apparent that it is the nonlinear shape of the relationship between the structural response limit state and model random variables, as shown in Figs. 4 and 5, that predicts the shift in the median, and which cannot accurately be captured by the linearization in FOSM or a linear response surface.

This nonlinear relationship depends on both the limit state of interest and the properties of the structure. For the 4-story example structure, the saturation of collapse capacity as a function of model random variables occurs because the structure has many possible failure modes, and an increase in a given model random variable tends to switch the failure mode. Hence, we don't see a large improvement in the collapse capacity as model random variables increase (see Fig. 4). Additional unpublished parametric studies by the authors indicate that "balanced designs," where the structure is not dominated by a single failure mode, tend to see a more significant shift of the median collapse capacity caused by the effects of modeling uncertainties. We observe below, for example, a smaller shift in the median for the one-story building, which has only one failure mode. For more discussion of the collapse failure modes for the case study structures see [8,12].

4.2. All case study structures

This same Monte Carlo and response surface method was used to investigate the effects of modeling uncertainties on the collapse fragility for five other reinforced concrete frame buildings. These effects are summarized for the case study structures in Tables 3 and 4, and the collapse probability distributions are illustrated in Fig. 8. As described earlier, the effect of incorporating modeling uncertainties is to shift the median collapse capacity and to increase the dispersion (σ_{ln}) of the collapse fragility. However, the extent of the change depends on the structure under consideration. Consideration of model uncertainties actually increases the median collapse capacity of the 12-story non-ductile reinforced concrete frame, which is atypical and contrary to the decrease observed for the 4-story ductile frame and all the other frame structures. The increase for the 12-story non-ductile frame occurs because the nonlinearities in the relationship between joint strength and collapse capacity are reversed from those shown in Fig. 4. For the 12-story frame, there is a very strong benefit from increasing the joint strength and moving the collapse mechanism out of the joints and into the beams, but there is a much smaller decrease in collapse capacity if the joint strength meta variable is decreased. The 1-story ductile structure has only a small shift in the median; this structure has essentially one possible collapse mode, a story mechanism in the first story.

Tables 3 and 4 illustrate the importance of accurately incorporating modeling uncertainties in the analysis. The base case, no consideration of modeling uncertainty, may be highly unconservative, and under-predicts the rate of collapse by a factor of 2.3 on average. The simplified FOSM approach may underestimate or overestimate the rate of collapse depending on the structure. Referring to Fig. 8, we observe that in many cases the left tail of the collapse fragility obtained by the FOSM (using the mean estimates approach) and proposed response surface based method are fairly close. However, when integrated with the hazard curve to obtain the mean annual frequency of collapse there are more significant differences between the FOSM/mean estimate approach and the response surface based approach in some cases. FOSM will also significantly underestimate the conditional probabilities of collapse for high probabilities of collapse. These differences are most significant when the relationship between the model random variables and the structural response is highly nonlinear and a shift in the median collapse capacity is likely. They are also more critical

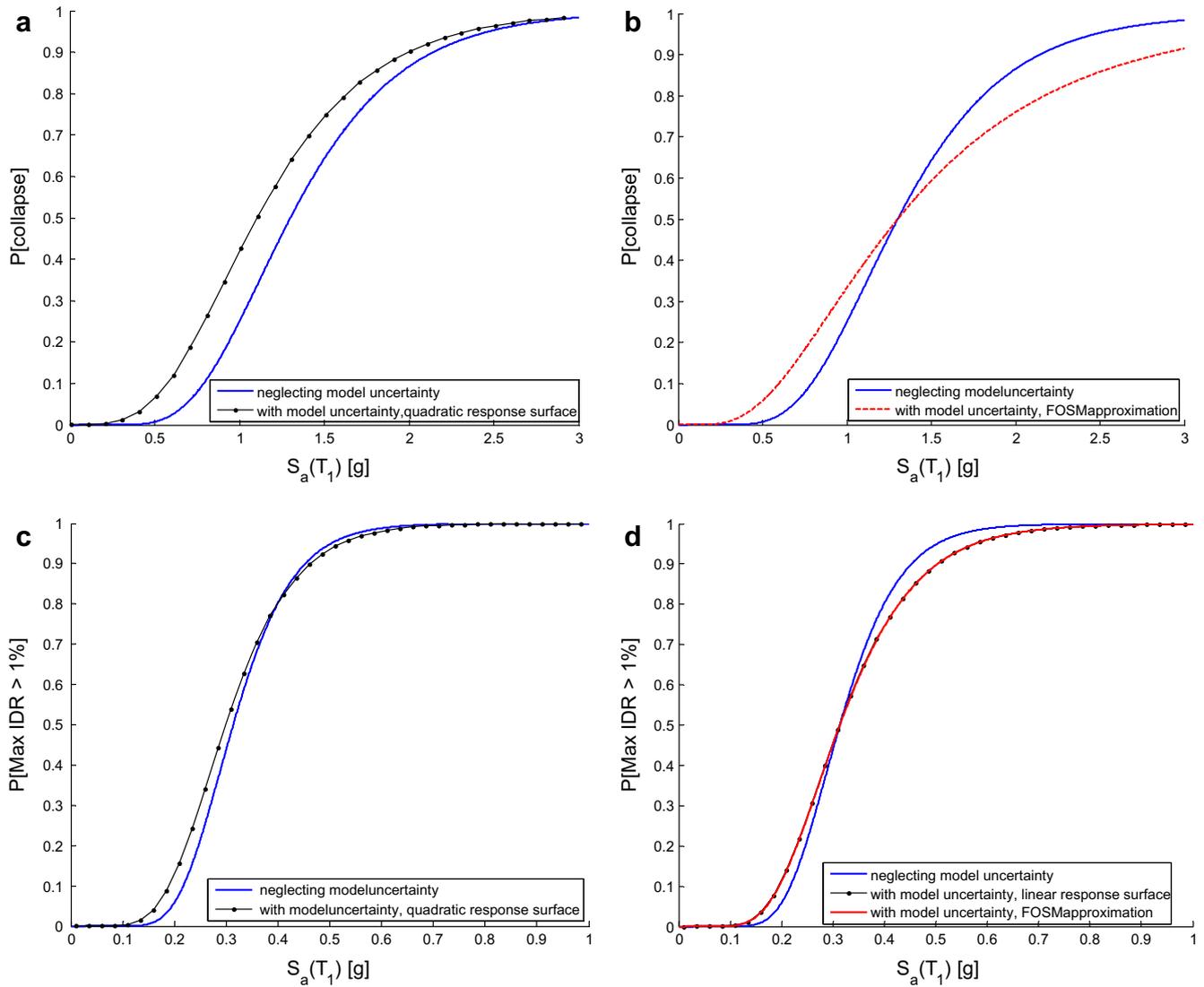


Fig. 7. Structural response fragilities representing the collapse limit state, obtained using: (a) quadratic (polynomial) response surface and (b) FOSM approximation, and the 1% interstory drift (IDR) limit state, obtained using (c) quadratic response surface and (d) FOSM approximation/linear response surface.

Table 3
Predicted effect of modeling uncertainties on median and dispersion (σ_{in}) of collapse fragility, comparing response surface based approach and FOSM with mean estimates

Number of stories	Frame ductility	Response surface		FOSM	
		Change in median (%)	Change in dispersion (%)	Change in median (%)	Change in dispersion (%)
1	Ductile	-4	12	0	25
4	Ductile	-19	19	0	35
12	Ductile	-9	10	0	28
2	Non-ductile	-18	28	0	39
4	Non-ductile	-7	12	0	30
12	Non-ductile	8	18	0	23

for structures, like the non-ductile RC frames, that have low collapse capacity relative to the MCE.

4.3. Effects of correlations between model random variables

In the results presented thus far the meta random variables are assumed to be uncorrelated. Since correlations are very difficult to quantify, it is important to evaluate the impact of the correlation assumptions on the effects of modeling uncertainties. In order to

examine the implications of these assumptions, two other sets of correlation assumptions are considered for the 4-story ductile frame. In the first case (Case I), the strength meta variables and the ductility meta variables are assumed to be correlated between beams and columns, but there is no correlation assumed between the strength and ductility parameters. In the second case (Case II), the beam meta variables (strength and ductility) are assumed to be correlated, as are the column meta variables, but the beam and column variables remain uncorrelated. The correlations only affect the Monte Carlo stage of the procedure, and they do not affect running of the nonlinear response analyses to conduct the sensitivity analyses and build the response surface. Therefore, it is relatively easy to vary the correlation model assumptions and obtain new results. The ease with which correlations can be studied is another benefit of the response surface based approach. To vary correlation assumptions in a full Monte Carlo approach, these dependencies would need to be included at the structural analysis stage, a painstaking process for each of the hundreds of Monte Carlo realizations needed. Thus the response surface approach is particularly well-suited to research applications, such as those presented here, where correlation assumptions are still being investigated.

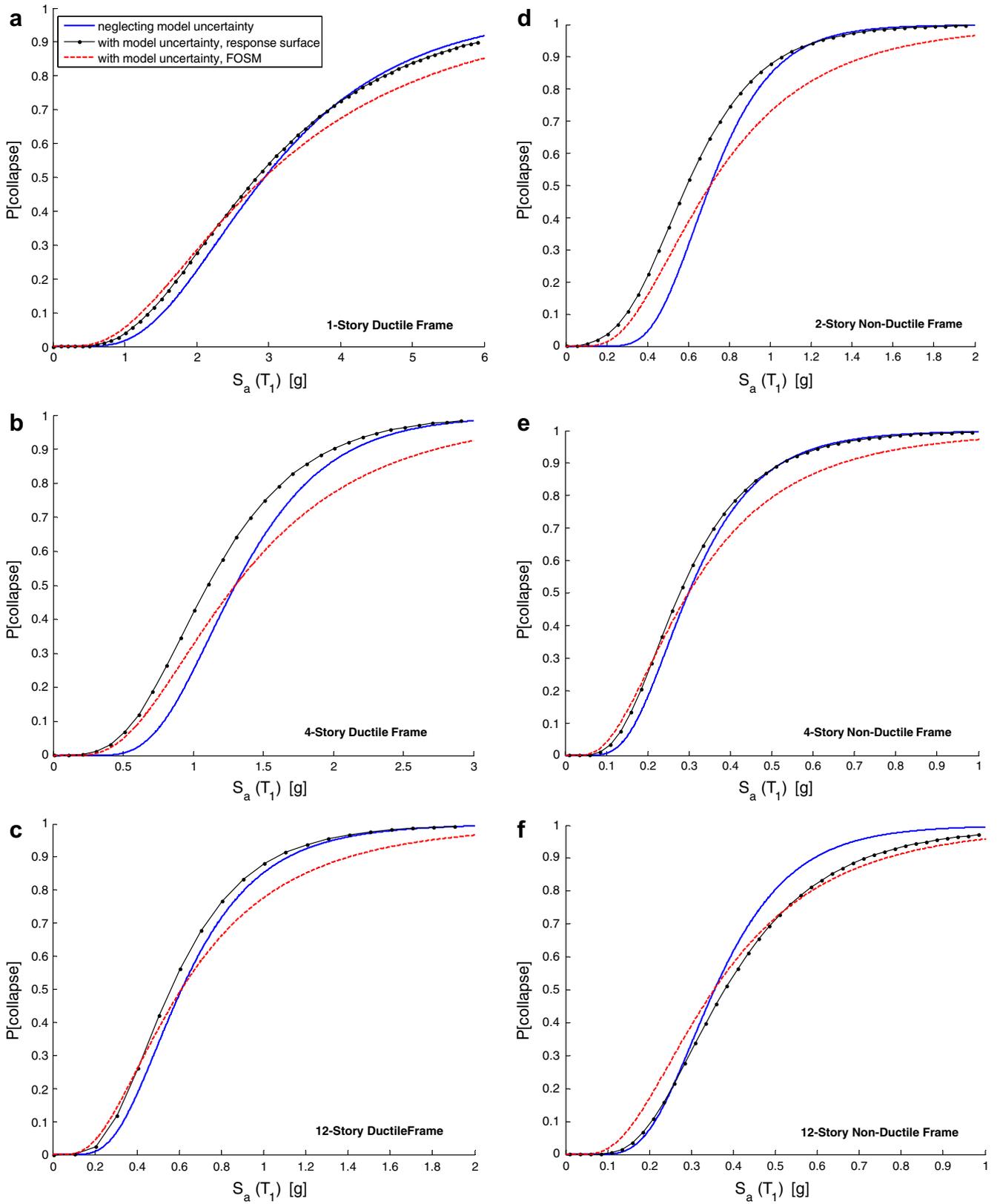


Fig. 8. Collapse fragilities obtained for reinforced concrete frames.

For Case I, full correlation between the meta variables leads to a 6.4% increase in the median collapse capacity from the baseline uncorrelated case [from 1.10 to 1.17 g], thus reducing the overall effect of modeling uncertainties on the outcome. These results,

summarized in Table 5, suggests that the relative difference in beam and column strength and beam and column ductility is a larger factor in determining collapse capacity than the absolute values. In Case II, as the assumed correlation between the meta

Table 4
Effect of modeling uncertainties on conditional probabilities and mean annual frequency of collapse ($\lambda_{\text{collapse}}$), comparing the response surface and FOSM methods

Number of stories	Frame ductility	$P[\text{Collapse} \text{MCE}]$			$A_{\text{collapse}} (\times 10^{-4})$		
		No consideration of modeling uncertainty	FOSM with mean estimates, where $\sigma_{\text{In,modelling}} = 0.45$	Response surface method	No consideration of modeling uncertainty	FOSM with mean estimates, where $\sigma_{\text{In,modelling}} = 0.45$	Response surface method
1	Ductile	0.09	0.16	0.14	1.2	4.1	1.6
4	Ductile	0.11	0.21	0.26	1.7	6.1	5.9
12	Ductile	0.26	0.32	0.33	6.7	17	12
2	Non-ductile	0.73	0.64	0.80	15	36	44
4	Non-ductile	0.82	0.74	0.83	62	111	89
12	Non-ductile	0.62	0.58	0.54	22	46	24

Table 5
Parametric study of element level correlation assumptions on collapse fragility for 4-story ductile frame

Correlation assumptions	Collapse fragility	
	Median ($Sa(T_1)$ [g])	σ_{In}
ρ		
(a) Case I		
0 ^a	1.10	0.48
0.5	1.12	0.47
1	1.17	0.47
(b) Case II		
0 ^a	1.10	0.48
0.5	1.05	0.51
1	1.01	0.55

^a Uncorrelated meta random variables; these are the results presented previously.

random variables decreases the median collapse capacity decreases and the dispersion (σ_{In}) increases. At higher levels of correlation it becomes more likely that beam behavior is either very good (in terms of both strength and ductility) or very bad in relation to column behavior. Since poor behavior tends to decrease the collapse capacity more than good behavior increases it, the median reduces with increasing Case II correlation. It is difficult to quantify correlations of this sort, but the Case II correlations (relating beam strength for example, to beam ductility) are not supported by the currently available data (e.g. [8]). Thus, the Case II correlation study is more for illustration than practical application. On the other hand, the Case I correlations (similar properties among similar members in a frame) are more likely, but the Case I correlation effects are relatively small and assuming zero correlation tends to be conservative. Negative correlations were also examined, but these are unsubstantiated by experimental data.

To probe the effects of structural level correlation assumptions on modeling uncertainties, we considered another case where the meta variable definitions were revised to examine a situation in which correlations between certain beams and columns were imposed to intentionally accentuate a critical collapse state. As initially defined, the meta variables are assumed to be perfectly correlated over every element in the building, e.g. beam strength of the 1st-floor beams is correlated to the beams in all the other floors. To further examine building level correlation assumptions in the study of the 4-story ductile frame structure, additional meta variables were created for the 3rd-floor beam strength (BS3) and the 2nd-story column strength (CS2). These meta variables are assumed to be uncorrelated from the other strength meta variables in the structure. Since the 4-story ductile frame structure often fails in a story mechanism in the second story (see [8] for more details), these additional meta variables were chosen to represent an upper bound on the effects of spatial correlation assumptions in the frame. To incorporate the two new meta variables, additional

sensitivity analyses were run, a new response surface was created, and the Monte Carlo procedure was repeated. Based on these analyses, relaxation of the full correlation assumption by the addition of two meta random variables lead to a 10% decrease in the median collapse capacity of the structure, as compared to the default case where the beam and column properties were assumed to be perfectly correlated in the building. Given the fact that the situation considered (3rd-floor beams and 2nd-story columns uncorrelated from the other random variables) is intentionally pessimistic for this frame, the change in median is relatively modest. If each story, or each column, were treated separately, or with partial correlation assumptions, the effects of relaxing the correlation assumptions would likely be much smaller.

5. Simplified method

This study demonstrates the importance of appropriately treating structural modeling uncertainties in collapse performance assessments. However, the response surface based procedure requires significant computational and analysis time, necessitating running between 30 and 95 sensitivity analyses (depending on the number of meta random variables), as well as the creation of the response surface, and generation of Monte Carlo realizations. Of these, the sensitivity analyses are the most time consuming. Depending on the level of complexity of the structural model, the number of earthquakes used in incremental dynamic analysis, and the available computing power, each sensitivity analysis could take 5–20 h of computing time. This level of effort may not be warranted for all problems, and it is therefore desirable to develop a simplified method that can be used to approximate the effects of modeling uncertainties.

The proposed simplified method is capable of estimating both the shift in the median and the increase in the dispersion (σ_{In}) due to modeling uncertainties. We call this method ASOSM, for approximated second order second moment. The method requires running sensitivity analyses, though fewer than required for the response surface based method. Beyond the mean model, the sensitivity studies involve running nonlinear response analyses for each of the key random variables scaled to $\pm 1.7\sigma$. For the ductile reinforced concrete frames with four meta variables, this requires the mean model analysis, plus eight additional analyses in which each meta variable is increased or decreased independently (i.e. $X_i^+ = \mu_{X_i} + 1.7\sigma_{X_i}$ and $X_i^- = \mu_{X_i} - 1.7\sigma_{X_i}$). In this sense, the method is similar to the analyses required for the FOSM assessment.

The logarithmic standard deviation of the response fragility, including the effects of modeling uncertainties, can be computed following the standard first order (FOSM) approach. The gradients used in FOSM computations should represent the average slope about the mean i.e.,

Table 6

Comparison of predicted dispersion (σ_{ln}) of the collapse fragility when record-to-record and modeling uncertainties are included, using the response surface based approach and ASOSM

Number of stories	Frame ductility	σ_{ln} (response surface)	σ_{ln} (ASOSM)	Error ^a (%)
1	Ductile	0.58	0.58	0
4	Ductile	0.48	0.46	-4
12	Ductile	0.52	0.52	0
2	Non-ductile	0.47	0.39	-16
4	Non-ductile	0.50	0.49	-2
12	Non-ductile	0.49	0.47	-3

^a ASOSM compared to response surface approach.

$$\frac{\partial g(X)}{\partial X_i} = \frac{\Delta \mu_{ln, Sa, col}}{\Delta X_i} = \frac{\mu_{ln, Sa, col}(X_i^+) - \mu_{ln, Sa, col}(X_i^-)}{X_i^+ - X_i^-}, \quad (3)$$

where $g(X)$ is the collapse capacity. Note that Eq. (3) differs from some of the FOSM calculations reported earlier, which used a maximum or one-sided gradient. After the gradients are calculated, the dispersion associated with modeling uncertainties is computed from the following equation for n random variables,

$$\sigma_{ln, mod}^2 = \left[\sum_{i=1}^n \sum_{j=1}^n \left[\frac{\partial g(X)}{\partial X_i} \frac{\partial g(X)}{\partial X_j} \right] \rho_{ij} \sigma_i \sigma_j \right] \quad (4)$$

and combined with the record-to-record uncertainties using SRSS,

$$\sigma_{ln, total}^2 = \sigma_{ln, mod}^2 + \sigma_{ln, RTR}^2. \quad (5)$$

As shown in Table 6, the resulting σ_{ln} shows very good agreement with those obtained from the response surface based procedure for the six reinforced concrete frame buildings.

The shift in the median is predicted based on the nonlinearities in the relationship between structural response and the model random variables (again, refer to Fig. 4). The response asymmetry is given by the parameter Δ^+/Δ^- ,⁷

$$\Delta^+/\Delta^- = \frac{\hat{m}^+/\hat{m}}{\hat{m}/\hat{m}^-}, \quad (6)$$

where \hat{m} is the median capacity of the model with mean model parameters, and

$$\hat{m}^+ = \frac{1}{n} \sum_{i=1}^n m_{X_i + x\sigma_i} \quad \text{and} \quad \hat{m}^- = \frac{1}{n} \sum_{i=1}^n m_{X_i - x\sigma_i}, \quad (7)$$

represent the average of the median collapse capacities when the model random variables are perturbed individually to $+1.7\sigma$, and -1.7σ , respectively, and n is the number of perturbed analyses performed (equal to the number of model random variables). Using data from the six reinforced concrete frame example structures, the resulting shift in the median collapse capacity can be calculated by the following,

$$\frac{\hat{m}_{mod}}{\hat{m}} = 0.64(\Delta^+/\Delta^-) + 0.36. \quad (8)$$

This equation was obtained from linear regression of the results for the collapse capacity of the six case study reinforced concrete frame structures ($R^2 = 0.97$), as shown in Fig. 9.⁸ The median value of the

⁷ Eq. (6) is equivalent to: $\frac{\exp(\mu_{ln}^+ - \mu_{ln})}{\exp(\mu_{ln} - \mu_{ln}^-)}$.

⁸ It is also possible to run sensitivity analyses at $\pm 1\sigma$ (instead of $\pm 1.7\sigma$), as is more typical. However, this tends to slightly under-predict the dispersion in some cases (due to nonlinearities, particularly in the negative direction) if used in Eqs. (3)–(5). If used to predict the median, it is suggested that Eq. (8) be replaced with $\frac{\hat{m}_{mod}}{\hat{m}} = 0.83(\Delta^+/\Delta^-) + 0.17$ ($R^2 = 0.92$). However, this is based on fewer data points than Eq. (8), and the statistical fit is not as good.

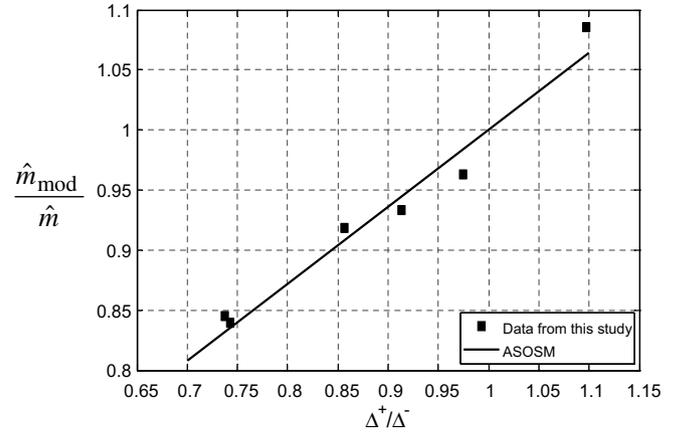


Fig. 9. Prediction of the shift in median associated with model uncertainties, as a function of Δ^+/Δ^- , a measure of the degree of nonlinearity in the relationship between model random variables and the limit state function. ASOSM provides good agreement with the data from the response surface method.

Table 7

Comparison of predicted median collapse capacity using different approaches for incorporating model uncertainties

Number of stories	Frame ductility	$\frac{\hat{m}_{mod}}{\hat{m}}$ (Eq. (8))	Median (without model uncertainty) [g]	Median predicted (ASOSM) [g]	Median predicted (response surface) [g]	Error ^a (%)
1	Ductile	0.98	2.95	2.90	2.84	2
4	Ductile	0.84	1.31	1.09	1.10	-1
12	Ductile	0.91	0.61	0.55	0.56	-1
2	Non-ductile	0.83	0.71	0.59	0.60	-2
4	Non-ductile	0.94	0.30	0.28	0.28	1
12	Non-ductile	1.06	0.35	0.37	0.38	-2

^a ASOSM compared to response surface approach.

fragility, including the effects of model uncertainties as predicted by ASOSM, is computed by multiplying the ratio, $\frac{\hat{m}_{mod}}{\hat{m}}$, from Eq. (8), by the median obtained in the original fragility, where only record-to-record uncertainties are considered. Note that if the relationship between a random variable and collapse capacity is linear, then $\Delta^+/\Delta^- = 1$, and thus median capacity is unchanged as would be expected, and as predicted by FOSM. Comparisons of ASOSM and the response surface based approach are tabulated in Table 7 for the six reinforced concrete frames. Since Eq. (8) has been derived from the results of this study it has not been validated for other structural systems (e.g. steel frames, reinforced concrete walls, etc.), but it should provide a reasonable approximation for other systems. The coefficients in Eq. (8) may need to be re-examined for prediction of other limit states.

This method is capable of capturing the significant decrease in the median collapse capacity observed for the 4-story ductile structure, and 2-story non-ductile structure, as well as the increase observed for the 12-story non-ductile structure. Note that once the sensitivity analyses for the FOSM assessment have been completed, no additional analyses are needed to compute Δ^+/Δ^- . Thus, this simplified approach provides a significant savings in computational time as compared to the response surface based method. For the case with four random variables, the response surface method requires 33 sets of IDA analyses, while the simplified method requires only 9 (twice the number of random variables, plus the mean model). The advantage of the simplified analysis is even larger when more model random variables are investigated.

Since the measure of nonlinearity, Δ^+/Δ^- , is obtained by varying each modeling random variable individually, the simplified method may miss some of the complex interactions between the random variables that are captured by the response surface. If Eqs. (8) and (5) predict that modeling uncertainties have a significant effect on the collapse fragility, the more complete response surface based method may be warranted.

6. Conclusions

In this study, we propose a procedure for incorporating structural modeling parameter uncertainties into probabilistic collapse risk assessments and other predictions of structural response. To accomplish this expediently and accurately, we advocate using Monte Carlo sampling with a response surface. The response surface is a multivariate function representing the relationship between the model random variables and a structural response parameter of interest (e.g. interstory drift, collapse capacity, etc.). Once the response surface is created from the results of sensitivity analyses, Monte Carlo methods are used to sample the model random variables and the structural response is predicted using the response surface, avoiding time consuming nonlinear simulations. The outcome of this process is a structural response fragility that incorporates both the uncertainty in the structural modeling parameters and in the ground motion.

We illustrate this method by applying it to reinforced concrete frame buildings, though the approach developed here is widely applicable. From the case study of RC frames we observe the following:

- Neglecting the effects of modeling uncertainties is *unconservative* in almost all cases.
- Incorporating modeling uncertainties increases the dispersion (σ_{ln}) in the response fragility, and also *shifts the prediction of the median*. The median of the response fragility typically decreases, and may decrease by as much as 20%.
- Modeling uncertainties have greater impact when the key modeling parameters are more uncertain.
- Modeling uncertainties have greater impact when the relationship between model parameters and structural response is highly nonlinear.

The importance of incorporating modeling uncertainties in the analysis is dependent upon both the structure and limit state of interest. The final two bulleted observations serve to explain why this study finds that modeling uncertainties have a more significant effect on performance predictions compared to previous studies. For one, the model variables important for predicting collapse include parameters related to component deformation capacity and post-capping (softening) behavior, which are highly uncertain. In addition, the relationship between the model random variables and collapse capacity is typically nonlinear, due in part to the many possible collapse modes in frame structures (and that these collapse modes may alternate depending on the values of the model random variables). These case studies demonstrate that comprehensive assessment of collapse risk requires careful propagation of modeling uncertainties.

The response surface method proposed here improves upon the often used FOSM approach because it is able to capture the effects of nonlinearities in the relationship between model random variables and the limit state function. This improvement is crucial for nonlinear limit states like collapse. We also show that this improvement is important when predicting mean annual rates of exceeding a limit state, or predicting conditional probabilities of exceedance when the probabilities are large. FOSM, however,

may be an adequate approximation for predicting conditional probabilities of exceedance when the probabilities are small, and, correspondingly, for predicting mean rates of exceedance that are dominated by the lower tail of the collapse fragility.

To remedy FOSM's potential deficiencies, but to avoid the extra effort needed for the response surface approach, we also propose a simplified method, termed ASOSM (approximate second order second moment). ASOSM uses FOSM to predict the increase in fragility's logarithmic standard deviation, but also provides a method for predicting the potential shift in the median of the limit state fragility. As a result, it will provide more accurate predictions of the mean rate of limit state exceedance and conditional probabilities in the upper tail of the distribution, and can serve as a diagnostic tool to investigate the importance of modeling uncertainties in the assessment process. Once sufficient analyses have been run for FOSM, ASOSM does not require any additional time history simulations.

These results point more generally to the importance of appropriately characterizing and propagating uncertainties in performance-based earthquake engineering. Since simplified approaches may have a large effect on calculated risks, the accuracy of simplifying assumptions should be considered with care when the results will impact important decisions.

Acknowledgements

This work has been supported by the PEER Center through the Earthquake Engineering Research Centers Program of the National Science Foundation (under award number EEC-9701568) and the Applied Technology Council through funding of the ATC-63 project by FEMA. Additional support for the first author was provided through the National Science Foundation Graduate Research Fellowship program and a Stanford Graduate Fellowship. The authors would also like to acknowledge the constructive comments and suggestions by participants at the workshop on Risk Communication and Acceptance held at Stanford University in March, 2007, as well as by Helmut Krawinkler, Eduardo Miranda and Charlie Kircher.

References

- [1] Porter KA, Beck JL, Shaikhutdinov RV. Sensitivity of building loss estimates to major uncertain variables. *Earthq Spectra* 2002;18(4):719–43.
- [2] Lee TH, Mosalam KM. Seismic demand sensitivity of reinforced concrete shear-wall building using FOSM method. *Earthq Eng Struct Dyn* 2005;34:1719–36.
- [3] Ibarra L, Krawinkler H, 2003. Global collapse of frame structures under seismic excitations. Blume Center TR 152, Stanford University; 2003.
- [4] Deierlein GG. Overview of a comprehensive framework for earthquake performance assessment. In: Proceedings of international workshop on performance-based seismic design concepts and implementation (Bled, Slovenia); 2004.
- [5] Vamvatsikos D, Cornell CA. Incremental dynamic analysis. *Earthq Eng Struct Dyn* 2002;31:491–514.
- [6] Pacific Earthquake Engineering Research Center. OpenSees <opensees.berkeley.edu>; 2006.
- [7] Ibarra LF, Medina RA, Krawinkler H. Hysteretic models that incorporate strength and stiffness deterioration. *Earthq Eng Struct Dyn* 2005;34:1489–511.
- [8] Haselton CB. Assessing seismic collapse safety of modern reinforced concrete frame buildings. Ph.D. thesis, Stanford University; 2006.
- [9] Applied Technology Council. Recommended methodology for quantification of building system performance and response parameters, ATC-63 75% draft. Redwood City, CA: FEMA; 2007.
- [10] Deierlein GG, Liel AB, Haselton CB, Kircher CA. Assessing building system collapse performance and associated requirements for seismic design. In: SEAOC convention (Tahoe, CA); 2007.
- [11] Goulet CA, Haselton CB, Mitrani-Reiser J, Beck JL, Deierlein GG, Porter KA, et al. Evaluation of the seismic performance of a code-conforming reinforced-concrete frame building – from seismic hazard to collapse safety and economic losses. *Earthq Eng Struct Dyn* 2007;36(13):1973–97.
- [12] Liel AB. Assessing the collapse risk of California's existing reinforced concrete frame structures: Metrics for seismic safety decisions. Ph.D. thesis, Stanford University; 2008.

- [13] DerKiureghian A. Non-ergodicity and PEER's framework formula. *Earthq Eng Struct Dyn* 2005;34:1643–52.
- [14] Esteva L, Ruiz SE. Seismic failure rates of multistory frames. *J Struct Eng* 1989;115(2):268–83.
- [15] Aslani H. Probabilistic earthquake loss estimation and loss disaggregation in buildings. Ph.D. thesis, Stanford University; 2005.
- [16] Baker JW, Cornell CA. Uncertainty propagation in probabilistic seismic loss estimation. *Struct Saf* 2008;30(3):236–52.
- [17] Helton JC, Davis FJ. Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems. Sandia National Laboratories; 2001. p. 75.
- [18] Rubinstein RY. Simulation and the Monte Carlo method. New York: John Wiley and Sons; 1981.
- [19] Porter K, Cornell C, Baker J. Propagation of uncertainties from IM to DV. In: Krawinkler H, editor. Van Nuys Hotel Building Testbed report: Exercising seismic performance assessment. Pacific Earthquake Engineering Research Center (PEER 2005/11); 2005.
- [20] Zhang J, Ellingwood B. Effects of uncertain material properties on structural stability. *J Struct Eng* 1995;121(4):705–16.
- [21] Pinto P, Giannini R, Franchin P. Seismic reliability analysis of structures. Pavia, Italy: IUSS Press; 2005.
- [22] Val D, Bljoger F, Yankelevsky D. Reliability evaluation in nonlinear analysis of reinforced concrete structures. *Struct Saf* 1997;19(2):203–17.
- [23] DerKiureghian A, Ditlevsen O. Aleatory or epistemic? Does it matter? *Struct Saf* (2008), doi:10.1016/j.strusafe.2008.06.020.
- [24] Cornell CA, Jalayer F, Hamburger R, Foutch DA. Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. *J Struct Eng* 2002;128(4):525–33.
- [25] Ellingwood B. Quantifying and communicating uncertainty in seismic risk assessment. *Struct Saf*, this issue, doi:10.1016/j.strusafe.2008.06.001.
- [26] Benjamin JR, Cornell CA. Probability, statistics and decision for civil engineers. New York: McGraw-Hill; 1970.
- [27] Mitra N, Lowes LN. Evaluation, calibration and verification of a reinforced concrete beam–column joint model. *J Struct Eng* 2007;133(1):105–20.
- [28] Liel AB, Haselton CB, Deierlein GG. The effectiveness of seismic building code provisions on reducing the collapse risk of reinforced concrete moment frame buildings. In: Proceedings of the 4th international conference in earthquake engineering; 2006.
- [29] Panagiotakos TB, Fardis MN. Deformations of reinforced concrete members at yielding and ultimate. *ACI Struct J* 2001;98(2):135–48.