Midterm Exam
Math 19
July 26, 2017

Name: 

Instructor: Lecture Time: 

Time: 2.5 hours

Do not open this exam until you are told to do so.

To receive full credit, show an appropriate amount of work for each problem. Graders must be able to see not only your answer but how you obtained it. This work could be a sequence of algebraic calculations or a couple sentences, whichever feels most appropriate for the problem.

Failure to follow these instructions will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason.
- You are bound by the Stanford Honor Code, which stipulates among other things that you may not communicate with anyone other than the instructors/CAs during the exam, or look at anyone else’s solutions.

I understand and accept these instructions.

Signature: 

Grade table (for instructor use only)

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1. (12 points) On the axes provided, sketch the graph of a function \( f \) that satisfies all of the conditions below. Label any asymptotes in your sketch. (No justification is required in this problem.)

- \( f \) is continuous on \((-\infty, -2)\) and \((-2, \infty)\)
- \( \lim_{x \to -2^-} f(x) = \infty \)
- \( \lim_{x \to -2^+} f(x) = -\infty \)
- \( f(x) > 0 \) for \( x < -2 \)
- \( f(0) = 1 \)
- \( \lim_{x \to -\infty} f(x) = 0 \)
- \( \lim_{x \to \infty} f(x) = -\infty \)
2. (12 points) Evaluate each of the following limits, providing brief justification. If the limit is infinite, specify whether it is $+\infty$ or $-\infty$.

(a) \[ \lim_{x \to 0} \left( \frac{1}{x\sqrt{1 + x}} - \frac{1}{x} \right) \]

(b) \[ \lim_{x \to 0^+} \arctan(e^{1/x}) \]
(c) \( \lim_{x \to \infty} \frac{1 + x^{4/3} + e^{-x}}{2 + x^{4/3}} \)

(d) \( \lim_{x \to \infty} (\sqrt{x^4 + 6x^2} - x^2) \)
3. (12 points) Let $f$ be the following function:

$$f(x) = \begin{cases} 
  x^2 e^x & \text{if } x < 0 \\
  0 & \text{if } x = 0 \\
  \tan(x) & \text{if } x > 0.
\end{cases}$$

(a) Show that $f$ is continuous on the interval $(-\infty, \pi/4)$.

(b) Is $f$ continuous on the interval $(-\infty, \infty)$? Briefly explain.
4. (12 points) Let $f(x) = \frac{e^{-x}(x^2 - 4)}{x^2 - 2x}$.

(a) Find all vertical asymptotes of $f$, or explain why none exist. Give complete reasoning.

(b) Find all horizontal asymptotes of $f$, or explain why none exist. Give complete reasoning.
5. (12 points) Mark each statement below as true or false, by circling either TRUE or FALSE. In this problem, no justification is necessary.

(a) For any function \( f \): If \( f(2) \) exists, then \( \lim_{x \to 2} f(x) \) exists.

TRUE FALSE

(b) For any function \( f \): If \( f'(2) \) exists, then \( \lim_{x \to 2} f(x) \) exists.

TRUE FALSE

(c) For any function \( f \): If \( \lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) \), then \( f(x) \) is continuous at \( x = 3 \).

TRUE FALSE

(d) For any function \( f \): If \( f(0) = 0 \), then \( f'(0) = \lim_{x \to 0} \frac{f(x)}{x} \).

TRUE FALSE

(e) For any function \( f \): If \( f \) has a vertical asymptote at \( x = 3 \), then \( f'(3) \) does not exist.

TRUE FALSE

(f) There is a continuous function \( g(x) \) with \( g(0) = 2 \) and \( \lim_{x \to 0} g(x^2) = 1 \).

TRUE FALSE
6. (12 points) Calculate the following derivatives, showing your work. You do not need to simplify your answers.

(a) \( \frac{d}{dx} \arctan(1/x) \)

(b) \( \frac{d}{dx} \frac{e^x \ln(x)}{x^2 + 1} \)
(c) \( \frac{d}{dx} x^2 \cos(x^2) \)
7. (12 points) Prove that there is a real number $x$ satisfying $e^x = \sin(x)$. 
8. (12 points) For constants $a$, $b$, and $c$, let $f(x) = \begin{cases} \ ax + b & \text{if } x < 0 \\ \ c & \text{if } x = 0 \\ \ x^2 \sin(1/x) & \text{if } x > 0. \end{cases}$

(a) Find necessary and sufficient conditions on $a$, $b$, $c$ so that $\lim_{x \to 0} f(x)$ exists. Give complete reasoning.

(b) Find necessary and sufficient conditions on $a$, $b$, $c$ so that $f(x)$ is continuous at $x = 0$. Give complete reasoning.
(c) Find necessary and sufficient conditions on $a, b, c$ so that $f(x)$ is differentiable at $x = 0$. Give complete reasoning.