Exam 1  
Math 19  
October 12th, 2015

Name: ____________________________

Instructor: ________________________  Lecture Time: __________

Time: 90 minutes

Do not open this exam until you are told to do so.

To receive full credit, show an appropriate amount of work for each problem. Graders must be able to see not only your answer but how you obtained it.

Failure to follow these instructions will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason.
- You are required to sit in your assigned seat.
- You are bound by the Stanford Honor Code, which stipulates among other things that you may not communicate with anyone other than the instructors/CAs during the exam, or look at anyone else’s solutions.

I understand and accept these instructions.

Signature: ________________________

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1. (15 points)
   
   (a) Find $\cos(\tan^{-1} x)$ for $x > 0$.

   (b) For $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, when is $\sin \theta = \sin \left(\frac{2\pi}{3}\right)$?

   (c) Find $x$ such that $\log_4 24 - \log_4 x = 1$. 
2. (20 points) For this question, let \( f(x) = 1 + \sqrt{4 + 2x} \) and \( g(x) = \ln(x) \).

(a) What are the domain and range of \( f(x) \)?

(b) Find \( f^{-1}(x) \). What are the domain and range of \( f^{-1}(x) \)?
(c) Find $g^{-1}(x)$. What are the domain and range of $g^{-1}(x)$?

(d) Find $(f \circ g^{-1})(x)$. What are the domain and range of $(f \circ g^{-1})(x)$?
3. (15 points) Consider the function \( f : \mathbb{R} \to \mathbb{R} \) defined by

\[
 f(x) = \begin{cases} 
 B(x - 3)^2 + 5 & \text{if } x < 2, \\
 2 & \text{if } x = 2, \\
 x^3 & \text{if } x > 2,
\end{cases}
\]

where \( B \) is some real number. Your answer for parts (a) and/or (b) may involve \( B \).

(a) Compute \( \lim_{x \to 2^+} f(x) \) or show it does not exist.

(b) Compute \( \lim_{x \to 2^-} f(x) \) or show it does not exist.

(c) Is there a value of \( B \) so that \( \lim_{x \to 2} f(x) \) exists? If so, find that value and explain your choice. If not, explain why not.
4. (18 points) Compute the following limits if they exist. If they do not exist, explain why.

(a) \( \lim_{x \to 4} \frac{x^2 - 7x + 12}{x - 4} \)

(b) \( \lim_{x \to \infty} e^{\frac{5x+3}{x^2+2}} \)

Hint: you may use the fact that \( \lim_{x \to \infty} e^{f(x)} = e^{(\lim_{x \to \infty} f(x))} \) if \( \lim_{x \to \infty} f(x) \) exists.
(c) \( \lim_{x \to \infty} \left( \sqrt{x^2 + 3x + 4} - x \right) \)
5. (20 points) Consider the function \( f(x) = \frac{3x - 2}{4x + 1} \).

(a) (5 points) Find the \( x \) and \( y \) intercept(s) of the graph \( y = f(x) \).

(b) (10 points) Find all the vertical asymptotes of the graph. For each asymptote \( x = a \), compute \( \lim_{x \to a^+} f(x) \) and \( \lim_{x \to a^-} f(x) \).
(c) (5 points) Calculate \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \) if they exist. If they do not exist, explain why.
6. (12 points) In answering the following questions, assume that the function \( J : \mathbb{R} \to \mathbb{R} \)
has a range of \(( -\infty, \infty )\) and is a one-to-one function. Write \( J^{-1} : \mathbb{R} \to \mathbb{R} \) for the inverse function.

For each of the following, determine if the statement is True, False or write “not enough information” if there is not enough information given to determine the answer.

(a) Suppose \( J(-32) = 18 \). Then \( J^{-1}(18) = -32 \).

(b) \( J(x) \) is an odd function.

(c) \( J(x) \) is an even function.

(d) \( \sin(J(x)) \) is a one-to-one function.