Midterm 1
Thursday, 10/14/2010

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.

- This is a closed-book exam. No calculators or other electronic aids will be permitted. You are allowed to have one 8.5 by 11 sheet of paper containing handwritten notes on both sides.

- In order to receive full credit, you must show all of your work and justify your answers. Your answer should be clearly labeled.

- If you need extra room, use the back sides of each page. Staple any scratch paper to your exam.

- Please sign the following:

> “On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination.

Full Name: ________________________________
Circle your lecture time: 9:30 / 11:00

Signature: ________________________________

Please leave the following table blank for the grader.

1. _________ (/10 points)
2. _________ (/35 points)
3. _________ (/10 points)
4. _________ (/10 points)
5. _________ (/10 points)
6. _________ (/10 points)

Total. _________ (/85 points)
1. (10 points) Consider the function $y = f(x) = \frac{2x+1}{1-x}$.

   (a) (2 points) What is the domain of $f$? Express your answer as an interval.

   (b) (4 points) Compute the inverse of $f^{-1}$. 

(c) (2 points) What is the range of $f$? Express your answer as an interval.

(d) (2 points) What is the domain of

$$\ln \left( \frac{2x + 1}{1 - x} \right)$$?

Express your answer as an interval.
2. (35 points) Compute the limit algebraically. You must show your work at a level appropriate for the particular problem at hand. Do not use L’Hopital’s rule, if you know it. If a limit does not exist, show why.

(a) \[
\lim_{x \to -1} \frac{4x^2 + 4x}{x^2 - 3x - 4}.
\]

(b) \[
\lim_{x \to 1} \frac{x - 1}{x^3 - 1}.
\]
(c) \[ \lim_{t \to 0} \left( \frac{1}{t} - \frac{1}{t^2 + t} \right). \]

(d) \[ \lim_{x \to 1} e^{2x} - \ln(x) + x^2 + 2x + 1 + \sin(\pi x). \]
(e) 
\[ \lim_{h \to 0} \frac{1 - \sqrt{1 + h}}{h} . \]

(f) 
\[ \lim_{x \to 0} x^4 \sin \left( \frac{\pi}{x} \right) . \]

(g) 
\[ \lim_{t \to 1} f(t), \]
where 
\[ f(t) = \begin{cases} 
-t + 2, & t < 1 \\
2, & t = 1 \\
t^2 + \log(t), & t > 1 
\end{cases} \]
3. (10 points) This question is about continuity.

(a) (2 points) State the definition of continuity of $y = f(x)$ at $x = 3$.

(b) (3 points) At what points is the function

$$f(x) = \begin{cases} \frac{x}{x-1}, & x < 0 \\ e^{x^2} + 3, & x \geq 0 \end{cases}$$

continuous? State your answer as an interval.

(c) (5 points) Consider the following function:

$$h(x) = \begin{cases} x^2 + cx + 2, & x < -1 \\ \sqrt{x + 2} + c, & x \geq -1 \end{cases}$$

Is there any value of $c$ for which $h(x)$ is continuous on $(-\infty, \infty)$? If so, find $c$. Otherwise, explain why not.
4. (10 points) This question is about the Intermediate value theorem.

(a) (2 points) State the Intermediate value theorem precisely.

(b) (3 points) Sketch the graph of a function that shows the statement of the theorem is false if you omit the hypothesis that the function is continuous. Explain your graph.

(c) (5 points) Prove that there is a real-valued solution to the equation

$$\sqrt{x} = 1 - x.$$
5. (10 points) Consider the parametric equations given by

\[
\begin{align*}
  x(t) &= \sin(t) \\
  y(t) &= \cos^2(t),
\end{align*}
\]

for \(0 \leq t \leq \pi\).

(a) Find the Cartesian equations of the curve.

(b) Sketch as accurately as possible the curve traced out by the parametric equations. Label your axes carefully.
6. (10 points) Determine if the following statements are true or false. No justification is needed.

(a) (2 points) The domain of $\log_7(1 - x^2)$ is $x < 1$.

(b) (2 points) $g(x) = \sqrt{x^2 + 9}$ is a polynomial.

(c) (2 points) $f(x) = \frac{1}{x^2 + 2x + \sqrt{2}}$ is a rational function.

(d) (2 points) The following equation holds for all real $x$:

$$\frac{x^2 + 2x + 1}{x + 1} = x + 1.$$ 

(e) (2 points) If $f$ is one-to-one then $f^{-1}(x) = \frac{1}{f(x)}$. 